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The intervalling effect bias in beta: A note

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Based on a comprehensive sample of domestic securities traded on the Brussels Stock Exchange, this paper points out the intervalling effect in the estimated betas and examines the speed of convergence of these. The results reveal that the estimated betas seem to converge to their asymptotic values and that their value depends on what day the differencing interval starts. It also appears that the magnitude of the intervalling effect is inversely related to the market value of the firms.

1. Introduction

An important issue related to the systematic risk or beta coefficient of a security is its sensitivity to the length of the differencing interval used to measure the returns. This effect, which is called the intervalling effect on estimated betas, has received considerable interest from the academic community, and several methods for adjusting the bias in the estimated betas have so far been put forward by Scholes and Williams (1977), Levhari and Levy (1977), Dimson (1979) and Cohen et al. (1983a, b). The concern of this paper is to underscore the intervalling effect in the betas of a large sample of the Brussels Stock Exchange (BSE) for three periods, and to examine how these betas converge to an asymptotic value when the differencing interval used to measure the returns is lengthened.

The impact of the length of the differencing interval used to measure the returns on the estimated betas was first shown by Pogue and Solnik (1974). Using samples from seven European countries, including Belgium, they found that the daily beta estimates depend on the length of the differencing interval. The intervalling effect bias in beta has been ascribed by Cohen et al. (1983a, b) to the friction in the trading process. Infrequent trading or, more generally, delays in the adjustment of a security price to a change in information induce cross serial correlation in the security returns and subsequently autocorrelation in the market index returns. According to the theory of Cohen et al. the expected magnitude of the price-adjustment delays
is related to the thinness of the securities: thinner securities have greater adjustment delays than frequently traded securities. Cohen et al. also demonstrated that thin securities have a downward bias in their betas for short differencing intervals, while relatively frequenly traded securities have an upward bias.

2. Sample and test methodology

2.1. The sample

The data consist of the daily returns of 250 domestic securities traded on the spot market of the BSE, which roughly represents the complete spot market of the BSE. The time period covered is from January 1977 to December 1985. The returns, 2,213 for the whole period, are continuously compounded returns. They are calculated as the difference between the natural logarithms of two consecutive closing prices, $R_t = ln(P_t) — \ln(P_{t-1})$. They are corrected for all capital adjustments and they incorporate dividends. Alongside the returns, the market value of the outstanding shares of the securities as well as their volume of trading have also been collected. The returns of the portfolio composed of the 250 securities, weighted by the market value of these, are used as market index returns.

The total nine-year period is divided into three three-year subperiods of 738 (1977 to 1979), 735 (1980 to 1982) and 740 daily returns (1983 to 1985). In order to avoid data problems due to the listing and delisting of securities, the securities have been selected on the basis of their continuous presence on a whole subperiod. Therefore the number of securities for each of the subperiods is respectively reduced to 153, 180 and 170 securities.

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2.2. *Test methodology*

We assume that the security returns are generated by the Market Model

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \quad t = 1, \ldots, T,$$

(1)

where β_i, the security beta, measures the change in R_{it} as a result of a change in the market index return R_{mt}, α_i measures the change in R_{it} that is independent of a change in R_{mt}, and ε_{it} is the random error term. According to this model, neither α_i nor β_t depend on the length of the differencing interval used to calculate the returns.

The estimates of α_i and β_i obtained using an ordinary least square regression, are however strongly dependent on the length of the differencing interval [Pogue and Solnik (1974), Hawawini (1980) and Cohen et al. (1983a, b)]. Hawawini (1980) demonstrated that when continuous returns are used, the value of a security beta for any particular length L of differencing interval is:

$$\beta_i(L) = \beta_i(1) \frac{L + \sum_{s=1}^{L-1} (L-s) \frac{\rho_{im}^{+s} + \rho_{im}^{-s}}{\rho_{im}^{0}}}{L + 2 \sum_{s=1}^{L-1} (L-s) \rho_m^s},$$

(2)

where ρ_{im}^{0}, ρ_{im}^{+s}, ρ_{im}^{-s} are, respectively, the intertemporal cross-correlation coefficient of order 0, + s (lead) and — s (lag) between the returns, measured on a one-day differencing interval, of security *i* and the market, and ρ^{s}_{m} is the autocorrelation of order *s* on the market daily returns.

It follows from this equation that the systematic risk will be invariant to the length of the differencing interval L only if there is no intertemporal cross-correlation between the returns of a security and the market, and if the market returns are not autocorrelated. Therefore, as the intertemporal cross-correlation and the market autocorrelation generally decrease with the order of the lag, the value of the OLS security beta approaches an asymptotic value when the differencing interval is lengthened.

$$\beta_i = \lim_{\substack{L \to \infty \\ T \to \infty}} \beta_i(L).$$

(3)

In order to examine the speed of convergence of the beta coefficient of each security *i* when the differencing interval is lengthened, the beta is estimated for a finite set of differencing interval lengths L,

$$R_{iLt} = \hat{\alpha}_{iL} + \hat{\beta}_{iL} R_{mLt} + \hat{\varepsilon}_{iLt}, \quad \text{for } L = 1, \ldots, 30 \text{ and } t = 1, \ldots, T,$$

(4)

where R_{iLt} and R_{mLt} are, respectively, the returns of security *i* and the market index, measured over a differencing interval of L days, L varying from one day to thirty days.

At this stage, a correction of the $\beta_i(L)$ is necessary to better discern the convergence of the beta coefficients. Corhay (1988) noticed indeed that the value of beta coefficients depends on the manner daily prices are juxtaposed to calculate returns on intervals longer than one day. Since a return for a specific interval length is measured as the difference in logarithm between two well-defined daily prices, any price move, whatever its magnitude, that occurs and is wiped out between these two days does not enter into the calculation of the return, nor, consequently, into the estimation of the beta. On the other hand, substantial moves that systematically occur on the day returns are measured have an impact on estimated betas. Corhay showed, for example, that the beta coefficients of Belgian stocks exhibit a seasonal pattern. Betas estimated using Monday to Monday weekly returns are always larger than those estimated using Friday to Friday weekly returns. Therefore the correction consists in running the regression L times for an interval length of L and in calculating an average beta coefficient. Such procedure allows us to avoid too high and too low estimated beta coefficients which would be due only to the juxtaposition of the daily prices. The regression is run a first time with returns of interval length L calculated using the complete series of daily returns. Then the first daily return is deleted, the returns of interval length *L* are recalculated with the remaining observations and the regression is run again and so forth until it is run L times.[1] So the regression model becomes:

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$$R_{iLt} = \hat{\alpha}_{iLn} + \hat{\beta}_{iLn} R_{mLt} + \hat{\varepsilon}_{iLt},$$

for $L = 1, \ldots, 30$ $t = 1 + n - 1, \ldots, T$ and $n = 1, \ldots, L.$

(5)

For each interval length, the average beta $\bar{\hat{\beta}}_{iL}$, as well as the standard deviation $\sigma(\hat{\beta}_{iL})$ of the betas are then calculated: [2]

$$\bar{\hat{\beta}}_{iL} = \sum_{n=1}^{L} \hat{\beta}_{iLn} \Big/ L \qquad \sigma(\hat{\beta}_{iL}) = \sqrt{\sum_{n=1}^{L} \frac{(\hat{\beta}_{iLn} - \bar{\hat{\beta}}_{iL})^2}{L}}.$$

(6)

The speed of convergence of the betas to their asymptotic value is examined. Given the number of securities, the number of periods in the study, all individual security results cannot be presented in this note. Therefore the results are presented for 10 portfolios and the sample as a whole, as well as for the individual securities composing portfolios 1 and 10. The number of securities in each portfolio for the subperiods is given in the tables.[3] In order to test differences between the means of the size portfolio betas, an analysis of variance is carried out on the individual betas and their standard deviations of the 10 portfolios, as well as on the individual betas of portfolios 1 and 10.

The portfolios are value weighted portfolios and they are constructed on the basis of the market value of the securities. The market value of a security is measured at the midpoint of a subperiod; it is the natural logarithm of the value, in millions of Belgian francs, of the outstanding shares of the security. The betas for portfolios formed on the basis of the volume of trading of the securities, as well as on the ratio volume of trading to the number of their outstanding shares, were also calculated, but as their results do not significantly differ from those obtained with the market value, they are not presented.[4] These three variables are related to the thinness of the securities. On the one hand, one can expect that larger firms, having a larger volume of transaction and about which the public is generally better informed, have a shorter delay in their price adjustment than smaller firms. On the other hand, trading securities having a high degree of rotation certainly presents some advantages to the investor who can more easily and more quickly dispose of the shares.

3. Empirical results

The results for the three subDeriods are presented in tables 1, 2 and 3. The values of the average betas $\bar{\hat{\beta}}_{iL}$, as well as their standard deviation $\sigma(\hat{\beta}_{iL})$,
are summarized in the tables for the ten market value formed portfolios, as well as for the whole sample, and for lengths of differencing interval, L=1,2,3,4,5,8,10,12,16,22 and 30.[5] Individual beta coefficients of portfolios 1 and 10 and *F*-test statistics of the analysis of variance are also reported in the tables.

There is no intervalling effect on the whole sample and its average beta is always close to one. This is because the sample used in the study almost represents the entire spot market of the BSE. As for the average betas of the ten size portfolios, there is an intervalling effect. The effect is quite large for small differencing intervals and it tends to decrease when it is lengthened. Our results tend therefore to confirm the asymptotic behavior of the security betas as demonstrated by Hawawini (1980) and Cohen et al. (1983a, b). The direction of the intervalling effect is negative for the first portfolio, composed of the largest firms, while it is on the average positive for the other nine, and its magnitude is inversely related to the market value of the firms. Besides, all *F*-test statistics resulting from the analysis of variance between the individual betas of the ten portfolios are statistically significant at the five per cent level, whatever the length of the differencing interval, which leads to the rejection of equality between the means of the size portfolio betas. Concerning the comparison between portfolios 1 and 10, the values of the *F*-tests are even higher. Therefore, firms with smaller market value appear to have on the average lower beta coefficients than large firms. It can, however, be observed that both F-test statistics decrease slightly but remain statistically significant when the differencing interval is lengthened.

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Table 1 Beta coefficients: Period 1977-1979.

Portfolio.	Number of stocks	Average market value		Differencing interval *L* 1	2	3	4	5	8	10	12	16	22	30
1	17	14,970	(1)	1.167	1.134	1.114	1.099	1.087	1.063	1.056	1.051	1.040	1.033	1.015
			(2)		0.057	0.090	0.099	0.094	0.123	0.117	0.135	0.141	0.150	0.185
2	15	3,656	(1)	0.567	0.665	0.737	0.791	0.827	0.885	0.904	0.919	0.944	0.971	1.020
			(2)		0.088	0.103	0.105	0.136	0.152	0.137	0.165	0.206	0.238	0.274
3	15	1,214	(1)	0.473	0.600	0.686	0.741	0.772	0.836	0.849	0.856	0.898	0.915	0.953
			(2)		0.107	0.147	0.157	0.144	0.169	0.158	0.211	0.191	0.234	0.293
4	15	776	(1)	0.507	0.617	0.667	0.709	0.745	0.827	0.849	0.865	0.897	0.965	1.097
			(2)		0.098	0.157	0.151	0.183	0.187	0.209	0.192	0.246	0.269	0.350
5	15	481	(1)	0.463	0.620	0.701	0.752	0.789	0.855	0.853	0.876	0.953	1.018	1.105
			(2)		0.145	0.154	0.177	0.191	0.204	0.211	0.238	0.288	0.317	0.360
6	15	280	(1)	0.255	0.381	0.472	0.538	0.582	0.647	0.675	0.695	0.748	0.810	0.897
			(2)		0.144	0.184	0.188	0.204	0.211	0.244	0.278	0.302	0.340	0.426
7	15	194	(1)	0.156	0.217	0.267	0.321	0.354	0.435	0.469	0.498	0.562	0.626	0.714
			(2)		0.091	0.142	0.137	0.198	0.163	0.204	0.232	0.228	0.290	0.413
8	15	103	(1)	0.156	0.157	0.196	0.210	0.220	0.281	0.303	0.330	0.374	0.378	0.389
			(2)		0.111	0.169	0.165	0.187	0.197	0.212	0.197	0.236	0.278	0.326
9	15	55	(1)	0.150	0.286	0.372	0.430	0.467	0.503	0.520	0.530	0.598	0.697	0.835
			(2)		0.178	0.201	0.201	0.197	0.195	0.248	0.248	0.292	0.338	0.450
10	16	27	(1)	0.075	0.111	0.176	0.232	0.285	0.357	0.396	0.421	0.503	0.536	0.547
			(2)		0.161	0.169	0.247	0.251	0.265	0.270	0.317	0.282	0.332	0.508
All stocks	153	2,329	(1)	0.977	0.984	0.989	0.994	0.995	0.996	0.996	0.997	1.000	1.005	1.010
			(2)		0.070	0.101	0.108	0.111	0.136	0.130	0.151	0.163	0.180	0.220
F-test betas (10 portf.)				8.39	8.08	7.47	6.95	6.42	5.20	4.65	4.32	3.51	3.28	3.10

F-test standard deviations (10 portf.)			<u>176</u>	1.74	<u>3.48</u>	<u>2.58</u>	<u>3.30</u>	<u>1.04</u>	<u>3.22</u>	<u>2.75</u>	<u>3.69</u>	<u>3.07</u>
Individual beta	coefficients	Differencing interval L										
Stock	Market value	1	2	3	4	5	8	10	12	16	22	30
Portfolio 1												
1	48,837	2.747	2.448	2.251	2.094	2.026	1.840	1.776	1.705	1.599	1.498	1.364
2	35,213	1.217	1.206	1.217	1.212	1.177	1.101	1.077	1.045	1.006	0.964	0.856
3	29,925	0.830	0.771	0.762	0.738	0.715	0.696	0.693	0.708	0.718	0.677	0.631
4	18,634	0.948	0.904	0.863	0.881	0.880	0.935	0.947	1.003	1.009	1.048	1.120
5	15,094	0.685	0.720	0.760	0.779	0.777	0.771	0.779	0.793	0.783	0.771	0.708
6	14,462	0.580	0.676	0.725	0.766	0.762	0.758	0.761	0.754	0.779	0.830	0.909
7	13,358	0.311	0.354	0.396	0.425	0.435	0.504	0.520	0.523	0.557	0.635	0.775
8	11,228	1.078	1.153	1.139	1.133	1.074	1.033	1.028	1.037	1.057	1.141	1.250
9	9,418	0.862	0.882	0.849	0.939	1.016	1.116	1.110	1.113	1.087	1.090	1.069
10	8,215	0.452	0.596	0.695	0.749	0.771	0.892	0.898	0.915	0.991	1.086	1.211
11	8,061	0.526	0.567	0.584	0.581	0.579	0.600	0.596	0.606	0.607	0.575	0.539
12	7,803	0.981	1.050	1.153	1.228	1.249	1.305	1.315	1.329	1.285	1.244	1.148
13	7,500	0.339	0.447	0.510	0.544	0.596	0.635	0.665	0.698	0.720	0.712	0.719
14	7,235	1.041	1.096	1.109	1.191	1.231	1.258	1.299	1.343	1.428	1.491	1.543
15	7,213	0.414	0.482	0.515	0.551	0.612	0.742	0.814	0.876	0.979	1.072	1.188
16	6,287	0.396	0.596	0.675	0.725	0.742	0.806	0.849	0.873	0.923	1.020	1.218
17	6,000	0.560	0.653	0.748	0.792	0.831	0.917	0.970	1.013	1.108	1.207	1.261
Portfolio 10												
1	38	-0.017	0.14	0.079	0.236	0.396	0.624	0.796	0.986	1.389	1.360	1.409
2	38	0.220	0.293	0.456	0.522	0.600	0.638	0.758	0.778	0.739	0.806	0.754
3	37	-0.070	-0.085	-0.124	-0.177	-0.166	-0.054	-0.040	-0.027	0.099	0.120	0.045
4	37	-0.113	-0.019	0.264	0.519	0.639	0.705	0.731	0.768	0.920	1.021	1.254
5	37	-0.036	-0.013	0.223	0.414	0.593	0.881	0.976	0.952	1.133	1.192	1.305
6	36	0.192	0.093	0.054	0.019	0.050	0.049	0.040	0.083	0.155	0.151	0.025
7	35	-0.026	0.060	0.192	0.168	0.130	0.114	0.168	0.203	0.210	0.340	0.383
8	33	-0.006	— 0.116	-0.090	-0.076	-0.054	0.005	-0.010	-0.050	- 0.100	-0.158	-0.287
9	27	0.015	0.011	0.007	0.018	0.056	0.157	0.216	0.251	0.364	0.434	0.392
10	26	0.060	0.179	0.141	0.077	-0.034	-0.283	-0.351	-0.395	- 0.457	-0.344	-0.296
11	24	0.376	0.532	0.536	0.542	0.576	0.682	0.678	0.704	0.743	0.720	0.710
12	23	0.284	0.217	0.335	0.423	0.563	0.743	0.682	0.585	0.596	0.456	0.369
13	16	0.155	0.177	0.174	0.242	0.257	0.452	0.533	0.601	0.666	0.739	0.847
14	15	0.291	0.381	0.417	0.459	0.486	0.445	0.451	0.515	0.548	0.565	0.583
15	13	0.274	0.290	0.316	0.377	0.362	0.258	0.303	0.345	0.345	0.466	0.669
16	3	-0.166	-0.072	-0.032	-0.054	-0.192	-0.595	-0.737	-0.783	- 0.850	-0.990	-1.122
F-test betas (portf. 1 versus portf. 10)		<u>24.25</u>	<u>32.23</u>	<u>35.24</u>	<u>34.42</u>	<u>29.90</u>	<u>24.09</u>	<u>20.62</u>	<u>19.19</u>	<u>13.64</u>	<u>12.94</u>	<u>11.24</u>

(1) and (2) are the means of the average individual beta $\hat{\beta}_{iL}$ and the individual standard deviation $\sigma(\hat{\beta}_{iL})$ of the stocks forming a portfolio, respectively. F-test statistics significant at the five percent level are underlined.

Table 2 Regression statistics: Period 1980-1982.

Portfolio	Number of stocks	Average market value		Differencing interval L										
				1	2	3	4	5	8	10	12	16	22	30
1	18	11,271	(1)	1.146	1.117	1.100	1.087	1.077	1.062	1.055	1.050	1.043	1.031	1.018
			(2)		0.049	0.080	0.105	0.077	0.098	0.072	0.103	0.111	0.097	0.130
2	18	2,456	(1)	0.762	0.846	0.893	0.926	0.948	0.970	0.981	0.991	0.998	1.012	1.020
			(2)		0.070	0.070	0.081	0.083	0.095	0.110	0.112	0.109	0.128	0.149
3	18	867	(1)	0.508	0.588	0.636	0.674	0.697	0.735	0.748	0.758	0.771	0.796	0.816
			(2)		0.094	0.089	0.114	0.113	0.116	0.113	0.126	0.137	0.143	0.163
4	18	428	(1)	0.372	0.441	0.496	0.535	0.571	0.642	0.663	0.665	0.679	0.692	0.695
			(2)		0.104	0.138	0.131	0.140	0.152	0.144	0.165	0.192	0.198	0.207
5	18	242	(1)	0.270	0.303	0.345	0.381	0.412	0.495	0.544	0.575	0.625	0.707	0.788
			(2)		0.046	0.102	0.080	0.096	0.114	0.120	0.145	0.158	0.187	0.164
6	18	157	(1)	0.126	0.184	0.226	0.262	0.287	0.318	0.336	0.356	0.385	0.440	0.507
			(2)		0.071	0.082	0.073	0.100	0.115	0.138	0.136	0.156	0.160	0.145
7	18	101	(1)	0.144	0.180	0.227	0.269	0.297	0.366	0.389	0.408	0.436	0.500	0.557
			(2)		0.064	0.078	0.095	0.109	0.124	0.155	0.137	0.149	0.173	0.207

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8	18	56	(1)	0.062	0.083	0.110	0.131	0.145	0.181	0.212	0.231	0.270	0.351	0.436
			(2)		0.055	0.062	0.092	0.088	0.096	0.126	0.128	0.148	0.159	0.171
9	18	36	(1)	0.108	0.161	0.204	0.240	0.258	0.305	0.334	0.360	0.407	0.504	0.608
			(2)		0.080	0.082	0.092	0.083	0.111	0.137	0.144	0.163	0.196	0.180
10	18	14	(1)	0.085	0.092	0.108	0.136	0.159	0.221	0.260	0.277	0.293	0.328	0.371
			(2)		0.067	0.074	0.089	0.108	0.117	0.144	0.156	0.164	0.193	0.198
All stocks	180	1,563	(1)	0.992	0.991	0.992	0.993	0.992	0.992	0.991	0.990	0.989	0.987	0.983
			(2)		0.057	0.081	0.102	0.083	0.100	0.084	0.108	0.116	0.110	0.139
F-test betas (10 portf.)				20.08	20.52	20.07	18.60	17.42	13.83	12.06	11.14	9.32	6.63	4.40
F-test standard deviations (10 portf.)					1.88	2.49	1.96	2.85	2.04	2.01	1.58	1.67	3.64	2.73
Individual beta coefficients				Differencing interval L										
Stock	Market value			1	2	3	4	5	8	10	12	16	22	30
Portfolio 1														
1	53,018			1.445	1.260	1.158	1.103	1.069	1.044	1.013	0.988	0.951	0.884	0.808
2	25,067			1.660	1.557	1.489	1.421	1.386	1.330	1.308	1.292	1.310	1.322	1.342
3	17,839			1.257	1.253	1.229	1.215	0.219	1.235	1.240	1.239	1.267	1.282	1.321
4	16,183			1.231	1.217	1.214	1.193	1.157	1.085	1.086	1.088	1.079	1.052	0.985
5	9,728			0.901	1.010	1.100	1.164	1.179	1.228	1.241	1.249	1.259	1.296	1.329
6	9,266			0.429	0.539	0.643	0.740	0.802	0.834	0.837	0.834	0.780	0.717	0.643
7	8,732			1.374	1.374	1.346	1.317	1.294	1.258	1.226	1.221	1.213	1.159	1.135
8	7,901			0.701	0.849	0.903	0.943	0.950	0.986	0.981	1.003	1.059	1.083	1.064
9	7,254			-0.030	0.017	0.097	0.133	0.156	0.192	0.213	0.229	0.230	0.253	0.282
10	5,951			1.281	1.276	1.249	1.234	1.230	1.151	1.142	1.152	1.160	1.237	1.346
11	5,867			1.521	1.506	1.481	1.460	1.440	1.348	1.301	1.285	1.248	1.242	1.230
12	5,760			0.490	0.531	0.594	0.652	0.689	0.789	0.824	0.842	0.817	0.807	0.836
13	5,248			0.662	0.842	0.956	1.001	1.011	1.019	1.051	1.052	1.047	1.105	1.191
14	5,234			1.271	1.330	1.285	1.239	1.220	1.144	1.136	1.142	1.171	1.266	1.378
15	5,170			0.597	0.720	0.836	0.913	0.982	1.090	1.144	1.170	1.159	1.103	1.040
16	5,096			0.529	0.608	0.652	0.655	0.650	0.615	0.619	0.649	0.653	0.637	0.591
17	4,890			0.791	0.877	0.900	0.925	0.950	0.968	0.977	0.969	0.975	1.005	1.011
18	4,680			0.600	0.640	0.684	0.695	0.696	0.711	0.758	0.763	0.780	0.835	0.960
Portfolio 10														
1	24			0.138	0.127	0.164	0.209	0.256	0.351	0.389	0.413	0.440	0.472	0.520
2	24			0.165	0.186	0.186	0.195	0.201	0.272	0.323	0.336	0.334	0.377	0.461
3	22			0.112	0.011	0.069	0.138	0.179	0.309	0.358	0.382	0.355	0.324	0.416
4	21			-0.163	-0.227	-0.311	-0.298	-0.257	-0.172	-0.129	-0.084	-0.055	-0.020	-0.008
5	19			0.051	0.128	0.188	0.225	0.220	0.167	0.119	0.093	0.061	0.023	-0.106
6	19			0.459	0.475	0.445	0.456	0.470	0.438	0.449	0.440	0.433	0.439	0.440
7	18			0.117	0.254	0.331	0.381	0.428	0.516	0.590	0.598	0.648	0.754	0.865
8	16			0.062	0.086	0.119	0.141	0.152	0.222	0.307	0.341	0.400	0.504	0.640
9	16			0.120	0.185	0.185	0.192	0.212	0.276	0.347	0.389	0.441	0.564	0.669
10	12			0.031	0.010	0.013	0.004	0.002	-0.026	-0.061	-0.087	-0.095	-0.161	-0.207
11	12			- 0.058	-0.072	-0.083	-0.063	-0.047	0.099	0.216	0.265	0.382	0.517	0.671
12	11			0.118	0.083	0.121	0.134	0.100	0.042	0.019	-0.004	-0.073	-0.060	-0.038
13	9			-0.187	-0.231	-0.230	-0.231	-0.203	-0.060	0.015	0.068	0.171	0.207	0.217
14	9			0.026	0.037	0.043	0.164	0.280	0.528	0.652	0.734	0.759	0.883	0.981
15	9			-0.058	-0.074	-0.087	-0.097	-0.094	-0.073	-0.072	-0.091	-0.113	-0.168	-0.243
16	7			0.280	0.355	0.396	0.394	0.363	0.264	0.242	0.199	0.148	0.106	0.049
17	6			-0.030	-0.061	-0.011	0.021	0.071	0.233	0.273	0.308	0.323	0.310	0.362
18	2			-0.036	0.158	0.291	0.446	0.551	0.797	0.855	0.925	1.118	1.460	1.704
F-test betas (portf. 1 versus portf. 10)				56.23	70.12	82.22	85.33	85.47	74.07	**65.94**	60.62	48.35	21.24	20.79

(1) and (2) are the means of the average individual beta $\bar{\beta}_{iL}$ and the individual standard deviation $\sigma(\hat{\beta}_{iL})$ ***of*** *the stocks forming a portfolio, respectively. F-test statistics significant at the five percent level are underlined.*

Table 3 Regression statistics: Period 1983-1985.

Portfolio	Number of stocks	Average market value	Differencing interval *L*										
			1	2	3	4	5	8	10	12	16	22	30

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1	17	22,996	(1)	1.229	1.173	1.138	1.114	1.098	1.068	1.058	1.051	1.050	1.052	1.054
			(2)		0.035	0.038	0.078	0.081	0.085	0.074	0.076	0.082	0.092	0.121
2	17	6,381	(1)a	0.807	0.877	0.919	0.945	0.958	0.987	0.996	0.996	0.986	0.982	0.974
			(2)		0.047	0.077	0.099	0.094	0.114	0.119	0.118	0.128	0.127	0.181
3	17	2,330	(1)	0.651	0.742	0.791	0.821	0.841	0.867	0.874	0.883	0.885	0.879	0.878
			(2)		0.064	0.102	0.102	0.104	0.111	0.118	0.128	0.120	0.148	0.187
4	17	1,144	(1)	0.366	0.446	0.519	0.569	0.608	0.695	0.728	0.747	0.779	0.795	0.823
			(2)		0.073	0.093	0.120	0.101	0.116	0.134	0.111	0.143	0.137	0.162
5	17	661	(1)	0.418	0.526	0.580	0.608	0.637	0.699	0.714	0.723	0.756	0.781	0.794
			(2)		0.072	0.106	0.127	0.145	0.162	0.184	0.166	0.179	0.214	0.238
6	17	373	(1)	0.208	0.234	0.276	0.319	0.377	0.502	0.545	0.562	0.572	0.582	0.598
			(2)		0.088	0.107	0.117	0.126	0.142	0.155	0.144	0.168	0.182	0.224
7	17	216	(1)	0.144	0.200	0.246	0.296	0.340	0.429	0.459	0.487	0.513	0.550	0.584
			(2)		0.089	0.086	0.100	0.130	0.141	0.150	0.175	0.165	0.214	0.255
8	17	143	(1)	0.119	0.170	0.222	0.263	0.297	0.363	0.395	0.408	0.440	0.490	0.503
			(2)		0.086	0.101	0.117	0.129	0.157	0.163	0.162	0.181	0.237	0.259
9	17	74	(1)	0.160	0.188	0.230	0.260	0.292	0.339	0.352	0.346	0.358	0.368	0.374
			(2)		0.075	0.102	0.118	0.160	0.171	0.189	0.187	0.181	0.226	0.273
10	17	25	(1)	0.018	0.026	0.037	0.052	0.060	0.080	0.101	0.117	0.120	0.134	0.158
			(2)		0.091	0.130	0.149	0.168	0.187	0.181	0.257	0.218	0.245	0.321
12	170	3,434	(1)	1.041	1.028	1.021	1.015	1.011	1.005	1.002	0.999	0.999	1.001	1.002
			(2)		0.042	0.054	0.087	0.088	0.097	0.091	0.092	0.099	0.109	0.143
F-test betas (10 portf.)				18.68	19.77	19.51	18.50	17.04	14.12	12.92	12.06	11.10	9.24	8.00
F-test standard deviations (10 portf.)					1.29	3.70	2.20	3.35	3.65	5.08	8.21	5.00	6.64	4.57

Individual beta coefficients		Differencing interval *L*										
Stock	Market value	1	2	3	4	5	8	10	12	16	22	30
Portfolio l												
1	111,071	1.816	1.580	1.449	1.384	1.334	1.271	1.257	1.263	1.284	1.308	1.345
2	48,755	1.630	1.481	1.404	1.322	1.265	1.170	1.131	1.089	1.060	1.029	1.030
3	35,145	1.158	1.135	1.132	1.127	1.137	1.086	1.041	1.001	0.962	0.949	0.936
4	26,537	0.987	1.088	1.148	1.181	1.216	1.245	1.250	1.279	1.295	1.315	1.290
5	20,180	1.271	1.320	1.304	1.260	1.244	1.281	1.311	1.350	1.375	1.347	1.298
6	19,161	0.550	0.614	0.649	0.679	0.695	0.741	0.758	0.770	0.792	0.788	0.753
7	16,005	0.915	0.922	0.909	0.902	0.890	0.844	0.822	0.776	0.738	0.744	0.758
8	14,953	0.431	0.535	0.579	0.626	0.669	0.704	0.729	0.746	0.775	0.797	0.793
9	12,090	0.948	1.043	0.997	0.957	0.938	0.951	0.943	0.942	0.950	0.951	0.934
10	11,713	1.320	1.443	1.507	1.527	1.508	1.442	1.422	1.387	1.335	1.338	1.373
11	11,416	1.246	1.324	1.312	1.311	1.313	1.329	1.308	1.282	1.252	1.205	1.198
12	11,234	0.272	0.227	0.261	0.277	0.291	0.277	0.258	0.244	0.237	0.250	0.292
13	10,975	1.025	1.057	1.100	1.097	1.078	1.032	1.040	1.030	1.052	1.042	1.005
14	10,967	0.231	0.268	0.299	0.333	0.358	0.422	0.463	0.493	0.503	0.517	0.510
15	10,485	0.355	0.380	0.390	0.414	0.433	0.468	0.498	0.513	0.521	0.539	0.533
16	10,252	0.580	0.678	0.752	0.784	0.788	0.707	0.680	0.631	0.608	0.592	0.513
17	9,992	0.739	0.811	0.845	0.863	0.885	0.935	0.952	0.921	0.901	0.893	0.906
Portfolio 10												
1	42	- 0.162	-0.199	-0.171	-0.125	-0.102	-0.081	-0.038	0.001	-0.008	0.021	0.034
2	42	0.087	0.193	0.166	0.129	0.087	-0.025	-0.068	-0.083	-0.110	-0.123	-0.128
3	42	-0.230	-0.101	-0.014	0.028	0.074	0.200	0.284	0.331	0.317	0.292	0.191
4	40	-0.049	-0.118	0.125	0.122	0.118	0.084	0.159	0.220	0.272	0.305	0.420
5	32	0.139	0.081	-0.002	-0.064	-0.130	-0.437	-0.481	-0.477	-0.421	-0.298	-0.196
6	30	0.046	-0.055	0.008	0.134	0.232	0.421	0.458	0.499	0.524	0.569	0.628
7	28	0.161	0.175	0.212	0.286	0.328	0.439	0.390	0.312	0.185	0.028	-0.011
8	26	0.002	-0.092	-0.145	-0.210	-0.230	-0.232	-0.243	-0.237	-0.112	-0.043	-0.067
9	26	-0.100	-0.085	-0.070	-0.037	-0.026	0.078	0.134	0.205	0.245	0.407	0.599
10	23	-0.224	-0.242	-0.280	-0.310	-0.316	-0.278	-0.272	-0.277	-0.318	-0.353	-0.336
11	20	0.265	0.155	0.169	0.148	0.052	-0.006	-0.021	-0.065	-0.191	-0.346	-0.323
12	19	0.370	0.390	0.472	0.523	0.553	0.576	0.560	0.523	0.522	0.552	0.545
13	14	0.091	0.059	0.022	-0.017	-0.075	-0.173	-0.186	-0.163	-0.101	-0.016	0.061
14	13	0.097	0.147	0.236	0.311	0.422	0.654	0.767	0.835	0.815	0.874	0.918
15	10	0.307	0.202	0.150	0.157	0.195	0.282	0.302	0.353	0.411	0.429	0.420
16	10	0.039	-0.186	-0.215	-0.174	-0.121	0.158	0.286	0.298	0.282	0.174	-0.002
17	9	0.165	0.178	0.250	0.330	0.408	0.527	0.602	0.633	0.666	0.702	0.791
F-test betas (portf. 1 versus portf. 10)		49.37	62.72	66.82	66.85	64.87	50.07	44.21	40.40	40.02	37.25	32.16

(1) and (2) are the means of the average individual beta $\bar{\hat{\beta}}_{iL}$ *and the individual standard deviation* $\sigma(\hat{\beta}_{iL})$ *of the stocks forming a portfolio,* respectively. F-test statistics significant at the five percent level are underlined.

Such results suggest that the small firm effect put in evidence by Banz (1981) on monthly data can only be partially explained by the bias in beta estimate, as this bias tends to disappear when returns are calculated on long intervals. A look at the individual beta coefficients reveals that there are more or less two or three very large securities of the first portfolio having an upward bias. So, only very large firms have an slight upward bias while all others, especially small firms, have a downward bias. Concerning individual beta coefficients of the smallest market value portfolio, different patterns can be observed. Although the intervalling effect is on the average positive, few security betas are decreasing with the length of the interval. Some negative coefficient can even be noticed, whatever the value of L. The values of the $\sigma(\hat{\beta}_{iL})$ reveal that the volatility of the unadjusted betas for a given length of differencing interval, is quite strong in all size portfolios.[6] It appears that small firm portfolio betas are more volatile than betas of the large firm portfolios.[7] Furthermore, the hypothesis of equality of the beta volatility, tested by an analysis of variance, is rejected for most differencing intervals at the five per cent level. The volatility also tends to increase continuously with the length of the differencing interval.[8] It can be concluded that the method of adjustment for the volatility of the betas used in this study at least eliminates the likelihood of having peculiar values of the estimated systematic risk for a given differencing interval length.

5. Conclusion

This note shows that the choice of a differencing interval length to measure the returns has an important impact on the magnitude of the estimated security betas. The results of this study, which is carried out on a comprehensive sample of the Brussels Stock Exchange and on three adjacent periods of three years, indicate that an intervalling effect bias is present in the estimated security betas for short differencing intervals. The bias in the betas is very important, especially for small market value securities, and it decreases when the differencing interval used to measure the returns is lengthened. The results also show that small firms have on the average lower

[6]The absence of value for $\sigma(\hat{\beta}_{iL})$ for a one-day differencing interval is due to the fact that there cannot be any fluctuation for a one day differencing interval.

[7]Because of thin trading, small firm prices are more chaotic. Therefore small firm returns for any interval length are more sensitive to the way prices are juxtaposed to calculate returns than those of larger firms, which in turn affects the estimated values of $\hat{\beta}_{iL}$.

[8]The increase in the volatility $\sigma(\beta_{iL})$ with the length of the differencing interval is caused on the one hand by the increase in the number of estimated $\hat{\beta}_{iL}$ with L, and on the other hand, since the number of returns decreases with L, by a lower confidence in the estimated values of $\hat{\beta}_{iL}$, both of which decrease the degree of freedom.

beta coefficients than large firms. Another interesting feature revealed by this study is the volatility of the estimated betas. It appeared indeed that the way the daily prices are juxtaposed to calculate returns of longer differencing intervals has also an effect on the values of the estimated betas.

Notes

[1]Deleting L-1 daily returns from the series decreases by a maximum of one the number of returns of interval length *L*.

[2]Because of the limited number of observations the estimate $\sigma(\hat{\beta}_{iL})$ is quite inefficient for short differencing intervals. It gives, however, an idea of the variation of the beta coefficient due to the juxtaposition of the daily prices in the sample.

[3]The number of securities in a portfolio for a particular subperiod is equal to the larger integer of the division of the number of securities by the number of portfolios. If there is a remainder it is allocated to the first and the tenth portfolios.

[4]The complete tables can be obtained on request.

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[5]The results for the other values of *L* are consistent with the values of L that are presented in the tables.

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