Aerospace & Mechanical Engineering

ECCOMAS Thematic Conference, Multibody Dynamics 2011
July 4-7, 2011, Brussels, Belgium

Modeling of contact between stiff bodies in automotive transmission systems

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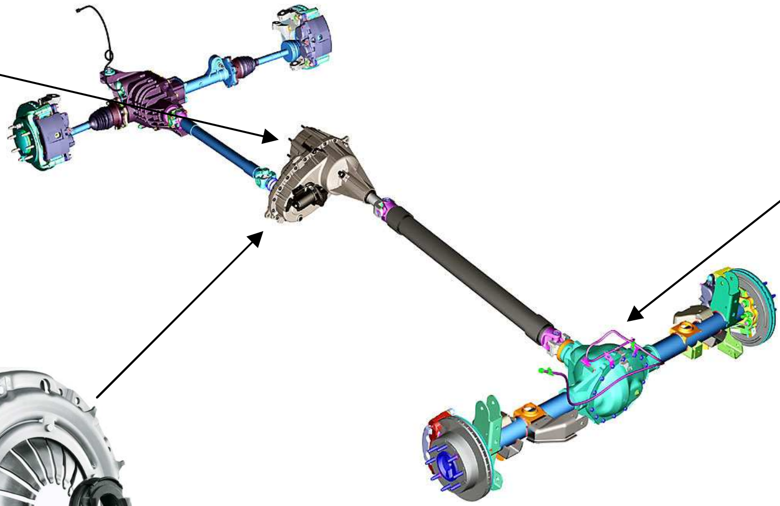
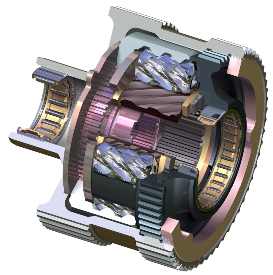
July 2011

Driveline modeling

Gear box





Differential

Clutch

Complex phenomena involved: backlash, stick-slip, contact, discontinuities, hysteresis, non linearities

➔ Numerical problems

In this work, TORSEN differential modeling and focus on contact formulation

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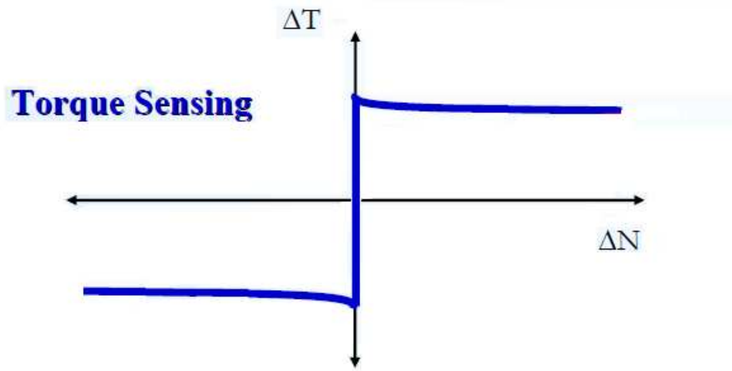
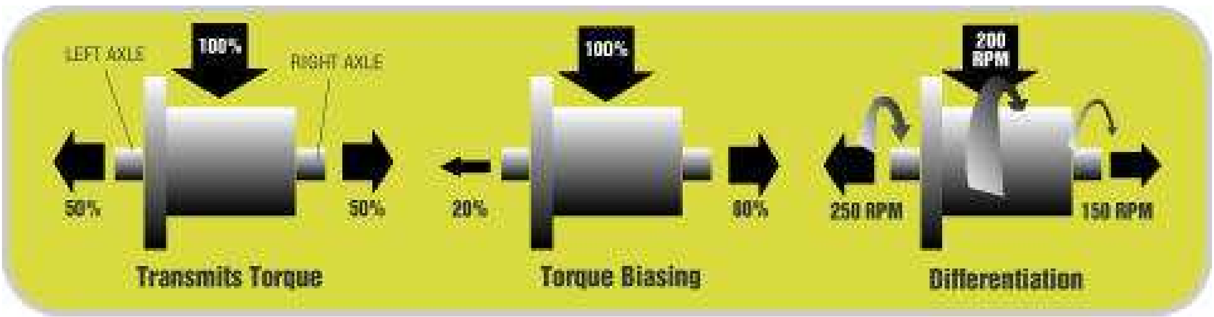
Outline

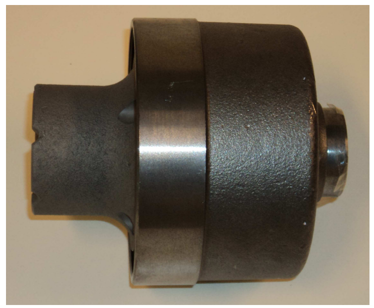
- Description of the application : Torsen differential
- Rigid/flexible contact model
  - Formulation
  - Numerical results for differential model
- Continuous impact modeling
  - Formulation
  - Numerical results for benchmark and differential model
- Conclusion

TORSEN differential

- Limited slip differential
  - Allow a variable torque distribution between the output shafts ➔ avoid spinning when ground adherence not sufficient on one driving wheel
  - Torque sensing before differentiation

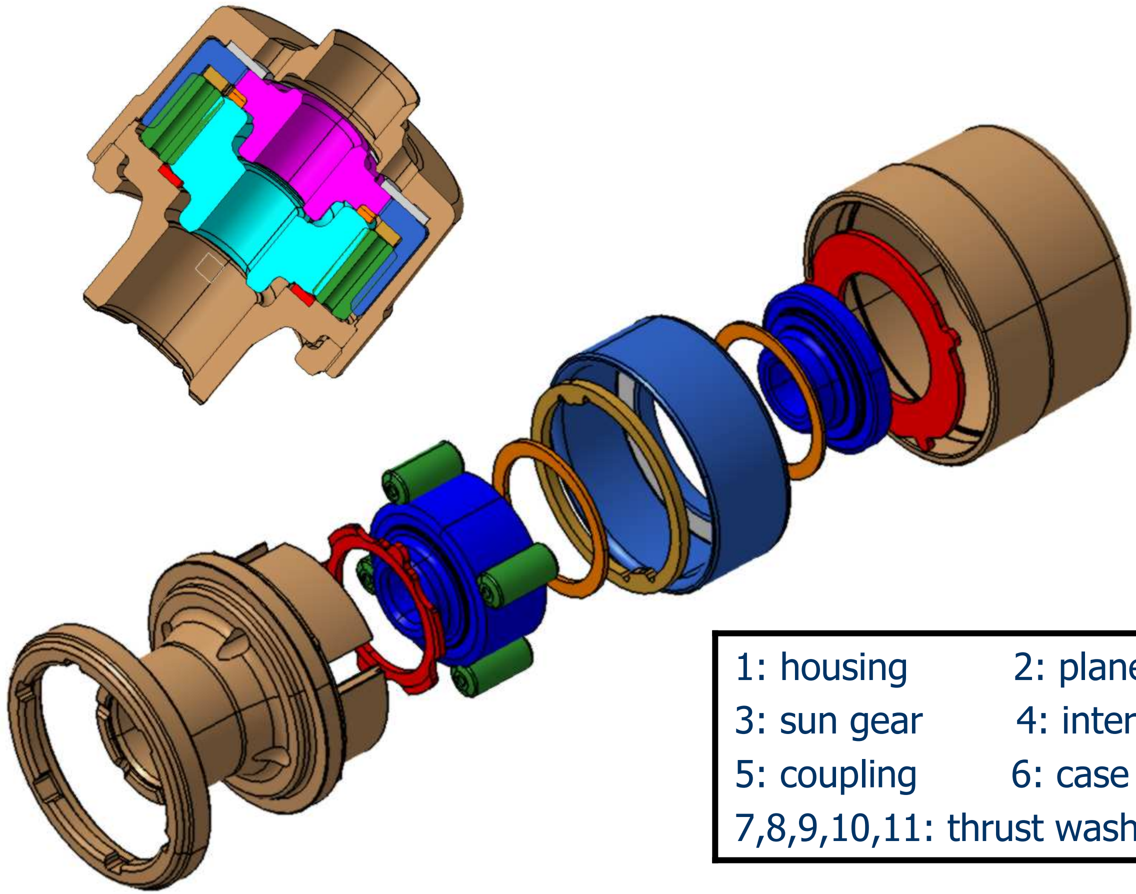


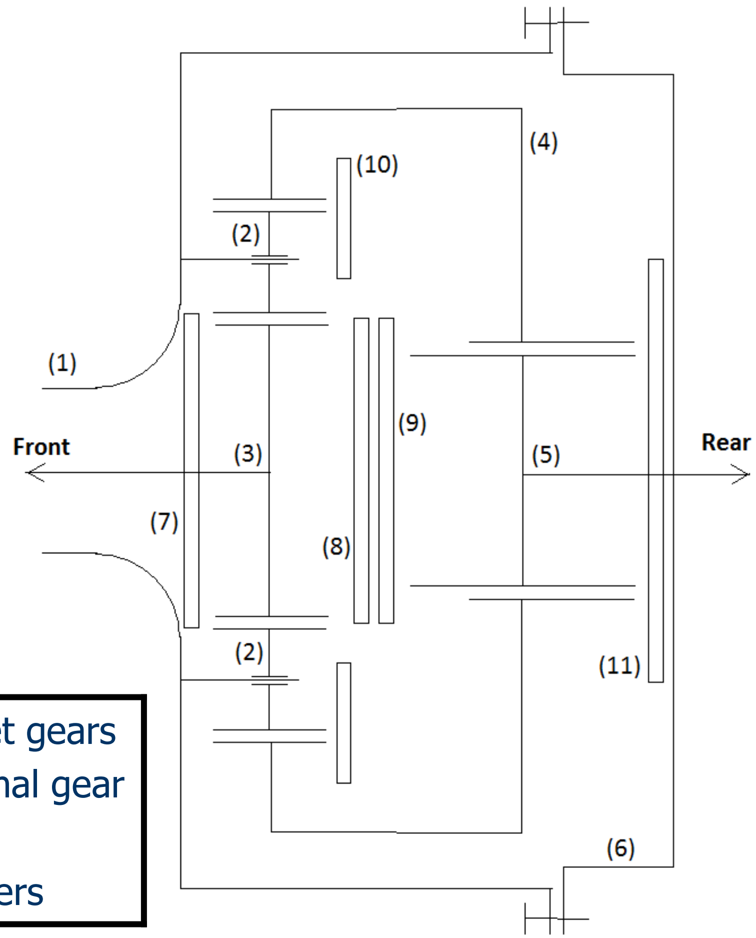




Type C Torsen

- Composed of gear pairs and thrust washers
- Locking due to relative friction between gears & washers
- 4 working modes





1: housing
2: planet gears
3: sun gear
4: internal gear
5: coupling
6: case
7,8,9,10,11: thrust washers

Contact element

- Features of contact element needed for TORSEN differential models :
  - Unilateral
  - Frictional
  - Robust to represent impact phenomenon

- Main contact formulations in nonlinear multibody systems simulation:
  - Continuous method
    - Lagrangian approach
    - Penalty
  - Instantaneous method (= non smooth)
    - Event-driven
    - Time-stepping

- Nonlinear finite element software : SAMCEF/MECANO

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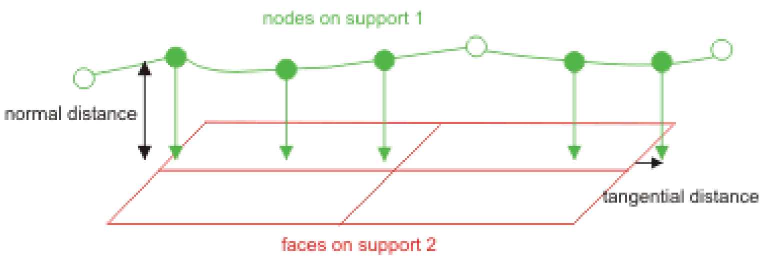
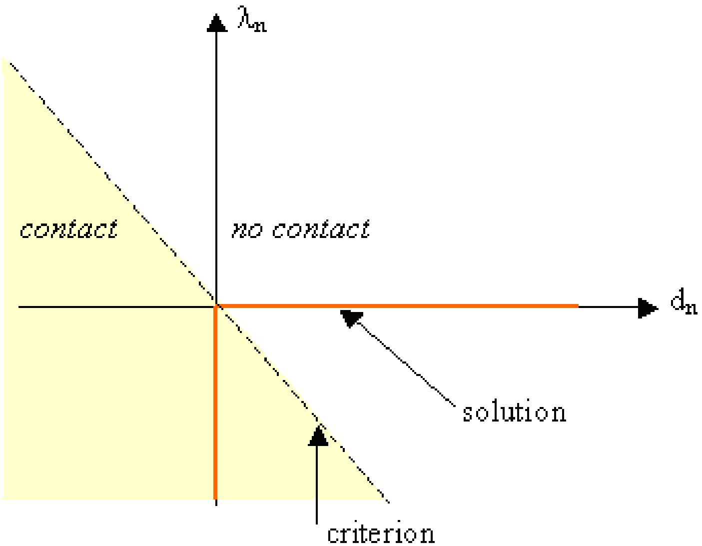
Augmented lagrangian method

- SAMCEF/MECANO : flexible/rigid or flexible/flexible contact
- 2 steps : - projection of slave nodes on master surface(s)



$$\begin{aligned} \delta d_n &= \underline{n}^T \; B \; \delta \underline{q} \\ \delta \Delta u_1 &= \underline{t}_1^T \; B \; \delta \underline{q} \\ \delta \Delta u_2 &= \underline{t}_2^T \; B \; \delta \underline{q} \end{aligned}$$

- definition of the contact condition



Contact criteria

$$\begin{aligned} \sigma_n &= k\lambda_n + p d_n \\ \sigma_{t_1} &= k\lambda_1 + p\Delta u_1 \\ \sigma_{t_2} &= k\lambda_2 + p\Delta u_2 \end{aligned}$$

(k = scaling factor , p=regularisation parameter)

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Augmented lagrangian method

- If $\sigma_n > 0$ ➔ no contact

$$\phi \equiv \lambda = 0$$

$$\delta \underline{q}^T \underline{F} = -(\delta\lambda_n \; k\lambda_n + \delta\lambda_{t_1} \; k\lambda_{t_1} + \delta\lambda_{t_2} \; k\lambda_{t_2})$$

- If $\sigma_n \leq 0$ ➔ contact

$$\phi \equiv d_n = 0$$

$$\delta \underline{q}^T \underline{F} = \delta d \; (p \; d + k \; \lambda_n) + \delta\lambda_n \; k \; d$$

- Included in the TORSEN differential model ➔ convergence problems due to impact phenomena (high relative axial speed at contact establishment)

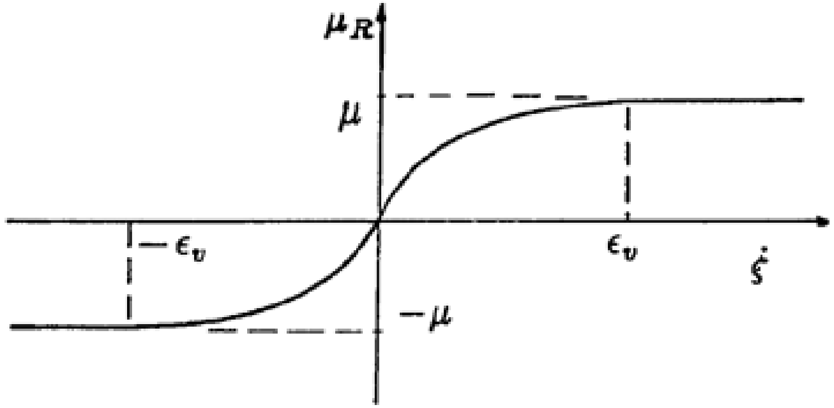
Penalty method

- Allow a small penetration between the two contacting bodies ➔ relax slightly the discontinuity
- Linear spring and damper

- Friction has been taken into account for this application

$$F_{fr} = \mu_R \, |F_{norm}|$$

Regularization to avoid discontinuities



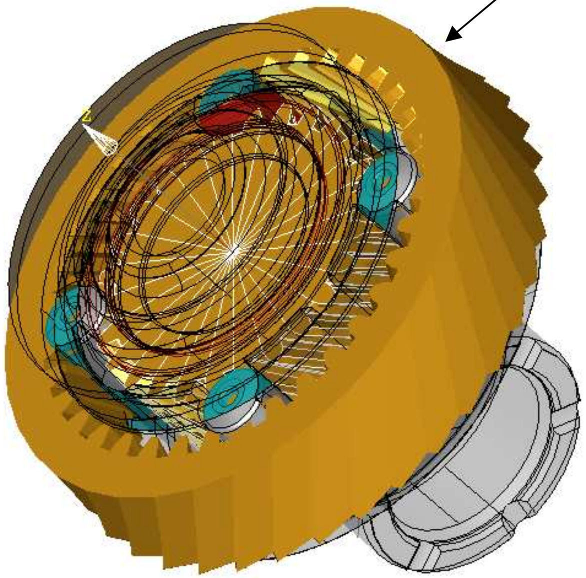
$$\mu_R(\dot{\xi}) = \begin{cases} \mu\left(2 - \frac{|\dot{\xi}|}{\epsilon_v}\right)\frac{\dot{\xi}}{\epsilon_v} & |\dot{\xi}| < \epsilon_v \\ \mu\frac{\dot{\xi}}{|\dot{\xi}|} & |\dot{\xi}| \geq \epsilon_v \end{cases}$$

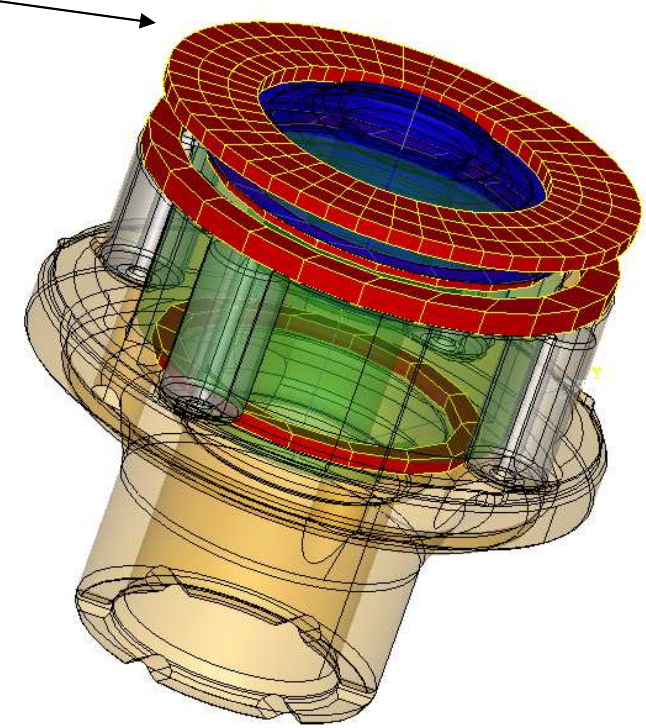
Model description

Assumptions: - joints between Planet gears and housing modeled as hinges

- planet gears and one thrust washer locked axialy
- contact SG/washer 3 and CPL/washer 4 neglected

- 15 bodies (*10 rigid, 5 flexible*), ≈8000 configuration parameters
- Constraints :
  - 8 gear elements
  - 5 contact relations
  - 4 hinges
  - 1 screw joint





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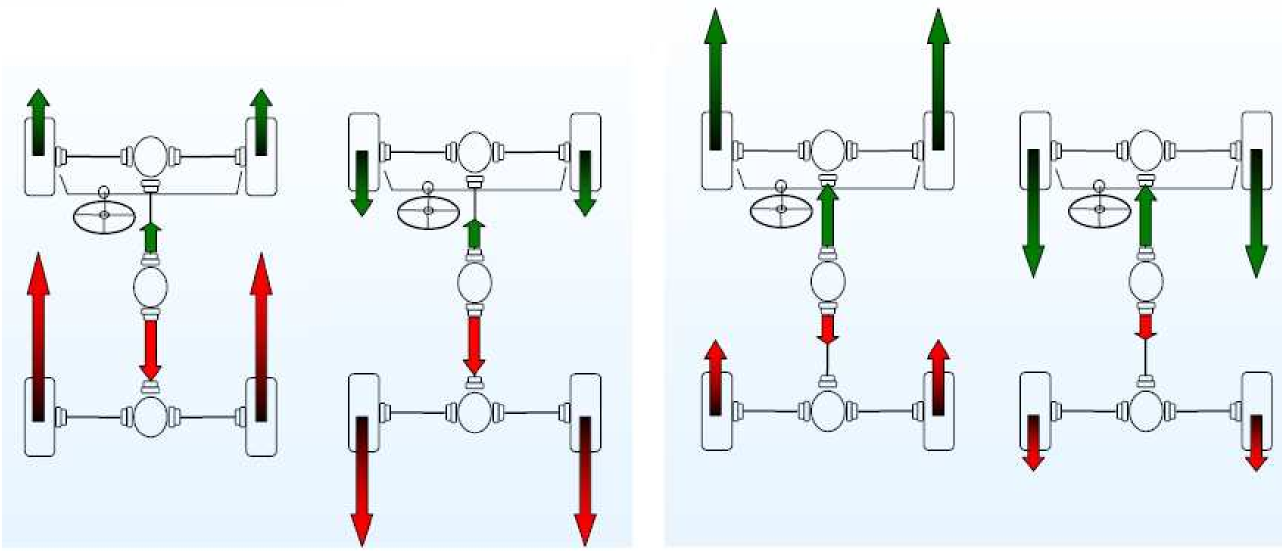
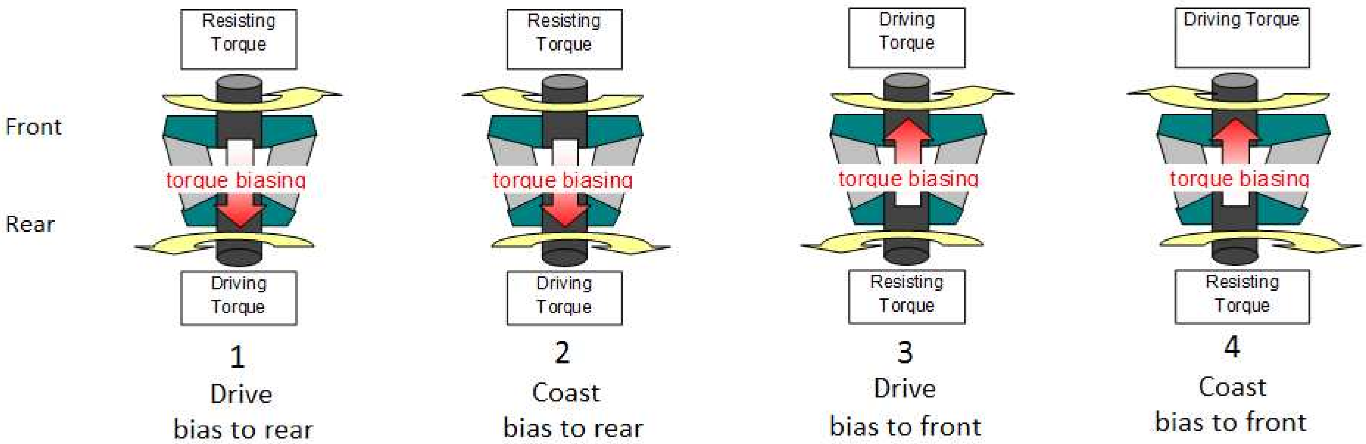
TDR computation for the 4 locking modes

- TDR : Torque Distribution Ratio

$$TDR = \frac{T_1}{T_2}$$





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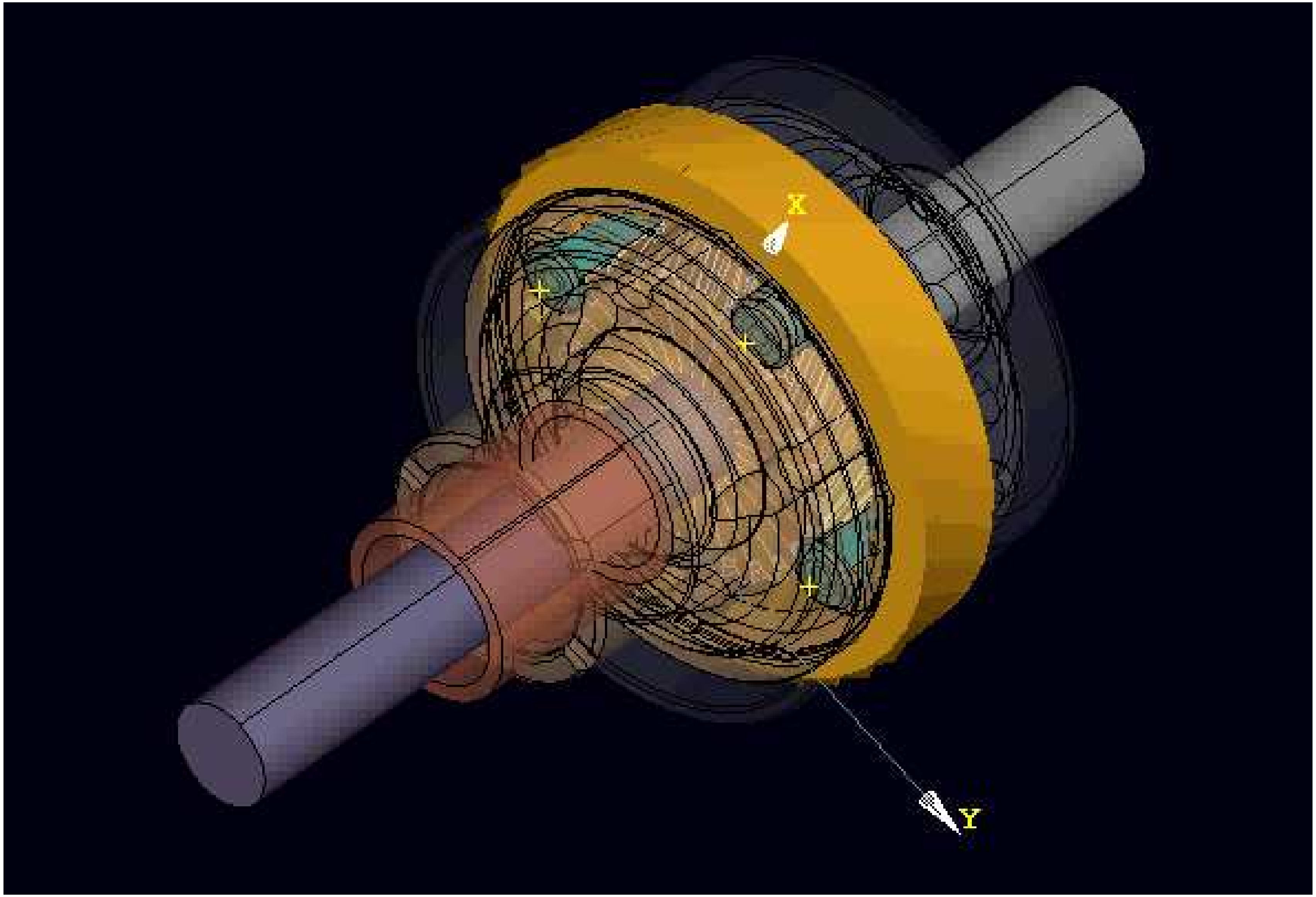
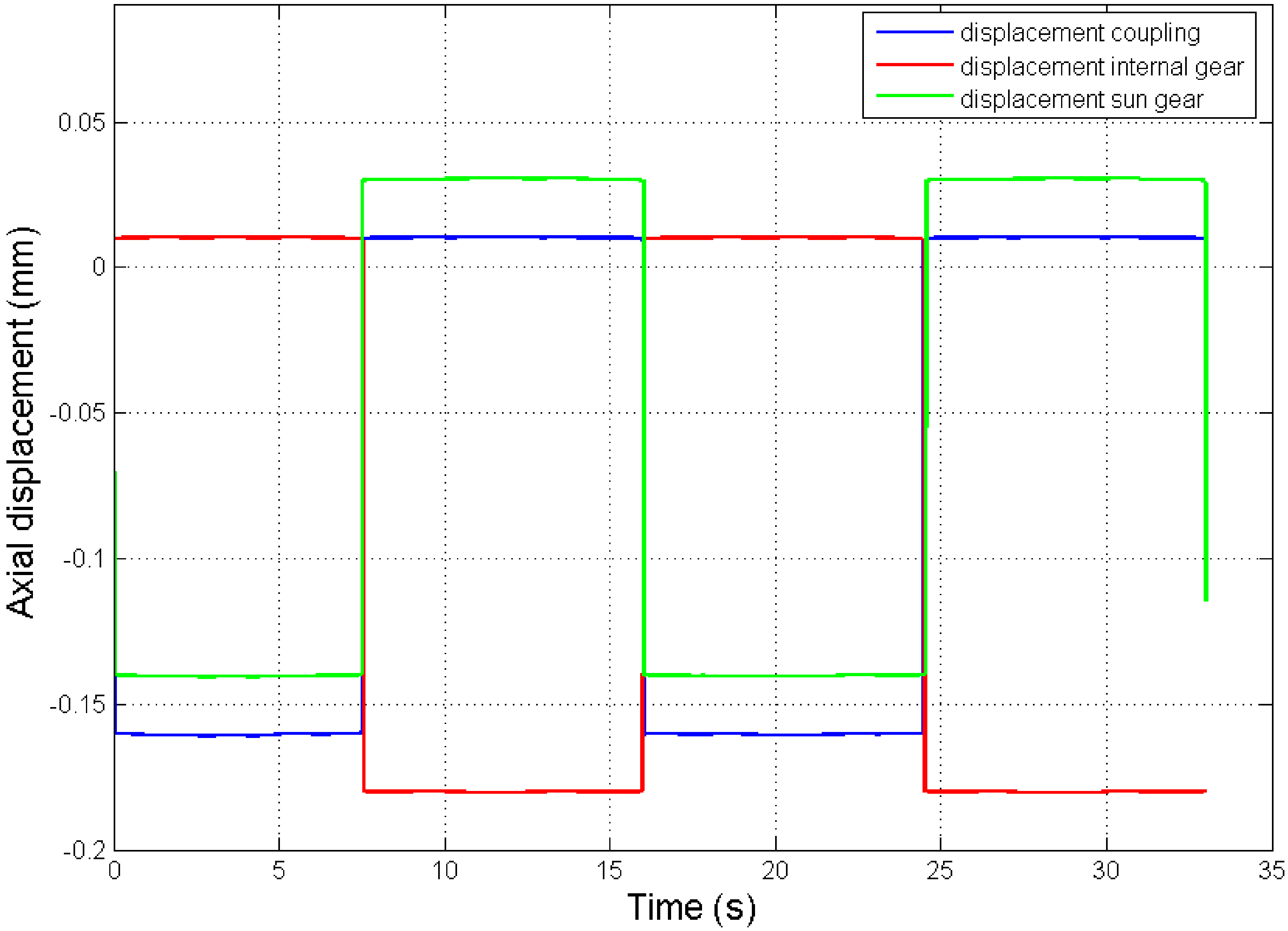
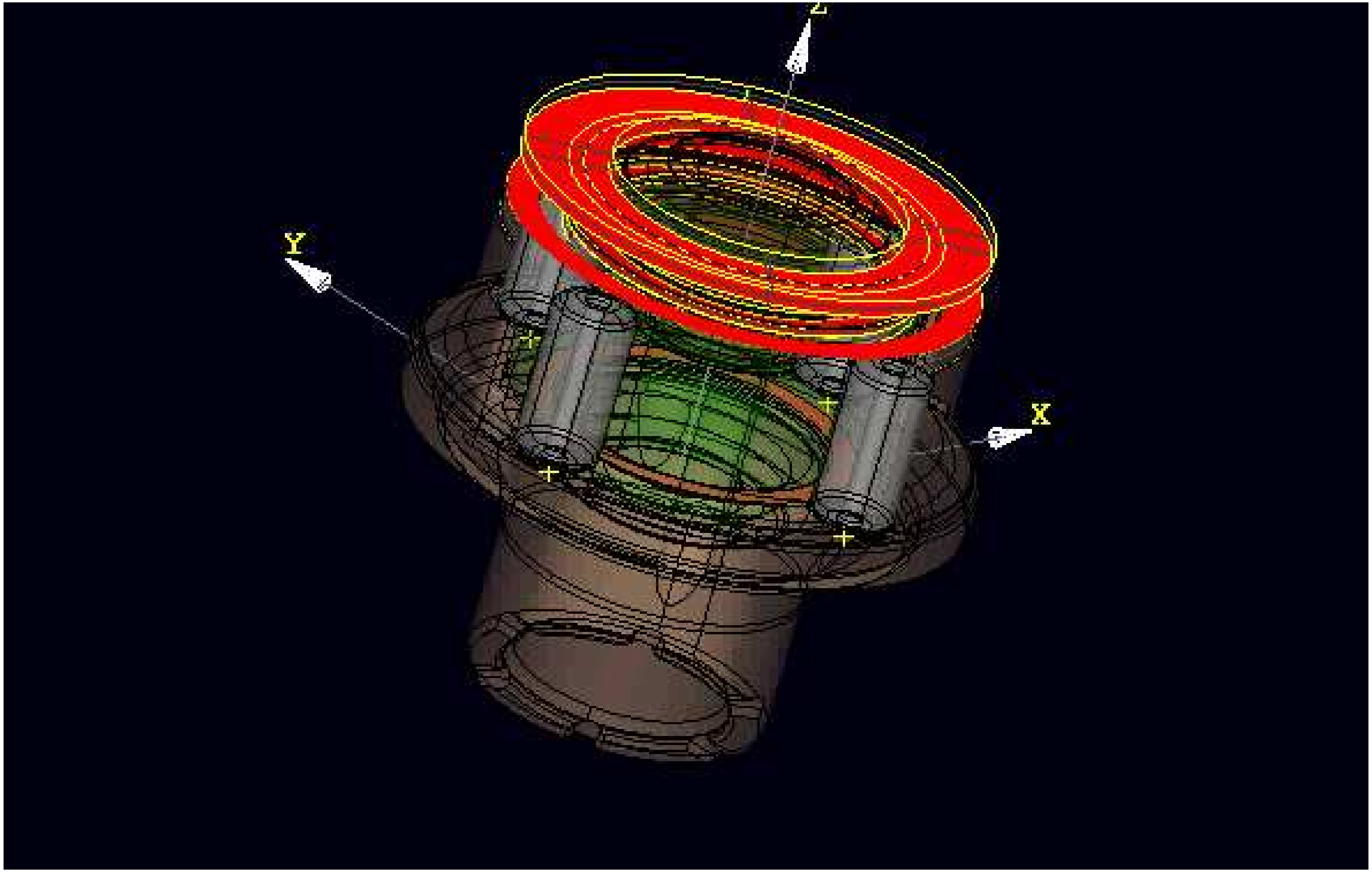
Configuration on vehicle

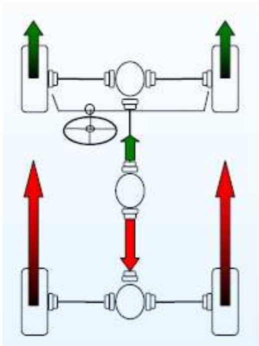


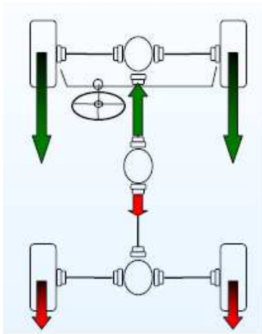
Axial displacements of gear wheels

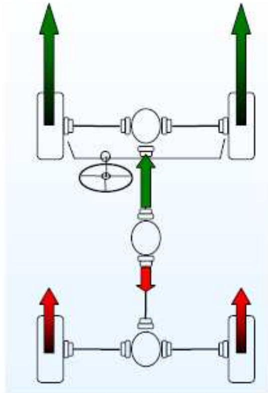


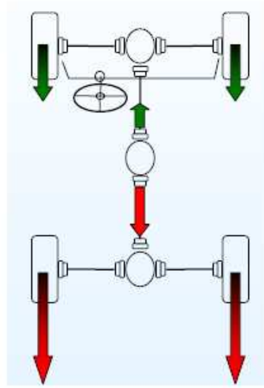
Contact pressure

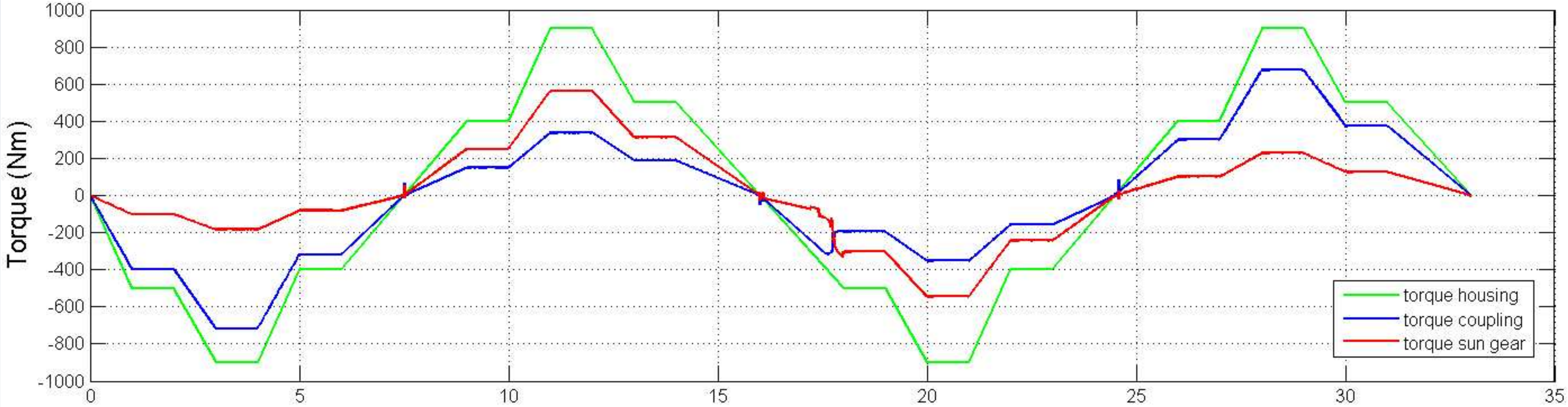


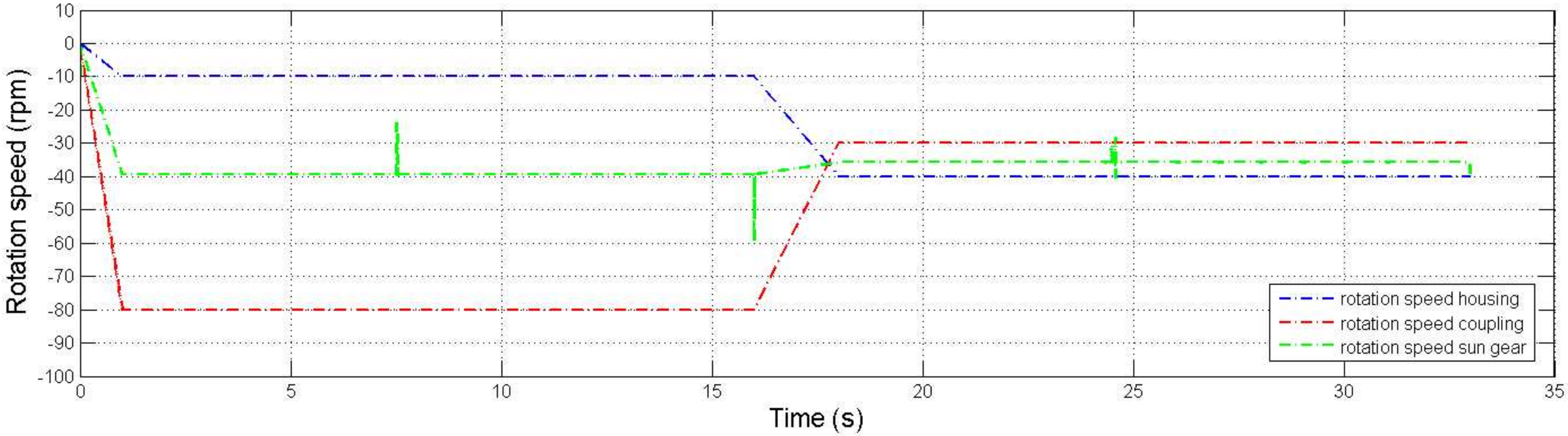








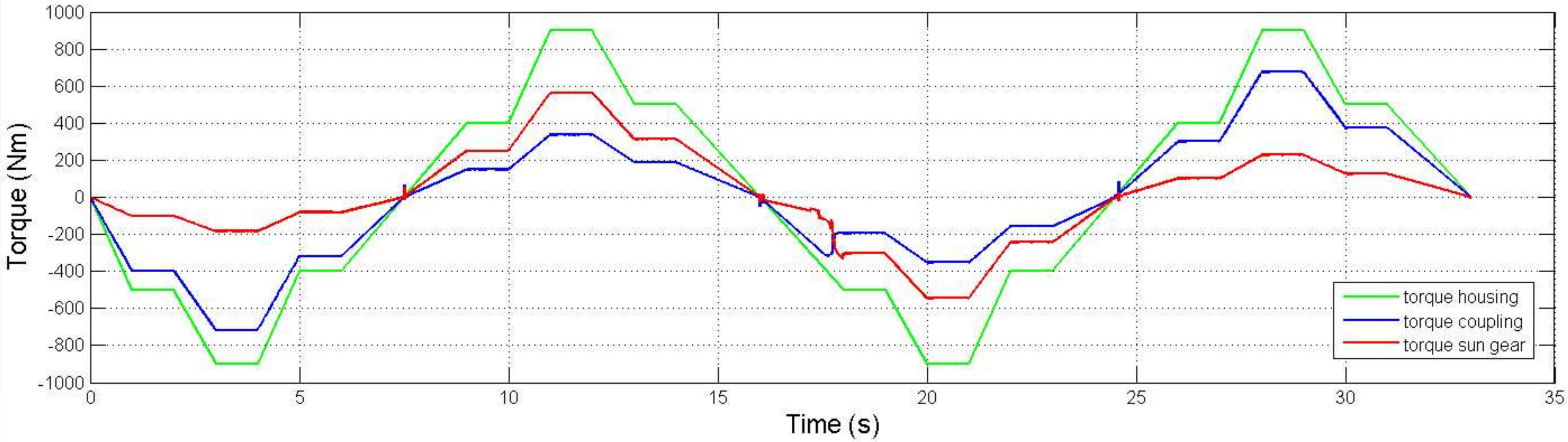

Torque (Nm)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
0
5
10
15
20
25
30
35
torque housing
torque coupling
torque sun gear


Rotation speed (rpm)
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
0
5
10
15
20
25
30
35
Time (s)
rotation speed housing
rotation speed coupling
rotation speed sun gear

Model validation

- Computation of Torque Distrubution Ratio and comparison with experimental data

$$TDR = \frac{T_{output\ shaft\ 1}}{T_{output\ shaft\ 2}}$$



mode	1 (Drive, rear)	2 (Coast, rear)	3 (Drive, front)	4 (Coast, front)
TDR simulation	3.9	2.94	1.56	1.65
TDR experimental	4.02	2.82	1.57	1.62

Contact element

- Global validation of the model but several drawbacks identified :
  - behavior at contact establishment (impact phenomenon) ➔using of very small time step (10^{-9} s)
  - Meshing of thrust washers (better choice: hexaedron elements)
  - Numerous variables (nodal coordinates, Lagrange multipliers) ➔ increase the computational time

- Solution: contact formulation define between rigid bodies and dedicated to impact problems

⟶ Continuous impact modeling

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Continuous impact modeling

- Based on a restitution coefficient:
  - Summarize the kinetic energy loss
  - Depend on shapes and material properties of colliding bodies and their relative velocity
  - roughly estimate by experince, determined by costly experiments or multi-scale simulations

- Several definitions (kinematic, kinetic, energetic)

$$
\begin{aligned}
e_N &= -\frac{\dot{g}_{n_e}}{\dot{g}_{n_s}} \\
e_P &= \frac{\Delta P_r}{\Delta P_c} = \frac{\int_{t_c}^{t_e} F \,\mathrm{d}t}{\int_{t_s}^{t_c} F \,\mathrm{d}t} \\
e_E^2 &= -\frac{T_r}{T_c} = -\frac{\int_{t_c}^{t_e} F \;\dot{g}_n \,\mathrm{d}t}{\int_{t_s}^{t_c} F \;\dot{g}_n \,\mathrm{d}t} = -\frac{\int_{h_c}^{h_e} F \,\mathrm{d}h}{\int_{h_s}^{h_c} F \,\mathrm{d}h}
\end{aligned}
$$

No energy loss

$(0 \leq e_i \leq 1)$

Total energy loss

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Continuous impact modeling

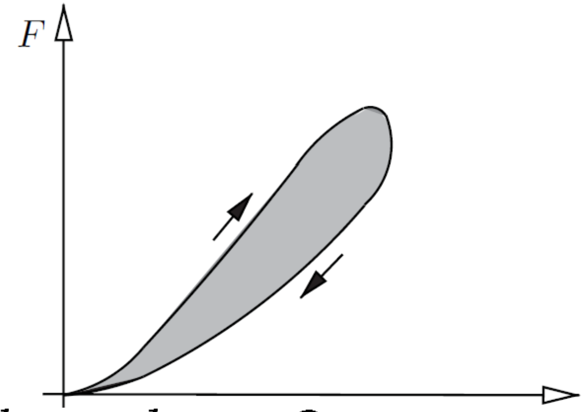
- Contact force law

$$F(h,\dot{h}) = k\ h^n + c\ h^n\ \dot{h}$$

$$c = \frac{3(1-e^2)}{4}\frac{k}{\dot{h}_s}$$

Restitution coefficient

- Hysteresis loop = kinetic energy loss during imapct



(R. Seifried)

- Friction Torque

$$M = 2\pi\ \mu_R\ \frac{F(h,\dot{h})}{S}\ \frac{r_{ext}^3 - r_{int}^3}{3}$$

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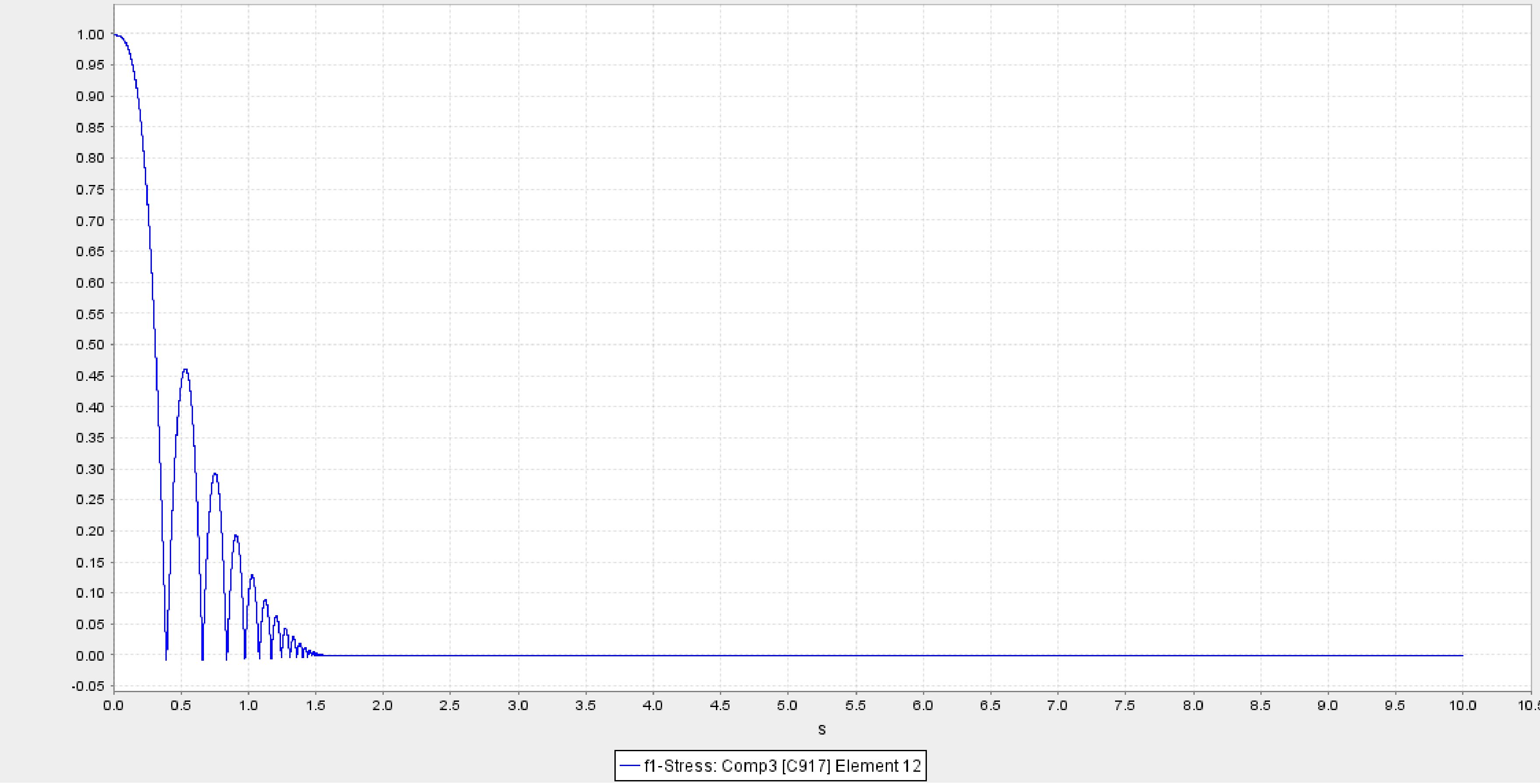
Benchmark

- Bouncing ball



(e=0.8, n=1,5, k=1e10 N/m, h_0=1 m,m=0,85 kg, a=10 m/s²)

TORSEN differential

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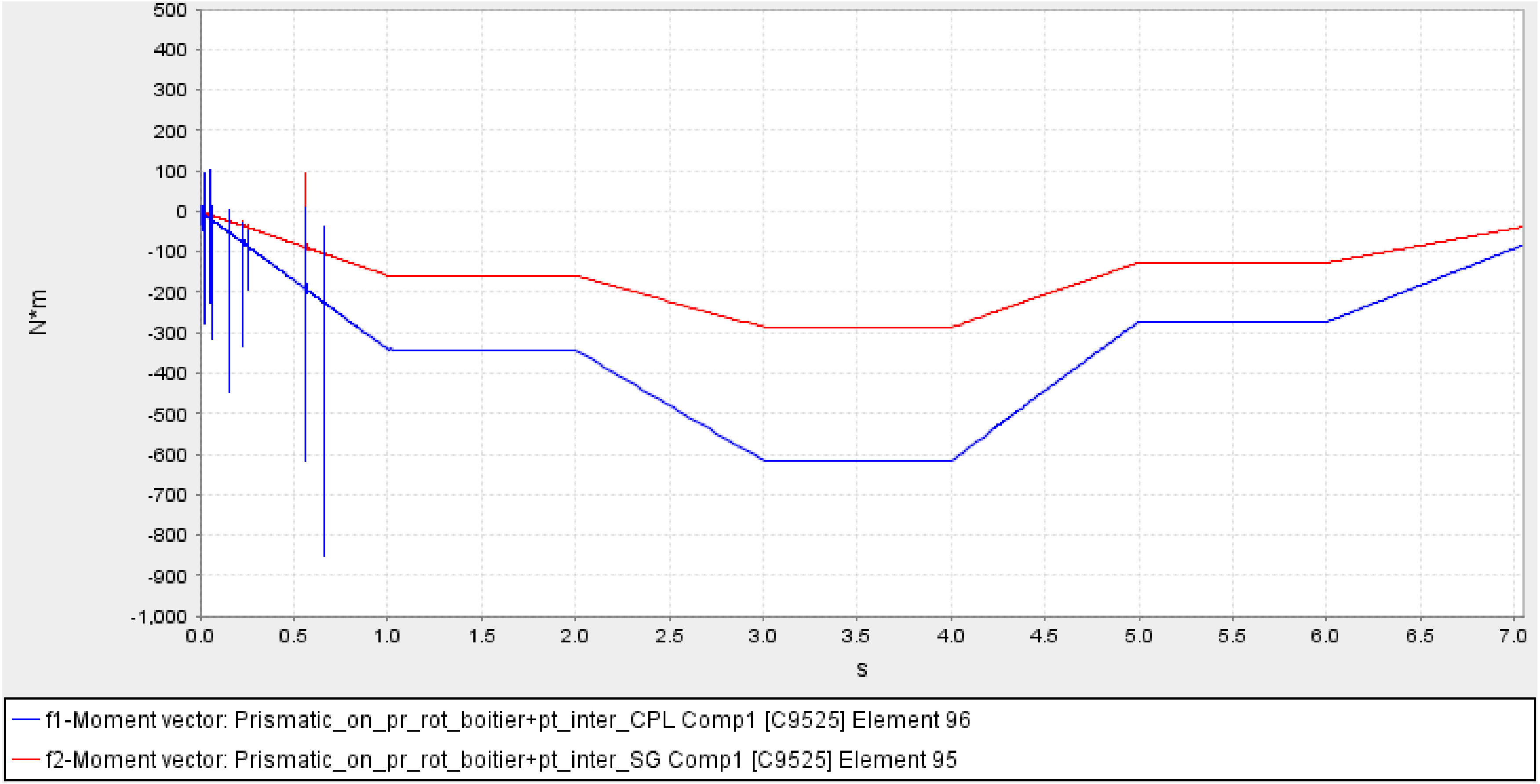
TORSEN differential

- Mode: Drive to rear

$$TDR = \frac{T_{output\ \ shaft\ \ 1}}{T_{output\ \ shaft\ \ 2}} \approx 3,92$$



Conclusion

- Continuous Contact element in TORSEN differential model :
  - Rigid/flexible contact :
    - Lagrange multiplier method ➔ convergence problems due to impacts phenomena
    - Penalty method ➔ simulation validated by experimental data
  - Rigid/rigid contact : continuous impact modeling
- Outlook :
  - Squeeze film modeling
  - Include differential model in complete vehicle model

Questions / Answers

Thank you for your attention !

References

R. Seifried, W. Schiehlen, and P. Eberhard. The role of the coefficient of restitution on impact problems in multi-body dynamics. *Proc. IMechE, Part K:J. Multi-body Dynamics*, 224:279–306, 2010.

Hamid Lankarani and Parviz Nikravesh. Continuous contact force models for impact analysis in multibody analysis. *Nonlinear Dynamics*, 5:193–207, 1994.

M. Géradin and A. Cardona. *Flexible multibody dynamics*. John Wiley & Sons, 2001.