Chapter 15

Conditional Heteroskedasticity in Stock Returns: International Evidence

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Knowledge of the distribution of security returns is important for both theoretical and empirical studies in finance. For instance, the distributional characteristics of stock returns have important implications for mean-variance portfolio theory, theoretical models of capital asset pricing, and valuation of contingent claims. Moreover, empirical tests of asset pricing models and the efficient market hypothesis make statistical inferences based on distributional assumptions of stock returns, and estimation of the variance is crucial for the pricing of derivative securities.

The most convenient assumption underlying the distribution of security returns is that of a multivariate normality with stationary parameters over time. Since the normal distribution is stable under addition, any portfolio of stocks will also be normally distributed. If it is also assumed that investors are risk averse, mean-variance portfolio theory can be derived. Moreover, the assumptions of normality and parameter stability are implied by most of the statistical methods usually employed in empirical financial studies.

Tests of the normality hypothesis were first reported by Osborne (1964). He investigated returns of stock indices over several time periods and concluded that they approximately follow normal processes. Other studies questioned the normality assumption and showed that distributions of returns have fatter tails and are more peaked than the normal distribution. Mandelbrot (1963) and Fama (1965) observed that the family of stable Paretian distributions whose members exhibit heavy tails conforms better to the distribution of stock returns.

These studies and many more recent ones in both U.S. and non-U.S. markets have shown that time series of daily stock returns exhibit a certain level of autocorrelation for short differencing intervals. However, it is generally accepted that changes in returns are uncorrelated over time for two reasons: First, because of the transaction costs, the pattern of the autocorrelations cannot be used in a profitable trading rule. Second, the significance of the sample coefficients is likely to be overstated because the empirical distributions of daily returns are different from Gaussian distributions.

A common and misleading assumption derived from the lack of serial correlation in stock returns is statistical independence. The lack of serial correlation leads to statistical independence only when stock prices follow a Gaussian stochastic process. This important fact is often neglected, and many models of stock price behavior simply assume independence. Because the empirical distributions of stock returns generally depart significantly from Gaussain distributions, the assumption of independence is incorrect. Another common and inade quate assumption concerning stock price behavior is the linearity of the return-generating process with parameters that are independent of the past realizations. There is, indeed, no reason to believe that series of stock returns are generated by a linear process. The choice of such an assumption relies only on the easiness of statistical analysis. Hinich and Patterson (1985) proved, for example, that stock return series are not made of a process of independent increments.

These two assumptions, independence and linearity, can be particularly questioned in speculative markets. Neftçi (1984) proved that there is no theoretical reason supporting these assumptions, and results of several empirical studies showed that linearity and intertemporal independence cannot be expected (Perry, 1982; Pindyck, 1984; Poterba and Summers, 1986; Akgiray, 1989).

There is yet no unanimity regarding the best stochastic return-generating model. The general conclusion, however, seems to be that stock returns are approximately uncorrelated, but not independent, and described by distributions with fatter tails. One of the most recently proposed classes of return-generating processes in the literature that can capture the temporal dependence of stock return series is the class of autoregressive conditional heteroskedastic processes introduced by Engle (1982) and its generalized version by Bollerslev (1986). These models allow for volatility clustering; that is, large changes are followed by large changes, and small by small, which has long been recognized as an important feature of stock returns behavior. Empirical studies showed, indeed, that such processes are successful in modeling various time series (French et al., 1987; Baillie and Bollerslev, 1989; Hsieh, 1989; Baillie and De Gennaro, 1990).

This class of models has been mainly applied to American stock markets. This chapter shows whether such models can adequately describe stock price behavior of capital markets that are much smaller and thinner than the American markets. To that end, nine countries throughout the world have been selected. The study of stock price behavior in these markets is interesting, for it can provide further evidence in favor of or against the use of this type of model for describing stock price behavior.

THE DATA

For this study, we have selected the indices of nine major stock markets representing three geographical areas: North America, Europe, and the Far East

The daily market indices were collected from DATASTREAM for the period 1 Jan 1980 to 30 Sep 1990. These indices are for the United States (Standard & Poor's Composite), Canada (Toronto Composite), the United Kingdom (FT All-shares), France (CAC General), Italy (Milan Banca), Japan (Nikkei Dow Jones), Australia (All Ordinary Shares), Singapore (Straits-Times), and South Korea (Composite). The daily returns of these market indices are continuously compounded returns. They are calculated as the difference in natural logarithm of the index value for two consecutive days, $R_t = \log P_t - \log P_{t-1}$.

STATISTICAL ANALYSIS

This section contains analysis of the distributional and time series properties of the stock market indices returns in the sample. A range of descriptive statistics are presented in Table 1. The results confirm the well-known fact that daily stock returns are not normally distributed, but are leptokurtic and skewed, whatever the country concerned. All distributions are negatively highly skewed, indicating that they are nonsymmetric, and they all exhibit high levels of kurtosis, meaning that they are more peaked and have fatter tails than normal distributions.

To test the hypothesis of whether returns are strict white noise, that is, random walk, the Box-Pierce test statistics up to lag 25 are calculated and presented in the table. This is a joint test that the first k autocorrelation coefficients are zero. Under the null hypothesis that the sample autocorrelations are not asymptotically correlated, the Box-Pierce statistic, $Q = n \sum_{i=1}^{k} \rho(i)^2$, has chi-square distribution with k degrees of freedom, where $\rho(i)$ is the ith autocorrelation. All values of Q are significant at the 5% level except that of the United States. This implies that the null hypothesis of strict white noise is rejected for eight countries, reflecting a rather long range of dependency in the returns series. However, it can be questioned whether this test accounts for the full probability distribution of the returns series, since heteroskedasticity can lead to the underestimation of the standard error, $\sqrt{1/n}$ of each sample, and therefore to the overestimation of the t and chi-square statistics. Diebold (1987) provides a heteroskedasticity-consistent estimate of the standard error for the ith sample autocorrelation coefficient:

$$S(i) = \sqrt{1/n(1 + \gamma_R^2(i)/\sigma^4} \quad (1)$$

where $\gamma_R^2(i)$ is the ith sample autocovariance of the square data and σ is the sample standard deviation of the data.

Using these adjusted standard errors, Diebold proposes an adjusted Box-Pierce statistic:

Table 1

Sample Statistics on Daily Returns Series

	Australia	Canada	France	Italy	Japan	Korea	Singapore	United Kingdom	United States
Mean ($\times 10^3$)	0.360	0.198	0.470	0.682	0.414	0.576	0.330	0.511	0.372
t (mean = 0)	1.674	1.198	2.399	2.526	2.274	2.476	1.337	2.991	1.815
Variance ($\times 10^3$)	0.130	0.077	0.107	0.204	0.093	0.151	0.171	0.082	0.118
Skewness	−6.11	−0.98	−1.51	−0.99	−1.69	−0.96	−3.19	−1.69	−3.52
Kurtosis	150.46	22.28	16.94	11.74	36.69	18.36	50.61	19.66	75.08
Autocorrelation	0.119	0.169	0.146	0.137	0.060	0.069	0.208	0.137	0.033
$Q(25)$	191.43	130.89	154.23	139.58	102.09	41.40	171.41	111.2	29.95
$Q^*(25)$	36.32	27.43	65.66	52.64	30.56	27.96	28.83	26.30	09.95

Note. Underlined numbers indicate 5% significance level.

$$Q^* = \sum_{i=1}^{k} \frac{\rho(i)}{S(i)^2} \tag{2}$$

which is asymptotically chi-square distributed with k degrees of freedom, under the null hypothesis of no serial correlation in the data. The values of Q^* in Table 1 are much lower than the nonadjusted ones. They are significant at the 5% level for France and Italy only. So, even after adjusting for heteroskedasticity, some significant autocorrelation remains in the series of returns for these two countries.

A comparison between the values of Q and Q^* suggests that the rejection of serial independence using Q, which is based on the standard testing procedure, is due to the presence of heteroskedasticity in the returns series. The presence of significant values of Q^* in the French and Italian returns indices, however, indicates that these returns series are not strict white noise processes. Furthermore, the fact that the first-order autocorrelation is significant for seven countries implies the rejection of white noise, that is, an uncorrelated process. Therefore, we must eliminate the serial correlation in the return series before searching for appropriate models that could account for heteroskedasticity in the returns. One way to do this is to apply Autoregressive Moving Average (ARMA) models.

ARMA MODELS

The class of univariate ARMA models might adequately represent the behavior of the stock returns. Therefore, several ARMA models were applied to the returns series. An AR(1) model appears to fit returns series best:

$$R_t = \Phi_0 + \Phi_1 R_{t-1} + \epsilon_t \tag{3}$$

The estimates of the above regression model for each country are presented in Table 2. To observe whether the residuals ϵ_t obtained from Eq. (3) are uncorrelated, we applied the same tests for normality and serial correlation as for the returns series. As before, the values of the first-order autocorrelation coefficients are presented in Table 2. The first-order autocorrelation coefficient for all countries is not significantly different from zero. Thus, the results indicate that the AR(1) provides an uncorrelated series of residuals.

The estimate of Φ_1 is statistically significant, except for the United States. The Dickey–Fuller test for unit roots indicates that Φ_1 is significantly less than one. The series of returns appear to follow a stationary random walk. The assumption of normality of the residuals can be rejected. The residual series appear to be leptokurtic and skewed. A comparison between values of Q and Q^* indicates that the residuals series still exhibit heteroskedasticity.

The presence of heteroskedasticity in stock prices and in the market model has already been documented (Morgan, 1976). But while previous studies fo-

Table 2
The Autoregressive Model

	Australia	Canada	France	Italy	Japan	Korea	Singapore	United Kingdom	United States
ϕ_0	0.003	0.002	0.004	0.006	0.004	0.005	0.003	0.004	0.004
ϕ_1	0.120	0.170	0.146	0.137	0.060	0.069	0.208	0.137	0.033
Fuller's test	2469.5	2329.1	2813.1	2781.8	2634.8	2610.6	2219.5	2815.5	2709.3
Residuals statistics									
Mean ($\times 10^3$)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Variance ($\times 10^3$)	0.128	0.750	0.105	0.201	0.093	0.015	0.016	0.080	0.117
Skewness	−5.87	−0.39	−1.42	−0.83	−1.50	−0.93	−2.68	−1.31	−3.44
Kurtosis	146.46	25.49	17.36	12.14	36.87	18.66	42.09	16.84	73.19
Autocorrelation	0.005	0.001	0.004	0.008	0.006	0.000	0.010	−0.005	0.001
$Q(25)$	139.84	48.91	76.32	92.09	92.14	23.67	44.59	43.95	26.80
$Q^*(25)$	27.84	21.86	34.54	38.34	29.22	19.27	17.17	17.67	9.32

Note. Underlined numbers indicate 5% significance level.

cused on unconditional heteroskedasticity, in this chapter, we use Engle's Autoregressive Conditional Heteroskedastic (ARCH) model, which focuses on conditional volatility movements. It is interesting to note that, according to Diebold et al. (1988), the presence of the ARCH effect appears to be generally independent of unconditional heteroskedasticity. Excess kurtosis observed in both returns and residuals series can be related to conditional heteroskedasticity; that is, its presence can be due to a time varying pattern of the volatility.

CONDITIONAL HETEROSKEDASTIC MODELS

The ARCH process imposes an autoregressive structure on the conditional variance, which permits volatility shocks to persist over time. In this process, the conditional error distribution is normal, with a conditional variance that is a linear function of past squared innovations. The model, denoted by ARCH(p), is as follows:

$$\epsilon_t \mid \Psi_{t-1} \sim N(0, h_t)$$
$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1}^2 \tag{4}$$

with $p > 0$, $\alpha_i > 0$, $i = 0, \ldots, p$, and where Ψ_t is the information set of all information through time t.

An important extension of the ARCH model is the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) process of Bollerslev (1986), denoted by GARCH (p, q). In this model, the linear function of the conditional variance includes lagged conditional variances as well. Equation (4) in the case of a GARCH model becomes

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j h_{t-j} \tag{5}$$

where $q \geq 0$ and $\beta_j \geq 0$, $j = 1, \ldots, q$. The parameters of a (G)ARCH model are obtained through a maximum likelihood estimation. Given the return series and initial values of ϵ_1 and h_1, for $1 = 0, \ldots, r$ and with $r = \max(p, q)$, the following log-likelihood function must be maximized for a normal distribution:

$$L(\phi \mid p, q) = -\frac{1}{2} T \ln(2\pi) + \sum_{t=r}^{T} \ln \frac{1}{\sqrt{h_t}} \exp \frac{-\epsilon_t^2}{2h_t} \tag{6}$$

where T is the number of observations; h_t is the conditional variance, defined by Eqs. (4) and (5) for the ARCH and GARCH models, respectively; and ϵ_t^2 are the residuals obtained from the appropriate linear regression model according to the country in consideration.

Because the values of p and q have to be prespecified in the model, we tested several combinations of p and q. The values of the maximized likelihood functions for all pairs of p and q are presented in Table 3. We also calculated the generalized likelihood ratio LR $= -2\{L(\phi_n) - (\phi_a)\}$ of the maximized likelihood functions under the null hypothesis, that is, the normal distribution, and the various alternative hypotheses. Under the null hypothesis, LR is chi-squared distributed with degrees of freedom equal to the difference in the number of parameters under the two hypotheses. Table 3 gives the values of the LR test for each model. It can be observed that the value of the LR test for all (G)ARCH models is statistically significant at the 1% level, which means that all of these models fit the data more likely than does the normal distribution.

To distinguish between an improvement in the likelihood function due to a better fit and an improvement due to an increase in the number of parameters, we also calculated Schwarz's order selection criterion, SIC $= -2L(\phi) + (\ln T)K$, where K is the number of parameters in the model. According to this criterion, the model with the lowest SIC value fits the data best. The SIC values are reported in Table 3. The GARCH (1, 1) model has the lowest SIC values for all countries except France. For France no GARCH model converged and ARCH(3) is the best. Table 4 contains the results of the best model fitting the series of returns for each country.

The sum of $\Sigma_{i=1}^{p} \alpha_i + \Sigma_{j=1}^{q} \beta_j$ in the conditional variance equations measures the persistence of the volatility. Engle and Bollerslev (1982, 1986) have shown that if this sum is equal to one, the GARCH process becomes an Integrated GARCH (IGARCH) process. Such an integrated model implies the persistence of a forecast of the conditional variance over all future horizons and an infinite variance of the unconditional distribution of ϵ_t. The sums of the parameters $\Sigma_{i=1}^{p} \alpha_i + \Sigma_{j=1}^{q} \beta_j$ for the appropriate (G)ARCH models were calculated. They are all less than unity, though rather close to one for some, which indicates a long persistence of shocks in volatility. This means that the process is second-order stationary and that the second moment exists. The unconditional variances of residuals, shown in Table 4, are, respectively, $\sigma_\epsilon^2 = \alpha_0/(1 - \Sigma_{i=1}^{3} \alpha_i)$ for France and $\sigma_\epsilon^2 = \alpha_0/(1 - \alpha_1\beta_i)$ for the other countries. The unconditional variance of returns are $\sigma_R^2 = \sigma_\epsilon^2/(1 - \Phi_1^2)$; they are reported in Table 4 as well.

CONCLUSIONS AND IMPLICATIONS

This chapter provides empirical support for the hypothesis that stock market indices of nine different countries exhibit nonlinear dependency that cannot be accounted for by the random walk model. Descriptive statistics and normality tests reveal that the distribution of returns is not normal, whatever the country concerned. It has further been shown that the residuals obtained after applying an AR(1) model, which accounts for the presence of serial correlation in the

Table 3
Maximum Log Likelihood for (G)ARCH Models

Model	p, q	Log likelihood	LR	SIC
		Australia		
Normal	—	11 142.20		
ARCH	(1, 0)	11 747.09	1 209.78	−23 486.24
ARCH	(2, 0)	11 791.97	1 299.54	−23 568.06
ARCH	(3, 0)	11 799.43	1 314.46	−23 575.04
GARCH	(1, 1)	11 815.91	1 347.42	−23 615.94
GARCH	(2, 1)	. . .		
GARCH	(1, 2)	11 815.75	1 347.10	−23 607.68
GARCH	(2, 2)	. . .		
		Canada		
Normal	—	11 896.45		
ARCH	(1, 0)	12 220.90	24 441.80	−24 433.86
ARCH	(2, 0)	12 320.07	24 640.14	−24 624.26
ARCH	(3, 0)	12 360.68	24 721.36	−24 697.54
GARCH	(1, 1)	12 429.54	24 859.08	−24 843.20
GARCH	(2, 1)	12 431.64	24 863.28	−24 839.46
GARCH	(1, 2)	. . .		
GARCH	(2, 2)	12 424.74	24 849.48	−24 817.73
		France		
Normal	—	11 406.50		
ARCH	(1, 0)	11 560.95	308.90	−23 113.96
ARCH	(2, 0)	11 664.35	515.70	−23 312.82
ARCH	(3, 0)	11 677.31	541.62	−23 330.80
GARCH	(1, 1)	. . .		
GARCH	(2, 1)	. . .		
GARCH	(1, 2)	. . .		
GARCH	(2, 2)	. . .		
		Italy		
Normal	—	10 506.12		
ARCH	(1, 0)	10 653.91	295.58	−21 299.88
ARCH	(2, 0)	10 757.64	503.04	−21 499.40
ARCH	(3, 0)	10 886.73	761.22	−21 749.64
GARCH	(1, 1)	10 992.68	973.12	−21 969.48
GARCH	(2, 1)	. . .		
GARCH	(1, 2)	. . .		
GARCH	(2, 2)	10 998.15	984.06	−21 964.55
		Japan		
Normal	—	11 608.42		
ARCH	(1, 0)	12 004.09	791.34	−24 000.24
ARCH	(2, 0)	12 175.28	1 133.72	−24 334.68

(*Table continues on next page*)

Table 3
Maximum Log Likelihood for (G)ARCH Models (*Continued*)

Model	*p, q*	Log likelihood	LR	SIC
		Japan (*continued*)		
ARCH	(3, 0)	12 209.14	1 201.44	−24 394.46
GARCH	(1, 1)	12 276.02	1 335.20	−24 536.16
GARCH	(2, 1)	12 273.66	1 330.48	−24 523.50
GARCH	(1, 2)	· · ·		
GARCH	(2, 2)	· · ·		
		Korea		
Normal	—	10 924.09		
ARCH	(1, 0)	11 006.52	164.86	−22 005.10
ARCH	(2, 0)	11 197.96	547.74	−22 380.04
ARCH	(3, 0)	11 204.59	561.00	−22 385.36
GARCH	(1, 1)	11 226.24	604.30	−22 436.60
GARCH	(2, 1)	11 228.10	608.02	−22 432.38
GARCH	(1, 2)	11 228.82	609.46	−22 433.82
GARCH	(2, 2)	· · ·		
		Singapore		
Normal	—	10 754.82		
ARCH	(1, 0)	11 220.27	1 428.30	−22 432.60
ARCH	(2, 0)	11 250.69	1 489.14	−22 485.50
ARCH	(3, 0)	11 263.74	1 515.24	−22 503.66
GARCH	(1, 1)	11 310.22	1 608.20	−22 604.56
GARCH	(2, 1)	· · ·		
GARCH	(1, 2)	11 309.73	1 607.22	−22 595.64
		United Kingdom		
Normal	—	11 789.03		
ARCH	(1, 0)	11 991.12	404.18	−23 974.30
ARCH	(2, 0)	12 046.90	515.74	−24 077.92
GARCH	(1, 1)	12 097.46	616.86	−24 179.04
GARCH	(2, 1)	12 091.50	604.94	−24 159.18
GARCH	(1, 2)	· · ·		
GARCH	(2, 2)	· · ·		
		United States		
Normal	—	11 280.33		
ARCH	(1, 0)	11 518.75	476.84	−23 029.56
ARCH	(2, 0)	11 550.89	541.12	−23 085.90
ARCH	(3, 0)	11 579.58	598.50	−23 135.34
GARCH	(1, 1)	11 628.24	695.82	−23 240.60
GARCH	(2, 1)	11 609.84	659.02	−23 195.86
GARCH	(1, 2)	11 623.64	686.62	−23 223.46
GARCH	(2, 2)	· · ·		

Note. · · · indicates where the optimization routine failed.

Table 4
GARCH Model Estimation

	Australia	Canada	France	Italy	Japan	Korea	Singapore	United Kingdom	United States
ϕ_0	0.464	0.203	0.688	0.806	0.750	0.535	0.707	0.678	0.566
ϕ_1	0.278	0.240	0.203	0.191	0.112	0.078	0.023	0.152	0.036
α_0	0.015	0.004	0.058	0.006	0.003	0.015	0.013	0.005	0.006
α_1	0.358	0.166	0.174	0.140	0.252	0.141	0.162	0.113	0.093
α_2			0.161						
α_3			0.098						
β_1	0.536	0.778	—	0.837	0.746	0.755	0.754	0.815	0.855
$\alpha_1 + \beta_1$	0.894	0.944	0.433[a]	0.976	0.997	0.896	0.916	0.928	0.948
$\sigma^2 (\times 10^3)$	0.144	0.072	0.103	0.268	0.214	0.141	0.155	0.074	0.109
$\sigma_R^2 (\times 10^3)$	0.156	0.076	0.107	0.278	0.230	0.141	0.156	0.076	0.110

Note. Underlined numbers indicate 5% significance level.
[a]$(\alpha_1 + \alpha_2 + \alpha_3)$ are parameters of the ARCH model.

returns, exhibit nonlinear dependence and non-normality. We tested various models belonging to the class of autoregressive conditional heteroskedasticity models. The results reveal that this class of models supersedes the random walk model. And among the different models the GARCH (1, 1) fits the data best for all countries except France, for which ARCH (3) fits better.

Conditional heteroskedasticity implies that the variance of the returns depends on information about recent returns. This can have important implications for research in finance. For example, GARCH models may be used to analyze the relationships between volatility and expected returns. In the Capital Asset Pricing Model, the expected return or risk premium can be influenced by the conditional second moment of returns. Volatility estimation is also an important feature for pricing derivative securities. GARCH models can indeed lead to better forecasts of volatility than alternative estimation methods.

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