ALE simulations using Metafor

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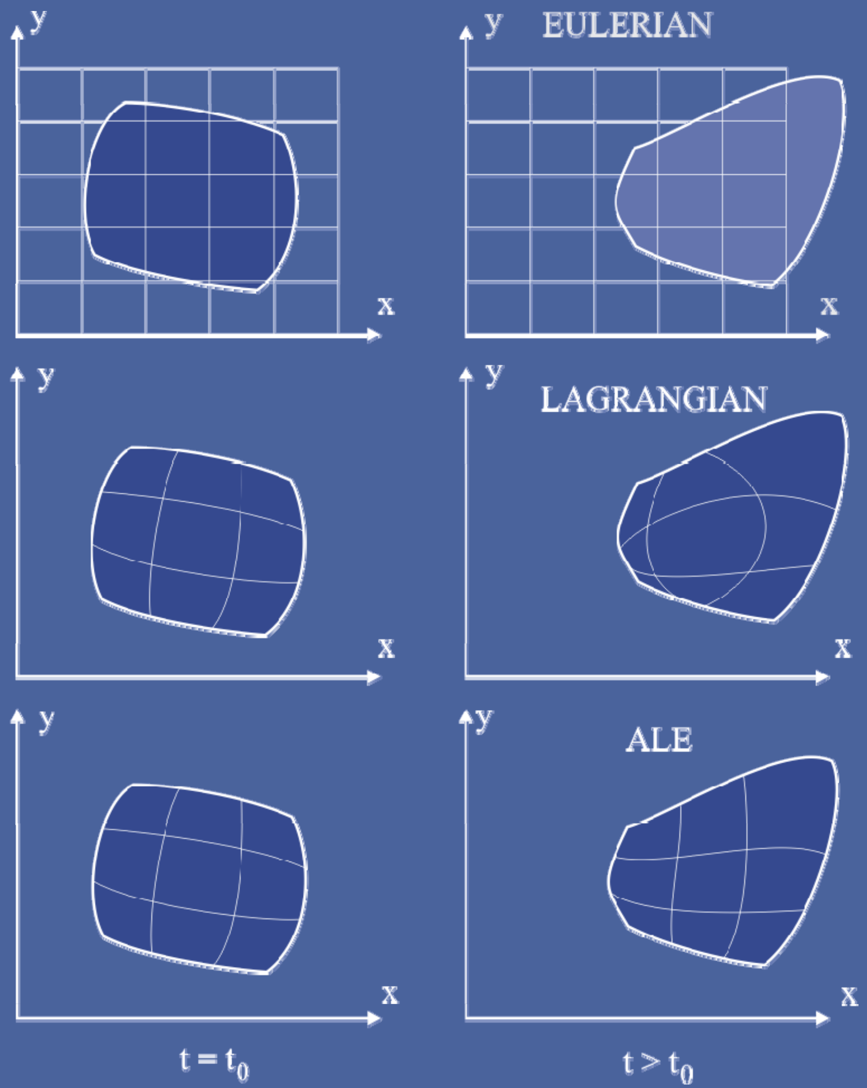
Introduction



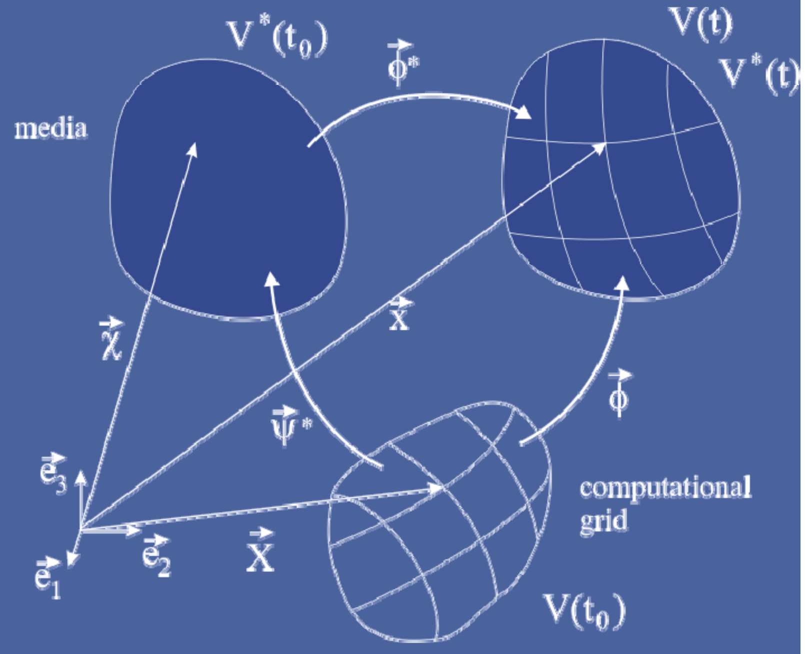
EULERIAN FORMALISM

- ✔ Undistorted mesh
- ✔ Ideal for stationary processes
- ✖ Free boundaries are difficult to follow

LAGRANGIAN FORMALISM

- ✔ Free boundaries are computed automatically
- ✔ History dependant materials are easier to handle
- ✖ The mesh can be rapidly distorted
- ✖ Large amount of finite elements are needed for the simulation of stationary processes

ALE Formalism – Operator Split



Complex problem

2 coupled problems :

- Equilibrium of the body in motion
- Motion of the mesh (minimization of distortion,…)

→ too complicated & too slow

Simplified problem

Time increments are divided into 2 steps :

- **LAGRANGIAN STEP**

  (Same as in Lagrangian formalism)

- **REMESHING & CONVECTIVE STEP**
  - Body is remeshed
  - Values stored at the Gauss points/nodes are updated

$$\left.\frac{\partial A}{\partial t}\right|_X = \left.\frac{\partial A}{\partial t}\right|_\chi + \vec{w}\cdot\vec{\nabla} A$$

$\vec{w} = \vec{v} - \vec{v}^*$: relative velocity

$\vec{v}$: material velocity

$\vec{v}^*$: mesh velocity

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ALE Formalism – Operator Split

Constitutive equation

$$\dot{\sigma} = \left.\frac{\partial \sigma}{\partial t}\right|_{\chi} + w_j \frac{\partial \sigma}{\partial x_j} = q$$

σ : stress tensor component, effective plastic strain, …

q : straining term and objectivity terms.

1. Lagrangian step

$$\left.\frac{\partial \sigma}{\partial t}\right|_{\chi = X} = q$$

2. Convective step

$$\left.\frac{\partial \sigma}{\partial t}\right|_{\chi} + w_j \frac{\partial \sigma}{\partial x_j} = 0$$

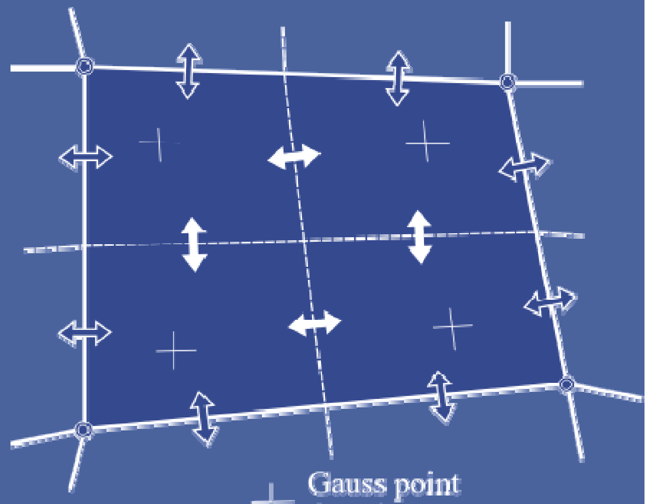
Well-known convection equation

MAIN DIFFICULTY :

σ is not defined at the nodes of the mesh but at the Gauss points of each element (discrete values)

Godunov update

- One FINITE VOLUME element around each Gauss point.
- σ is assumed to be constant on each finite volume.



Weak form

$$\int_\Omega \omega_I \left(\frac{\partial \sigma}{\partial t} + \frac{\partial (\sigma\, w_i)}{\partial x_i} \right) d\Omega = \int_\Omega \omega_I\, \sigma \frac{\partial w_i}{\partial x_i}\, d\Omega$$

$$\Rightarrow \frac{\partial \sigma_s}{\partial t} = -\frac{1}{2\,V_s} \sum_{i=1}^{4} f_i \left(\sigma_i^c - \sigma_s\right)\left(1 - \alpha\ sign\ f_i\right)$$

upwind parameter

$$f_i = \int_{S_{si}} w_k\, n_k\, dS$$

Explicit time discretization

$$\sigma_s^{n+1} = \sigma_s^n - \frac{\Delta t}{2\,V_s} \sum_{i=1}^{4} f_i \left(\sigma_i^c - \sigma_s\right)\left(1 - \alpha\ sign\ f_i\right)$$

V_s : volume of the cell *s*

σ_s : value of σ in the cell *s*

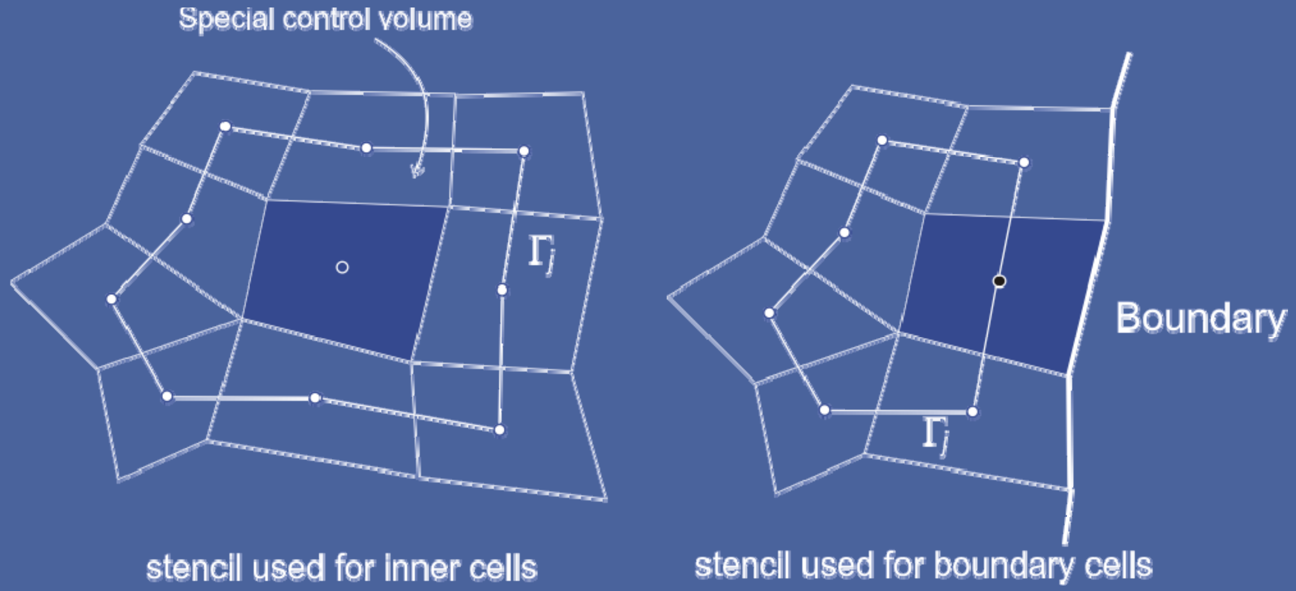
σ_i^c : value of σ in the cells surrounding the cell *s*

Higher order convection scheme

- Constant reconstruction → Linear reconstruction
- An approximation of the gradient has to be computed using the data of the given cell's neighborhood (Green-Gauss linear reconstruction)

$$\breve{\nabla}\sigma_j = \frac{1}{\Gamma_j}\oint_{\Gamma_j} \sigma\, \mathbf{n}\, d\partial\Gamma_j \quad \Rightarrow \quad \breve{\nabla}\sigma_j = \frac{1}{\Gamma_j}\sum_{i=1} \frac{\sigma_i + \sigma_{i+1}}{2}\, l_i\, n_i$$

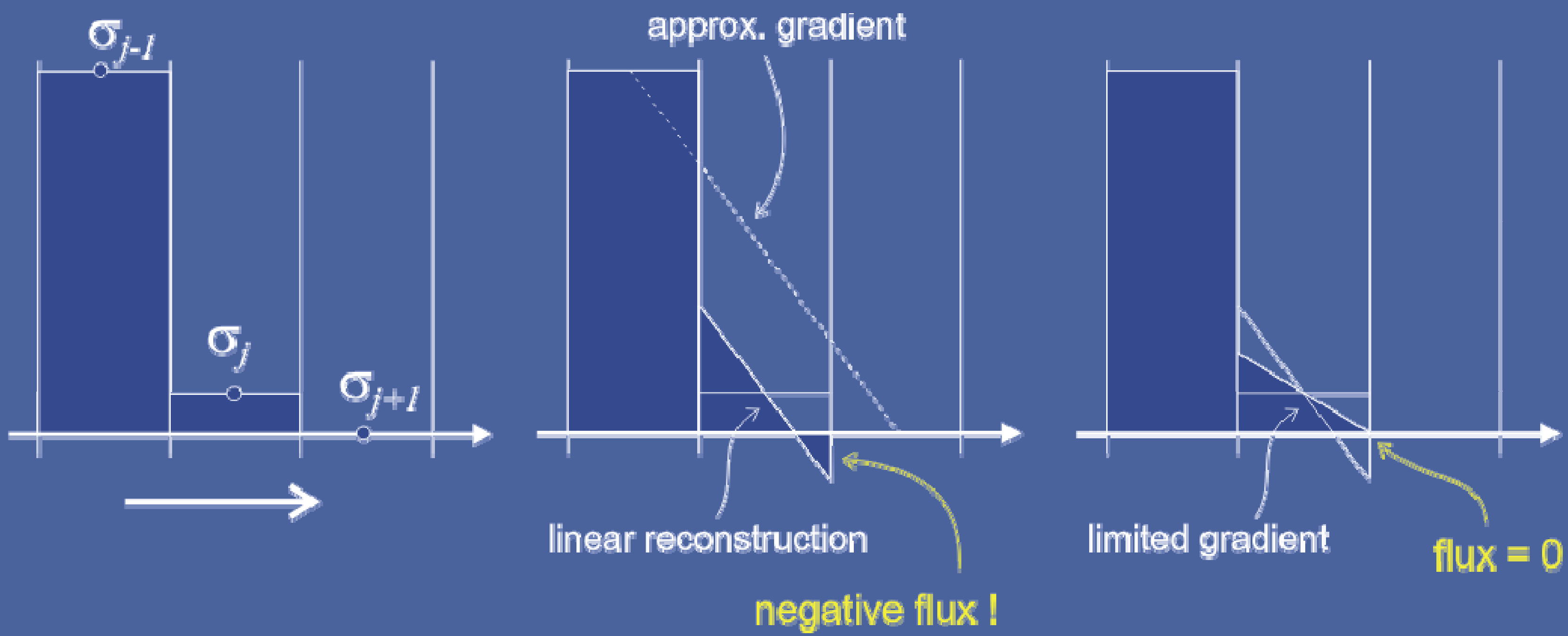
using a trapezoidal rule



2nd order scheme on structured grids – 1st order on unstructured grids

Preserving monotonicity

- The approximated gradient is used for the computation of the flux in the Godunov update equation.
- However, this leads to oscillations and a limiter (Barth & Jespersen) has to be introduced to preserve monotonicity (TVD scheme)



Constant vs Linear Reconstruction

Constant reconstruction

- Monotonic
- Fast (one loop over edges of the mesh)

- Good results on regular meshes (first order)
- Time dependent phenomenon's are smoothed (large artificial diffusion).

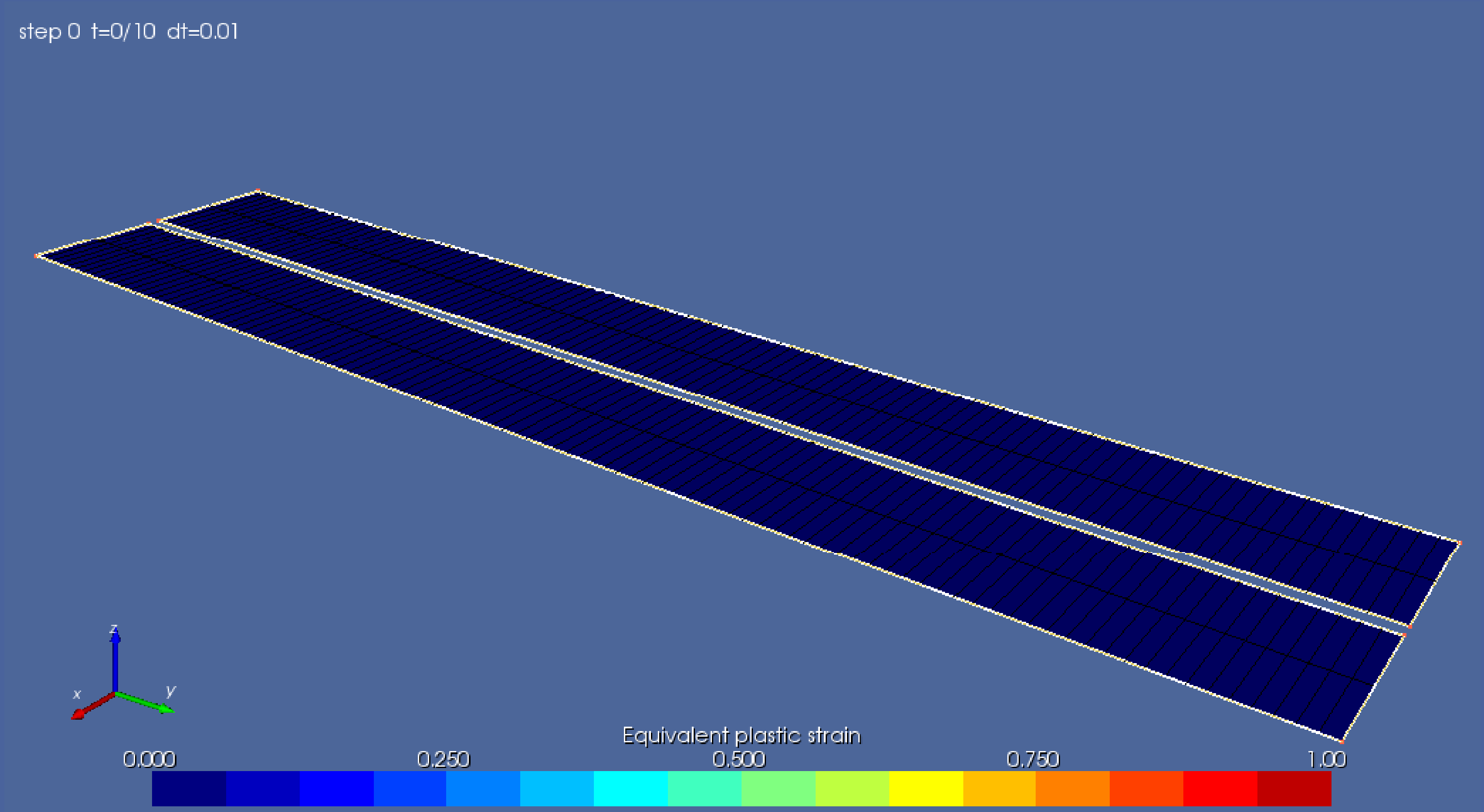
⇒ good scheme for stationary processes on regular meshes

Linear reconstruction

- Needs complex TVD limiters
- Rather slow or memory consuming
  (gradients have to be stored or computed many times)

- Good results on all kind of meshes (up to second order)
- Less artificial diffusion (better results for time dependent problems)

Constant vs Linear Reconstruction



Basic rezoning strategies

Nodes displacements are prescribed by the user. Methods depend on the CAD entity they belong to :

Nodes on vertices

- Can be Eulerian, Lagrangian or boundary (see further)

Nodes on edges

- Boundary curves are remeshed using a spline going though the Lagrangian positions of the nodes.
- 2D or 3D methods are used for interior curves

Nodes on sides

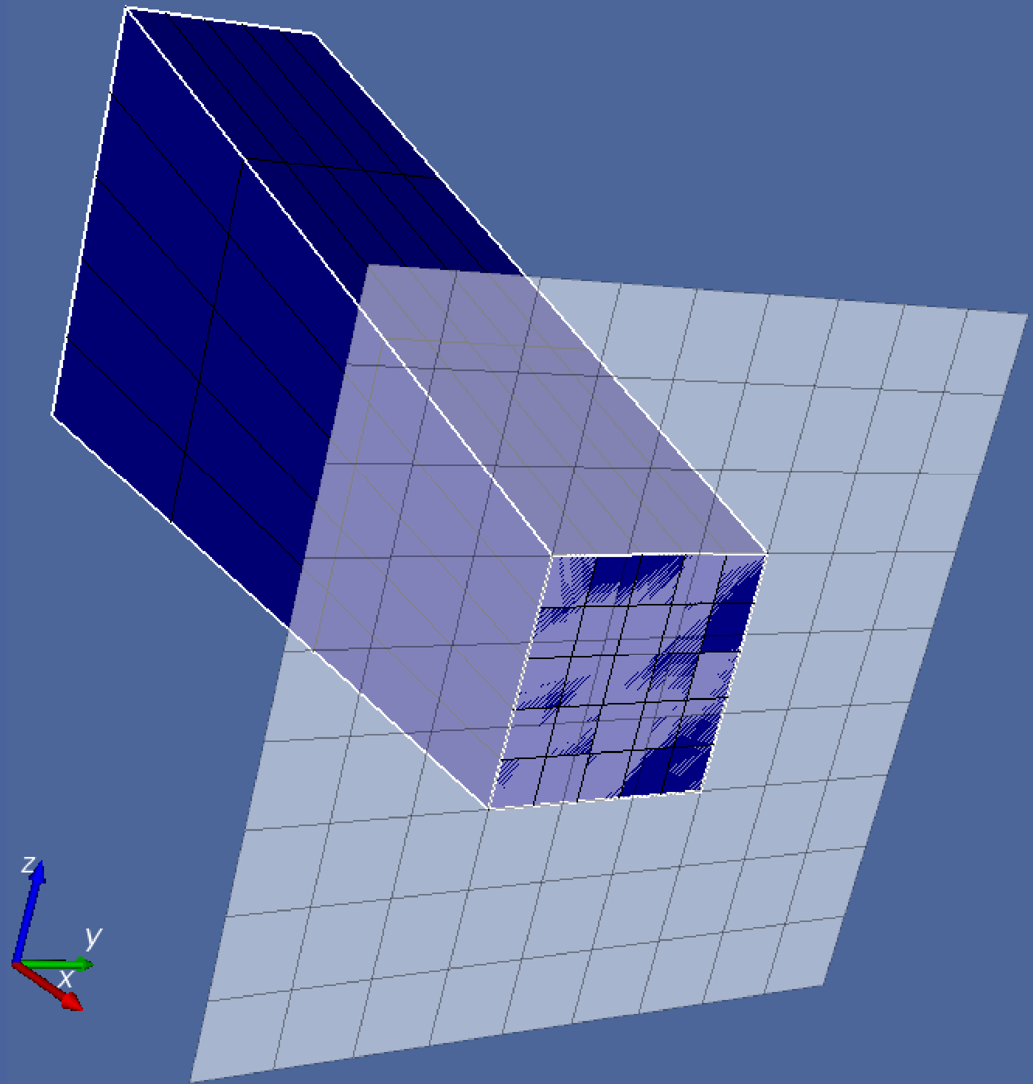
- Planar Boundary sides are remeshed a Transfinite Interpolation Mesher or smoothed using 2D smoothers (Laplacian, Giuliani, Isoparametric, etc).
- Curved boundaries are smoothed using a spline (see further)
- 3D methods are used for interior sides

Nodes on volumes

- Volumes are remeshed a Transfinite Interpolation Mesher or smoothed using 3D smoothers (Laplacian, Giuliani).

Rezoning strategies – Eulerian boundaries

- Allow to prevent the body from crossing a given surface.
- Boundary of a « quasi-Eulerian » region.
- Useful for stationary processes (rolling, roll forming, …)

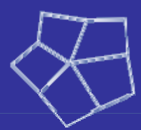
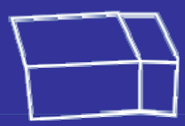


Rezoning strategies – 3D surfaces

Remarks about surface smoothing

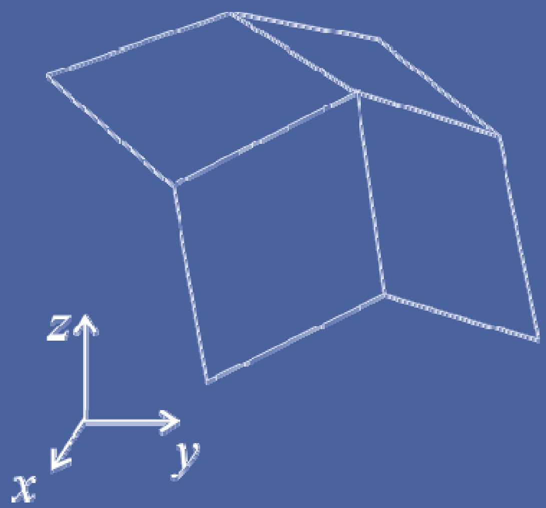
- Traditional methods (Laplacian, transfinite meshers) need to be extended in the case of 3D surfaces
- Methods must be able to manage a non uniform & unstructured surface mesh (graded elements).
- Mixing methods is interesting.

Available 2D methods

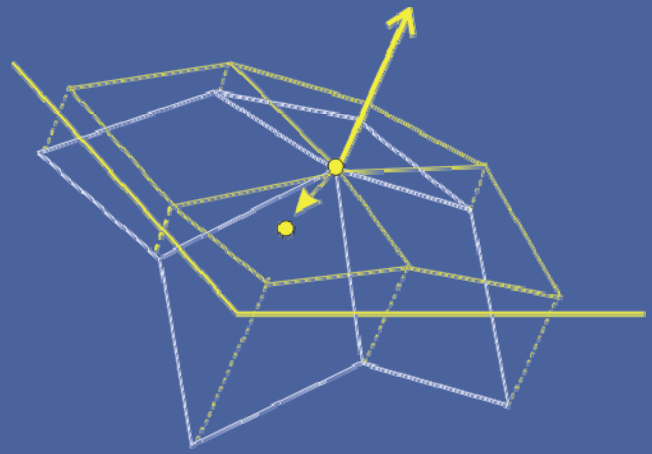
	Unstructured 	Graded 
Laplacian smoothing	✓	✗
Weighted Laplacian smoothing	✓	✓
Iso-parametric smoothing	✗	✓
« Area-pull » method	✓	✗
Equipotential smoothing	✗	✓
Giuliani smoothing	✓	✗

Rezoning strategies – 3D surfaces

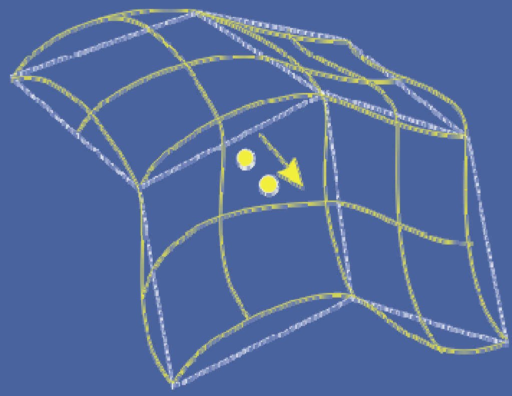
Extension to « 3D surfaces »



- Initial mesh



- Tangent plane computation
- 2D smoothing in this plane



- Projection on a spline surface

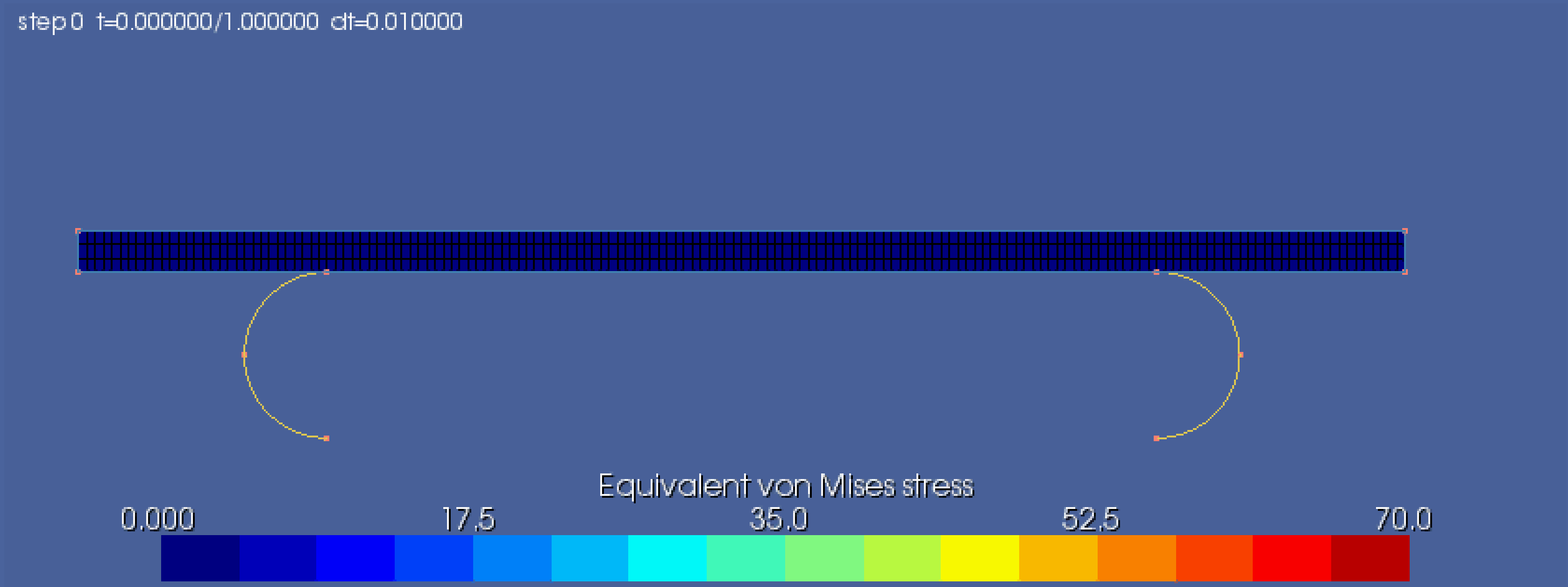
Blending methods

2D methods are used together using a weighting factor.

Contact with friction

Issues

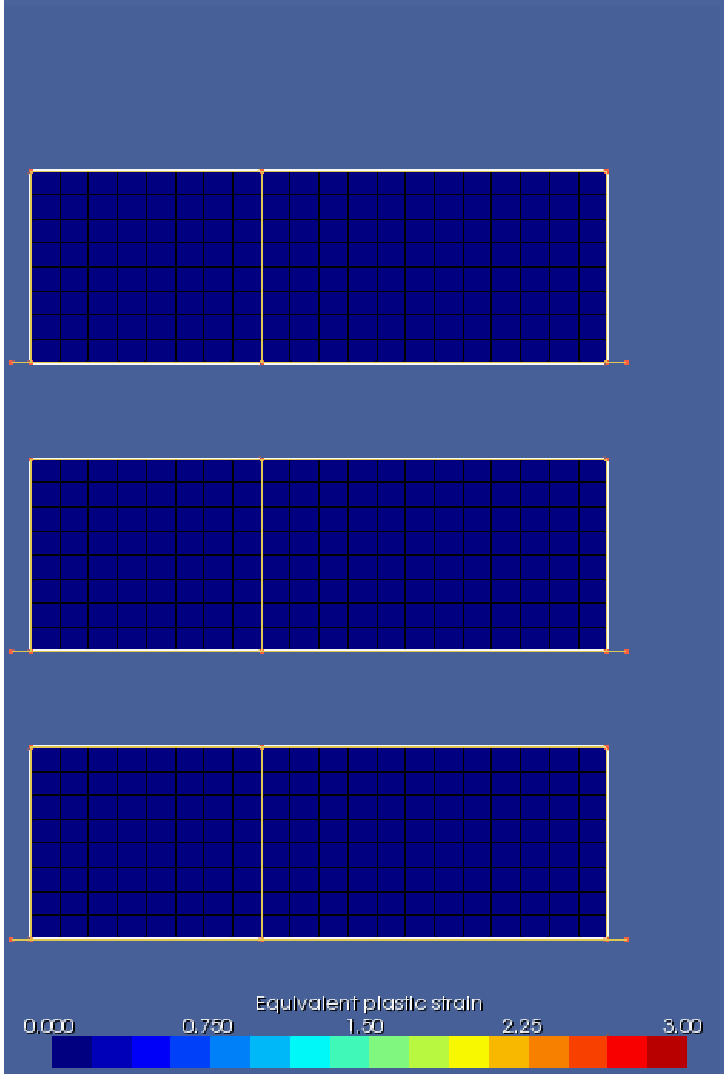
- Geometry : smoothing the contact boundary is difficult (how to deal with nodes in contact before remeshing and free after it?, …)
- Physics : the contact history (friction) must be convected like the volumic GP values. For now, the gap is computed from the value of the friction force after rezoning.



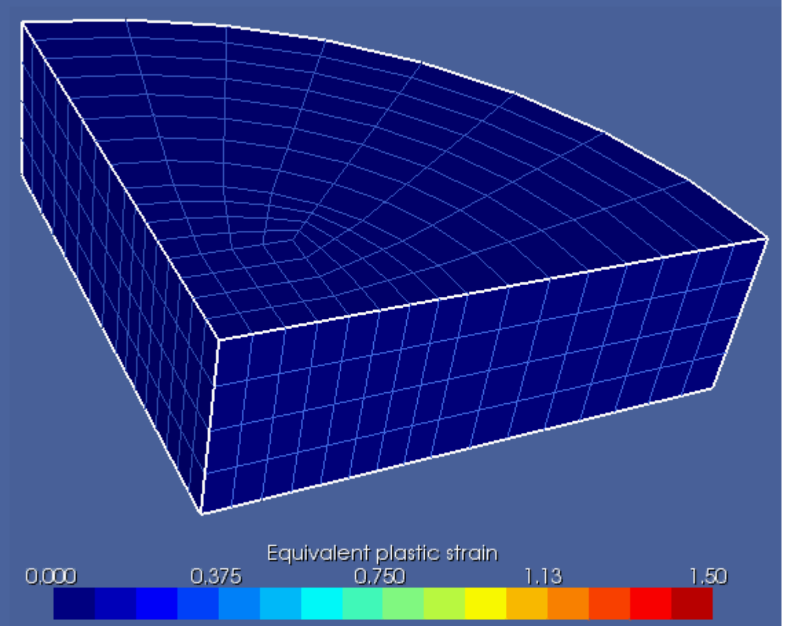
ALE applications

1. Coining
2. Wire extrusion
3. Extrusion
4. Machining
5. Rolling
6. Roller leveling
7. Roll forming

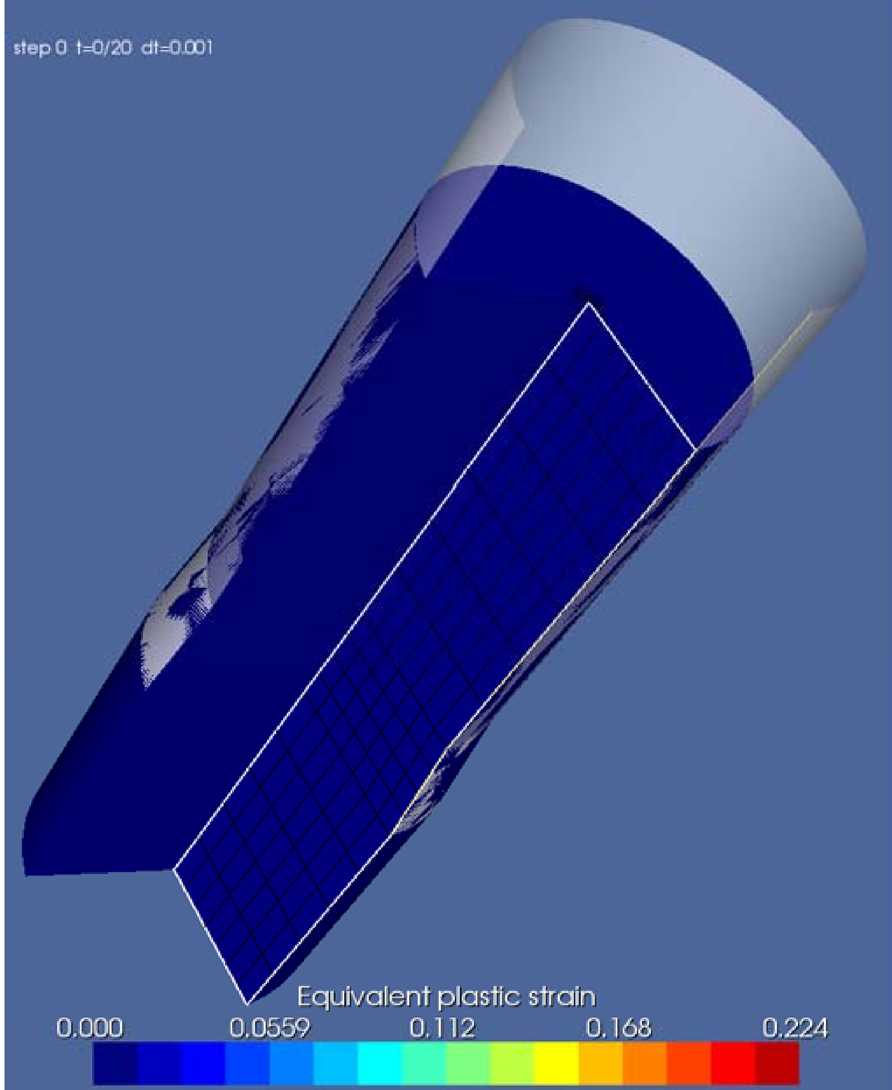
Coining of a cylindrical block using ALE



- ALE formalism is used for avoiding mesh distortion
- Upper boundary nodes are constantly moved on the free surface to keep the boundary curved shape
- Height of the workpiece is reduced by 60%
- Results must be axisymmetric

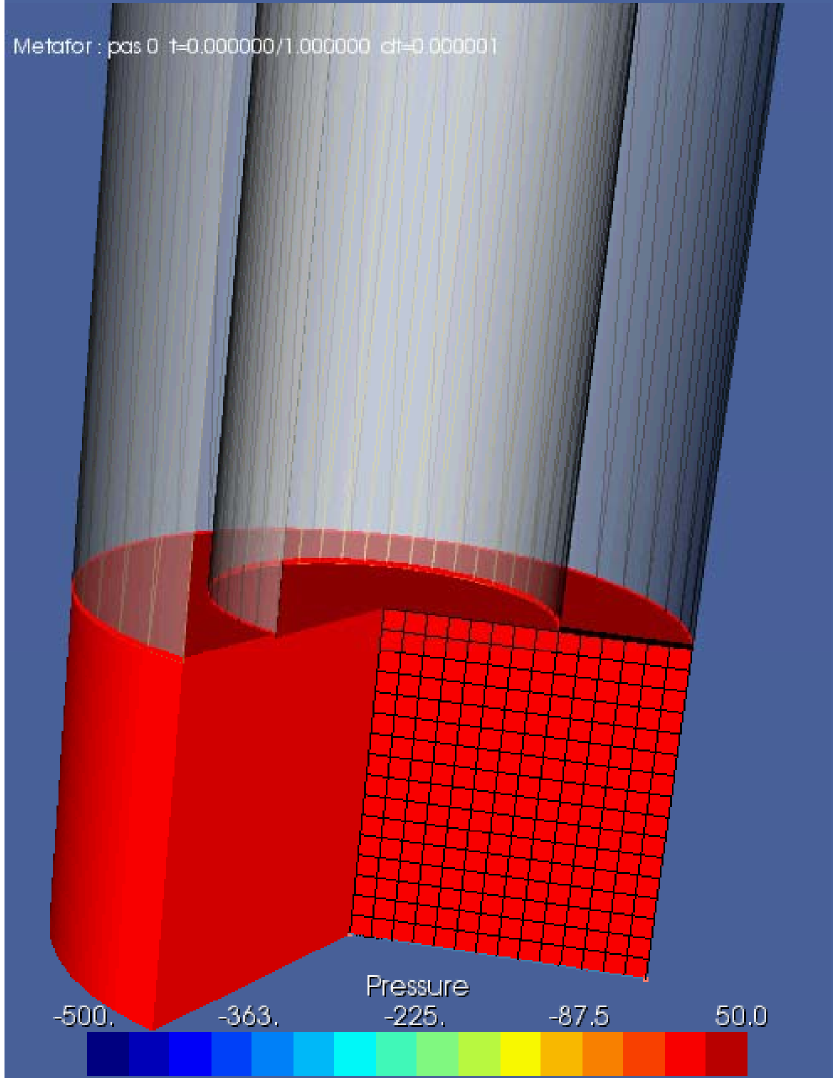


Stationary wire extrusion using ALE formalism

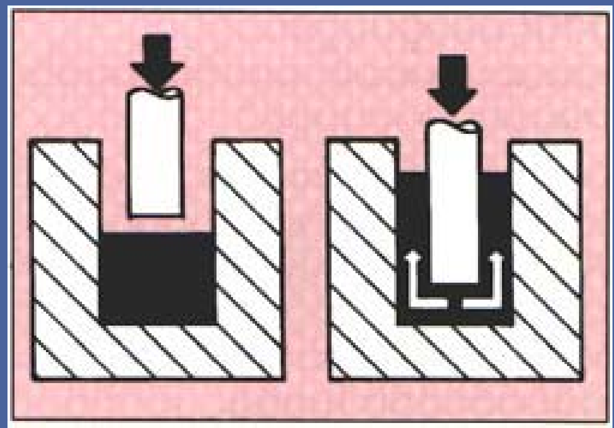


- The ALE formalism is used for simulating this stationary process.
- The mesh is Eulerian (every nodes are moved back to their initial position after each step). This avoids to consider a very large Lagrangian mesh.
- The problem is axisymmetric.
- The wire is pulled from the top through a rigid die.
- The penalty method is used for the contact between the die and the wire.
- After a brief transient state, the stationary state is reached.

Backward extrusion using ALE

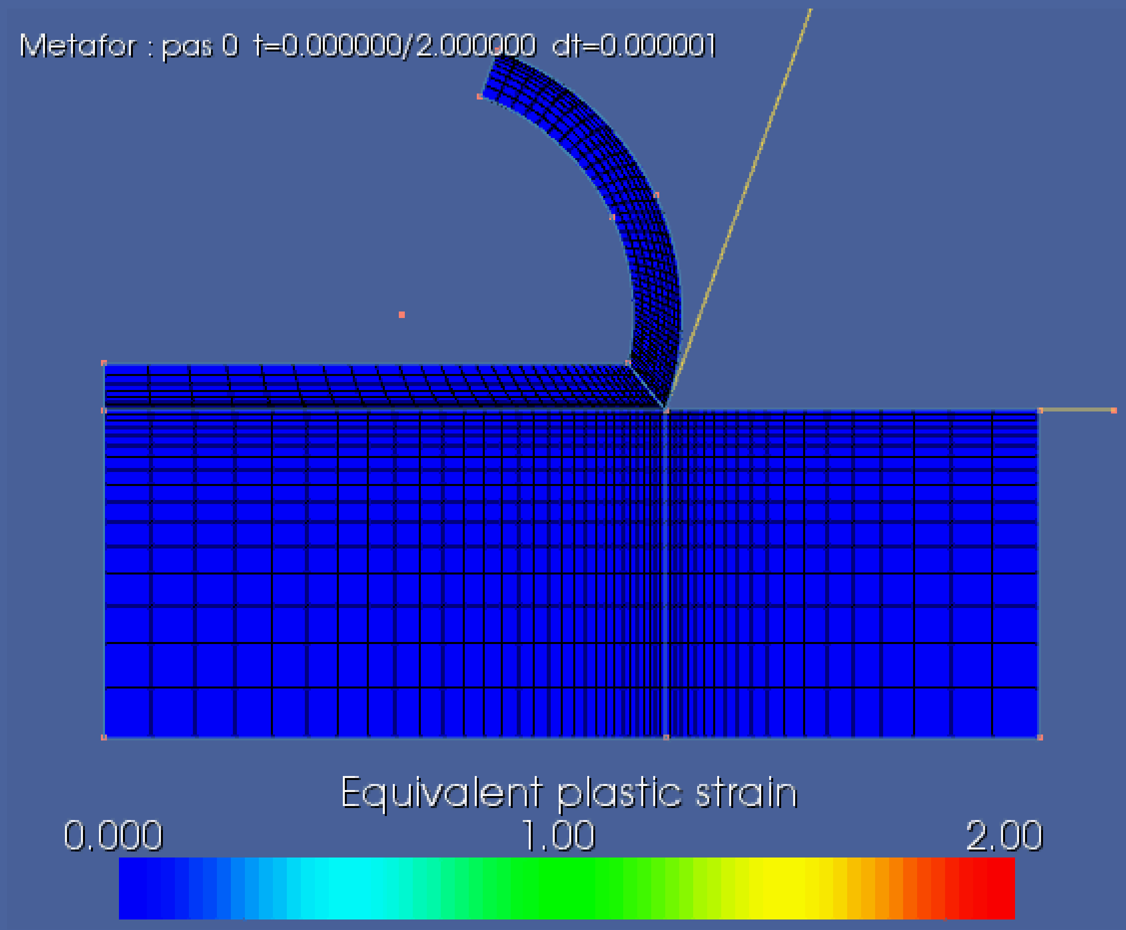


- A cylindrical piece of metal is pushed through a rigid tool.
- A very thin mesh is placed where the solid is supposed to go out.
- During the process, this flat mesh grows due to the convection and the main mesh shrinks.
- The shape of the mesh remains good and no remeshing is needed.

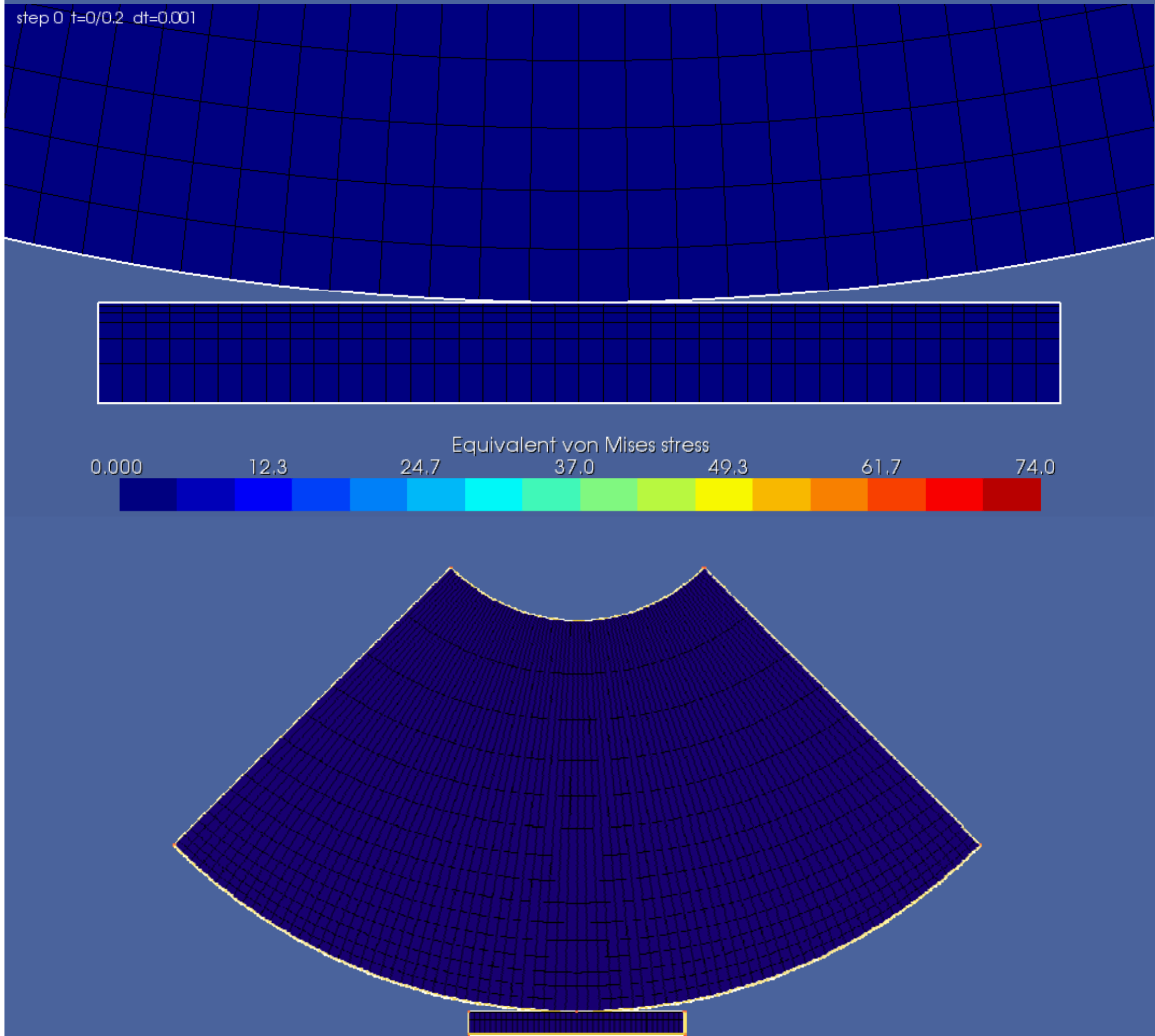


Machining using ALE

- A tool cuts and divides a piece of metal into two parts
- A guess of the final stationary shape of the chip is used as initial mesh.
- The final shape (chip's width) is automatically computed by the ALE method
- The mesh is refined near the crack.
- The model could be highly improved with an appropriate cracking model.



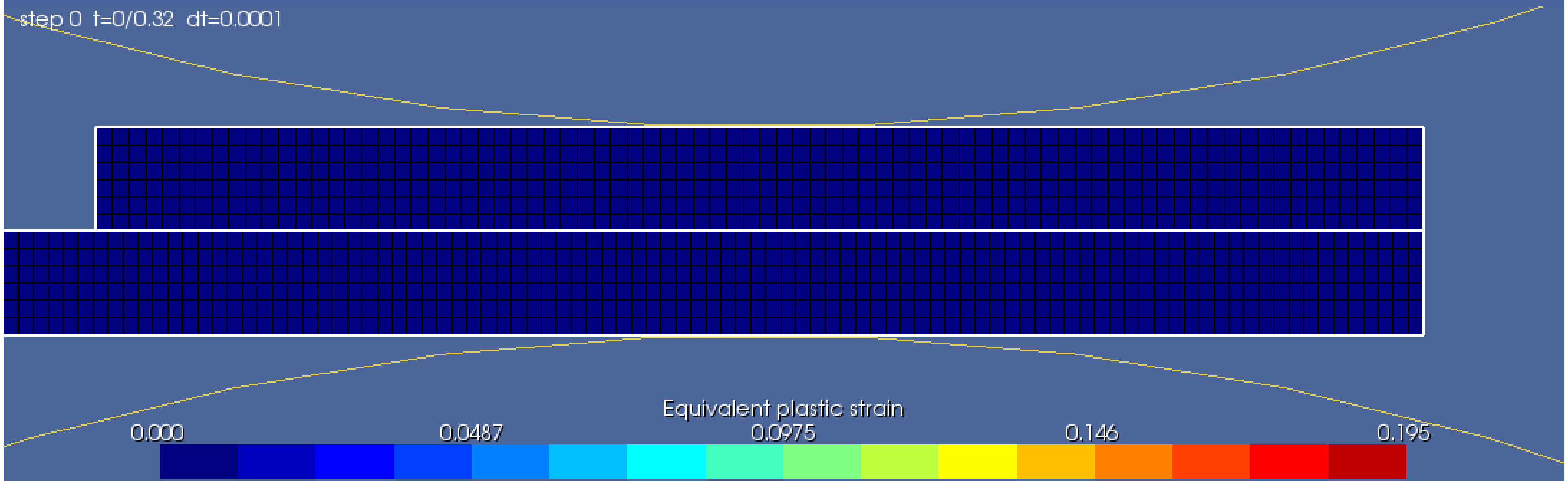
Cold Rolling process using ALE formalism



- Only the interesting part of the problem is meshed, thanks to the ALE formalism.
- The mesh is Eulerian in the rolling direction and Lagrangian in the transverse direction.
- The stationary state is reached by first clamping the sheet between the rolls and secondly making them rotate around their axis.
- The rolls are rigid and the sheet is thick.
- The free surface of the sheet in the outlet zone is automatically computed using spline remeshing.
- Eulerian convection of the Gauss points values is performed using a 1st order Finite Volume algorithm.

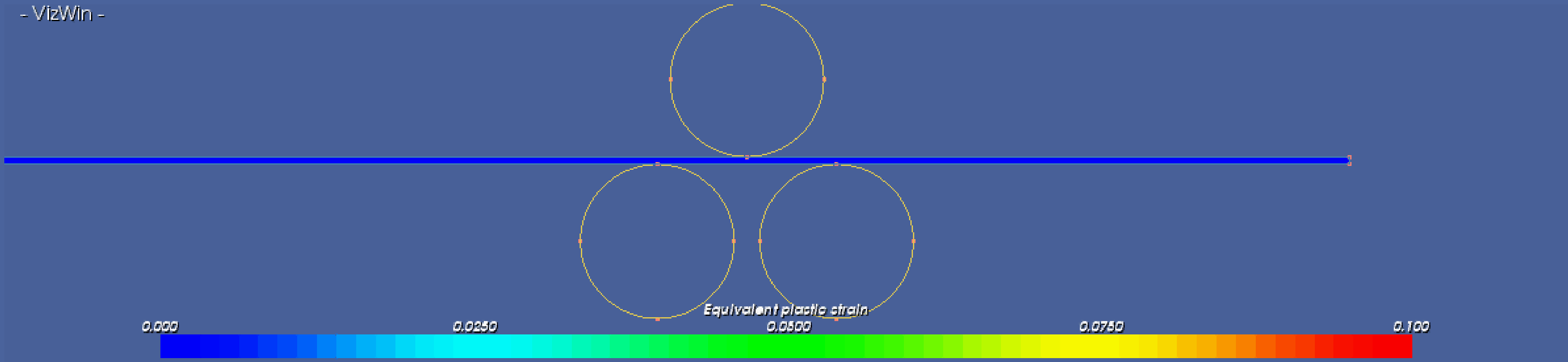
Cold Rolling process using ALE formalism

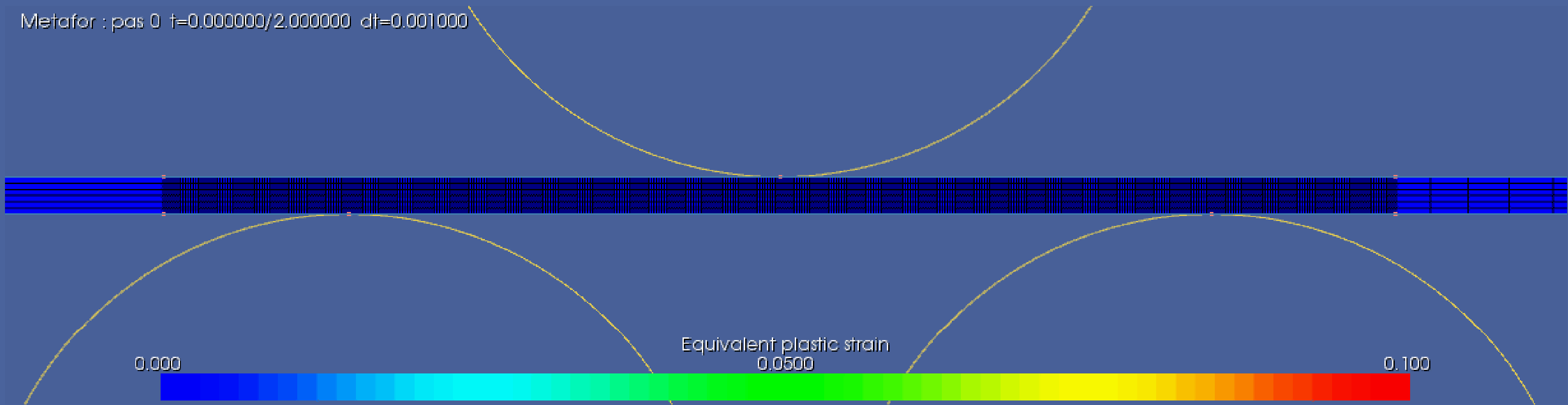
Comparison between ALE (above) and Lagrangian (below) formalisms



Roller levelling

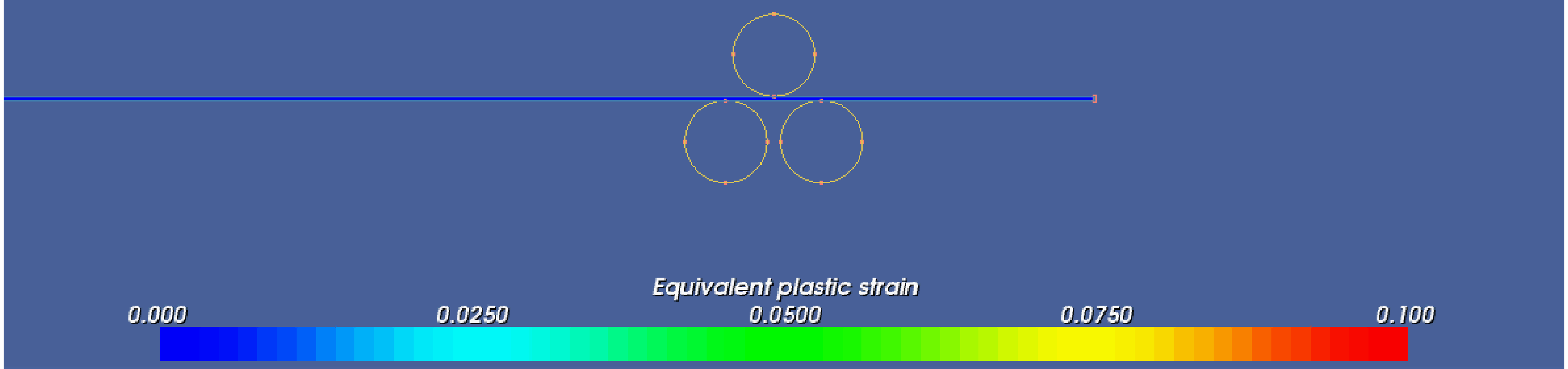
- A sheet is driven through 3 cylinders.
- Small contact areas require a fine mesh
- ALE keeps a fine mesh between the cylinders.
- CPU : 50min (ALE) vs 2 days (Lagrangian)



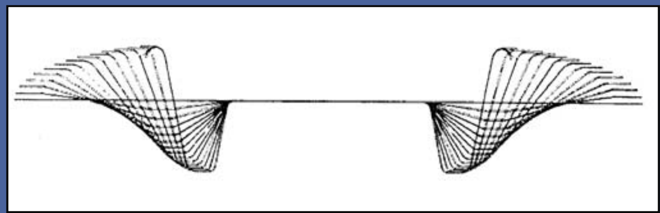


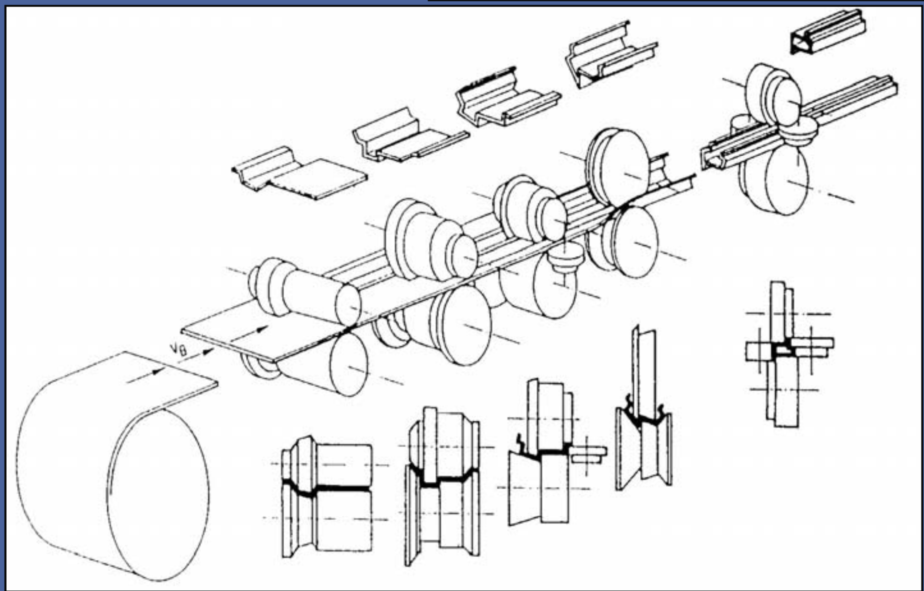
Roller levelling - springback

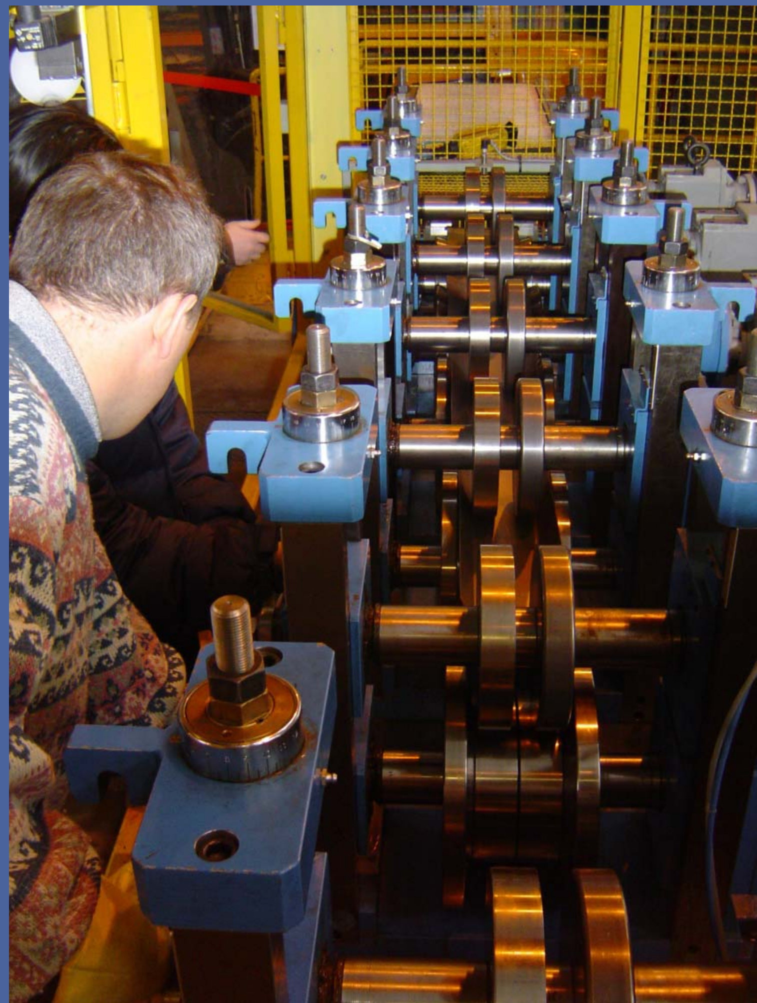
- Part #1 : quasi-static problem
- Part #2 : springback modelling : loading and tools are removed in a second dynamic phase.



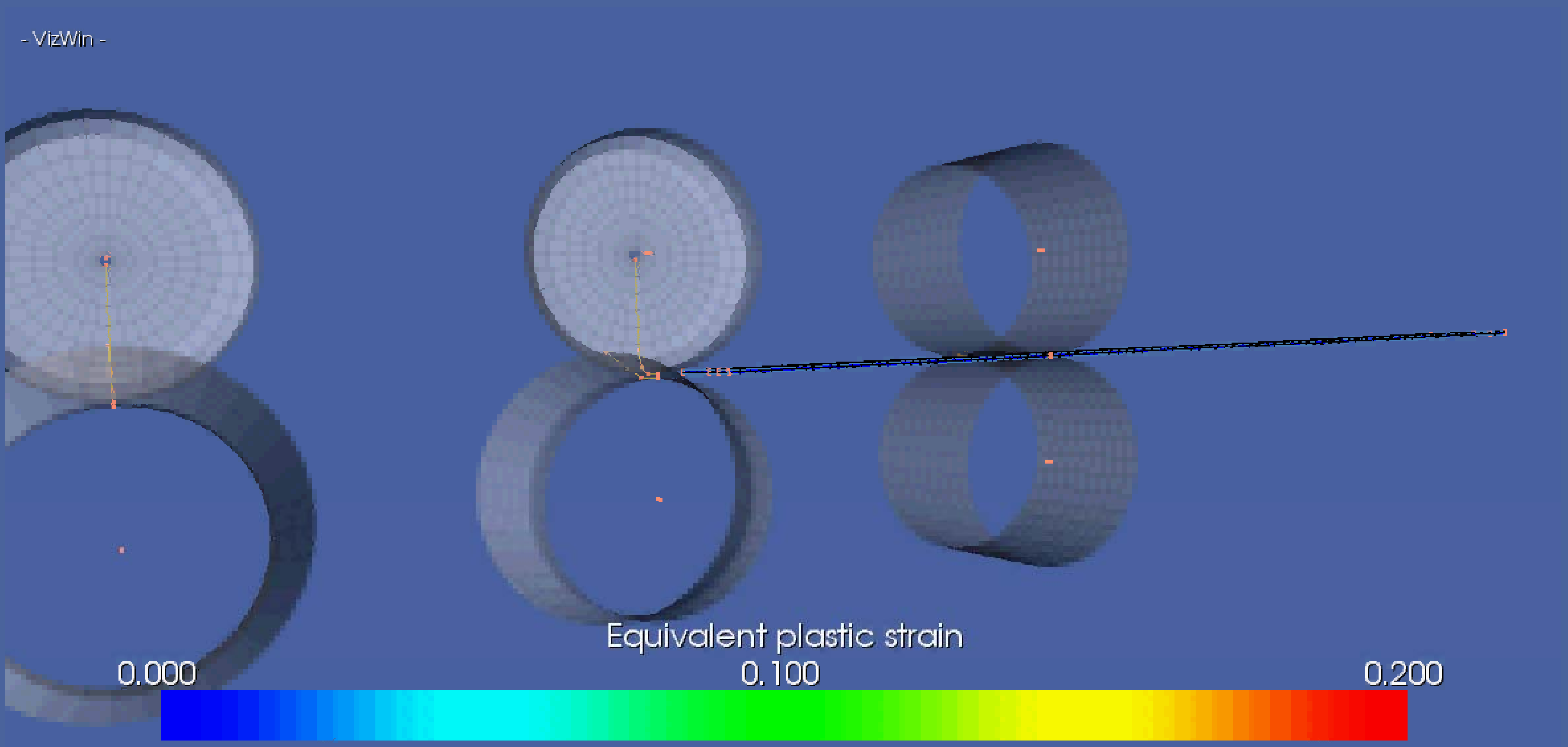
Roll forming







Roll forming – Lagrangian simulation



Roll forming – ALE simulation