

## Simulation of metal forming processes :

### Numerical aspects of contact and friction

## LTAS-MCT in a few words

### People

- Laboratoire des Techniques Aéronautiques et Spatiales (Aerospace Laboratory)
- Milieux Continus & Thermomécanique (Continuum Media & Thermomechanics)
- Professor M. Hogge – J.-P. Ponthot
- 3 staff scientists - 3 PhD students – 3 research engineers

### Research interests

- Thermal modeling
- Process simulation with large strains : deep drawing, spring-back, rolling, crash, impact, superplasticity, etc.
- Main industrial partners: Cockerill-Sambre/Arcelor (Steel maker), SABCA (Ariane 5 components), GOOD YEAR (tire company), SONACA (Airbus components), SNECMA (Aero engines manufacturer).

### Software

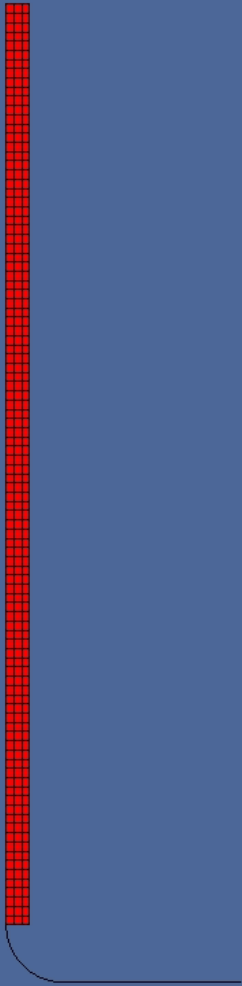
- Home made finite element software : Metafor

## Scope

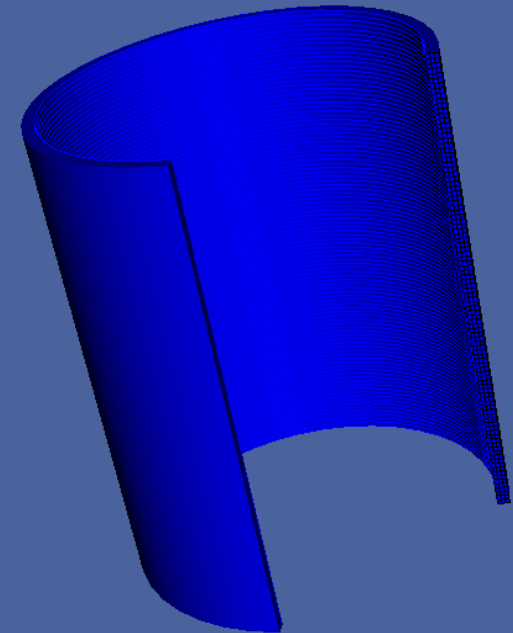
- Introduction
- Part I : Geometrical aspects
- Part II : Contact algorithms
- Part III : Friction
- Part IV : Numerical examples
- Conclusions

## Introduction - Example

- Thermomechanical simulation of a cylindrical shock damper device
- The problem is 2D axisymmetric
- The penalty method is used for managing contact between the metal and the rigid die



Metafor : pas 0 t=0.000000/0.001250 dt=0.000001



*Equivalent plastic strain*

0.000 0.188 0.375 0.563 0.750



## Introduction - Nonlinear mechanics

### Sources of nonlinearities :

- Geometry (large displacements – large rotations)
- Material (nonlinear elasticity, plasticity, visco-plasticity)
- Thermo-mechanical coupling
- Contact with friction

### Quasi-static solver :

- Load is applied incrementally
- For each time step, a Newton Raphson (N-R) procedure is used for solving the equations

$$\mathbf{f}_{int} = \mathbf{f}_{ext}$$

- For each iteration of N-R., a linearized system is solved

$$\mathbf{K}^n \Delta \mathbf{x}^n = -\mathbf{r}^n \quad \mathbf{x}^{n+1} = \mathbf{x}^n + \Delta \mathbf{x}^n$$

$\mathbf{K}^n$  : tangent stiffness

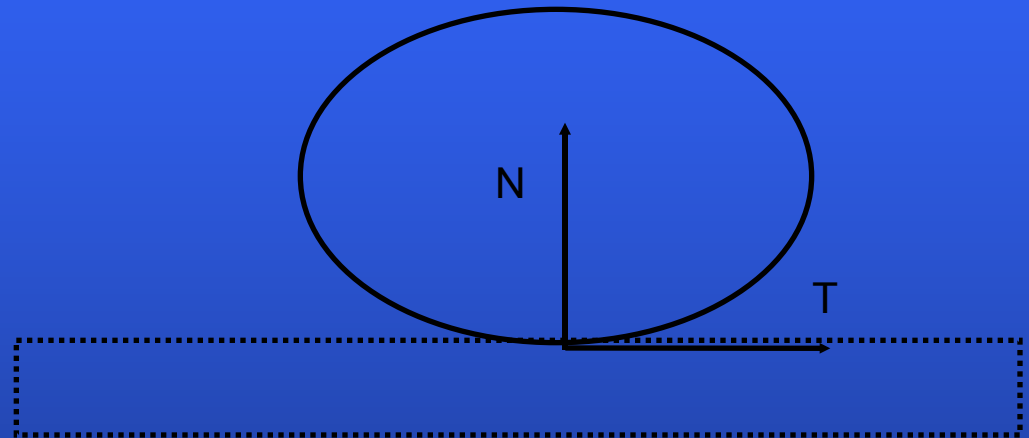
$\mathbf{r}^n$  : residual

## Introduction - Contact

### Physical problem

➤ Non-penetration conditions

$$\begin{aligned}
 U_N &\geq 0 & U_N N &= 0 \\
 N &\geq 0
 \end{aligned}$$



➤ Friction conditions

either  $u_T > 0$  and  $-\vec{T} = \mu \vec{D}$

or  $u_T = 0$  and  $-\vec{T} < \mu \vec{D}$

$$\mu = \mu(N, x, \dots)$$

Friction law

$$\vec{D} = \frac{\vec{U}_T}{|\vec{U}_T|}$$

Sliding direction

## Introduction - Contact

### Numerical procedure

#### Main problem :

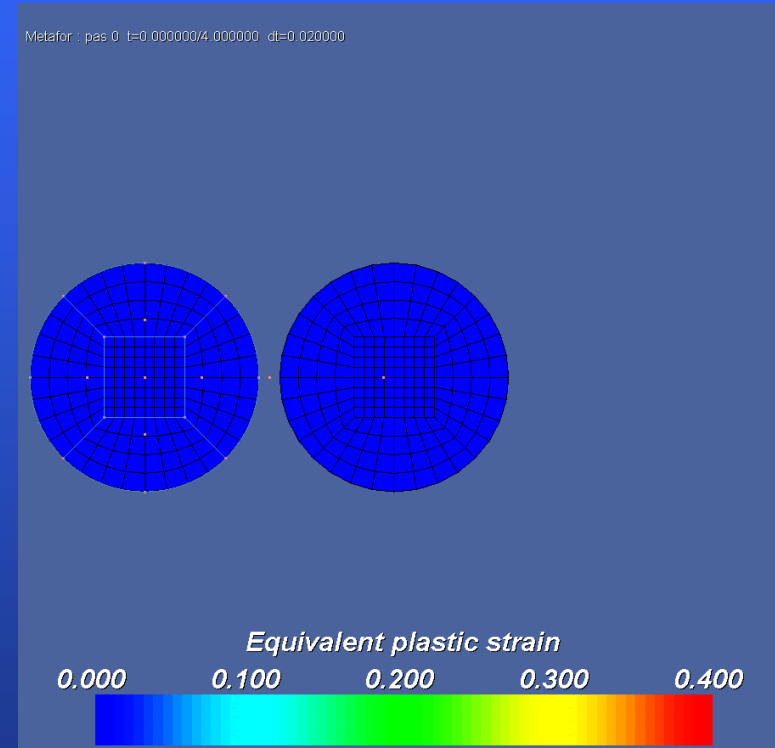
- The contact area is not known at the beginning

#### Discretization :

- Bodies are meshed
- Problem reduced to the contact between two meshed lines (2D) or surfaces (3D)

#### Algorithm :

- first step : contact search (geometrical problem)
- second step :
  - normal force computation (contact problem)
  - tangential force computation (friction problem)



## Introduction - Contact types (1)

### Contact with a rigid tool

- Rigid tool defined by a CAD file (curved curves – surfaces)
- Problem reduced to the interaction between a node and an analytical surface.

Avoid the discretization of the geometry (exact geometry)

No mesh needed (less d.o.f. needed)

The size of the stiffness matrix remains small in some cases

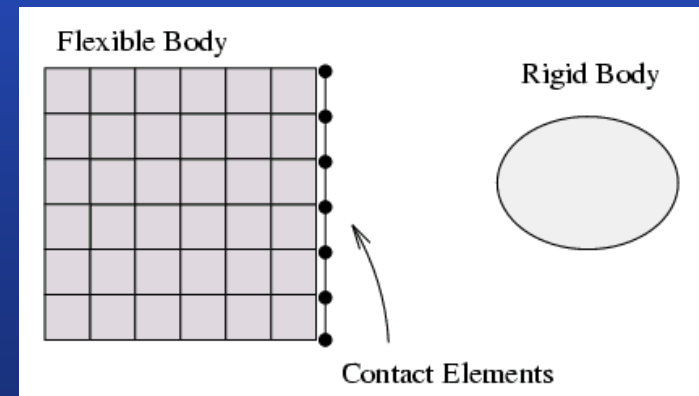
Avoid normal discontinuities (good convergence expected)

Available in Metafor

Contact search can be very difficult (in case of complex surfaces)

No strain/stress/temperature fields available in the tool.

Not available in Cast3M





## Introduction - Contact types (2)

Contact between two deformable bodies / self contact

- More general problem
- Problem reduced to the interaction between :
  - A set of “slave” nodes (from the “slave” line/surface)
  - A set of “master” segments (from the “master” line/surface)

Stress/strain/temperature fields available everywhere

Geometry is easy to handle (straight segments – planar surfaces)

Available in Cast3M and Metafor

Normal discontinuities need special attention

Contact search can be a long

The size of the stiffness matrix is highly increased

## Part I – Contact elements

### Three usual strategies

#### Single step :

- One line/surface is the master (set of segments)
- One line/surface is the slave (set of nodes)
- Slave nodes cannot go through the master lines

Interesting in the case of similar mesh sizes with low curvature

Small number of contact elements

Penetrations (see next slide)

#### Double step :

- Master and slave lines are permuted every time steps

Try to avoid the problems of single step with the same low number of elements

Bad convergence

#### Modified double step :

- Each line is simultaneously master and slave

No penetration

High number of contact elements

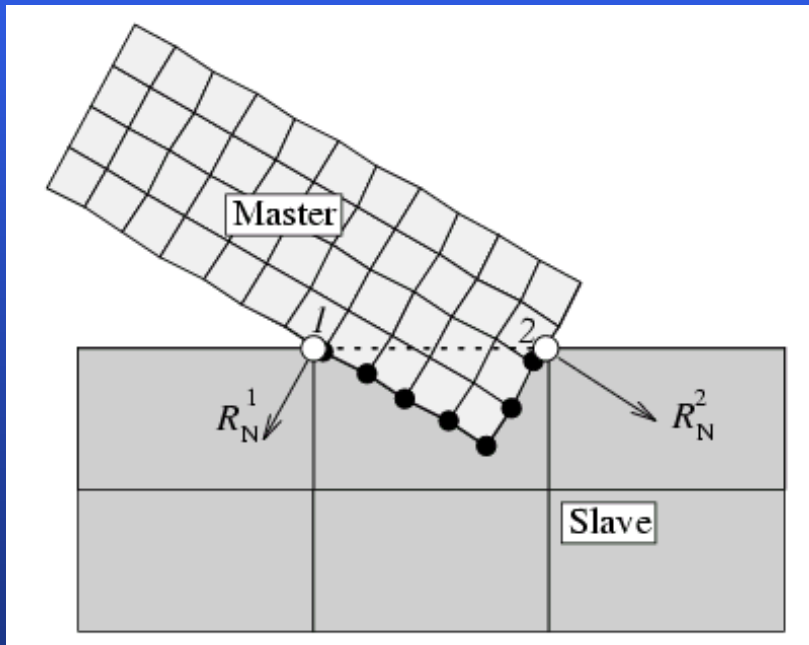
Problem may be over-constrained

Computed pressure oscillates

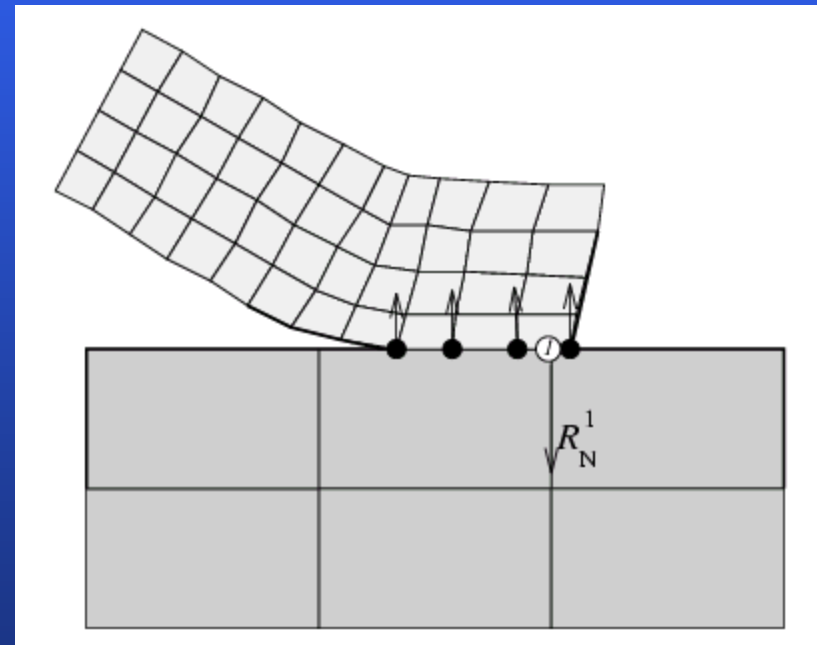
## Part I – Contact elements

Problems using the “single step” strategy

Single step



Double step



## Part I - Geometrical aspects (1)

### Problem definition

#### Hypothesis :

- Time step  $n$ , iteration  $i$ .
- The displacement field is given

#### Problem :

- Which nodes are “in contact” (inside a forbidden area/volume) ?
- Which set of nodes/segments (2D) or nodes/surfaces (3D) are involved ?
- Which contact elements must be activated for the next step of the algorithm ?

## Part I - Geometrical aspects (2)

### Method and problems

#### Method :

- Each contact node is projected on each segment/surface (on the CAD definition)
- Inside/Outside test (surroundedness test)
- A gap between the node is computed
- Contact occurs if the gap is negative

#### Usual problems :

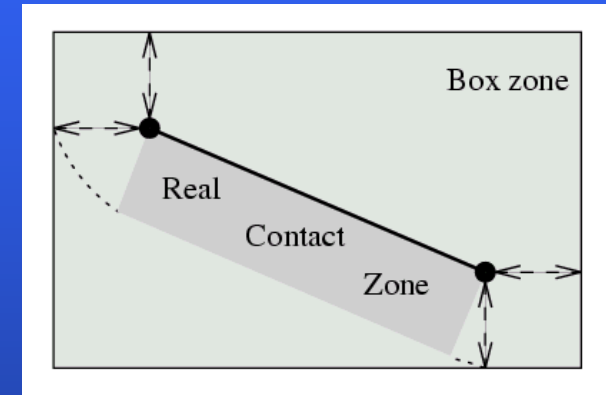
- More than one projection is obtained
- One projection found but outside
- Too many tests (too slow)
- No projection but the node is inside (normal discontinuities)
- Projection algorithm doesn't converge (singular points on NURBS curves, surfaces)
- Surroundedness test is difficult in case of complex curved faces (including holes)

## Part I - Geometrical aspects (3)

Too many tests !

Solution used in Metafor :

- Each test is first performed using a bounding box
- The bounding box moves with the curve/surface
- If contact occurred with a given segment, this segment is first tested, then the neighbors, then the others.



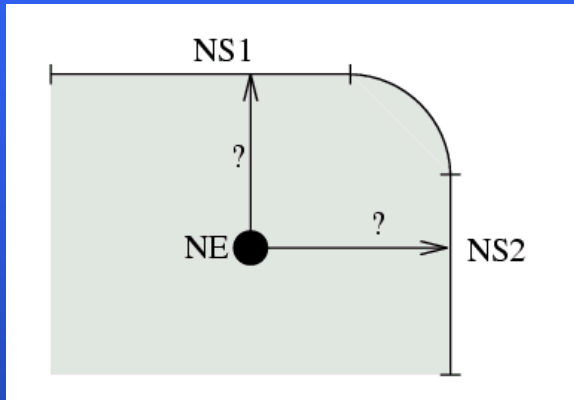
Solution used in Cast3M :

- The node must belong to the circle centered on the segment (2D)
- A region is defined by planes in 3D

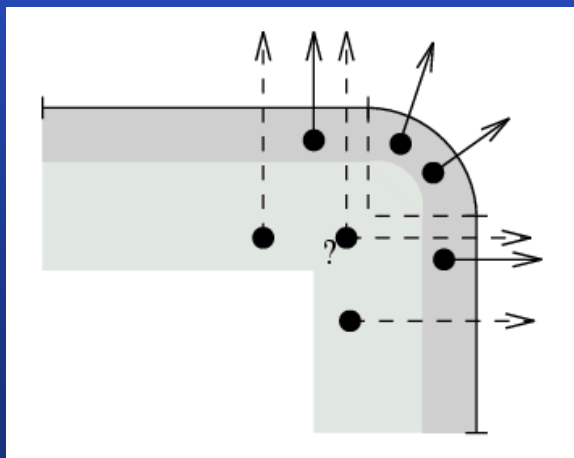
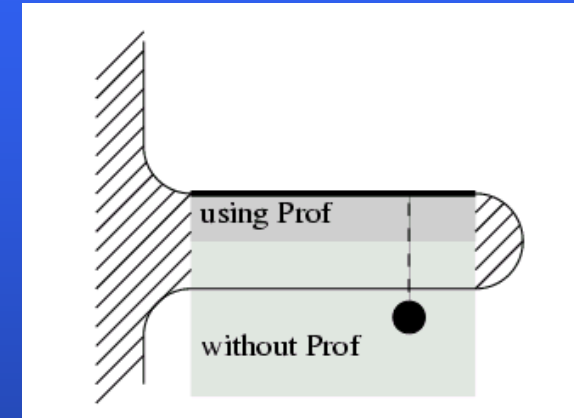
## Part I - Geometrical aspects (4)

### Projection problems

More than one projection is obtained



One projection found but outside



Solution in Metafor :

- An additional parameter is used : prof
- Prof = maximum gap allowed

Solution in Cast3M :

- Previous “circle” test is used in 2D
- 3D ?

## Part I - Geometrical aspects (5)

### Inside/Outside test

Problem formulation :

- A projection has been found on the surface. Does it belong to the face ?

Remarks :

- Problem must only be solved when using complex CAD surfaces (rigid tools).
- Tools are defined using BREP (Boundary Representation)
- Face = Surface + Outside boundary + Set of holes (inside boundaries)

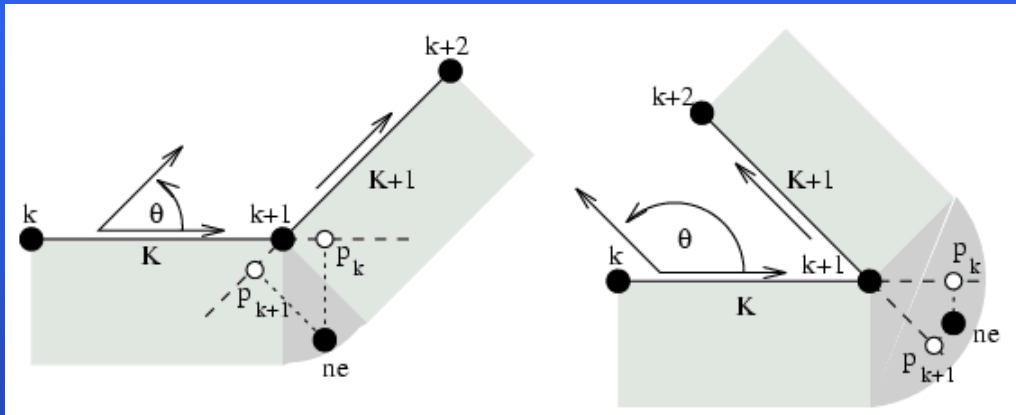
Algorithm :

- We work on the surface's parameter plane (2D)
- A outer point is computed.
- A test line is drawn from the outer point to the point to be tested
- N Intersections between the line and the boundary are computed
- If N is even, the point is outside
- In case of failure, the algorithm is restarted with another outer point



## Part I - Geometrical aspects (7)

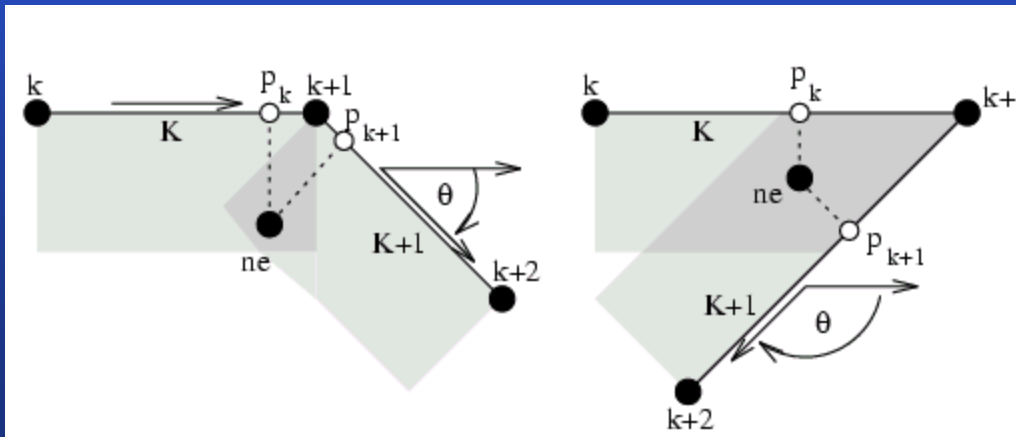
### Dealing with normal discontinuities



➤ Projection outside the boundaries must be allowed

➤ The normal vector is smoothed using various equations involving:.

- The gap(s)
- The angle  $\theta$
- The projections of the node



## Part II – Contact Algorithms

### Problem definition

Hypothesis :

- Contact search has been completed successfully.

Problem :

- Computation of the normal force.

2 main methods :

- Lagrange multipliers (used by Cast3M)
- Penalty method (used by Metafor) & variants (Augmented Lagrangian)

## Part II – Lagrange multipliers

### General method

- Constrained minimization of the (linearized) total potential energy :

$$\min\{q \in V_r\} \left( \frac{1}{2} q^t K q - q^t F \right) \quad V_r = \{q : Aq = b\}$$

- New potential energy ( introducing Lagrange Multipliers  $\lambda$  ) :

$$\Omega' = \frac{1}{2} q^t K q - q^t F + \lambda^t (Aq - b) \quad \frac{\partial \Omega'}{\partial q} = 0 \quad Kq - F + A^t \lambda = 0$$

$$\frac{\partial \Omega'}{\partial \lambda} = 0 \quad Aq = b$$

- New linear system to be solved :

$$\begin{pmatrix} K & A^t \\ A & 0 \end{pmatrix} \begin{pmatrix} q \\ \lambda \end{pmatrix} = \begin{pmatrix} F \\ b \end{pmatrix}$$

## Part II – Lagrange multipliers

Contact - Iterative loop needed

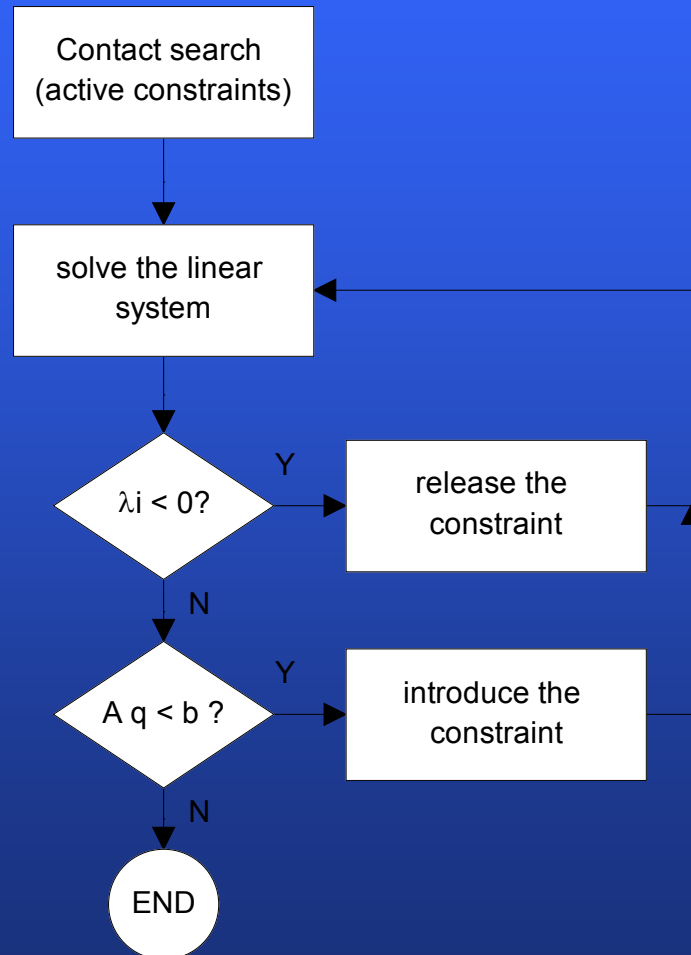
Contact problem :

$$Aq - b \geq 0$$

$$\lambda \geq 0$$

$$\lambda (Aq - b) = 0$$

- Convergence is not sure.
- Lots of variants can be made (depending on the way of managing introduction/release of the contact constraints)
- Usually converge in <10 iterations according to P. Verpeaux
- Not well documented in Cast3M



## Part II – Lagrange multipliers

### Main advantages

Constraints are solved exactly :

- No penetration (sometimes called “exact contact solution”)
- However curved boundaries must be discretized.

Contact is solved like any other constraints

- A stiffness matrix related to contact elements is built and added to the system
- Same methods used for Lagrange multipliers elimination.

No additional parameters :

- Very easy to use

## Part II – Lagrange multipliers

### Main difficulties (1)

The new system cannot be factorized using a  $L^T DL$  decomposition without row/column swapping

- Two sets of Lagrange multipliers are introduced :  $\lambda_1$  and  $\lambda_2$
- The energy becomes :

$$\frac{1}{2}q^t Kq - q^t F + (\lambda_1 + \lambda_2)^t (Aq - (b_1 + b_2)) - \frac{1}{2}(\lambda_1 - \lambda_2)^2$$

- Which leads to a system which can be factorized unless the unknowns are numbered in this order :  $\lambda_1, q, \lambda_2$

Contact constraints must be linear :

- Otherwise, very sophisticated algorithms must be used
- Linear elements in 2D (straight line segments)
- Planar surfaces in 3D (3 noded triangles)

## Part II – Lagrange multipliers

### Main difficulties (2)

Adds (a lot of) new unknowns :

- Bad method for sheet metal forming.
- Slow method
- However, the system can sometimes be reduced (writing some unknowns in terms of others).

No parameter to play with

- If the method doesn't work, no parameter can be tuned.

Oscillations in contact forces :

- The status of contact nodes may vary between “in contact” and “not in contact”
- Bad convergence

## Part II – Penalty method Method

Main idea :

- Some small penetrations are allowed.
- The normal force is proportional to the gap

Mathematical formulation :

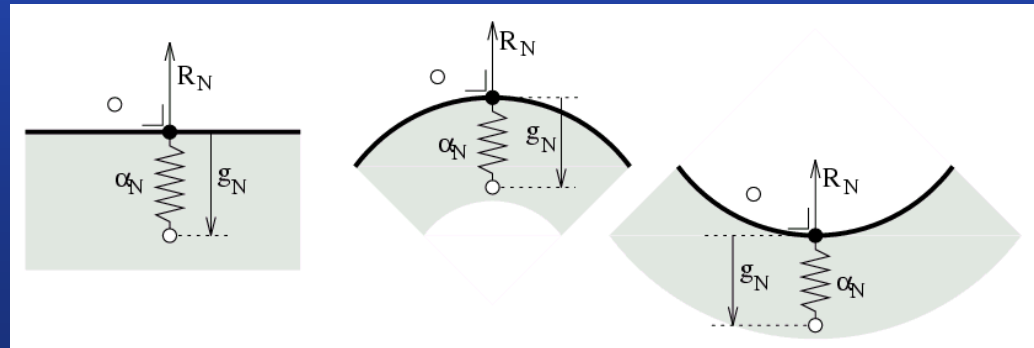
Equivalent to the Lagrange multiplier method where  $\lambda = \frac{1}{2} \alpha A q$   
 $\alpha =$  Penalty coefficient

$$\mathbf{R}_N = R_N \mathbf{n}$$

$$R_N = -\alpha_N f(g_N) \quad \text{if } g_N < 0$$

$$R_N = 0 \quad \text{if } g_N \geq 0$$

usually  $f(g_N) = g_N$





## Part II – Penalty method

### Main advantages

No additional d.o.f. :

- The linear system is solved very fast.
- Can manage a lot of nodes in contact (sheet metal forming)

Penalty parameter can be tuned :

- A small one leads to a fast computation with a large error
- A large one leads to a slow computation with a small error

Very easy to program :

- No sophisticated solver needed
- Curved boundaries are easy to take into account.

Fast convergence :

- Analytical tangent stiffness easy to compute
- Leads to quadratic convergence in the N-R algorithm

## Part II – Penalty method

### Main difficulties

An adequate penalty coefficient must be found

- A sensitivity analysis to this parameter is always needed
- Sometimes, no value can be found

Penetrations cannot be avoided

- May be a problem when dealing with small strain/stresses problems

## Part II – Penalty method

### Variants – Augmented Lagrangian

Main idea :

- A small penalty coefficient is used.
- The penetration is iteratively set to zero

Mathematical formulation :

Equivalent to the Lagrange multiplier method where

$$\lambda = \lambda^* + \frac{1}{2} \alpha A q$$

$\alpha$  = Penalty coefficient

$\lambda^*$  = known for a given iteration

$$\mathbf{R}_N = R_N \mathbf{n}$$

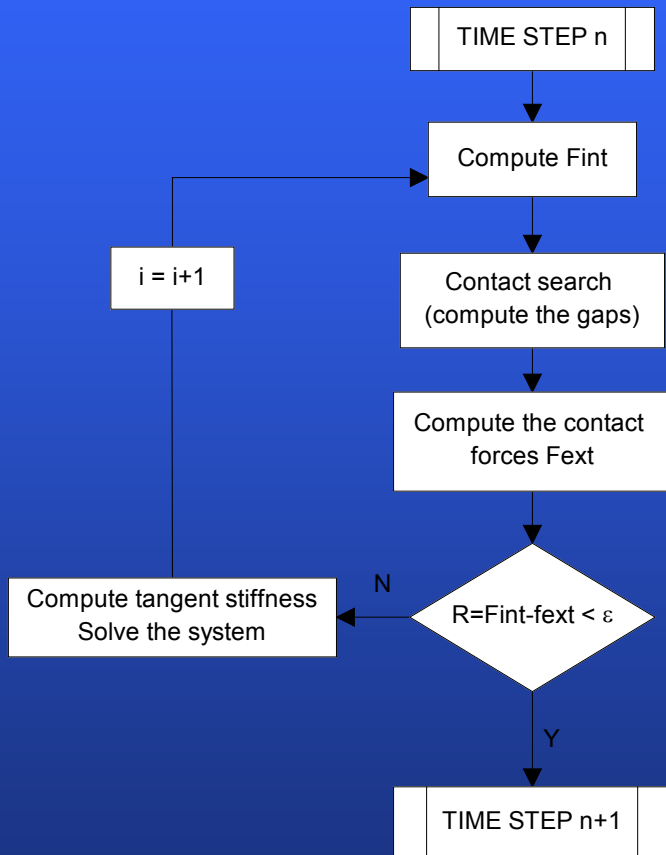
$$R_N = \lambda^* - \alpha_N f(\mathbf{g}_N)$$

$$R_N = 0 \quad \text{if } \lambda^* - \alpha_N f(\mathbf{g}_N) < 0$$

usually  $f(\mathbf{g}_N) = \mathbf{g}_N$

## Part II – Penalty method

Flowcharts for the penalty & Augmented Lagrangian methods



## Part III – Penalty method

### Notes about the contact problem

Tangent stiffness matrix :

- Can be easily computed.
- Is symmetric

Contact between two deformable bodies :

- The contact force must be splitted and applied to the master segment.
- Translation and rotation equilibrium must be satisfied.

Penalty coefficient :

- Depends on the problem
- Must be adapted if the mesh size is modified
- If element size is divided by 2,  $\alpha_N$  must be divided by 2 for obtaining the same gaps.

## Part III - Friction

### Problem definition

Hypothesis :

- Normal force has (or can) be computed by a given method (see before)

Problem :

- The node in contact may slide on the boundary
- Computation of the tangential force.
- One constitutive model is needed => friction law (usually Coulomb law)

$$\|\mathbf{R}_T\| < \mu \|\mathbf{R}_N\| \quad \text{If sticking contact}$$

$$\mathbf{R}_T = -\mu \|\mathbf{R}_N\| \frac{\Delta \mathbf{v}}{\|\Delta \mathbf{v}\|} \quad \text{If sliding contact}$$

2 main methods :

- Lagrange multipliers (used by Cast3M)
- Penalty method (used by Metafor) & variants (Augmented Lagrangian)

## Part III – Lagrange multipliers method

Variable substitution : (along the tangent direction – known in 2D – unknown in 3D)

$$\begin{array}{ccc}
 R_T < \mu R_N & \Delta v = 0 & \\
 R_T = \mu R_N & \Delta v > 0 & \\
 \end{array}
 \xrightarrow{R'_T = R_T - \mu R_N}
 \begin{array}{ccc}
 R'_T > 0 & \Delta v = 0 & \\
 R'_T = 0 & \Delta v > 0 & \\
 \end{array}$$

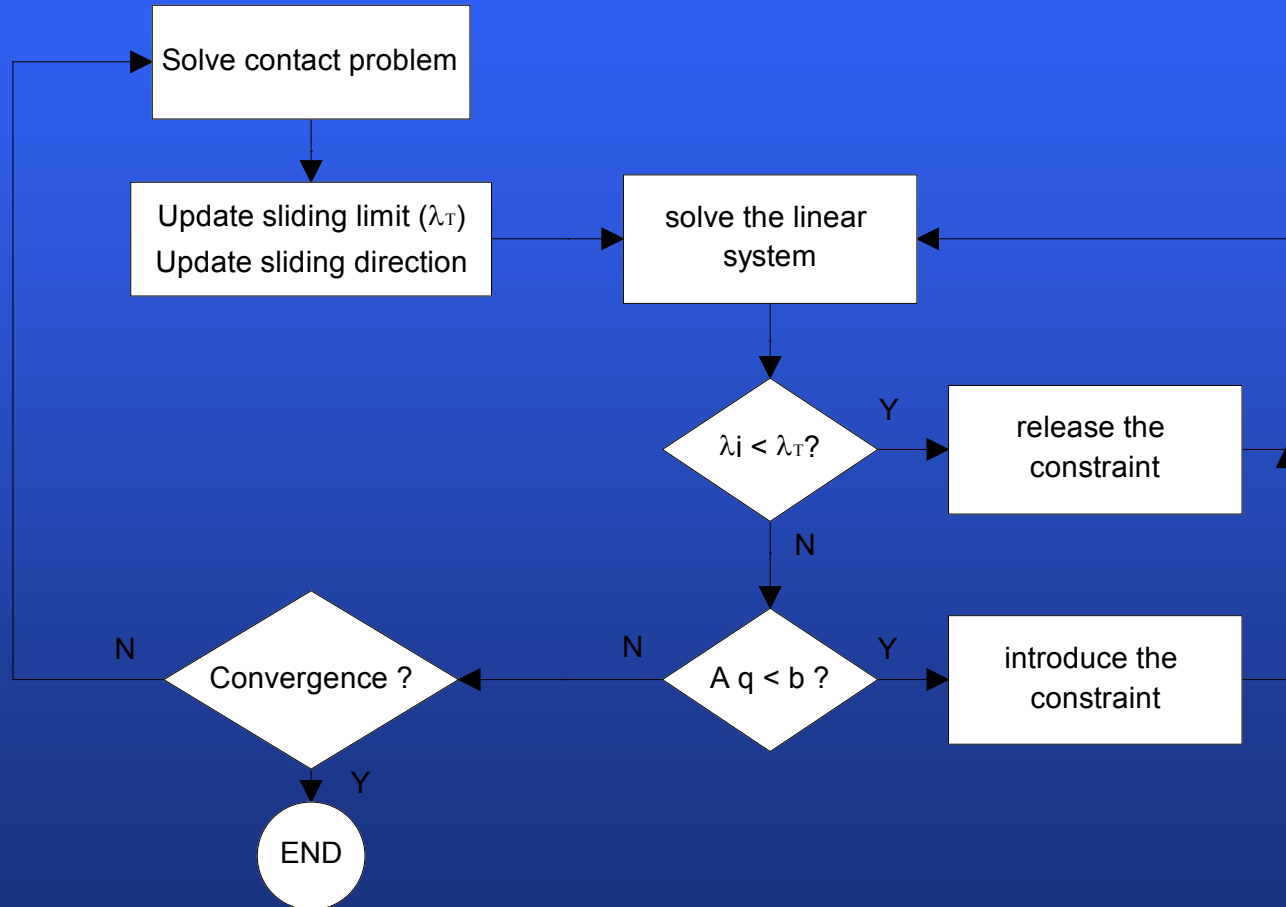
- The friction problem is similar to the contact problem.
- The same methods can be used

Remarks :

- Advantages and disadvantages of the method are similar to the contact problem
- The algorithm becomes very complicated (4 imbricated loops)

Time step loop, N-R loop, contact loop, friction loop

## Part III – Lagrange multipliers Flowchart





## Part III – Penalty method

### Problem definition

Main idea :

- A small sliding is allowed even for sticking nodes (tangential gap).
- The tangent force is proportional to this gap

Mathematical formulation :

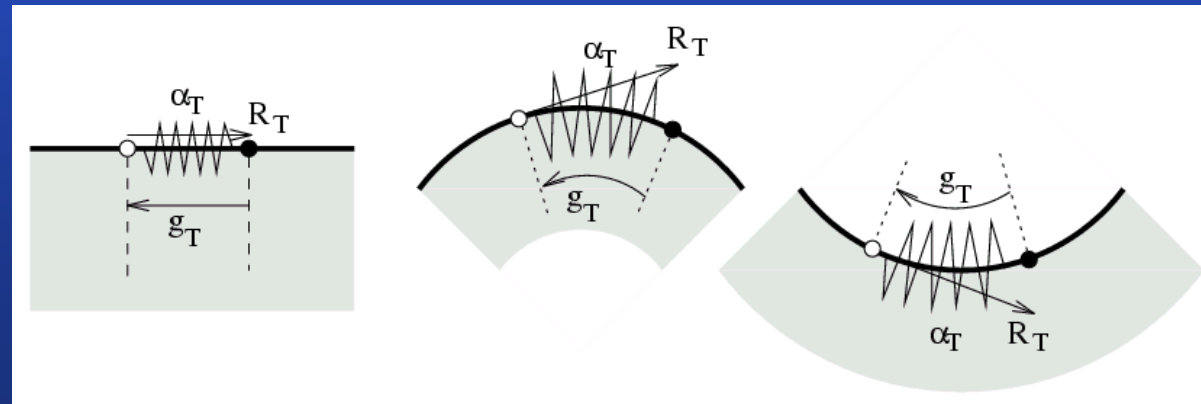
$$\mathbf{R}_T = R_T \mathbf{t}$$

$$R_T^* = -\alpha_T f(g_T)$$

$$\psi = R_T^* - \mu R_N$$

$$R_T = R_T^* - \langle -\psi \rangle \text{sign}(R_T^*)$$

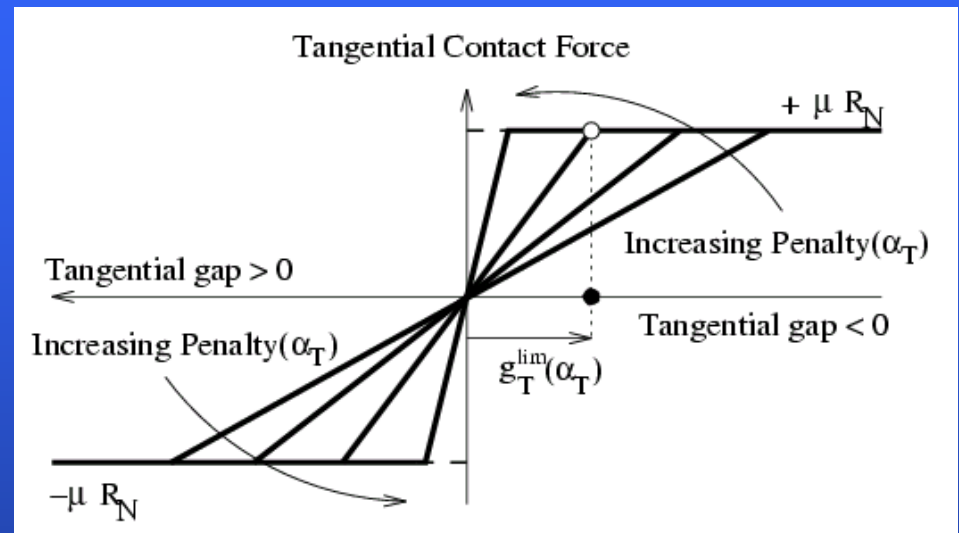
usually  $f(g_T) = g_T$



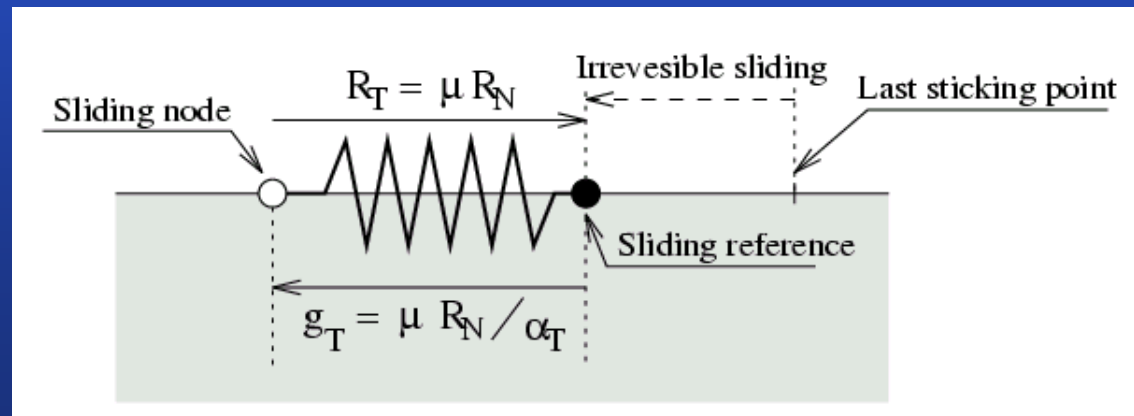
## Part III – Penalty method

### Regularized Coulomb's law

Coulomb's law becomes :



Spring model :



## Part III – Penalty method

### Notes about the friction problem

Tangent stiffness matrix :

- Can be easily computed.
- Is non symmetric due to friction : non symmetric solver needed

Computation of the first impact :

- First method : The slave node and the master segment is assumed to move with a constant velocity during a step. Impact problem reduces to an intersection problem.
- Second method : Friction is neglected during the first step when contact occurs.

Penalty coefficient :

- Usually 10 times lower than the normal penalty coefficient
- Leads to same gaps in normal and tangent direction

## Numerical examples

### Rigid Tools

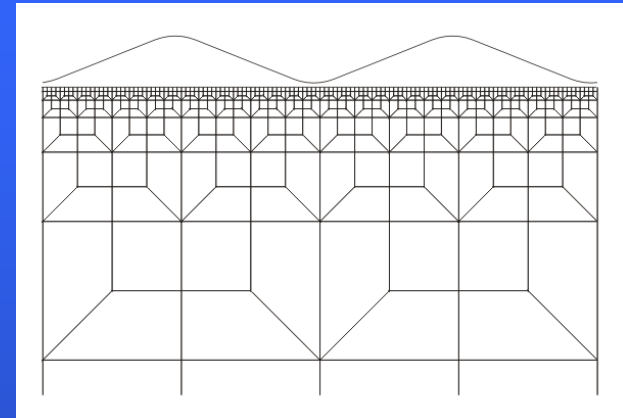
- Roughness transfer during skin-pass (2D)
- Nine's blankholder simulation (2D)
- Superplastic forming (3D)
- Tube Hydroforming (3D)

### Deformable bodies – self contact

- Rivet forming (2D)
- Dynamic buckling of a cylinder (2D/3D)
- Blade loss (3D)

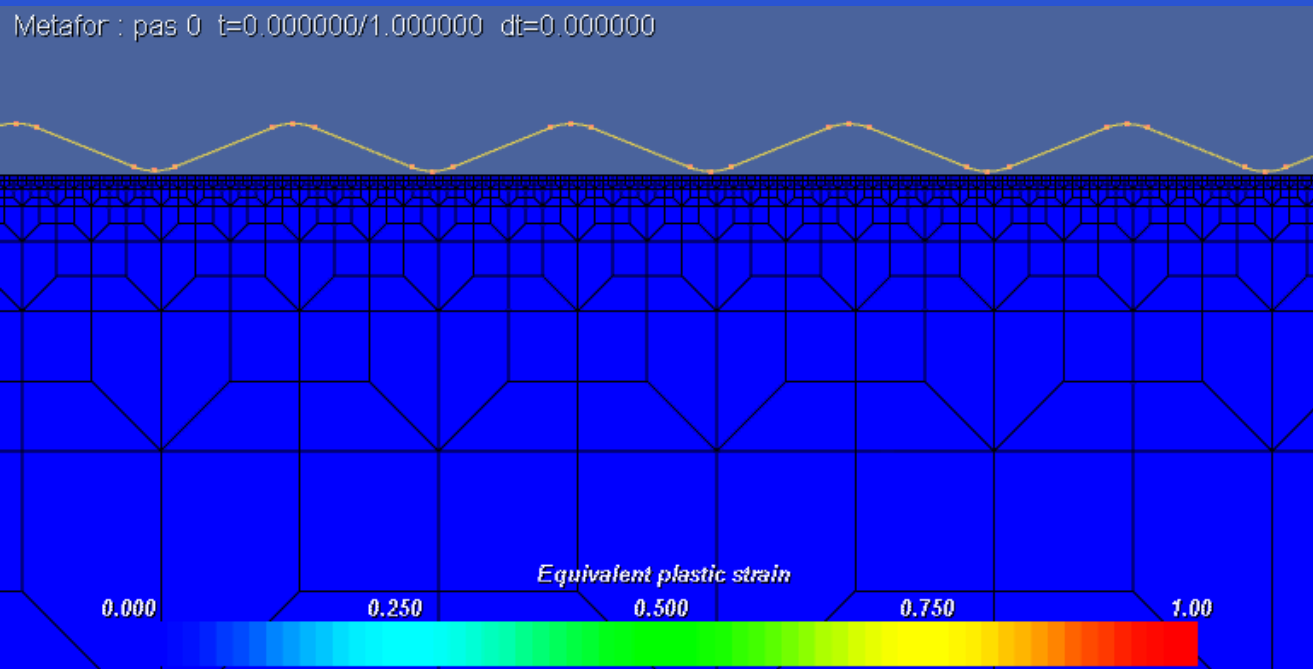
## Roughness transfer during skin-pass

- Smooth sheet & rough rigid roll.
- Material : steel with linear hardening law..
- smallest element edge =  $0.74 \mu\text{m}$



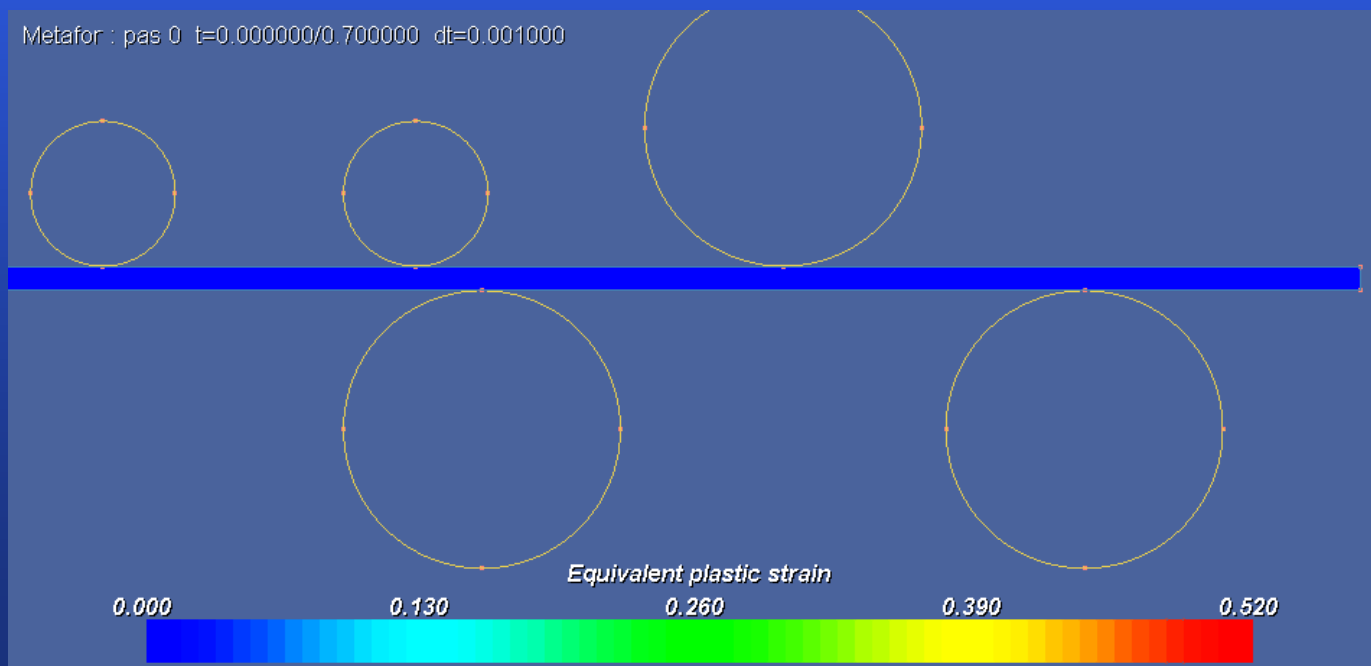
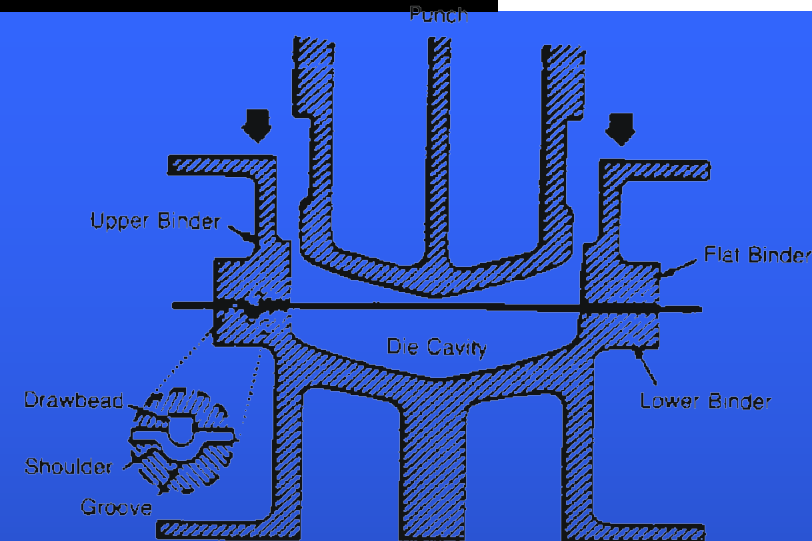
6  $\mu\text{m}$

30  $\mu\text{m}$

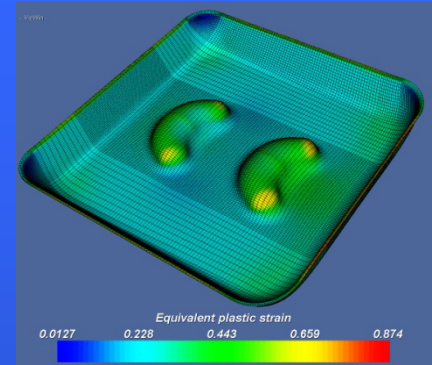
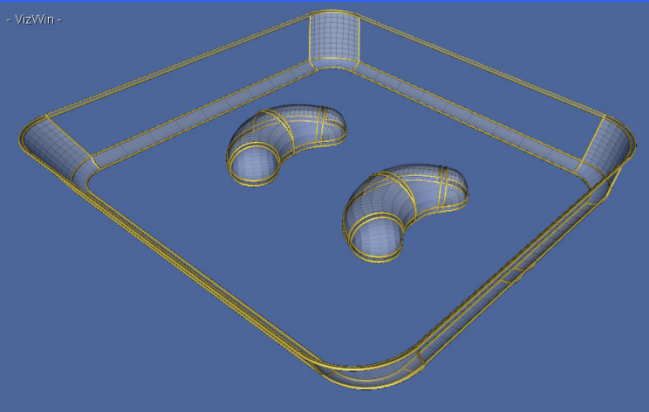


## Nine's blankholder simulation

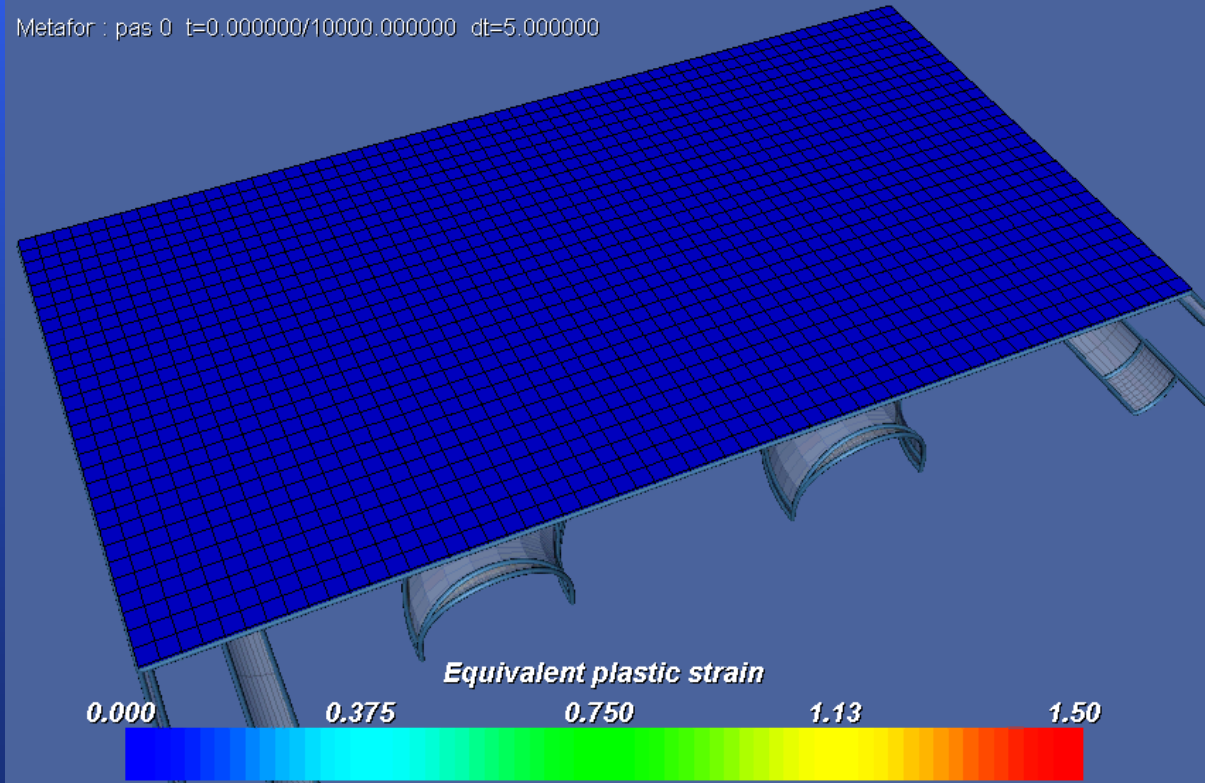
- Numerical simulation of an experimental device (used by Nine - 1992)
- Drawing and clamping forces are studied.
- Frictional component can be avoided if the rollers are free to rotate.



## Superplastic forming



Metafor : pas 0 t=0.000000/10000.000000 dt=5.000000

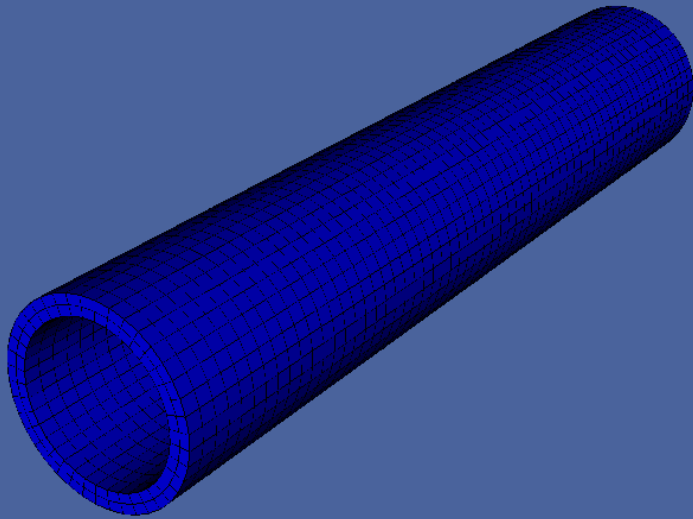


- Importation of an industrial geometry including Nurbs curves & surfaces.
- Superplastic Forming (coarse mesh) with optimization of the applied pressure cycle.

## Tube Hydroforming

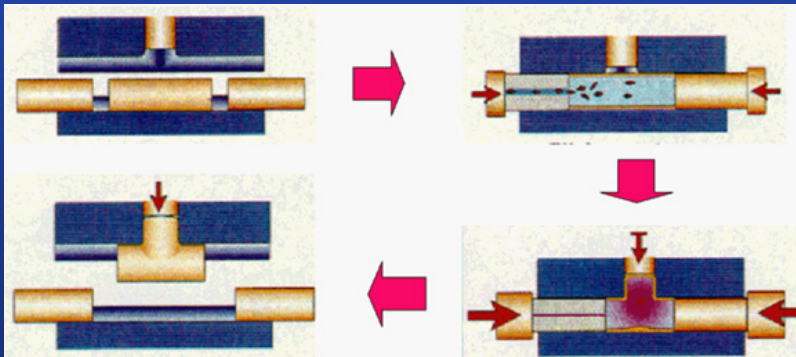
- Thermomechanical hydroforming of a tube.
- Material with thermo-elastoplastic behavior.
- The exact geometry of the die has been imported from CATIA (including Nurbs surfaces).

Metafor : pas 0 t=0.000000/1.000000 dt=0.000010

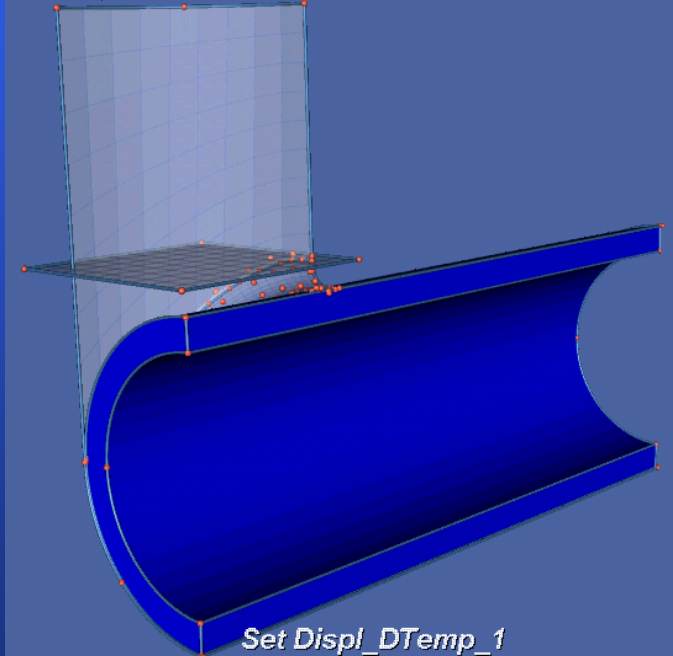


Equivalent plastic strain

0.000 0.420 0.840 1.26 1.68



Metafor : pas 0 t=0.000000/1.000000 dt=0.000000



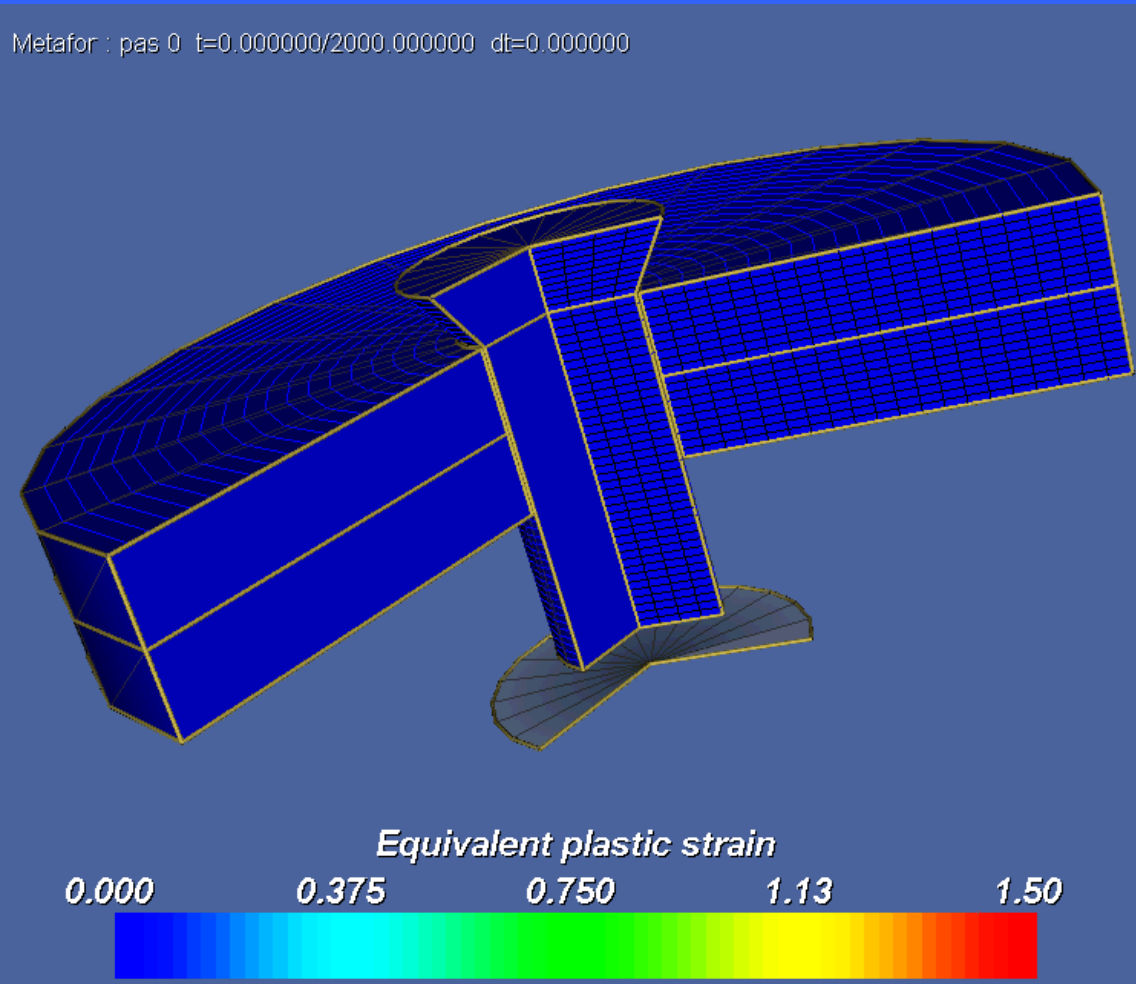
Set Displ\_DTemp\_1

0.000 7.50 15.0 22.5 30.0





## Rivet forming

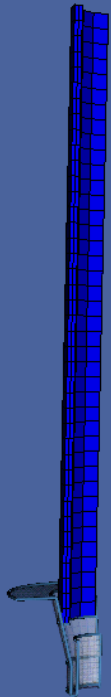


- Thermomechanical forming of a rivet for plates assembly.
- Material with thermo-elasto-viscoplastic behavior.
- Elastic, plastic and viscous behaviors are temperature dependent.
- Computation of the viscoplastic dissipation.
- Thermocontact interactions between the two plates and the rivet.
- Computation of the exchanged heat fluxes and of the frictional dissipation.

## Dynamic buckling of a cylinder

- 3D Simulation of the dynamic buckling of a cylinder.
- Contact between non smooth surfaces (resulting from discretisation).
- Friction is taken into account between the die and the cylinder.

Metafor : pas 0 t=0.000000/0.015000

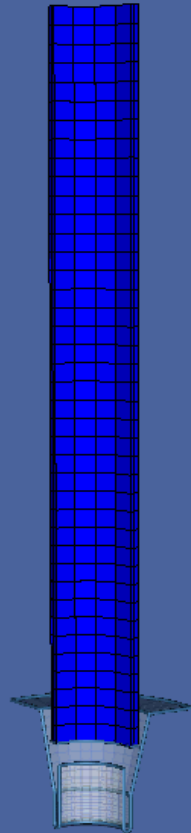


J2

0.000 0.250 1.000-007 1.500-007 2.500-007



Metafor : pas 0 t=0.000000/0.030000

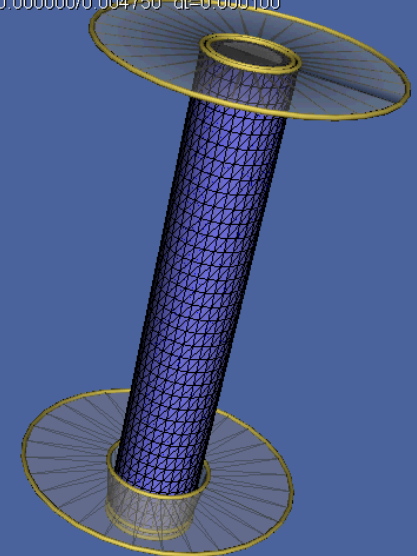


J2

0.000 0.750 1.500-007 2.250-007 3.000-007



Metafor : pas 0 t=0.000000/0.004750 dt=0.000100

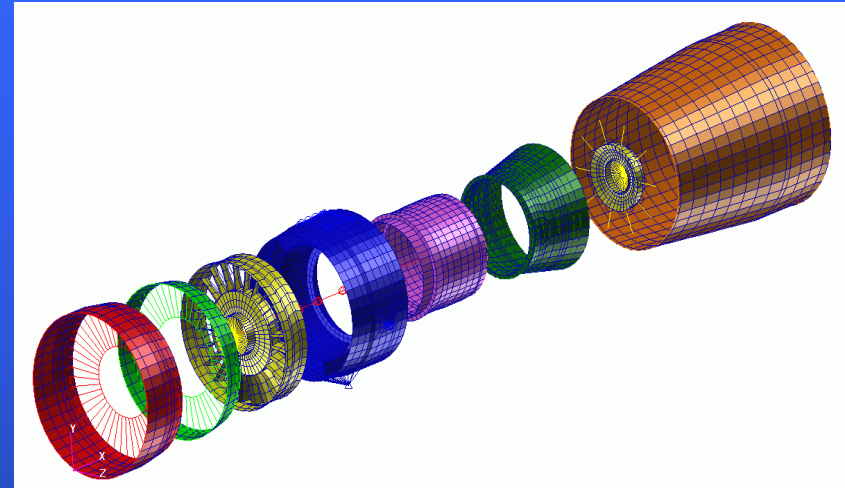
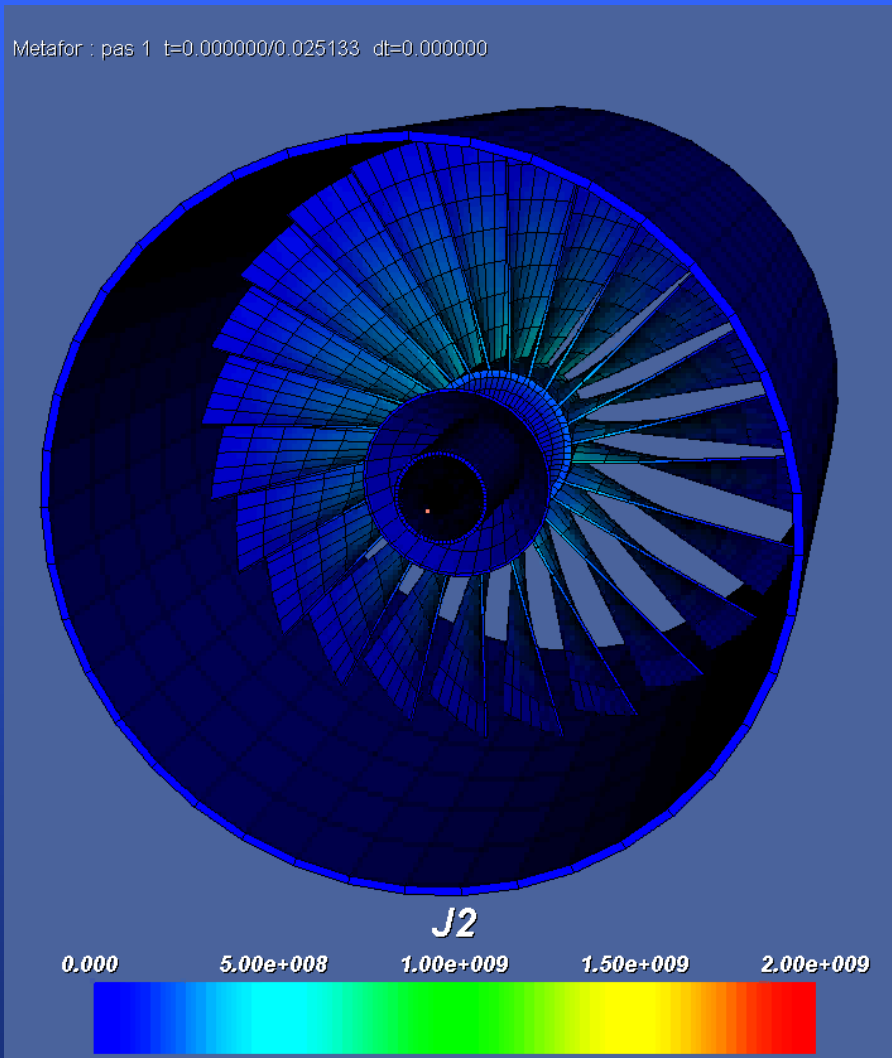


Equivalent plastic strain

0.000 0.250 0.500 0.750 1.00



## Blade loss



- Numerical simulation of an engine validation test
- Implicit algorithm (CPU time : 50 h – ev6 600MHz)
- (fixed) bearing & (moving) shaft connected by springs
- Blade/casing frictional contact interactions

## Conclusions

Two algorithms for managing contact have been presented

- The Lagrange Multiplier method seems to have many drawbacks for the simulation of metal forming processes

More tests could be done in Cast3M

A comparison with Metafor would be interesting

- The penalty method is useful for the simulation of metal forming processes.

This method could be implemented in Cast3M