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Abstract

Global imbalances are considered as one of the main culprits of the financial crisis which started in the United States in 2007. This paper aims to build a two-country deterministic growth framework with overlapping generations to investigate the macroeconomic effects of global imbalances that originate from forced saving in one country. This framework allows us to study the existence of a dynamic equilibrium with global imbalances, the impact on the world interest rate, and the short-run and long-run welfare implications on the young and old generations in both countries. In particular, we show that global imbalances worsen the welfare of the young generations of both countries in the short run and can offset the potential gain of the international integration of capital markets.

Keywords: Balance of payments, Global imbalances, Growth, Overlapping generations
JEL Classification: E20, F21, F43, O16, O41

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1 Introduction

Among the root causes of the financial crisis which started in the United States in 2007, some point to the persistence of large global imbalances for a decade before the meltdown (Figure 1).[1] By global imbalances it is meant that the fast-growing emerging economies and the developing countries finance the current account deficits of the slow-growing advanced economies.[2] This is an anomaly. Countries with a young population and high economic growth rates, as is the case with emerging economies, are expected to experience current account deficits that can be financed by foreign saving seeking high-yield investments. Advanced economies with ageing populations and low growth prospects are expected to save more and, hence, experience current account surpluses. Saving is thus allocated where it is most productive and imbalances in current accounts reduce as diminishing returns on capital curb the growth rates of emerging economies over time. This international self-equilibrium mechanism is ineffective when the advanced economies rely on the saving of the export-led developing world to finance their growth and their external deficits, as has been the case since 1996.[3] This phenomenon is a new and serious challenge for the stability of the world economy. The integration of the countries of the former Soviet Bloc into the world goods and capital markets and the international integration of capital markets in the last twenty years have created a huge and a truly global capital market. In this international capital market, global imbalances such as those observed since 1996 can now occur, and there is no market mechanism to wipe them out over time. Some have expressed concern regarding the systemic risks these global imbalances could create for the world economy (IMF (2005), Krugman (2007) and Obstfeld and Rogoff (2007)). Bernanke (2005, 2007) expresses greater confidence regarding the ability of the market to gradually resolve the external imbalances.

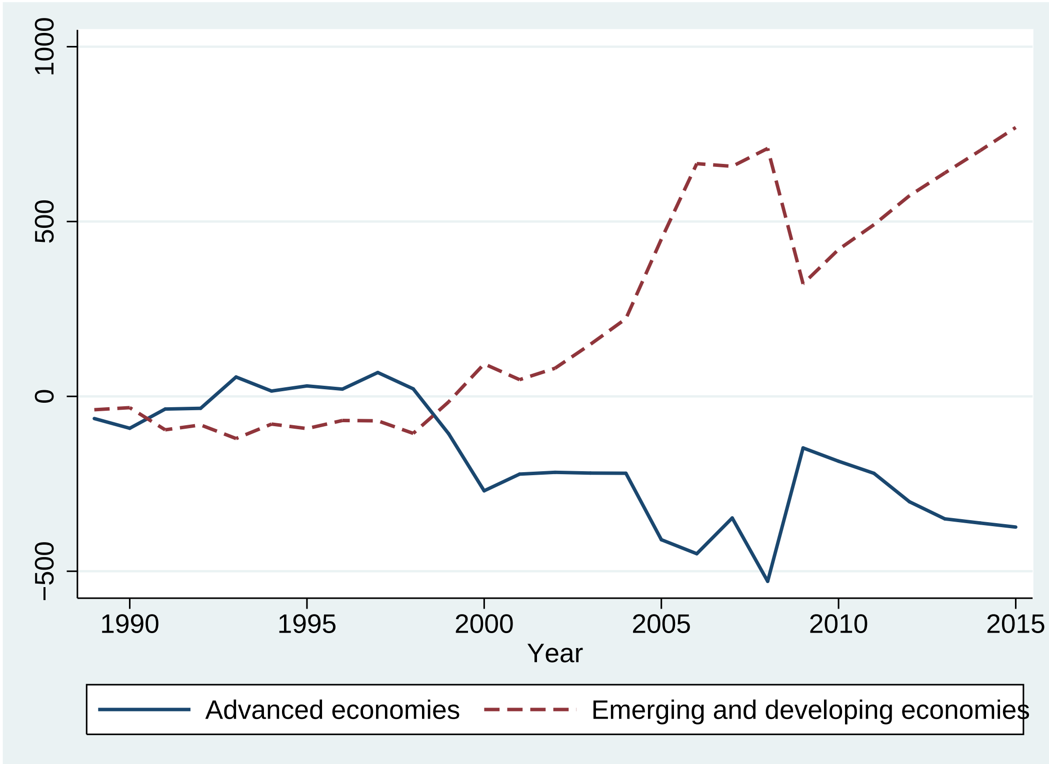
The origins of these global imbalances are also debated. Bernanke (2005) challenged the common view that was held at the time that the large U.S. current account deficit was due to the U.S. economic policies responsible for the low domestic saving and the frenzied consumption of foreign goods. He argued that the reversal in the current account positions of the emerging economies in the second half of the 1990s created a "global saving glut" allowing the cheap financing of the U.S. current account deficit and accounting for both

[1]This question is controversial. For instance, Bernanke (2009), BIS (2009), Obstfeld and Rogoff (2009) and Portes (2009) argue that the global imbalances are one of the causes or are a co-determinant of the financial crisis. Blanchard and Milesi-Ferretti (2010) think that the failures of the financial system are the trigger for the financial crisis and contributed to the widening of global imbalances. Laibson and Mollerstrom (2010) and Whelan (2010) challenge the link between global imbalances and the financial crisis.

[2]This definition is somewhat restrictive as global imbalances could result from the current account deficits and surpluses of any groups of countries, reflecting the disparities in economic and demographic trends. However, the term "global imbalances" has been used in the literature to characterize precisely the situation described by this definition.

[3]Blanchard and Milesi-Ferretti (2010) show that the ratio of the absolute value of the world current account balances to the world GDP was stable from 1970 to 1996 and started to increase sharply from then on.

the widening of global imbalances and the low level of real interest rates. Bernanke puts forward several reasons to explain this reversal. First, many developing and emerging economies modified their economic policies after the series of financial crises in the 1990s so as to yield current account surpluses and build foreign exchange reserves in order to reduce the financial liquidity risk in case of a sudden change in foreign investors' behaviour ("disruptive adjustment"). Second, other countries, such as China, maintained their export-led growth policy by preventing their currency from appreciating. Third, the rise in oil prices during the last decade inflated the income of oil-exporting countries and, hence, increased their level of saving. Finally, the deep and liquid U.S. financial markets provided a highly attractive heaven for this foreign saving glut. All these factors contributed to increasing saving in developing countries and enabling the U.S. and other industrial countries to afford to live on credit. In order to account for global imbalances, Caballero et al (2008) add to Bernanke's hypothesis the underdevelopment of financial markets in emerging countries. In their model, increasing saving in emerging countries does not find sufficient sound local stores of value and, therefore, a rising proportion of saving flows to the perceived better U.S. financial markets. Thus, the widening of the U.S. current account deficit, the decline in world interest rates and the increase in U.S. assets within global portfolios are not an anomaly but rather a global equilibrium resulting from the capital flows in asset markets. The "global saving glut" hypothesis has nevertheless been questioned by a number of authors stressing the fact that there has been little evidence of excess supply at the world level (IMF 2005). Laibson and Mollerstrom (2010) further argue that the inflows of foreign capital into the U.S. due to the saving glut should have increased the U.S. investment rate, but this is not reflected in the U.S. data. Calibrating a behavioural model with an exogenous asset bubble for the U.S., the authors show that the rise in asset and real estate prices creates a perceived wealth effect for consumers, leading to a consumption boom and an associated decrease in the saving rate. Consistent with the U.S. data, their model shows that investment is not affected, saving is lower, and therefore, the U.S. current account deficit increases. Based on a dataset of 43 countries between 1990 and 2005, Aizenman and Jinjarak (2009) confirm the positive relationship between current account balances and real estate valuation across countries. Taylor (2009) lies in between these two stances. He does not dispute the fact that there was a saving glut outside the advanced economies, which is consistent with the widening of global imbalances. However, as the data show that this saving glut was not big enough to create an excess supply of saving at the world level, he argues that the observed low level of real interest rates could not be a consequence of the rise in saving in emerging economies but rather the result of the Federal Reserve's policy. By maintaining the federal funds rate below the Taylor rule for too long, this policy combined with U.S. government interventions in the real estate market contributed significantly to the sheer size of the real estate bubble, accounting for both the consumption boom and the current account deficit in the U.S. Although Bernanke (2005) concludes that the world outside the U.S. is responsible for global imbalances and Taylor (2009) blames U.S. policies, we think that these two views are not mutually exclusive and even reinforce each other. This paper adopts Bernanke's hypothesis that global imbalances emerged as a sudden increase



Source: IMF, World Economic Outlook, April 2010
Country groups information: see IMF 2010 WEO database.
IMF estimates for data after 2008.

Figure 1: Current account balance in billions U.S. dollars (1990-2015)

in saving in the developing countries. This increase is the result of government policy in these countries, either because they pursue an export-led growth strategy or because they do not want to be dependent on foreign saving. In both cases, government intervention leads to forced saving. Our main objective is to build a growth framework flexible enough to study different issues associated with the possibility that capital flows from fast-growing to slow-growing economies. The model extends Buiter (1981) by considering a two-country overlapping generations (OLG) model with forced saving – represented by an exogenous increase in the preference parameter – in the fast-growing country. A series of propositions are derived such as the existence of a dynamic equilibrium with global imbalances, the impact on the world interest rate, and the short-run and long-run welfare implications on the young and old generations in both countries. In particular, we want to analyse the combination of the effect of the international integration of capital markets studied by Buiter (1981) and the effect of global imbalances. We demonstrate that global imbalances worsen the welfare of the young generations of both countries in the short run and can offset the potential gain of the international integration of capital markets.

The paper is organized as follows. Section 2 defines the two-country overlapping generations model and presents the dynamic equilibrium in autarky and in open economy. Section 3 analyses the steady-state current account balances when tastes and population growth rates differ across countries. Section 4 introduces global imbalances in the two-country model, and studies the existence of an intertemporal equilibrium and the impact on the world interest rate and transition growth. Section 5 presents the implications of global imbalances on the short-run and long-run welfare of the young and old generations in both countries. Finally, section 6 concludes.

2 A Two-Country Model

2.1 Setup

We consider a discrete-time deterministic model of an economy consisting of two countries, A and B, producing the same good under perfect competition from date $t = 0$ to infinity. The model builds on Diamond (1965). Each country is populated by overlapping generations living for two periods. When young, individuals supply inelastically one unit of labour to the firms, receive a wage and allocate this income between consumption and saving. When old, they retire and consume the return on their saving. The labour market is perfectly competitive within the national borders while physical capital moves freely across countries. The representative firm in each country produces a single aggregate good using a Cobb-Douglas technology of the form

$$Y_{i,t} = A_i K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \quad i = A, B, \tag{1}$$

where $K_{i,t}$ is the stock of capital, $L_{i,t}$ is the labour input, and A_i is a technological parameter of country i at time t. We assume that physical capital fully depreciates after one period. At time t, the representative firm of country i has an installed stock of capital $K_{i,t}$, chooses the labour input paid at the competitive wage $w_{i,t}$, equal to the marginal product of labour, and maximizes its profits

$$\pi_{i,t} = A_i k_{i,t}^{\alpha} - w_{i,t}, \tag{2}$$

where $\pi_{i,t} = R_{i,t} k_{i,t}$ are the profits per worker distributed to the owners of the capital stock, the interest factor $R_{i,t}$ is equal to the marginal product of capital, and $k_{i,t} \equiv K_{i,t}/L_{i,t}$ is the capital-labour ratio.

The representative agent of country i maximizes a logarithmic additively separable utility function

$$U_i = \ln c_{i,t} + \beta_i \ln d_{i,t+1} \tag{3}$$

subject to the budget constraints

$$\begin{aligned} c_{i,t} + s_{i,t} &= w_{i,t} & (4) \\ d_{i,t+1} &= R_{i,t+1} s_{i,t}, & (5) \end{aligned}$$

where $c_{i,t}$ is consumption when young and $s_{i,t}$ is individual saving at time t. When old, the individuals consume $d_{i,t+1}$. The parameter $\beta_i > 0$ is the psychological discount factor in country i. We assume that this parameter may have different values across countries. The maximization of (3) with respect to (4) and to (5) yields the optimal level of individual saving:

$$s_{i,t} = \frac{\beta_i}{1+\beta_i}(1-\alpha)A_i k_{i,t}^{\alpha}. \tag{6}$$

Individual saving depends only on the marginal product of labour and the preference parameter β_i. The lower β_i, ceteris paribus, the higher the preference for the present and the lower the level of saving.

2.2 The Autarkic Equilibrium

If capital is not mobile across countries, the two-country growth model is equivalent to the autarkic model. The equilibrium in the national labour market is given by the equality between the national supply and demand for labour. Since the labour supply is inelastic and the production function exhibits constant returns to scale, the national equilibrium wage is equal to the marginal product of labour. The equilibrium in the goods market in country i at time t is given by the national income accounts identity:

$$Y_{i,t} = L_{i,t} c_{i,t} + L_{i,t-1} d_{i,t} + I_{i,t}, \qquad i = A, B \tag{7}$$

The aggregate output is equal to total consumption at time t in country i. Full depreciation of the current capital stock implies $I_{i,t} = K_{i,t+1}$. The equilibrium in the national capital market is given by the equality between national investment and saving:

$$K_{i,t+1} = L_{i,t} s_{i,t}. \tag{8}$$

The capital stock per worker in country i accumulates according to

$$(1+n_i)k_{i,t+1} = \frac{\beta_i}{1+\beta_i}(1-\alpha)A_i k_{i,t}^{\alpha}, \tag{9}$$

where $n_i = \left(\frac{L_{i,t+1}}{L_{i,t}} - 1\right)$ is the population growth rate of country i. The autarkic equilibrium admits a unique globally stable steady state:

$$\bar{k}_i = \left(\frac{\beta_i(1-\alpha)A_i}{(1+\beta_i)(1+n_i)} \right)^{\frac{1}{1-\alpha}}. \tag{10}$$

The level of the steady state capital stock per worker of country i increases with the psychological discount factor β_i and with the level of the technological parameter A_i, and decreases when the population growth rate increases. Each country converges to its own steady state income per capita, which is entirely determined by national parameters. The growth rate of the autarkic economy is given by

$$\frac{dk_{i,t+1}}{dk_{i,t}} = \frac{\alpha\beta_i(1-\alpha)}{(1+\beta_i)(1+n_i)} A_i k_{i,t}^{\alpha-1}. \tag{11}$$

This growth rate is a positive function of β_i and A_i, and a negative function of $k_{i,t}$ and n_i. At the steady state, the growth rate of the capital stock per worker is zero.

2.3 The Open-Economy Equilibrium

Capital is now allowed to move from one country to another in a frictionless international capital market. The equilibrium in the labour market is the same as in the autarkic equilibrium, since labour is immobile across countries. The equilibrium in the world goods market at period t is given by the world income accounts identity:

$$Y_{A,t} + Y_{B,t} = L_{A,t}c_{A,t} + L_{A,t-1}d_{A,t} + L_{B,t}c_{B,t} + L_{B,t-1}d_{B,t} + I_{A,t} + I_{B,t}, \tag{12}$$

where the world output is equal to the aggregate consumption of the young and the old generations and the aggregate investment in both countries. Full depreciation of the current capital stock in each country implies $I_{A,t} = K_{A,t+1}$ and $I_{B,t} = K_{B,t+1}$.

It is assumed that the owners of the capital stock at date $t = 0$ in both countries cannot move this stock from one country to the other. The integration of capital markets thus occurs at date $t = 1$. The equilibrium in the international capital market, once capital is mobile across countries, derives from (12) and yields:

$$K_{A,t+1} + K_{B,t+1} = L_{A,t}s_{A,t} + L_{B,t}s_{B,t}. \tag{13}$$

The perfect mobility on the international capital market makes domestic and foreign assets perfect substitutes. At the world level, total investment must equal total saving. The equilibrium in the capital market requires that the returns to capital are equal in both countries:

$$\frac{k_{A,t+1}}{k_{Bt+1}} = \left(\frac{A_A}{A_B} \right)^{\frac{1}{1-\alpha}}. \tag{14}$$

By using Equations (6), (13) and (14), we can compute the intertemporal equilibrium with perfect foresight in each country:

$$k_{A,t+1} = \frac{1-\alpha}{\phi}\left(\frac{A_A}{A_B}\right)^{\frac{1}{1-\alpha}}\left(\frac{\beta_A L_{A,t} A_A k_{A,t}^{\alpha}}{1+\beta_A} + \frac{\beta_B L_{B,t} A_B k_{B,t}^{\alpha}}{1+\beta_B}\right) \quad (15)$$

$$k_{B,t+1} = \frac{1-\alpha}{\phi}\left(\frac{\beta_A L_{A,t} A_A k_{A,t}^{\alpha}}{1+\beta_A} + \frac{\beta_B L_{B,t} A_B k_{B,t}^{\alpha}}{1+\beta_B}\right), \quad (16)$$

where $\phi = \left(L_{A,t+1}\left(\frac{A_A}{A_B}\right)^{\frac{1}{1-\alpha}} + L_{B,t+1}\right)$.

The two-country intertemporal equilibrium admits a unique globally stable interior steady state characterized by:

$$\bar{k}_A = \left[\frac{1-\alpha}{\phi}\left(\frac{A_A}{A_B}\right)^{\frac{1}{1-\alpha}}\left(\frac{\beta_A L_{A,t}}{1+\beta_A}A_A\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\beta_B L_{B,t}}{1+\beta_B}A_B\right)\right]^{\frac{1}{1-\alpha}} \quad (17)$$

$$\bar{k}_B = \left[\frac{1-\alpha}{\phi}\left(\frac{\beta_A L_{A,t}}{1+\beta_A}A_A\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\beta_B L_{B,t}}{1+\beta_B}A_B\right)\right]^{\frac{1}{1-\alpha}} \quad (18)$$

The level of $\bar{k}_i$ increases with an increase in the psychological discount factor of both countries and the level of the domestic technological parameter, and decreases with an increase in the growth rate of the population of both countries and the level of the foreign technological parameter. At the steady state, the capital stock per worker and hence the income per capita remains constant. The transition dynamics in the two countries are governed by the following two equations:

$$dk_{A,t+1} = \frac{\alpha(1-\alpha)}{\phi}\left(\frac{A_A}{A_B}\right)^{\frac{1}{1-\alpha}}\left[\left(\frac{\beta_A L_{A,t} A_A}{(1+\beta_A)k_{A,t}^{1-\alpha}}\right)dk_{A,t} + \left(\frac{\beta_B L_{B,t} A_B}{(1+\beta_B)k_{B,t}^{1-\alpha}}\right)dk_{B,t}\right] \quad (19)$$

$$dk_{A,t+1} = \frac{\alpha(1-\alpha)}{\phi}\left[\left(\frac{\beta_A L_{A,t} A_A}{(1+\beta_A)k_{A,t}^{1-\alpha}}\right)dk_{A,t} + \left(\frac{\beta_B L_{B,t} A_B}{(1+\beta_B)k_{B,t}^{1-\alpha}}\right)dk_{B,t}\right]. \quad (20)$$

The capital stock per worker in both countries at time $t+1$ is a positive function of $k_{A,t}$ and $k_{B,t}$. At the steady state, the growth rate of the capital stock per worker is zero in both countries.

3 The Balance of Payments

In an open two-country world, a country can finance domestic investment by foreign saving. The difference between domestic investment and domestic saving is equal to the current account balance. In other words, a country can spend more or less than it produces. The national income accounts identity of country i in this two-country economy is

$$Y_{i,t} + R_t(L_{i,t-1}s_{i,t-1} - K_{i,t}) = L_{i,t}c_{i,t} + L_{i,t-1}d_{i,t} + K_{i,t+1} + G_{i,t}, \tag{21}$$

where $Y_{i,t}$ and $R_t(L_{i,t}s_{i,t} - K_{i,t+1})$ are the Gross Domestic Product (GDP) and the net factor income from abroad respectively, and the sum of the two is the Gross National Income (GNI) of country i at time t. On the right hand side of the identity, $G_{i,t}$ is the difference between domestic spending on foreign capital and foreign spending on domestic capital. In this model of one single good, where there is no trade in consumption goods and there are no unilateral transfers, $G_{i,t}$ is the current account balance of country i at time t. This is simply the difference between the factor income from abroad and the factor income payments to the foreign country. In intensive form, taking into account the fact that $y_{i,t} = w_{i,t} + R_t k_{i,t}$, the current account balance is equal to

$$g_{i,t} = w_{i,t} + \frac{L_{i,t-1}}{L_{i,t}}R_t s_{i,t-1} - c_{i,t} - \frac{L_{i,t-1}}{L_{i,t}}d_{i,t} - \frac{L_{i,t+1}}{L_{i,t}}k_{i,t+1}, \tag{22}$$

or, equivalently, since $d_{i,t} = R_t s_{i,t-1}$,

$$g_{i,t} = s_{i,t} - \frac{L_{i,t+1}}{L_{i,t}}k_{i,t+1}. \tag{23}$$

Without loss of generality, we focus on country A. The conditions on the current account balance per worker are as follows:

$$g_{A,t} \lesseqqgtr 0 \;\; \texttt{if} \;\; \frac{k_{A,t}}{k_{B,t}} \lesseqqgtr \left[\frac{L_{A,t+1}L_{B,t}}{L_{A,t}L_{B,t+1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}. \tag{24}$$

The current account balance of country A is an increasing function of $k_{A,t}$, β_A, and the population growth rate of country B, and a decreasing function of $k_{B,t}$, β_B and the population growth rate of country A. When capital is free to move from one country to another,

$$g_{A,t} \lesseqqgtr 0 \;\; \texttt{if} \;\; \left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) \lesseqqgtr \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right). \tag{25}$$

Condition (25) is also the condition for $g_A \lesseqqgtr 0$ at the steady state. We now establish the following two propositions:

Proposition 1 *Buiter (1981) In a two-country model with overlapping generations living for two periods, a country i experiences a current account deficit (surplus) at the steady state if, for identical population growth rates across countries, the preference parameter β_i is lower (higher) than that of the foreign country.*

Proof: From condition (25) it is straightforward to show that $g_{A,t} \lesseqgtr 0$ if $\left(\frac{\beta_A}{1+\beta_A}\right) \lesseqgtr \left(\frac{\beta_B}{1+\beta_B}\right)$ and the growth rates of population are identical.

Under autarky, the level of the steady-state capital stock per worker is an increasing function of β, the psychological discount factor (see Equation (10)). Assuming that two countries are identical in all respects except in the preference parameter β, a country populated with more impatient consumers (lower β) will have a lower $\bar{k}$ and a higher steady-state capital return than the country populated with more patient consumers. If capital markets are integrated, the country with impatient consumers will attract foreign investment owing to a higher capital return up to the point where capital returns are equal. Therefore, this country will have a current account deficit at the steady state.

Proposition 2 *In a two-country model with overlapping generations living for two periods, a country i experiences a current account deficit (surplus) at the steady state if, for identical tastes across countries, the growth rate of country i's population is higher (lower) than that of the foreign country.*

Proof: From condition (25) it is straightforward to show that $g_{A,t} \lesseqgtr 0$ if $\left(\frac{L_{A,t}}{L_{A,t+1}}\right) \lesseqgtr \left(\frac{L_{B,t}}{L_{B,t+1}}\right)$ and $\beta_A = \beta_B$.

Under autarky, the level of the steady-state capital stock per worker is a decreasing function of the population growth rate (see Equation (10)). Assuming that two countries are identical in all respects except in their demographic patterns, a country with a fast-growing population will have a lower $\bar{k}$ and a higher steady-state capital return than the country with a slow-growing population. If capital markets are integrated, the country with the higher capital return will attract foreign investment up to the point where capital returns are equal. Therefore, the country with the fast-growing population will record a current account deficit at the steady state.[4]

Let us now consider that one country, say country A, has a higher value for the preference parameter than country B but has a higher population growth rate. If country A has a

[4] Empirical studies find that countries with low dependency ratios tend to experience current account surpluses and countries with high fertility rates and young populations tend to experience current account deficits (Higgins (1998) and IMF (2004) for instance).

sufficiently higher population growth rate, this country will have a current deficit at the steady state. Hence, the following proposition:

Proposition 3 *In a two-country model with overlapping generations living for two periods, a country, say country A, experiences a current account deficit at the steady state if, for $\beta_A > \beta_B$, its population growth rate verifies the following condition:*

$$\frac{L_{A,t+1}}{L_{A,t}} > \frac{\beta_A(1+\beta_B)L_{B,t+1}}{\beta_B(1+\beta_A)L_{B,t}}. \tag{26}$$

Proof: Again, this results derives easily from condition (25).

This proposition stresses the fact that, even in a country with thrifty consumers, the level of the preference parameter may not be sufficiently high to compensate for the negative effect of a higher population growth rate on its current account. The higher the differential in population growth rates across countries, the higher the differential in the preference parameters must be.

4 A Two-Country Model with Global Imbalances

In this section, we consider a two-country world in which country A is a developing economy and country B is an advanced economy. The development gap is captured by the technological parameter and the initial capital stocks per worker: $A_A < A_B$ and $k_{A,0} < k_{B,0}$. We will also assume that the government of country A intervenes whenever the market outcome yields a current account deficit. Its intervention is represented by a constraint in the consumer's optimization programme and is evidenced by a change in the value of the parameter β_A so as to generate a current account balance positive or null. The government's policy can thus be interpreted as forced saving. This section is organized as follows. First, we define an intertemporal equilibrium with global imbalances. Second, the conditions for country A's government intervention are established. Third, we study the existence of an intertemporal equilibrium with global imbalances. Fourth, we examine the real interest rate when there are global imbalances. Finally, we verify whether the equilibrium result with government intervention improves or worsens the welfare of the young and the old generations.

4.1 Intertemporal Equilibrium with Global Imbalances: Definition

Given $A_A < A_B$ or/and $k_{A,0} < k_{B,0}$, an intertemporal equilibrium with global imbalances is a sequence of temporary equilibria that satisfies $g_{A,t} \geqslant 0$ for all $t \geqslant 0$.

Table 1: Current account potential trajectories and conditions for country A

	$g_{A,0}$	$g_{A,1}$	$g_{A,2}$	$\cdots$	$g_{A,\infty}$	Condition
Case 1	0	0	0	0	0	if $\frac{k_{A,0}}{k_{B,0}} = \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) = \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 2	0	+	+	+	+	if $\frac{k_{A,0}}{k_{B,0}} = \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) > \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 3	0	-	-	-	-	if $\frac{k_{A,0}}{k_{B,0}} = \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) < \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 4	+	0	0	0	0	if $\frac{k_{A,0}}{k_{B,0}} > \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) = \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 5	+	+	+	+	+	if $\frac{k_{A,0}}{k_{B,0}} > \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) > \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 6	+	-	-	-	-	if $\frac{k_{A,0}}{k_{B,0}} > \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) < \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 7	-	0	0	0	0	if $\frac{k_{A,0}}{k_{B,0}} < \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) = \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 8	-	+	+	+	+	if $\frac{k_{A,0}}{k_{B,0}} < \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) > \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$
Case 9	-	-	-	-	-	if $\frac{k_{A,0}}{k_{B,0}} < \left[\frac{L_{A,1}L_{B,0}}{L_{A,0}L_{B,1}}\left(\frac{A_A}{A_B}\right)^{\frac{\alpha}{1-\alpha}}\frac{\beta_B(1+\beta_A)}{\beta_A(1+\beta_B)}\right]^{\frac{1}{\alpha}}$ *and* $\left(\frac{L_{A,t}}{L_{A,t+1}}\right)\left(\frac{\beta_A}{1+\beta_A}\right) < \left(\frac{L_{B,t}}{L_{B,t+1}}\right)\left(\frac{\beta_B}{1+\beta_B}\right)$

4.2 Country A's Government Intervention

From Equations (24) and (25), we can identify nine potential trajectories for g_A, the current account balance per worker in the developing economy. Assuming that international capital integration is achieved at $t = 1$, Table 1 displays these nine potential trajectories as well as the conditions under which they arise. By assumption, the government of country A intervenes whenever the current account balance is negative. Three cases (7, 8 and 9) are mainly of interest since the government of country A can intervene at the initial date to avoid the current account deficit yielded by the market. In cases 7 and 8, the government can intervene only at $t = 0$, since the current account balance is nonnegative for $t > 0$. Cases 7 and 8 can thus be grouped together. In case 9, the government can intervene at all times. Cases 3 and 6 can be omitted as they match case 9 when the international integration of capital markets is achieved.

4.3 Existence of an Intertemporal Equilibrium with Global Imbalances

After identifying the conditions under which the government of country A intervenes to guarantee nonnegative current account balances, we can now address the question of whether an intertemporal equilibrium with global imbalances exists. As already mentioned, we define an intertemporal equilibrium with global imbalances by a sequence of temporary equilibria in which the current account balance of country A is never negative. We study the existence condition and determine the policy response of the government to ensure nonnegative current account balances. The model is identical to the one defined in Section 2 with an integrated international capital market except for country A's consumer's optimization programme of country A. In the present model, the representative agent of country A maximizes a logarithmic additively separable utility function

$$U_A = \ln c_{A,t} + \beta_A \ln d_{A,t+1} \tag{27}$$

subject to the following three constraints

$$\begin{aligned} c_{A,t} + s_{A,t} &= w_{A,t} && (28)\\ d_{A,t+1} &= R_{A,t+1} s_{A,t}, && (29)\\ g_{A,t} &\geqslant 0, && (30) \end{aligned}$$

where inequality (30) is the government's constraint imposed on the consumers of country A. If $g_{A,t} \geqslant 0$ is verified at each period, then the decision to save by the individuals is given by (6) and the government does not intervene. If $g_{A,t} < 0$, the government acts on β_A to guarantee $g_{A,t} \geqslant 0$. As a consequence, focusing on the three cases of interest defined in Section 4.2, the government modifies β_A at $t = 0$ only for cases 7 and 8 and at each period for case 9.

Proposition 4 *In a two-country model with overlapping generations living for two periods, an intertemporal equilibrium with global imbalances exists if and only if, for all* $t \geqslant 0$*,*

$$\beta_A \geqslant \left[\left(\frac{k_{A,t}}{k_{B,t}} \right)^{\alpha} \frac{L_{A,t} L_{B,t+1}}{L_{A,t+1} L_{B,t}} \left(\frac{A_B}{A_A} \right)^{\frac{\alpha}{1-\alpha}} \frac{1+\beta_B}{\beta_B} - 1 \right]^{-1}. \tag{31}$$

Proof: $g_{A,t} \geqslant 0$ for all $t \geqslant 0$ if condition (24) is verified. The necessary value for β_A derives from this condition.

If the expression in square brackets is positive, then the threshold given by condition (31) increases with the increase in the population growth rate of country A. If condition (31) is not satisfied by the preference parameter of country A's representative consumer, then country A's government intervenes to impose a value for β_A that satisfies this condition.

Proposition 4 establishes that, with a perfect integrated capital market, the global imbalances are an equilibrium result when the fast-growing economy displays a sufficiently higher propensity to save than the slow-growing economy. The larger the difference between the preference parameters across countries, the larger global imbalances. This higher propensity to save in the fast-growing economy may result from the consumer preferences or from forced saving imposed by government policies. In the former case, the equilibrium is a pure market outcome. The lack of social insurance or the uneasy access to credit can explain why the propensity to save is higher in emerging countries. If this is caused by forced saving, global imbalances are the result of a government's intervention. Self-insurance against disruptive adjustments in the balance of payments is generally put forward to account for such a public policy. Empirically, the IMF (2005) study shows that the saving rate declined in advanced economies and increased in emerging and oil-producing economies at the end of the 1990s, yielding a reversal in current account balances in emerging economies and leading to large global imbalances. This reversal can be explained by government intervention in emerging economies after the Asian financial crisis.

4.4 The Interest Rate

The (gross) interest rate $R_{i,t+1}$ is the rental rate of capital of country i at time $t+1$. When the capital markets are integrated, $R_{A,t+1} = R_{B,t+1} = R_{t+1}$. If β_A does not satisfy condition (31), the government intervenes, β_A increases and the new interest rate is lower.

Proposition 5 *In a two-country model with overlapping generations living for two periods, the interest rate of the integrated capital market decreases, ceteris paribus, when country A's government intervenes to satisfy condition (31).*

Proof: If β_A does not satisfy condition (31), the government intervenes, β_A increases and so does the capital stock per worker, $k_{A,t+1}$. Therefore, due to the diminishing returns to capital, the rental rate of capital of country A decreases. Country B's capital becomes more attractive and consumers of country A invest in country B up to the point where the equality $R_{A,t+1} = R_{B,t+1}$ is restored. In the end, the interest rate is lower than before country A's government intervention.

Real interest rates have gradually declined in the world over the last two decades to levels not seen since the 1970s. A number of variables such as the weak labour force growth in rich countries and demographic changes in the world can account for this evolution Desroches and Francis (2010). However, the emergence of global imbalances at the end of the 1990s likely contributed to maintaining world real interest rates at low levels Bernanke (2005).

4.5 Transition Dynamics and Comparative Statics

If country A's government has to intervene in period t to satisfy condition (31), this affects either the growth rate or the steady state level of the capital stock per worker in both countries.

Proposition 6 *In a two-country model with overlapping generations living for two periods, country A's government intervention in period t to satisfy condition (31) implies, ceteris paribus, a higher growth rate of the capital stock per capita in both countries if the economies are on the transition path to the steady state. Otherwise, the intervention results in a higher steady-state level of capital stock per worker if the economies have reached their steady state in that period.*

Proof: The transition dynamics in the two countries are governed by Equation (19) for country A and Equation (20) for country B. It is straightforward to show that, if the preference parameter of country A increases in period t, the growth rate of the capital stock per worker (and hence of the income per worker) between the generations t and $t+1$ increases, ceteris paribus, in both countries along their transition path to the steady state. If the economies are already at their steady state in period t, then Equations (17) and (18) show that a higher β_A implies a higher level of capital stock per worker in both countries.

5 Global Imbalances and Welfare Analysis

The previous section showed that a higher β_A imposed by country A's government yields a higher transition income growth rate or a higher steady-state income per worker in both countries. It now remains to find how the welfare of the young and the old generations is modified when β_A is increased in the short and the long run. Without loss of generality, we will assume that the population growth rates are equal across countries.

5.1 Short-Run Welfare Analysis

Short-run welfare analysis refers to the first two periods of the economy. Two changes may occur during this timespan: the integration of the capital markets and country A's government intervention on β_A. First, let us assume that the capital markets of both countries integrate between $t = 0$ and $t = 1$.[5]

Proposition 7 *In a two-country model with overlapping generations living for two periods, the welfare of the old generations of both countries born at $t = -1$ is unaffected by the international capital integration, while the young generation born at $t = 0$ of the country with the higher (lower) psychological discount factor is raised (reduced).*

Proof: See Buiter (1981).

The result is obvious for the old generations as their welfare is determined by the past capital stock per worker. For the young generations, the result depends on the effect on the interest rate of the integration of capital markets. Since the capital stock per worker is higher in the country with the higher psychological discount factor, its interest rate is lower than in the foreign country, and therefore increases when capital markets are integrated. An increase in the interest rate has no effect on the saving decision of the young agents, as their preferences are logarithmic, but it does raise the level of their utility. The opposite is true for the country with the lower psychological discount factor. In that country, the integration of capital markets results in a decrease in the interest rate and yields a lower level of utility for the young generation.

Second, when capital is free to move from one country to another, country A's government imposes an increase in β_A whenever it is required to avoid a current account deficit. The welfare effect of this intervention is established in the following proposition:

Proposition 8 *In a two-country model with overlapping generations living for two periods and with integrated capital markets, the welfare of the old generation of both countries born at $t = -1$ is unaffected by the increase in β_A imposed by country A's government. In contrast, the welfare effect on the young generation born at $t = 0$ in both countries is negative.*

Proof: As previously, the welfare of the old generations is determined by the past capital stock per worker and is not affected by a change in β_A. For the young generations, the proof is given in the Appendix.

Proposition 7 considers the welfare effect on the young and the old generations when the economy moves from autarky to open economy between $t = 0$ and $t = 1$. Proposition 8 compares the welfare of the young and the old generations in an open economy without

[5] We assume that the owners of capital at $t = 0$ cannot move this capital from country to country.

government intervention and in an open economy with government intervention. The result is unambiguous: the welfare of the old generations in both countries is unchanged, while the young generations in both countries are worse off. Before analysing the welfare effect of country A's government intervention in the long run, it would be interesting to study the welfare effect on the young and the old generations when the economy moves from autarky to open economy with government intervention between $t = 0$ and $t = 1$.

Proposition 9 *In a two-country model with overlapping generations living for two periods, the welfare of the old generations born at* $t = -1$ *is unaffected as the economy moves from autarky to an open economy with country* A*'s government intervention. In contrast, the effect on the welfare of the young generation born at* $t = 0$ *in country* A *is negative while it is ambiguous for country* B*.*

Proof: Proof for Proposition 9 can be easily derived from Propositions 7 and 8.

If we consider that the integration of capital markets and the government intervention of one country occur in the same period (between $t = 0$ and $t = 1$), then the welfare effect on the young and old generations that has to be considered relates to Proposition 9.

5.2 Long-Run Welfare Analysis

Long-run welfare analysis refers to the steady state. Buiter (1981) analyses the case of an economy moving from autarky to integrated capital markets. In this section, we study the welfare effect on the young and old generations in an open two-country economy, in which country A's government has to intervene at the steady state (case 9) in order to avoid current account deficits. In the standard OLG model, the maximum of welfare is attained when the competitive equilibrium coincides with the golden rule. Whereas this remains true for country B in our model, in country A, the welfare gain from moving closer to the golden rule may be offset by the welfare loss resulting from the modification of the intertemporal consumption allocation imposed by country A's government increase in β_A.

Proposition 10 *In a two-country model with overlapping generations living for two periods and with integrated capital markets, country* A*'s government intervention may result in an increase in the welfare of country* B*'s young and old generations at the steady state if and only if the market outcome without government intervention leads to capital under-accumulation. Otherwise, the welfare is unambiguously decreased.*

Proof: Country A's government intervention always results in a higher steady-state capital stock per worker in both countries. As a result, it can only be closer to the golden rule capital stock per worker in country B if the economy experiences under-accumulation of capital without government intervention. If the capital stock per worker is initially higher than the golden rule, it can only move away from the optimum.

Proposition 11 *The results of Proposition 10 remain true for country A. However, country A's generations undergo a specific welfare loss due to the imposed different intertemporal allocation of consumption. As a consequence, even an outcome closer to the golden rule can coincide with a lower level of welfare.*

Proof: See the proof for Proposition 10. In addition, we can show that for a given level of capital stock per capita ($\tilde{k}$), any change in the intertemporal consumption allocation results in a decreased welfare level. The intertemporal utility of the representative consumer in country A when the government intervenes is:

$$U = ln\left(\frac{1}{1+\beta_C}A(1-\alpha)\tilde{k}^{\alpha}\right) + \beta_A ln\left(\frac{\beta_C}{1+\beta_C}\alpha(1-\alpha)A^2\tilde{k}^{2\alpha-1}\right), \tag{32}$$

where β_A is the discount factor of the representative consumer and β_C is the discount factor imposed by the government. Since the utility function is concave, we know that utility attains a maximum under the budget constraint whenever $\beta_C = \beta_A$. If $\beta_C \neq \beta_A$, then utility is lower. Hence, for a given capital stock per worker, if the imposed discount factor, β_C, is higher than the representative consumer's preference parameter, β_A, the welfare of the generations of country A is decreased.

6 Conclusion

Since the end of the 1990s, the world economy has been characterized by large global imbalances, i.e. a situation in which the fast-growing economies finance the current account deficits of the advanced economies. The purpose of this paper is to build a growth model to study the equilibrium conditions for global imbalances to occur and to examine the welfare implications for the young and old generations in both countries. We extend the two-country overlapping-generations model of Buiter (1981) to analyse the "global saving glut" hypothesis Bernanke (2005) in a growth framework. A number of results are obtained. Propositions 1 to 3 derive the conditions for steady-state current account deficits (surplus) when two economies differ in their tastes, in their population growth rates, or both. Proposition 4 gives the condition for an intertemporal equilibrium with global imbalances to exist. Proposition 5 shows that a government's intervention in the fast-growing economy to avoid current account deficits implies a decrease in the world interest rate, while its effect on the transition growth rate and on the steady-state level of capital is positive (Proposition 6). Propositions 7 to 9 analyse the welfare effect of both the integration of capital markets and the government's intervention in the fast-growing country on the young and the old generations. While this effect is null for the old generations in both countries, it is negative for the young generation of the fast-growing economy and is ambiguous for the young generation of the slow-growing economy.

In the long run, the effect of the government's intervention in the fast-growing country on the welfare of the young and the old generations in the slow-growing country is positive if there is under-accumulation of capital and is negative otherwise (Proposition 10). The welfare effect on the young and the old generations of the fast-growing country is more complicated as the government of this country modifies their choice of intertemporal allocation of income. Proposition 11 shows that this welfare effect is negative. Overall, the decision of the government of the fast-growing economy to modify the propensity of its domestic residents to save has no impact on the old generation in the short run but it does decrease the welfare of all other generations in the short and long run.

This deterministic framework with perfect capital markets can be extended in a number of ways. For instance, the model is flexible enough to add capital market imperfections in order to study the relationship between global imbalances and imperfections in the capital markets in slow-growing economies. Another interesting line of research would be the possibility for the generations of the slow-growing economies to postpone the interest payments from period to period. Finally, a framework with uncertainty could allow for the analysis of possible disruptive adjustments in the balance of payments.

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7 Appendix

To prove that the welfare effect of country A's government intervention on country A's young generation born at $t = 0$ is negative (Proposition 8), it suffices to study the difference between the utility U_0 of the young generation resulting from the market outcome, and the utility U_0^C resulting from any other imposed psychological discount factor β_C, ceteris paribus. This difference can be expressed as:

$$\begin{aligned} D(\beta_C) = U_0 - U_0^C = ln\left(\frac{1}{1+\beta_A}A_A(1-\alpha)k_{A,0}^{\alpha}\right) + \beta_A ln\left(\frac{\beta_A}{1+\beta_A}A_A(1-\alpha)k_{A,0}^{\alpha}A_A\alpha k_{A,1}^{\alpha-1}\right) \\ -ln\left(\frac{1}{1+\beta_C}A_A(1-\alpha)k_{A,0}^{\alpha}\right) - \beta_A ln\left(\frac{\beta_C}{1+\beta_C}A_A(1-\alpha)k_{A,0}^{\alpha}A_A\alpha k_{C,1}^{\alpha-1}\right), \end{aligned} \quad (33)$$

where $D(\beta_C)$ is the difference between the utilities of the young generation without and with government intervention, and $k_{C,1}$ the per capita stock of capital in country A at $t = 1$ under the optimization based on β_C instead of β_A.

We can rewrite Equation (33) as:

$$\begin{aligned} D(\beta_C) &= ln\left(\frac{1+\beta_C}{1+\beta_A}\right) + \beta_A ln\left(\frac{\beta_A}{1+\beta_A}\frac{1+\beta_C}{\beta_C}\frac{k_{A,1}^{\alpha-1}}{k_{C,1}^{\alpha-1}}\right) \\ &= ln(1+\beta_C) - ln(1+\beta_A) + \beta_A ln\left(\frac{\beta_A}{1+\beta_A}\right) + \beta_A ln(1+\beta_C) - \beta_A ln(\beta_C) \\ &+ \beta_A(\alpha-1)ln\left(\frac{k_{A,1}}{k_{C,1}}\right). \end{aligned} \quad (34)$$

We know from Equations (15) that:

$$\frac{k_{A,1}}{k_{C,1}} = \frac{L_{A,0}\frac{\beta_A}{1+\beta_A}A_A k_{A,0}^{\alpha} + L_{B,0}\frac{\beta_B}{1+\beta_B}A_B k_{B,0}^{\alpha}}{L_{A,0}\frac{\beta_C}{1+\beta_C}A_A k_{A,0}^{\alpha} + L_{B,0}\frac{\beta_B}{1+\beta_B}A_B k_{B,0}^{\alpha}}. \quad (35)$$

Hence,

$$\begin{aligned} D(\beta_C) = \; & ln(1+\beta_C) - ln(1+\beta_A) + \beta_A ln\left(\tfrac{\beta_A}{1+\beta_A}\right) + \beta_A ln(1+\beta_C) - \beta_A ln(\beta_C) \\ & -\beta_A(1-\alpha)ln\left(L_{A,0}\tfrac{\beta_A}{1+\beta_A}A_A k_{A,0}^{\alpha} + L_{B,0}\tfrac{\beta_B}{1+\beta_B}A_B k_{B,0}^{\alpha}\right) \\ & +\beta_A(1-\alpha)ln\left(L_{A,0}\tfrac{\beta_C}{1+\beta_C}A_A k_{A,0}^{\alpha} + L_{B,0}\tfrac{\beta_B}{1+\beta_B}A_B k_{B,0}^{\alpha}\right). \end{aligned} \quad (36)$$

If $\beta_C = \beta_A$, then $D(\beta_C) = 0$. Whenever it is required, country A's government intervention results in a higher preference parameter than that of the representative consumer. Therefore, we merley have to compute the first derivative of $D(\beta_C)$ with respect to β_C and observe its sign when $\beta_C > \beta_A$. The derivative of $D(\beta_C)$ with respect to β_C is:

$$\frac{\partial D}{\partial \beta_C}(\beta_C) = \frac{\beta_C - \beta_A}{\beta_C(1+\beta_C)} + \frac{\beta_A(1-\alpha)L_{A,0}A_A k_{A,0}^{\alpha}}{(1+\beta_C)^2\left(L_{A,0}\frac{\beta_C}{1+\beta_C}A_A k_{A,0}^{\alpha} + L_{B,0}\frac{\beta_B}{1+\beta_B}A_B k_{B,0}^{\alpha}\right)}. \tag{37}$$

If $\beta_C > \beta_A$, Equation (37) is positive. Therefore, the utility of country A's young generation born at $t = 0$ diminishes when its government has to intervene (cases 7, 8 and 9) in order to avoid a current account deficit. We can conclude that the welfare of country A's young generation is lower in an open economy with country$A$'s government intervention than in an open economy without government intervention.

Let us now turn to the welfare effect of country A's government intervention on country B's young generation born at $t = 0$. Proposition 8 states that the welfare effect is also negative. The welfare of country B's young generation born at $t = 0$ is only affected through the modification of the world interest rate at $t = 1$. The saving decision of this young generation is unaffected by the modification of β_A but its consumption in the next period is influenced by the world interest rate at $t = 1$. An increase in β_A results in more saving in country A leading to a lower world interest rate at $t = 1$. This lower interest rate decreases the consumption of country B's young generation (born at $t = 0$) at $t = 1$ while leaving unchanged its consumption level at $t = 0$ due to the logarithmic form of the utility function. In total, the welfare of this young generation decreases. Consequently, any increase in β_A results in a decrease in the welfare of the young generation at $t = 0$ in country B.