

Forward observables at RHIC, the Tevatron run II and the LHC¹

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Abstract

We present predictions on the total cross sections and on the ratio of the real part to the imaginary part of the elastic amplitude (ρ parameter) for present and future pp and $\bar{p}p$ colliders, and on total cross sections for $\gamma p \rightarrow$ hadrons at cosmic-ray energies and for $\gamma\gamma \rightarrow$ hadrons up to $\sqrt{s} = 1$ TeV. These predictions are based on a study of many possible analytic parametrisations and invoke the current hadronic dataset at $t = 0$. The uncertainties on total cross sections, including the systematic theoretical errors, reach 1% at RHIC, 3% at the Tevatron, and 10% at the LHC, whereas those on the ρ parameter are respectively 10%, 17%, and 26%.

This report is based on ref. [1], which constitutes the conclusion of an exhaustive study [2] of analytic parametrisations of soft forward data at $t = 0$. As explained in V. V. Ezhela's contribution to these proceedings, this study has three main purposes. First of all, it helps maintain the dataset of cross sections and ρ parameters available to the community. Secondly, it enables us to decide which models are the best, and in which region of s . Finally, and this will be the main object of this report, it enables us to make predictions based on a multitude of models, and on all available data.

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The dataset of this study includes all measured total cross sections and ratios of the real part to the imaginary part of the elastic amplitude (ρ parameter) for the scattering of pp , $\bar{p}p$, $\pi^\pm p$, $K^\pm p$, and total cross sections for γp , $\gamma\gamma$ and $\Sigma^- p$. Compared with the 2002 Review of Particle Properties dataset [3], it includes the latest ZEUS points[4] on total cross sections, as well as cosmic ray measurements [5]. The number of points of each sub-sample of the dataset is given in Table 1.

The base of models is made of 256 different analytic parametrisations. We can summarize their general form by quoting the form of total cross sections, from which the ρ parameter is obtained via derivative dispersion relations. The ingredients are the contribution M^{ab} of the highest meson trajectories (ρ , ω , a and f) and the rising term H^{ab} for the pomeron.

$$\sigma_{tot}^{ab} = (M^{ab} + H^{ab})/s \quad (1)$$

The first term is parametrised via Regge theory, and we allow the lower trajectories to be partially non-degenerate, *i.e.* we allow one intercept for the $C = +1$ trajectories, and another one for the $C = -1$ [6]. A further lifting of the degeneracy is certainly possible, but does not seem to modify significantly the results [7]. Hence we use

$$M^{ab} = Y_+^{ab} \left(\frac{s}{s_0}\right)^{\alpha_+} \pm Y_-^{ab} \left(\frac{s}{s_0}\right)^{\alpha_-} \quad (2)$$

with $s_0 = 1 \text{ GeV}^2$. The contribution of these trajectories is represented by RR. As for the pomeron term, we choose a combination of the following possibilities:

$$H^{ab} = X^{ab} \left(\frac{s}{s_0}\right)^{\alpha_\varphi} + sP^{ab} \quad (3)$$

$$H^{ab} = s \left[B^{ab} \ln \left(\frac{s}{s_0}\right) + P^{ab} \right] \quad (4)$$

$$H^{ab} = s \left[B^{ab} \ln^2 \left(\frac{s}{s_1}\right) + P^{ab} \right] \quad (5)$$

with $s_0 = 1 \text{ GeV}^2$ and s_1 to be determined by the fit. The contribution of these terms is marked PE, PL and PL2 respectively. Note that the pole structure of the pomeron cannot be directly obtained from these forms, as multiple poles at $J = 1$ produce constant terms which mimic simple poles at $t = 0$. Furthermore, we have considered several possible constraints on the parameters of Eqs. (2-5):

Table 1: Summary of the quality of the fits for different scenarios considered in this report, for $\sqrt{s} \geq 5$ GeV: DB02Z – the 2002 Review of Particle Properties database with new ZEUS data, DB02Z-CDF – with the CDF point removed; DB02Z-E710/E811 – with E710/E811 points removed. The first line gives the overall χ^2/dof for the global fits, the other lines give the χ^2/nop for data sub-samples, the last line gives in each case the parameter controlling the asymptotic form of cross sections.

Sample	Number of points	DB02Z	DB02Z –CDF	DB02Z –E710/E811
total		0.965	0.964	0.951
total cross sections				
pp	111	0.84	0.90	0.90
$\bar{p}p$	57–59	1.15	1.12	1.05
π^+p	50	0.71	0.71	0.71
π^-p	95	0.96	0.96	0.96
K^+p	40	0.71	0.71	0.71
K^-p	63	0.62	0.62	0.61
Σ^-p	9	0.38	0.38	0.38
γp	37	0.58	0.58	0.58
$\gamma\gamma$	38	0.64	0.64	0.63
elastic forward Re/Im				
pp	64	1.83	1.83	1.80
$\bar{p}p$	11	0.52	0.52	0.53
π^+p	8	1.50	1.52	1.46
π^-p	30	1.10	1.09	1.14
K^+p	10	1.07	1.10	0.98
K^-p	8	0.99	1.00	0.96
values of the parameter B				
		0.307(10)	0.301(10)	0.327(10)

- degeneracy of the reggeon trajectories $\alpha_+ = \alpha_-$, noted $(RR)_d$;
- universality of rising terms (B^{ab} independent of the hadrons), noted $L2_u$, L_u and E_u [8];
- factorization for the residues in the case of the $\gamma\gamma$ and γp cross sections. If not otherwise indicated by the subscript nf , we impose $H_{\gamma\gamma} = \delta H_{\gamma p} = \delta^2 H_{pp}$;
- quark counting rules [9] to predict the Σp cross section from pp , Kp and πp , indicated by the subscript qc ;
- Johnson-Treiman-Freund [10] relation for the cross section differences, noted R_c .

All possible variations of Eqs. (2-5), using the above constraints, amount to 256 variants.

These variants are then fitted to the database, allowing for the minimum c.m. energy $\sqrt{s_{min}}$ of the fit to vary between 3 and 10 GeV. For $\sqrt{s} \geq 9$ GeV, 33 variants have an overall $\chi^2/d.o.f. \leq 1.0$ if one fits only to total cross sections, whereas 21 obeyed this criterion when one includes the ρ parameters in the data to be fitted to. One can try to lower the minimum energy of the fit, and one finds that for 11 models one can extend the minimum energy of the cross section fit to 4 GeV, and that of the combined fit of σ_{tot} and ρ to 5 GeV. Several parametrisations based on triple poles (RRPL2), double poles (RRPL) or simple poles (RRPE) are kept. The only notable candidate which seems to be ruled out is the popular simple-pole model (RRE) [11]. Its predictions for pp and $\bar{p}p$ nevertheless fall within our errors⁸.

After this selection is made, the remaining models are ranked. We measure some characteristics of the fits, namely: the number of parameters, the confidence level in the considered region, the size of the region where the model achieves a $\chi^2/dof \leq 1$ and the value of that χ^2/dof , the stability of the parameters when the minimum c.m. energy is changed, their stability with respect to the inclusion of the ρ data, the uniformity of the χ^2/dof for different processes and quantities, and finally the quality of the correlation matrix. All these features are important, and we have managed to measure them, introducing new statistical indicators [2, 13]. The ideal fit would be

⁸This conclusion could be affected by a re-calculation of the ρ parameter from *integral* dispersion relations – see O. V. Selyugin’s contribution to these proceedings.

the one with the least number of parameters, the biggest region of applicability, the best χ^2 , etc. Unfortunately, a single fit does not concentrate all these virtues. As the new indicators do not have (yet) a probabilistic interpretation and as all the parametrisations which fit are *a priori* acceptable, we choose the “best” model through a ranking procedure: for each feature, the models are ordered according to how well they perform. One then sums the position of each model for each indicator, and the model with least points is preferred. The advantage of this method, besides the fact that it automatically looks at many qualities of each fit, is that the best model is decided on the basis of automatic criteria, which do not depend on our own prejudice.

Following that procedure, the triple-pole parametrisation RRP_{nfL2u} [8, 12] gives the most satisfactory description of the data. This parameterization has a universal (u) $B \ln^2(s/s_0)$ term, a non-factorizing (nf) constant term and non-degenerate lower trajectories.

We are now in a position to evaluate several quantities of interest for future measurements. First of all, our best parametrisation can of course be used to predict σ_{tot} and ρ , with their statistical errors. We choose for this the parameters determined for a minimum c.m. energy $\sqrt{s_{min}} = 5$ GeV. For pp and $\bar{p}p$, the central value of this fit gives

$$\sigma_{tot}^{\bar{p}p,pp} = 43. s^{-0.46} \pm 33. s^{-0.545} + 35.5 + 0.307 \ln^2 \left(\frac{s}{29.} \right) \quad (6)$$

with all coefficients in mb and s in GeV^2 . We assign errors by using the full error matrix E_{ij} from the fit, and define

$$\Delta Q = \sum_{ij} E_{ij} \frac{\partial Q}{\partial x_i \partial x_j} \quad (7)$$

with $Q = \sigma_{tot}$ or ρ and x_i the parameters of the model. Our predictions are given in Table 2 and the corresponding 1σ region is shown as a dark band in Figs. 1 and 2.

One can now concentrate on the evaluation of systematic errors. The first source of these is the presence of contradictory data points in the database. One can see from Table 1 that $\sigma_{\bar{p}p}$, ρ_{pp} , $\rho_{\pi p}$ and ρ_{K+p} are not well fitted to. For the ρ parameters, this can partially be attributed to contradictions in the data, and partially to our use of derivative dispersion relations. For $\sigma_{\bar{p}p}$, this comes entirely from problems with the data. Of these, the most notable one is the disagreement at the Tevatron between the measurements of CDF

Table 2: Predictions for σ_{tot} and ρ , for $\bar{p}p$ (at $\sqrt{s} = 1960$ GeV) and for pp (all other energies). The central values and statistical errors correspond to the preferred model $RRP_{nf}L2_u$, fitted for $\sqrt{s_{min}} = 5$ GeV. The first systematic errors come from the consideration of two choices between CDF and E-710/E-811 $\bar{p}p$ data in the simultaneous global fits. The second systematic error corresponds to the consideration of the 21 parametrisations compatible with existing data.

\sqrt{s} (GeV)	σ (mb)			ρ		
100	46.37 ± 0.06	+0.11	+0.31	0.1058 ± 0.0012	+0.0028	+0.0024
		-0.03	-0.06		-0.0009	-0.0019
200	51.76 ± 0.12	+0.27	+0.43	0.1275 ± 0.0015	+0.0035	+0.0000
		-0.08	-0.15		-0.0011	-0.0023
300	55.50 ± 0.17	+0.39	+0.39	0.1352 ± 0.0016	+0.0038	+0.0000
		-0.12	-0.20		-0.0012	-0.0059
400	58.41 ± 0.21	+0.49	+0.28	0.1391 ± 0.0017	+0.0039	+0.0002
		-0.16	-0.23		-0.0013	-0.0087
500	60.82 ± 0.25	+0.58	+0.15	0.1413 ± 0.0017	+0.0040	+0.0006
		-0.19	-0.25		-0.0013	-0.0109
600	62.87 ± 0.28	+0.66	+0.03	0.1427 ± 0.0018	+0.0040	+0.0008
		-0.21	-0.26		-0.0013	-0.0125
1960	78.26 ± 0.55	+1.30	+0.08	0.1450 ± 0.0018	+0.0038	+0.0022
		-0.42	-1.95		-0.0013	-0.0226
10000	105.1 ± 1.1	+2.6	+0.60	0.1382 ± 0.0016	+0.0032	+0.0028
		-0.82	-8.30		-0.0011	-0.0324
12000	108.5 ± 1.2	+2.7	+0.70	0.1371 ± 0.0015	+0.0031	+0.0030
		-0.87	-9.20		-0.0011	-0.0332
14000	111.5 ± 1.2	+2.9	+0.80	0.1361 ± 0.0015	+0.0030	+0.0030
		-0.92	-10.2		-0.0011	-0.0337

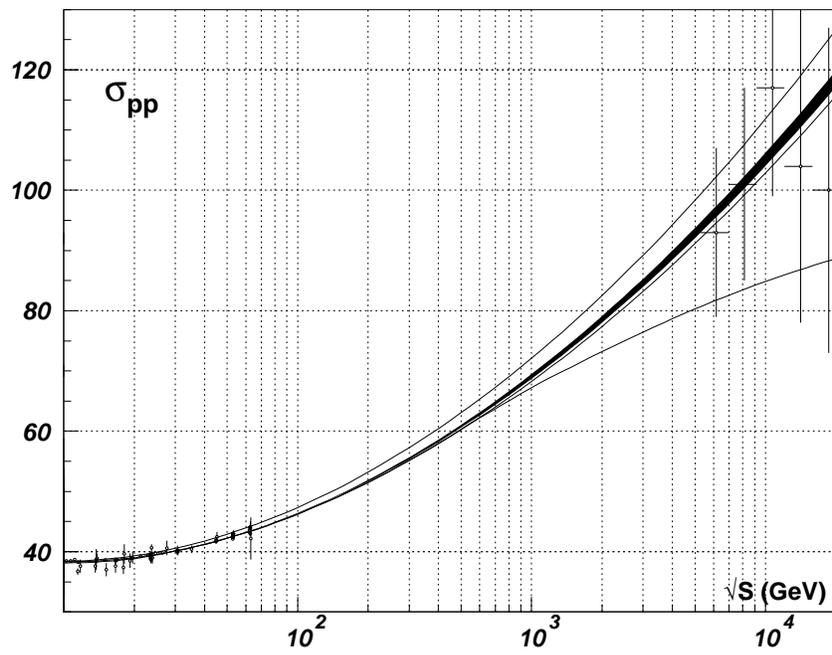


Figure 1: Predictions for total cross sections.

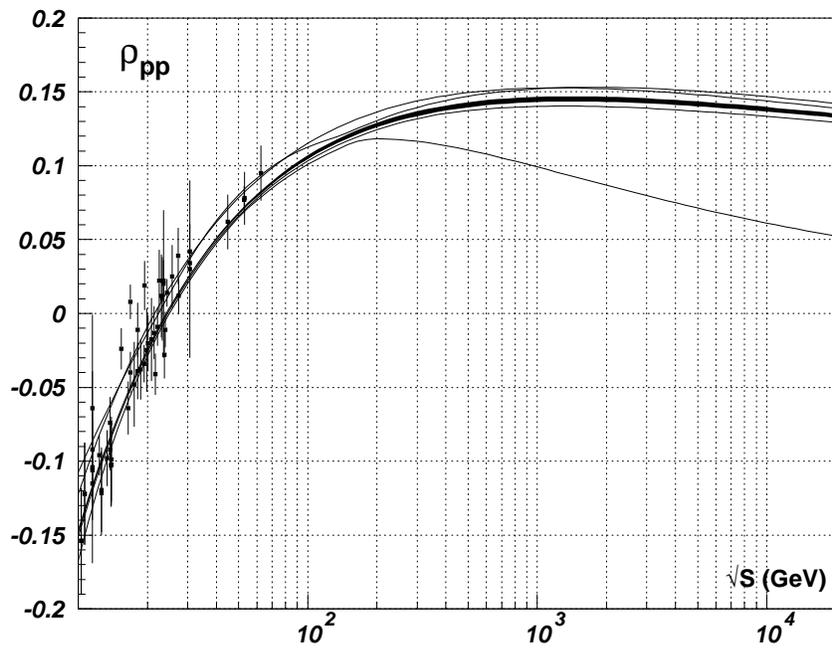


Figure 2: Predictions for the ρ parameter.

Table 3: Predictions for σ_{tot} for $\gamma p \rightarrow hadrons$ and for $\gamma\gamma \rightarrow hadrons$ for cosmic ray energies. We quote the central values, the statistical errors and the experimental systematic errors, defined as in Table 2.

p_{lab}^γ (GeV)	$\sigma_{\gamma p}$ (mb)	\sqrt{s} (GeV)	$\sigma_{\gamma\gamma}$ (μ b)
$0.5 \cdot 10^6$	0.24 ± 0.01	200	0.55 ± 0.03
	$+0.00$ -0.11		$+0.00$ -0.29
$1.0 \cdot 10^6$	0.26 ± 0.01	300	0.61 ± 0.04
	$+0.00$ -0.11		$+0.00$ -0.29
$1.0 \cdot 10^7$	0.33 ± 0.02	400	0.66 ± 0.04
	$+0.00$ -0.11		$+0.00$ -0.29
$1.0 \cdot 10^8$	0.42 ± 0.02	500	0.70 ± 0.05
	$+0.00$ -0.11		$+0.00$ -0.29
$1.0 \cdot 10^9$	0.52 ± 0.03	1000	0.84 ± 0.07
	$+0.00$ -0.11		$+0.00$ -0.29

and those of E710/E811. We show in Table 1 the effect of the removal of the CDF or of the E710/E811 measurements. First of all, one can see that the coefficient of the $\log^2 s$ changes by more than 1σ , and hence the predictions for the LHC are muddled by this discrepancy. We also see that our preferred parametrisation favours the CDF measurement, as its global χ^2/dof goes down when the E710/E811 point is removed. It must be noted that a similar situation exists for all \log^2 and simple pole parametrisations⁹. On the other hand, the dipole RRPL ($\log s$) parametrisations do prefer the lower value. The only thing we can do at present is indicate what shift the adoption of one point or the others would cause on our central value. This is given in Table 2 as a systematic experimental error, corresponding to the shift in the upper and lower 1σ allowed values¹⁰, and shown as the two curves closest to the central band in Figs. 1 and 2.

We can now, and maybe for the first time, give a reasonable estimate of the theoretical error. The idea is to choose a less constraining minimum

⁹The original Donnachie-Landshoff parametrisation did predict a low cross section at the Tevatron, but this was due to the use of a non-conventional (and non-probabilistic) χ^2 . Using the statistical definition of χ^2 leads to a rejection of the E710/E711 point.

¹⁰This definition differs from that of [1] by about 1 statistical error

energy for the fits (we take 9 GeV), and to consider the results of the 21 models that succeed in reproducing both σ_{tot} and ρ . This gives us 21 predictions with error bars. We can then define the theoretical systematic error by taking the distance between the highest (resp. lowest) values in the 1σ intervals with the 1σ central region. We give the resulting numbers as a third error in Table 2, and as the outer curves of Figs. 1 and 2.

Note that the systematic errors cannot be added in quadrature, and that the theoretical systematic error is an absolute shift from model to model, and does not have any probabilistic interpretation.

One can see that the errors on total cross sections are of the order of 1% at RHIC, 3% at the Tevatron and as large as 10% at the LHC. At RHIC, the systematic errors due to the Tevatron discrepancy and those due to theory are of comparable order. The value of the cross section is constrained by the $\bar{p}p$ data, and by the fact that we allow only one $C = -1$ contribution, which is well constrained by the overall fit. Very precise RHIC measurements, at the level of one in a thousand could shed light on the Tevatron discrepancy, and discriminate between models. Of course, the extrapolation to LHC energies presents the largest uncertainty and is dominated by systematic theoretical errors, with the double pole models (RRPL) giving a cross section significantly lower than the triple poles or the simple poles. A determination of the cross section at the 5% level could rule out one half of the models or the other.

The errors on the ρ parameter are much larger, reaching 10% at RHIC, 17% at the Tevatron and 26% at the LHC. This is due to the fact that experimental errors are bigger, hence less constraining, but this also stems from the incompatibility of some low-energy determinations of ρ [2], and from our use of derivative dispersion relations. Although integral dispersion relations have the potential to reduce the χ^2/dof , they have the inconvenient of introducing extra parameters (because they necessitate subtractions). Hence it is unlikely that a different theoretical treatment can reduce the errors. On the other hand, a re-analysis of some of the data could be envisaged. It should involve a combination of the information on total cross section with that on elastic hadronic cross sections, on electromagnetic form factors and on Regge trajectories (see V. V. Ezhela's contribution to these proceedings).

Finally, we can use the same approach to predict cross sections for cosmic photon studies. We show the results in Table 3, where we have given only

the experimental systematic error¹¹.

To conclude, we believe that we have pushed the database technology to the point where it can make predictions, and decide on which models or theories are the best. This is an example proof of the feasibility of the COMPETE program, and of its utility.

We have given here the best possible estimates for present and future pp and $\bar{p}p$ facilities. The central values of our fits and the corresponding statistical error give our “best guess” estimate. The systematic experimental errors tell us how much this guess could be affected by incompatible data. The theoretical systematic errors will tell us directly whether an experiment can be fitted by one of the standard analytic parametrisations, or whether it calls for new ideas.

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¹¹ note that the corresponding table in ref. [1] has a typo in the error bars which are systematically a factor 10 too small.

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