"Optimization of Stochastic Multi-Period Problems in Transportation"

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Joint work from
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HEC-ULg, QuantOM
Transportation models: as decisions making
Set of optimal decisions or optimal sequence of decisions
TSP, VRP, PDP...No past, no future

- Mono-period vs Multi-period (not periodic, not year)
- Independent and Subsequent and related
- Deterministic and Stochastic Information
- Data and Forecasts
- Parameters and Distribution laws
- Optimal solutions, Heuristic values and Policies
- Instances and Scenarios or Futures
- Values and Statistical performances
- P solvable, NP-Hard and "Intractable"

Contribution: A framework for experimentation
1. Outlines

- Multi-period problems
- Decision making under uncertainty
- Stochastic Optimization Techniques
- A Methodology
  1. Bounds
  2. A picture for manager
  3. Algorithms
  4. Results validation
- Case study (Vehicle-Load Assignment)
- Conclusions
2. Rolling Horizon

<table>
<thead>
<tr>
<th>[P1]</th>
<th>[P1..Pi]</th>
<th>[Pi+1..RH]</th>
<th>[RH+1..Pn]</th>
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<tbody>
<tr>
<td>Deterministic</td>
<td>Stochastic</td>
<td>Tail</td>
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</tr>
</tbody>
</table>

**Decision**: in P1

**Action**: on deterministic part \([P1...Pi]\) \(\Rightarrow\) feasible

**Case study**:
1. rolling horizon = 5 periods = 5 days = 1 week
2. periods deterministic 2 days, forecasts 3 days
3. action in P1

**Dynamism of the system**

- Decision in P1
- Actions (info out)
- Roll-over 1 period, updates (in)
  1. stochastic becomes deterministic \(Pi + 1 = Pi\)
  2. new stochastic info \(RH + 1 = RH\)
- Decision in P2 = P1
3. Solutions to a stochastic problem?

- Worst case (oversize solution)
- Chance constrained (95%)
- Robust to variation (Tree)
- Flexible: Easy to recover (Grass)
- Min or max Expected cost-profit ($E^*$)
4. Usual techniques

1. "Stochastic Programming" 2-stage, convex, continuous
2. "Markov Chain" states, actions, stability
3. "Approximated Dynamic Programming" ADP
4. "Sample Average Approximation" SAA (continuous)
5. Scenario tree, but rolling horizon

Integer + Discrete distribution laws + Tail
Curse of dimensionality $\Rightarrow$ Intractable!
10 trips per period $2^{10} = 1024$, 3 periods $2^{30}$

Conclusion: "Hard" to find $E^*$

Methodology:
Simulation and optimization over deterministic scenarios
Solve one, several, some scenarios = "futures"

Literature and algorithms as a brick

Which scenarios?
5.1 Bounds

**Oracle**: a posteriori, revealed info $O^*$
Infinite horizon value with deterministic info

**Real Bound** (Upper or lower) on $E^*$

**VPI**: Value of the Perfect Information $|O^*-E^*| \geq 0$

**Myopic**: Deterministic periods value $LO$
Bound (Lower or upper) on any policy with forecasts

Deterministic approximation : **Mean**

**Mean** $\implies$ **EVS**: Expected Value Solution

**VSS**: Value of the stochastic solution $|E^*-EVS| \geq 0$

**Rolling horizon**: Finite Oracle $O^*(RH)$

**VMPM**: Value of the multi-period model $|O^*(RH)-LO| \geq 0$

**VAI**: Value of the Available Information $|O^*(RH)-E^*| \geq 0$
5.2 A picture for a manager: max

Simulations => "Expected value of" : EO*, EO*(RH), ELO, EEVS, EVSS, EVAI, EVPI, EVMPM
5.3 Approximations of E*

- Solve a "good" single scenario (Mean, Mod)
- **Consensus (Cs)**:
  1. Solve $N$ scenarios
  2. Create a new solution with common decisions
- **Restricted Expectation (RE)**: Solve scenarios $i,j$ and cross-evaluate action $i$ over scenarios $j$
  1. Scenarios $i,j (\in N) \Rightarrow$ Solutions $i,j \Rightarrow$ Actions $i,j$
  2. Evaluate value of Action $i$ on Scenario $j$
  3. Cumulated value of Actions $i, j$
  4. Select the best action

**Questions**

- Reduced actions and state techniques?
- Scenarios generation?
- Stochastic solution from deterministic model?
- Deterministic solutions are elitist, no option in it
- CPU Time: $1, N + 1, N^2$
5.3 Approximations of $\mathbb{E}^*$

**Full tree**: Deterministic equivalent

⇒ One common action for all futures

Links: *Non-anticipativity constraints*
Action variables are equal in each scenario

In practice: Out of Memory, CPU Time, B & B

Approximation by a **Subtree** $(1 \ast ST \neq ST \ast 1)$

Subtree formulation often harder than a single scenario
5.4 Statistical validation and Robustness

**Statistical validation**: 
E* = Best policy we can found  
How to compare Policy 1 with Policy 2, E*1 vs E*2 ?

**Outclassment** = significant difference between means 
"*Paired sample comparison*"  
Hypothesis : $\mu_1 \neq \mu_2$, $\mu_1 > \mu_2$ ?  
Solve 30 scenarios by instance over an horizon 20 P  
Non Non-Normality check, confidence level, t-student...

**Robustness analysis**:  
Assumption : Distribution law is known in practice  
Test : Calibration law $\neq$ Real law (Cs, RE, Mean)
6.1 Vehicle-Load Assignment Problem

**Problem** Assign trip to truck FTL (PDP with selection)

**Decisions** : Wait, Move Empty, Load

**Objective** :
Maximize Profit (Load-Empty Moves-Waiting)

**Constraint** : loading if at place on time, no preemption

**Data** : [1,2] and forecasts on available loads [3,4,5]

**Stochasticity** : Availability [%] of a trip from A to B, start in [3,4,5]

**Distribution laws** [%] linked to :
1. Traveled distance (1, 2, 3, 4)
2. City size (B, M, S)

**Solution** : Network flow problem
6.2 A Picture for a scenario

A representation of the time-space (Periods, Cities)
6.3 Results: one example 150 loads 20 P

**TABLE:** Distribution laws linked to distance

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<tr>
<th>Info</th>
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<th>O*2</th>
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6.4 Preliminary conclusions

1. Dynamism is important : VMPM
2. VPI is high (68.4% + 30.7%)
3. No influence of graphs, distribution laws...
4. Subtree algorithm is usually the best
5. Subtree 30 often better than Subtree 10
6. Subtree never under-performs
7. EVS is the second best after subtree
8. the VSS is important 23.1%

Subsequent tests for the subtree:
Algorithmic parameter : calibration scenarios 10-30-50...
⇒ Subtree 50 (mean increases, variance reduces)
No statistical outclassment 50 vs 30, once 50 vs 10
CPU time increases "linearly", LP solution \( \cong \) IP
6.5 Robustness

**TABLE:** Robustness of distribution law parameter

<table>
<thead>
<tr>
<th>Info</th>
<th>LB</th>
<th>EVS</th>
<th>$TR^{30}_{30}$</th>
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</table>
6.6 Conclusions Case Study

1. VPI, VMPM, VSS are relevant
2. Independent of graph shape, size or distribution laws
3. Subtree is the best algo and others under-perform
4. Subtree with 30 or 50 scenarios is enough
5. By default, calibrate subtree for 50% availability (2nd best/3 and outclasses if reality is 50%)
6. Robustness: better to stick to distribution and approximate by the center
7. Less uncertainty on information closes the gap and reduces the VPI
7.1 Conclusions-contribution

- Importance of stochastic multi-period models
- Tool to measure the values of informations
- Understandable bounds for managers
- A toolbox of algorithms to tackle those problems
- A statistical validation of algorithms, outclassment
- A subtree solvable by a LP Solver in the case study
7.2 Perspectives

- Metaheuristics (many statistical issues)
- Subtree generation
- Exact: Column generation in subtree if hard
- Improve Cs and RE algorithms
- Improve calibration scenarios generation
- Repositioning strategy, LTL (PDP)
- Investigate the gap between VPI and VAI
- Compare with ADP
- Answer your questions, comments, remarks...