Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Natural and projectively equivariant quantizations

Fabian Radoux

Przemyśl

1 May 2007

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Introduction

- At the origin : $qp \to QP$

$$qp \to \tfrac{1}{2}(QP + PQ)$$

with $P = \partial_x$; $Q = x$.

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Introduction

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Introduction

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Quantization : $Q : \mathcal{S}(M) \mapsto \mathcal{D}_{\frac{1}{2}}(M)$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Quantization : $Q : \mathcal{S}(M) \mapsto \mathcal{D}_{\frac{1}{2}}(M)$

  Q : linear bijection

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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- $\nexists Q : Q(\Phi^* S) = \Phi^* Q(S) \; \forall \text{ local diffeomorphism } \Phi$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

"Flat" case :

- $\exists Q : Q(\Phi_g^* S) = \Phi_g^* Q(S) \; \forall g \in G$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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- $PGL(m+1, \mathbb{R})$ acts on $\mathbb{R}P^m$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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- $PGL(m + 1, \mathbb{R})$ acts on $\mathbb{R}P^m$

  $X \in sl(m + 1, \mathbb{R}) \mapsto X^*$ vector field on $\mathbb{R}^m$

  $\exists Q : L_X Q(S) = Q(L_X S) \; \forall X \in sl(m + 1, \mathbb{R})$

• Method of the Casimir operator :

$$C : \mathcal{S}(\mathbb{R}^m) \mapsto \mathcal{S}(\mathbb{R}^m) \quad ; \quad \mathcal{C} : \mathcal{D}(\mathbb{R}^m) \mapsto \mathcal{D}(\mathbb{R}^m)$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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$$\mathcal{C}(Q(S)) = \alpha Q(S) \text{ and } \sigma(Q(S)) = S$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

"Curved" case :

$\bullet Q(\nabla) : \mathcal{S}^3(M) \mapsto \mathcal{D}^3(M)$

$Q(\nabla) = Q(\nabla')$ if $\nabla' = \nabla + \alpha \vee id$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Conjecture : $Q(\nabla) : \mathcal{S}(M) \mapsto \mathcal{D}(M)$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Conjecture : $Q(\nabla) : \mathcal{S}(M) \mapsto \mathcal{D}(M)$ natural

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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$$\phi_t^* Q(\nabla^0)(S) = Q(\phi_t^* \nabla^0)(\phi_t^* S)$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- M. Bordemann method :

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• M. Bordemann method : $M \mapsto \tilde{M}$;

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• M. Bordemann method : $M \mapsto \tilde{M}$;

$$\nabla \mapsto \tilde{\nabla};$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• M. Bordemann method : $M \mapsto \tilde{M}$;

$$\nabla \mapsto \tilde{\nabla};$$

$$S \mapsto \tilde{S};$$

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$$\nabla \mapsto \tilde{\nabla};$$

$$S \mapsto \tilde{S};$$

$$Q(\widetilde{\nabla)(S}(f) = \tau(\tilde{\nabla})(\tilde{S})(\tilde{f})$$

• Questions :

- Questions : Critical values of δ

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Questions : Critical values of δ

Explicit formula

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Questions : Critical values of δ

    Explicit formula

    Uniqueness

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  Explicit formula

  Uniqueness

  Other differential operators

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- Cartan fiber bundles and connections

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

$$[\nabla] \to P \mapsto M$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

$$[\nabla] \to P \mapsto M$$

$$[\nabla] \to \omega : TP \mapsto \mathfrak{g}$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

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$$\omega_u : T_uP \mapsto \mathfrak{g} \text{ bijection } \forall u \in P$$

•Cartan fiber bundles and connections

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

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• Cartan fiber bundles and connections

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G_0-equivariant

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

• Cartan fiber bundles and connections

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G_0-equivariant

$$f \ G_1\text{-equivariant} \not\Rightarrow \nabla^\omega f$$

G_1-equivariant

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- The case of the densities (P. Mathonet, R.) :

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

- The case of the densities (P. Mathonet, R.) :

$$S \to p^* S \in \mathcal{C}^\infty(P, S_\delta^k(\mathbb{R}^m))_H$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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$$\omega \to Div^\omega = \sum_i i(\epsilon^i) L_{\omega^{-1}(e_i)}$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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$\langle p^* S, \nabla_s^{\omega^k} p^* f \rangle$ not G_1-equivariant !

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$$S \to p^* S \in \mathcal{C}^\infty(P, S^k_\delta(\mathbb{R}^m))_H$$

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One adds terms of lower orders in $p^* f$...

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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$\langle p^*S, \nabla_s^{\omega^k} p^*f \rangle$ not G_1-equivariant !

One adds terms of lower orders in p^*f...

One finds then :

$$Q_M(\nabla, S)(f) = p^{*^{-1}}(\sum_{l=0}^k C_{k,l} \langle Div^{\omega^l} p^*S, \nabla_s^{\omega^{k-l}} p^*f \rangle),$$

with $C_{k,l} = \frac{(\lambda + \frac{k-1}{m+1}) \cdots (\lambda + \frac{k-l}{m+1})}{\gamma_{2k-1} \cdots \gamma_{2k-l}} \begin{pmatrix} k \\ l \end{pmatrix}, \forall l \geq 1, \quad C_{k,0} = 1$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

•Other differential operators (P. Mathonet, R.) : method of
the Casimir operator :

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"Flat" case   "Curved" case

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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"Flat" case   "Curved" case

Affine quantization Q_{Aff} :   "Affine" quantization Q_ω :

$$\partial_i \qquad\qquad\qquad L_{\omega^{-1}(e_i)}$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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Affine quantization Q_{Aff} :   "Affine" quantization Q_ω :
$$\partial_i \qquad\qquad\qquad L_{\omega^{-1}(e_i)}$$

Application γ :   Application γ :

$\mathcal{L}_{X^h} Q_{Aff}(S)(f) =$ $\mathcal{L}_{h^*} Q_\omega(S)(f) =$

$Q_{Aff}((L_{X^h} + \gamma(h))S)(f)$ $Q_\omega((L_{h^*} + \gamma(h))S)(f),\ h \in g_1$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

•Other differential operators (P. Mathonet, R.) : method of the Casimir operator :

"Flat" case

"Curved" case

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Application γ :

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$\mathcal{L}_{X^h} Q_{Aff}(S)(f) =$
$Q_{Aff}((L_{X^h} + \gamma(h))S)(f)$

$\mathcal{L}_{h^*} Q_\omega(S)(f) =$
$Q_\omega((L_{h^*} + \gamma(h))S)(f), \; h \in g_1$

Casimirs C and $\mathcal{C}$

"Casimirs" C^ω and $\mathcal{C}^\omega$

$$\partial_i$$

$$L_{\omega^{-1}(e_i)}$$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Quantization :

$Q_{Aff}(Q(S))$, $Q(S)$ such that

if $C(S) = \alpha S$, then

$\mathcal{C}(Q(S)) = \alpha Q(S)$ and

"head" of $Q(S) = S$

Quantization :

$Q_\omega(Q(S))$, $Q(S)$ such that

if $C^\omega(S) = \alpha S$, then

$\mathcal{C}^\omega(Q(S)) = \alpha Q(S)$ and

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Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

Quantization :

$Q_{Aff}(Q(S))$, $Q(S)$ such that

if $C(S) = \alpha S$, then

$\mathcal{C}(Q(S)) = \alpha Q(S)$ and

"head" of $Q(S) = S$

Then :
$\mathcal{L} \circ Q = Q \circ L$

because

$[\mathcal{C}, \mathcal{L}] = 0$ and $[C, L] = 0$

Quantization :

$Q_{\omega}(Q(S))$, $Q(S)$ such that

if $C^{\omega}(S) = \alpha S$, then

$\mathcal{C}^{\omega}(Q(S)) = \alpha Q(S)$ and

"head" of $Q(S) = S$

Then :
$(L_{h^*} + \gamma(h)) \circ Q = Q \circ L_{h^*}$

because $[\mathcal{C}^{\omega}, L_{h^*} + \gamma(h)] = 0$

and $[C^{\omega}, L_{h^*}] = 0$

Natural and
projectively
equivariant
quantizations

Fabian Radoux

Introduction

Cartan fiber
bundles and
connections

The case of the
densities

Other differential
operators

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"head" of $Q(S) = S$

Then :
$\mathcal{L} \circ Q = Q \circ L$

because

$[\mathcal{C}, \mathcal{L}] = 0$ and $[C, L] = 0$

•Conclusion : "Flat" case
$$\partial_i$$

Quantization :

$Q_{\omega}(Q(S))$, $Q(S)$ such that

if $C^{\omega}(S) = \alpha S$, then

$\mathcal{C}^{\omega}(Q(S)) = \alpha Q(S)$ and

"head" of $Q(S) = S$

Then :
$(L_{h^*} + \gamma(h)) \circ Q = Q \circ L_{h^*}$

because $[\mathcal{C}^{\omega}, L_{h^*} + \gamma(h)] = 0$

and $[C^{\omega}, L_{h^*}] = 0$

"Curved" case
$$L_{\omega^{-1}(e_i)}$$