# MATHEMATICAL MODELLING OF AN ADVANCED ONCE-THROUGH SUB- OR SUPERCRITICAL HEAT RECOVERY STEAM GENERATOR

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*The design and the follow-up of a once-through circulation boiler differs from the design and the follow-up of a conventional boiler. A specific thermodynamic model has to suit very high pressure, sub- and supercritical steam properties. Mathematical models have to be adapted to account for the disappearance of the conventional economiser, boiler and superheater. Empirical equations corresponding to each part of the traditional boiler are no more possible. In a once-through heat recovery boiler the location of the boiling point is no more fixed. General equations have to be used for each tube of the boiler. The mathematical complexity as well as the number of equations is increased.*

*This paper presents one subcritical 180 bar once-through heat recovery boiler model. Comparison with usual boilers in terms of mathematical results are presented, including the description of a specific mathematical model, especially developed, using the Belsim-VALI software, to represent a once-through boiler.*

## THERMODYNAMIC MODEL

To estimate water and steam properties, we make use of "IAPWS Industrial Formulation for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1]. It replaces the previous industrial standard IFC-67. This formulation provides a very accurate representation of the thermodynamic properties of water and steam over a wide range of temperature and pressure with a formulation that is designed for fast computation.
The IAPWS Industrial Formulation 1997 consists of a set of equations for different regions which cover the following range of validity:

| | | |
|---|---|---|
| 0°C | < T < 800°C | p<1000 bar |
| 800°C | < T <2000°C | p<100 bar |

Figure 1 shows the 5 regions into which the entire range of validity of IAPWS-IF97 is divided.

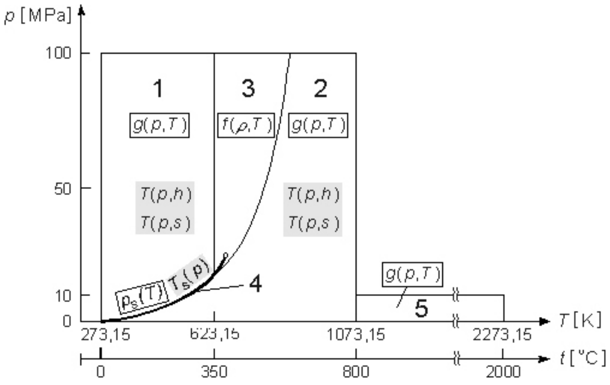



**Figure 1: IAPWS-IF97 range of validity**

## MATHEMATICAL MODEL

The following set of equations represents a boiler from the data reconciliation point of view. For each fluid (cold water and hot fumes) there is:

1. mass balance
$$x_{in} = x_{out} \qquad (1)$$
2. pressure balance
$$P_{out} = P_{in} - \Delta P \qquad (2)$$
3. pressure drop estimation
$$\Delta P = f(T, P, x_i, geometry) \qquad (3)$$
4. heat balance
$$H_{out} = H_{in} \pm Q \qquad (4)$$
5. heat transfer coefficient estimation
$$U = f(T, P, x_i, geometry) \qquad (5)$$

### Heat transfer

**Water.** Mathematical models for traditional boilers are usually based on empirical equations corresponding to each part of the boiler : the economizer, the boiler and the super heater. Those three parts of boiler are clearly separated thus it is not difficult to chose the right equation. In a once-through boiler (OTB) this separation is not so clear. We have first to estimate the flow pattern in the tube then to chose the equation to be used. "Liquid single phase" and "vapor single phase" are easily located with temperature and pressure data. According to Gnielinski [2] the following equations apply (for turbulent and hydrodynamically developed flow):

$l = d_i$

$$Nu = \frac{(\xi/8)(\mathrm{Re}_l - 1000)\,\mathrm{Pr}}{1 + 12{,}7\sqrt{(\xi/8)}\left(\mathrm{Pr}^{2/3} - 1\right)} \tag{6}$$

$$\xi = \frac{1}{\sqrt{\left(1{,}82\log_{10}\mathrm{Re} - 1{,}64\right)}}$$

$$Nu = \frac{\alpha * d}{\lambda} \tag{7}$$

During vaporization different flow patterns can be observed, for which the rate of heat transfer also differs.

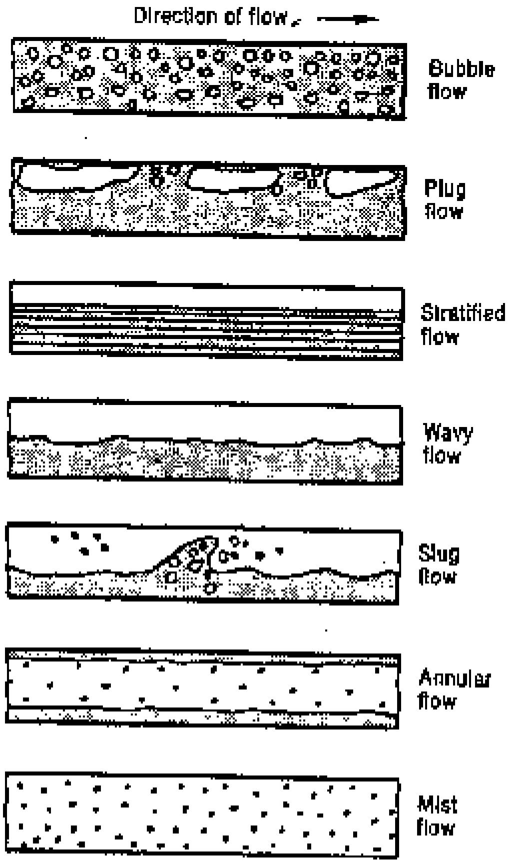



**Figure 2: flow patterns in horizontal tubes**

In stratified-wavy flow pattern incomplete wetting has an effect on the heat transfer coefficient. A reduction could appear for this type of flow pattern. Computing conditions where a change in flow pattern occurs is useful. A method to establish a flow pattern map in horizontal tube for given pressure and flow conditions is clearly exposed in VDI [2]. This method has been used in this study.

The heat transfer coefficient is estimated from numerous data. It is a combination of convective heat transfer coefficient and nucleate boiling heat transfer coefficient.

$$\alpha(z) = \sqrt[3]{\alpha(z)^3_{conv} + \alpha(z)^3_B} \tag{8}$$

$$\frac{\alpha(z)_{conv}}{\alpha_{lo}} = \left\{ \left[ (1-x) + 1.2x^{0.4}(1-x)^{0.01}\left(\frac{\rho_{liq}}{\rho_{vap}}\right)^{0.37} \right]^{-2.2} + \left[ \frac{\alpha_{go}}{\alpha_{lo}} x^{0.01} \left(1 + 8(1-x)^{0.7}\left(\frac{\rho_{liq}}{\rho_{vap}}\right)^{0.67}\right) \right]^{-2} \right\}^{-0.5} \tag{9}$$

$$\frac{\alpha(z)_B}{\alpha_{lo}} = \left\{ heatflow, pressure, roughness, geometry \right\} \tag{10}$$

In equations (9) and (10), $\alpha_{LO}$ is the heat transfer coefficient with total mass velocity in the form of the liquid and $\alpha_{GO}$ is the heat transfer coefficient with total mass velocity in the form of the vapor.

**Fumes.** There is no difference between the equations used for a conventional heat recovery boiler and a once trough heat recovery boiler. Main part of the heat transfer coefficient is the convective part (low fumes temperature). The effect of the turbulence has been introduced to reduce the heat transfer coefficient in the first few rows of the tube bundle.

The main difficulty to evaluate the heat transfer coefficient for the fume side comes from the fins that enhance the heat transfer, but could also produce other sources of resistance in the heat transfer, such as fouling on the surface of fins or inadequate contact between the core tube and the fin base.

There are two methods to evaluate the heat transfer coefficient:

- The first one is based on a general equation to evaluate the Nusselt number in cross flow over pipes and the efficiency of the fins. An apparent heat transfer coefficient is then computed with

$$\alpha_{app} = \alpha_f * \left[ \frac{A_{po}}{A} + \eta_f \frac{A_{fo}}{A} \right] \tag{11}$$

- The second one is based on empirical correlations derived from experimental data. For more than four banks in staggered arrangement :

$$Nu_d = 0.38\,\mathrm{Re}_d^{0.6} \left(\frac{A}{A_b}\right)^{-0.15} \mathrm{Pr}^{1/3} \tag{12}$$

It is not obvious to find the most appropriate correlation for a given fin geometry and tube bundle arrangement. The best is to ask finned tube manufacturers to provide their correlations for heat transfer coefficient and fin efficiency corresponding to the required finned tube.

**Overall heat transfer coefficient.** Finally the overall heat transfer coefficient is obtained from :

$$\frac{1}{\alpha}=\frac{1}{\alpha_{app}}+\frac{e}{\lambda*\frac{A_w}{A}}+\frac{1}{\alpha_i*\frac{A_i}{A}} \tag{13}$$

and we obtain the global heat transferred for each tube:

$$Q=\alpha*A*\Delta T_{sl}$$

$$\Delta T_{sl}=\frac{\left(T_{mf}-T_{w1}\right)-\left(T_{mf}-T_{w2}\right)}{\ln\left(\frac{\left(T_{mf}-T_{w1}\right)}{\left(T_{mf}-T_{w2}\right)}\right)} \tag{14}$$

$$T_{mf}=\frac{T_{f1}+T_{f2}}{2}$$

We call $\Delta T_{sl}$ "semi logarithmic temperature difference". It is the best compromise between pure logarithmic temperature difference that has no sense here (only one tube) and pure arithmetic temperature difference that does not allow to follow evolution of water properties along the tube.

**Pressure drop**

**Water**. The pressure drop in pipe flow is given by

$$\Delta P=\frac{f\cdot\rho\cdot\overline{V}^2}{2g}\frac{l}{d_i} \tag{15}$$

The coefficient f depends on the Reynolds number for flow within the tube. In laminar flow, the Hagen-Poiseuille law can be applied

$$f=\frac{64}{\mathrm{Re}} \tag{16}$$

In turbulent flow we can use the Blasius equation

$$f=\frac{0.3164}{\sqrt[4]{\mathrm{Re}}} \tag{17}$$

The main difficulty is the evaluation of water pressure drop during transition boiling. The pressure drop consist of three components : friction ($\Delta P_f$), acceleration($\Delta P_m$) and static pressure($\Delta P_g$). In once-through horizontal tubes boiler $\Delta P_g$=0. We used the Lockard-Martinelli formulation for friction

$$\left[\frac{\Delta P}{L}\right]_{2\,phases}=\left[\frac{\Delta P}{L}\right]_{liquid}\cdot\Phi^2_{ftt} \tag{18}$$

$$\Phi^2_{ftt}=1+\frac{20}{X}+\frac{1}{X^2}$$

$$X=\left(\frac{1-x}{x}\right)^{0,875}\left(\frac{\rho_{go}}{\rho_{lo}}\right)^{0,5}\left(\frac{\eta_{lo}}{\eta_{go}}\right)^{0,125} \tag{19}$$

$$X=\sqrt{\frac{\left[\frac{\Delta P}{L}\right]_{liquid}}{\left[\frac{\Delta P}{L}\right]_{vapor}}}$$

The acceleration term is defined with

$$\Delta P_m=G^2*\left|\frac{x^2}{\alpha*\rho_{vap}}+\frac{(1-x)^2}{(1-\alpha)*\rho_{liq}}\right|_{x_1}^{x_2} \tag{20}$$

In equation (19) α is the volume fraction of vapor (void fraction). It is recommended to discretise the tube in several short sections to obtain more accurate results!

**Fumes**. The pressure drop in tube bundle is given by

$$\Delta P=\frac{f\cdot\rho\cdot\overline{V}^2}{2}N_R \tag{21}$$

In this case the number of rows ($N_R$) play an important role in the pressure drop evaluation. The coefficient f is more difficult to compute from generalized correlations. The easiest way is once more to ask the finned tubes manufacturer to obtain accurate correlation.

**Example**. The comparison will be based on a 180 bar "one pressure steam" boiler.
The traditional boiler is composed with the economizer (22 rows, 2 rows for one pass), the vaporizer (12 rows, 6 rows for one pass) and the super heater (8 rows, 2 rows for one pass). There are 13 tubes in one row.
The OTB has the same structure with 42 rows, 2 rows for one pass.

WATER: 10.25 t/h Tin=44°C Tout = 500°C
FUMES: 72.5 t/h Tin=592°C Tout = 197°C

In VALI-Belsim software the simulation of the traditional boiler is done with 3 modules, each one corresponding to a "predefined" element of the boiler (economizer, vaporizer or super heater) whereas the simulation of the OTB needs 42 modules, one for each row of tubes. All these modules receive a name. In this case cold water enter boiler in ECOV36A and ECOV35A and leave it from SUPH02A and SUPH01A (remember there is two rows for one pass).

**Traditional boiler**. With traditional boiler equations we obtain information on temperature before and after each module. We also obtain the different heat transfer coefficients and some "extra" information as pressure drops, fluid velocity, etc.

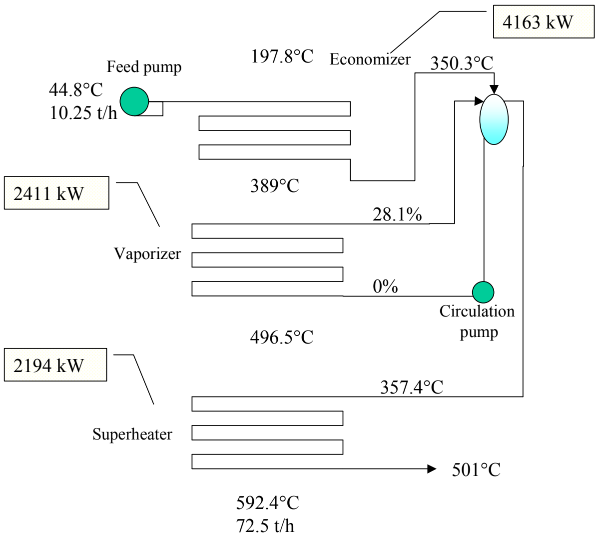
**Once-through boiler**. With the "OTB" formulation we obtain the same information for **EACH** tube.

We can visualize the fumes and water temperature evolution in regard with the load exchanged on each tube (figure 4). We clearly view that the vaporization appears around tube "ECOV10A" and is complete around tube "ECOV01A".

We can compute the flow pattern in each tube during vaporization (figure 5). The flow pattern map is following the Martinelli parameter X. Equation 19 shows that this parameter decreases when the vapor fraction increases.

The internal heat transfer coefficient can also be drawn (figure6) with a significant increase during the vaporization and a sudden decrease after it. We can notice that during vaporization a small decrease appears before the end of the vaporization. This is due to annular flow in the tube. It is interesting to design boiler knowing in advance that this type of flow pattern could occur in order to avoid tube super insulation. In figure 6 we present the internal transfer coefficient in comparison with those computed for the traditional boiler.

We can also draw the evolution of the water/vapor velocity in the tubes and the fumes velocity in the casing (figure 7).




| | Internal heat transfer coeff. | Overall heat Transfer coeff. | External heat transfer coeff. |
|---|---|---|---|
| | kcal/h/m2/K | kcal/h/m2/K | kcal/h/m2/K |
| ECONO | 3960 | 48.5 | 61.8 |
| VAPO | 20530.6 | 53.7 | 64 |
| SUPER | 2366.3 | 41.4 | 64 |

| | Fumes | | Water/steam | |
|---|---|---|---|---|
| | DP | Speed | DP | Speed |
| | mmH2O | m/s | bar | m/s |
| ECONO | 25.2 | 8.1 | 0.1 | 0.7 |
| VAPO | 19.7 | 9.5 | 0.1 | 1.7 |
| SUPER | 17.8 | 11 | 0.6 | 6.8 |

**Figure 3: Results from "traditional boiler" simulation**

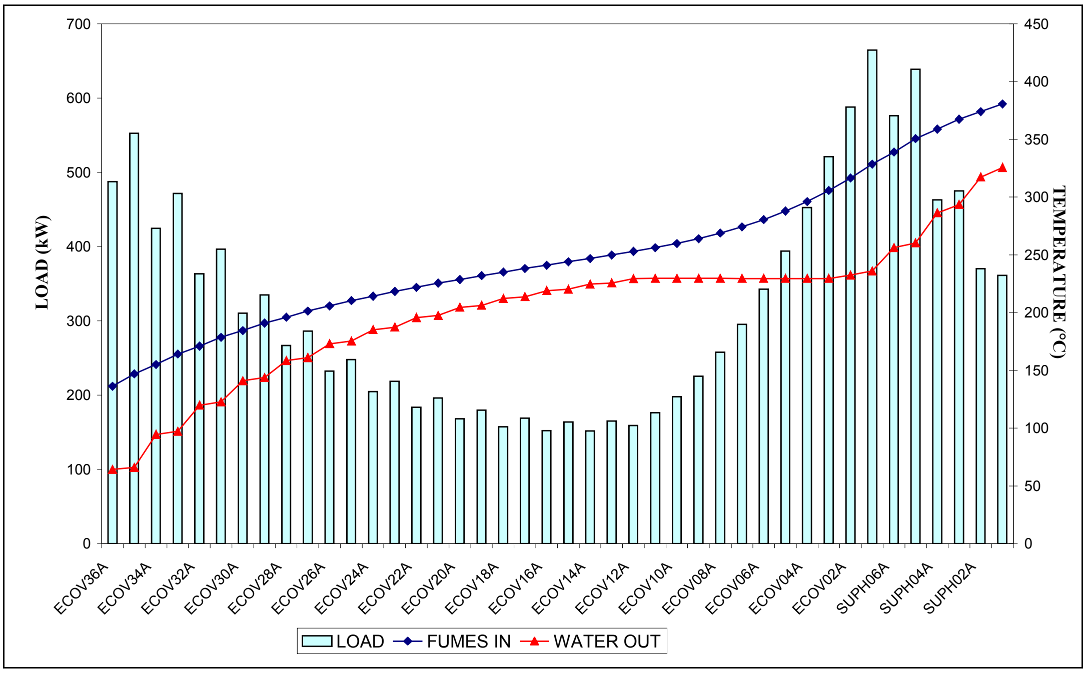



**Figure 4: Fumes and water temperature evolution through out the boiler with the "OTB" mathematical formulation**

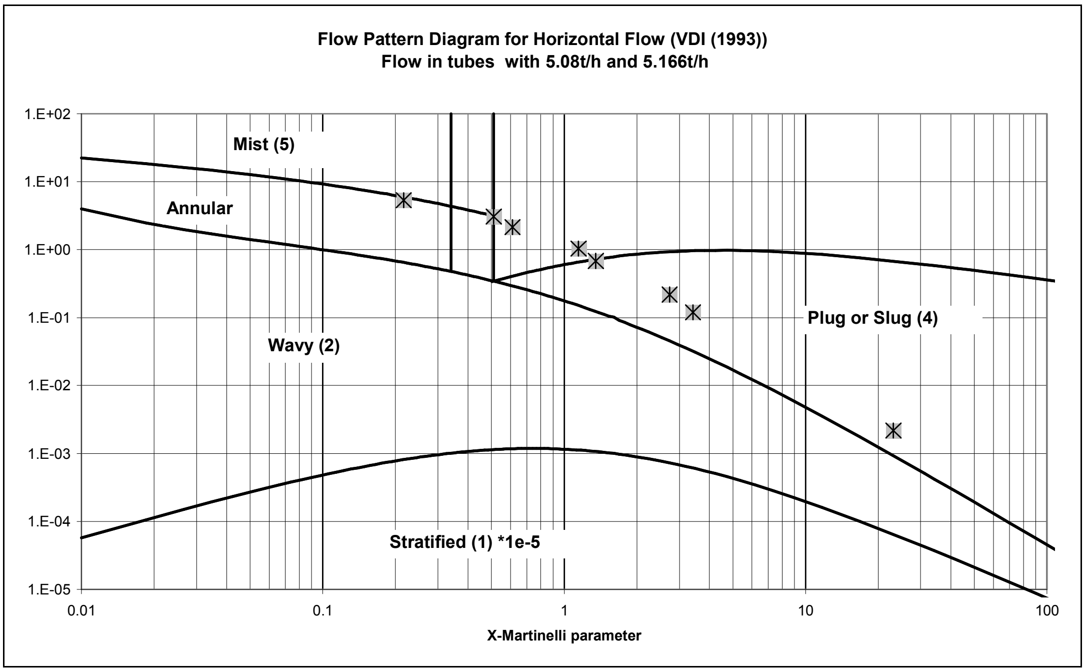



**Figure 5: Flow pattern in the boiling zone**

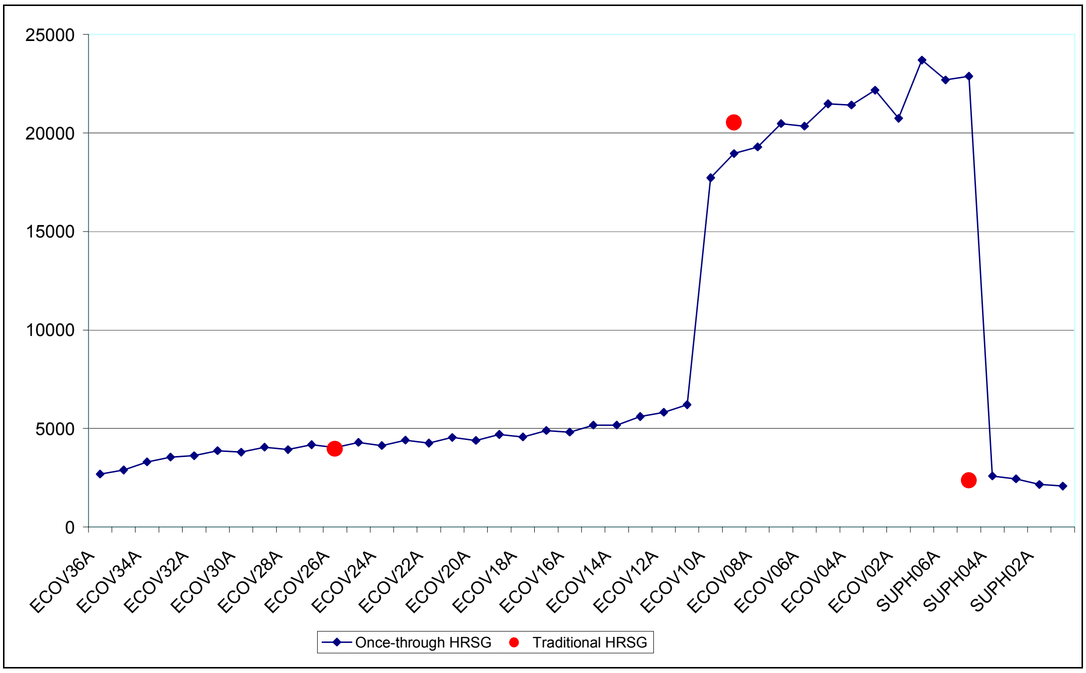



**Figure 6 : Internal heat transfer coefficient evolution in the once through boiler compared to average coefficients of a conventional steam generator (large dots)**

A very important advantage of this type of modeling is the computation of the water flow in parallel row of tubes. In this case there are two rows for one pass and the water flow IS NOT EQUALLY distributed. A difference appears due to non symmetric heating of each row. In this case there is 48.1% of the total flow in the warmer row.

Finally we have to point out that the use of VALI-Belsim software for boilers allows the design as well the follow up of different boiler types. Naturally some small improvements could be done to use the same mathematical model for modeling traditional boiler with the different advantages listed before.

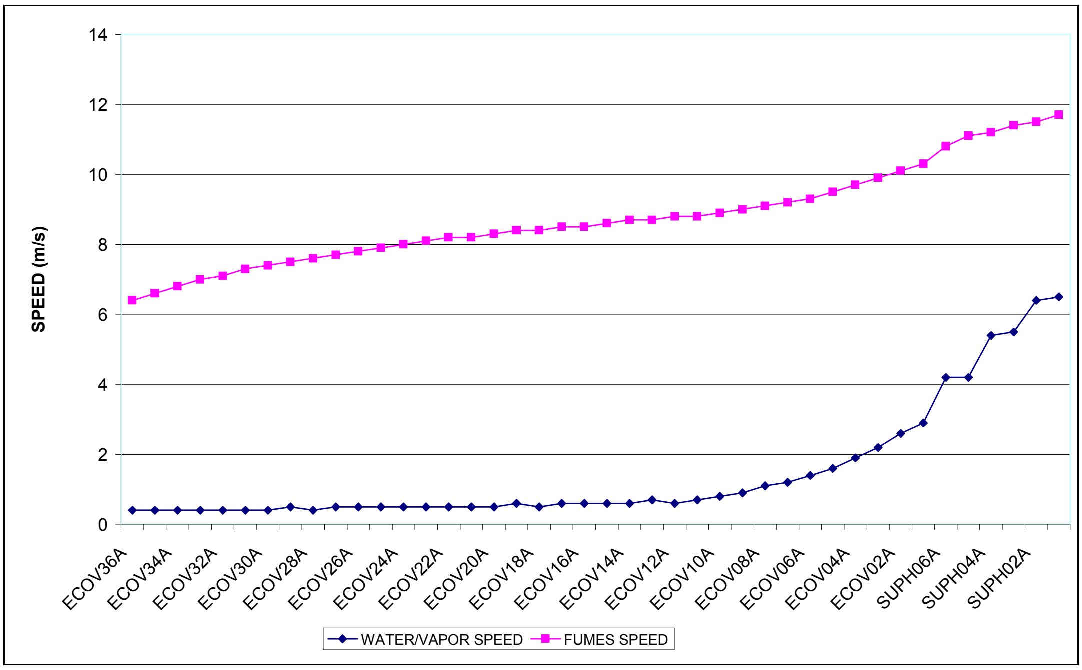



**Figure 7: Water/vapor and fumes velocity in the boiler**

## Nomenclature

| | |
|---|---|
| $A$ | total area of outer surface (m²) |
| $A_b$ | bare tube outside surface area (m²) |
| $A_{fo}$ | fin outside surface area (m²) |
| $A_i$ | inside surface area (m²) |
| $A_{po}$ | free area of tube outer surface (m²) |
| $A_w$ | mean area of homogeneous tube wall |
| $c_p$ | specific heat capacity at constant pressure (J/kg/K) |
| $d_i$ | tube internal diameter (m) |
| $\Delta P$ | pressure drop (bar) |
| f | pressure drop coefficient |
| G | mass flux (kg/m²/s) |
| H | enthalpy flow (kW) |
| $N_R$ | number of rows in the bundle |
| Nu | Nusselt number $Nu_l = \frac{\alpha \cdot l}{\lambda}$ |
| P | pressure (bar) |
| Pr | Prandl number $\Pr = \frac{c_p \cdot \eta}{\lambda}$ |
| Q | exchanged heat (kW) |
| Re | Reynolds number $\mathrm{Re}_l = \frac{\rho \cdot w \cdot l}{\eta}$ |
| T | temperature (K) |
| $\overline{V}$ | fluid velocity (m/s) |
| x | vapor mass fraction |
| $x_i$ | component flow rate (kg/s) |
| $\alpha$ | heat transfer coefficient (kW/m²/K) |
| $\alpha(z)$ | local heat transfer coefficient |
| $\lambda$ | thermal conductivity (W/m/K) |
| $\rho$ | density (kg/m³) |
| $\eta$ | dynamic viscosity (Pa.s) or (kg/m/s) |
| $\eta_f$ | fin efficiency |

**indices**

| | |
|---|---|
| liq | liquid |
| f | fumes |
| go | saturated vapor |
| in or 1 | inlet |
| lo | saturated liquid |
| out or 2 | outlet |
| sl | semi logarithmic |
| vap | vapor |
| w | water |


## Acknowledgements

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## REFERENCES


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[2]D. Steiner, 1993, *VDI heat atlas*, VDI-Verlag, Düsseldorf, Germany, HBB 1-23