Multi-period vehicle assignment problem with stochastic transportation order availability

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1 Industrial problem

The present work investigates a Dynamic Vehicle Allocation problem (DVAP) which is faced by major forwarding companies active in road transportation. A company owning a limited fleet of vehicles wants to maximize its operational profit over an infinite horizon divided into equal periods (days). The profit stems from revenues for transporting full truckloads (FTL) and from costs derived from waiting idle and moving unladen. A decision leading to a set of actions is made at every period of time and is based on the dispatcher’s information over a restricted horizon, called rolling horizon, as it evolves subsequently period per period. The data provided by the customers concern their prospective loads or requirements for transportation: locations of departure and destination cities, and a unique pick-up period for each transportation order. Moreover, the dispatcher has data regarding travel times between cities, current location and status (unladen or loaded) of trucks. This is known for sure and represents the deterministic part of the problem.

The stochastic component of the problem arises from the uncertainty on the requirement for the transportation orders. More precisely, the availability of each order can be either confirmed, or denied, a few periods ahead of the loading period (meaning that clients confirm their order, which the carrier may still decide to fulfill, or not). For projected orders in the remote part of the rolling horizon, the dispatcher only knows the order confirmation probability which represents the stochastic transportation order availability. The decision problem faced by the dispatcher in each period is to accept or to reject orders, then to assign the selected orders to trucks, taking into account prospective and confirmed orders as well as the availability and current location of trucks. Rejected orders are supposed to be sub-contracted at no cost while bringing no profit.

In practice, as a rule, trucking orders are communicated by the dispatching center to the drivers and to the customers on the eve of the loading period at the latest. The loading decisions are made when all available orders are known for the next day. In the present work, it is slightly different, decisions are supposed to be taken at the early morning of the current decision period when the information for the current day are all known. So, this process is performed with short regular periods, typically equal to days. This results in a planning context which is different from an on-line setting, where the system generates a decision whenever a piece of information changes, or a model with long periods of time, such as in maritime transportation, wherein the accuracy of the probabilistic forecasts may evolve over several weeks.
2 Problem specific features and scientific contributions

Within the literature on transportation problems, many developments focus on finding one approximate or exact solution to a single period deterministic problem such as the vehicle routing problem (VRP) or the pick-up and delivery problem (PDP), including their variants: limited or unlimited capacity, single vehicle or fleet, time windows, etc. (See [2] for a detailed review.) In such problems, the temporal dimension is usually almost absent: it may be restricted, for instance, to an upper bound on the duration of a tour, or to time windows at customer locations. In other cases, such as when designing bus or train schedules, the decision-maker may restrict his attention to the computation of periodic schedules. Even in this case, where the problem takes several periods into account, a unique decision is made once and for all. Yet, in other decision frameworks, the time dimension cannot be so easily disregarded. In particular, long-haul transportation firms usually have to cope with a rolling decision process, typically unfolding over days, where deterministic or uncertain forecasts over subsequent periods are available from customers, and where decisions can be possibly postponed or anticipated. This framework leads to a decision process consisting of successive dependent optimization phases. The present research aims at describing a strategy to tackle such problems, and techniques to select a performing algorithmic policy $\pi^*$. 

We should also stress that the stochastic component of the problems that we handle here is not quite standard. Indeed, in transportation models, the uncertain or stochastic element frequently relates to a parameter value such as customer demand or travel time which can be reasonably modeled by continuous probability distributions (see [3] for an introduction). More importantly, even though the realization of these random variables may affect the cost or the feasibility of the solutions, it does not modify the set of decision variables. In contrast with this situation, we consider here discrete, Bernoulli random variables (representing the availability of transportation orders) whose realization profoundly affects the structure of the available solutions. In fact, owing to these discrete distributions, the optimization techniques for the model to be solved are scenario-based. Therefore, some decision variables may typically appear in, or disappear from the models associated with these various deterministic scenarios depending on the binary values assumed by these random variables (as in [4] for an application).

These binary values determine the problem size for every scenario and the set of potential decisions. So, each scenario represents a particular set of orders available for transportation within the rolling horizon. This set obviously contains the orders confirmed in the deterministic part of the horizon, as well as the orders associated with the specific scenario at hand. Consequently, the decision to be made, i.e., the problem to be solved, changes significantly with each scenario, and in every period. This is by far different from a slight variation in the values assumed by numerical parameters, such as those that might result in scenarios associated with realizations of continuous random variables.

3 Scenario-based algorithmic strategies

This problem is computationally difficult owing to the large number of possible realizations of the random variables, and to the combinatorial nature of the decision space. The methodology is based on optimizing decisions for deterministic scenarios. By solving the assignment problem for a single or a sample of scenarios, we aim at finding actions per decision period leading to profitable expected value policies $\mu^*_\pi$ in the long run. Several policies $\pi^*_m$ are generated in this way from simple heuristics or more complex approaches:
**Deterministic approximations by single scenarios**

1. **Expected Value Scenario** based on the expected reward for each order,
2. **Modal Value Scenario** assuming that orders are confirmed only if their probability of availability is $\geq 50\%$.
3. **Optimistic scenario** assuming that all transportation orders are confirmed,
4. **Pessimistic scenario** (myopic deterministic bound $O_{RH}^*$) assuming that none projected order is confirmed.

**Multiple scenarios approaches (see [5])**

1. **Consensus algorithm**: a subset of scenarios randomly generated are independently solved. Then, the most frequent decisions among these solutions are recorded in order to generate a promising consensus solution using a dedicated aggregating procedure.
2. **Restricted Expectation algorithm**: a subset of scenarios randomly generated are independently solved. Then, each partial solution for the present decision period is successively applied in the other scenarios. Successively, the cumulated values of the solutions taking into account this modified decision period are computed over all scenarios. The solution with the least expected cost is selected.
3. **Subtree algorithm** : from network flow models and adding non-anticipativity constraints, it is possible to derive a formulation over subtrees of scenarios. Therefore, a subset of randomly generated scenarios is solved at once and the solution for the decision period is selected.

Similar approaches have proved effective for other problems; see, e.g., [1].

Using the a-posteriori information over the rolling horizon $O_H^*$ and the overall horizon $O_T^*$, deterministic optimization models are used to compute bounds allowing for performance evaluation and value of information analysis (see figure 1).

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**Figure 1: Bounds and values of information**
4 Results and contributions

Test are performed on various instances featuring different numbers of loads, graph sizes, sparsity, and probability distributions. Performances are compared statistically over paired samples to assess the significance of the observed differences among algorithmic policies $\pi_i$.

Our main contribution in this work consists in testing the effectiveness of a generic, practical methodology for tackling such multi-period transportation problems with discrete stochastic forecasts.

As an additional contribution, we also provide illustrative numerical estimations and discussions for the expected value of the multi-period model (EVMPM), the expected value of the perfect information (EVPI), the expected value of the accessible information (EVAI), the expected value of the tail information (EVTI), the expected value of the expected value solution (EEVS) and the expected value of the stochastic solution (EVSS) (see figure 1).

Finally, we analyze the robustness of the algorithm with respect to the accuracy of the estimation of the probability distributions, and we discuss the sensitivity of the performance of the most promising algorithm with respect to the length of the rolling horizon.

Results show that the subtree algorithm outperforms statistically (i.e. $\pi^\star$) other algorithmic policies and closes by 2/3 on average the gap between the myopic bound $O_{RH}^\star$ and the bound $O_{H}^\star$ using the a-posteriori information over the rolling horizon. Moreover, the subtree algorithm calibrated with a 50% probability for the availability of orders remains efficient, i.e. robust, when under or over estimating the real availability of orders.

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References