

An update on the evolution of double parton distributions

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We consider double parton distributions in the general case in which the virtualities of the interacting partons are different. We elaborate the corresponding evolution equations and their extension to next-to-leading logarithmic accuracy.

Keywords: Multiple parton interactions, double parton distributions, QCD evolution equations

I. INTRODUCTION

In hadron-hadron collisions it is often assumed that final states containing high-mass systems or high transverse momentum jets are generated by a single hard scattering which involves one parton from each colliding hadron. The possibility, however, of multiple hard scatterings should be considered as well. One might therefore consider the case in which two hard interactions occur within the same hadron-hadron collision as an approximation to the full multiple parton interactions contributions. Several experimental results indeed support this possibility and are based on the analysis of the four-jet [1–3] and $\gamma + 3$ jets channels [4, 5]. Multiple parton interactions has been first modelled and included in modern Monte Carlo event generators [6–8]. Very recently detailed phenomenological investigations on double parton scattering have appeared in the literature. They focus on the four-jet [9, 10], double-inclusive-forward-pion production [13], same-sign W [11] and Z plus jets [12] final states. The efforts to identify processes which could be maximally sensitive to the contributions of double-parton scattering (DPS) is driven by two main interests. On the one hand a careful assessment of phase-space region where DPS events might impact searches for new physics is needed. On the other hand a genuine understanding of hadron structure in high energy

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collisions in terms of multi-partons distributions would emerge from these studies. Most of the predictions reported in the phenomenological analysis are based on the simplified model in which double parton distributions (DPD) are supposed to be the product of single-parton distributions. This assumption is indeed reasonable given the regime of low parton fractional momenta presently accessible at hadron colliders. Such an assumption simply disregards any longitudinal-momentum and flavour correlation between the two interacting partons from each hadron, so that each one evolves according to standard DGLAP equations [14]. The main virtue of such an approach is that it is technically appealing since numerous single parton distributions sets are available. The scale dependence of double-parton distributions has been worked out in Ref. [15]. With respect to standard single-parton distributions evolution equations (DGLAP), they do contain an additional term which is responsible for dynamical correlation between the interacting partons. Quite recently a new set of double parton distributions has been obtained by means of numerical integrations of the DPD evolutions equations. The initial conditions are such that DPD preserve under evolution a number of momentum and flavour sum rules [16]. The evolution equations elaborated in Ref. [15] however assume that both the interacting parton have the same virtualities.

Numerical studies [10, 12] and the arguments given in Ref. [17] indeed indicate that the characterizing scale for double parton scattering is the transverse momentum of the final state products. One may therefore consider the production of a gauge boson of mass $M^2 = Q_2^2$ in the first hard scattering associated with jets produced in the second hard scattering and characterized by the jet transverse momentum $P_t^2 = Q_1^2$. We indicate with Q_1^2 and Q_2^2 the factorization scales for the two hard processes. The low P_t^2 regime, with $P_t^2 \ll M^2$, for which we expect significant contributions from DPS events, is not covered by evolution equations proposed in Ref. [15]. The first purpose of this paper is to obtain DPD evolutions equations for different virtualities of the interacting partons. Then we consider the extension of the formalism beyond the leading logarithmic approximation. By using jet calculus rules we work out the inhomogeneous term at next-to-leading order accuracy and connect the real two-loops splitting functions arising in DPD evolution equations to the one appearing in fracture functions evolution equations at the same level of accuracy. Our main results are all framed within the Jet Calculus formalism since it proves to be an efficient tool for calculating multi-parton distributions properties and only an *ab initio* calculation could bring these findings on a firmer ground.

This paper is organized as follows. In Sec. II we review the basics of Jet Calculus formalism and recover known results on DPD. In Sec. III we work out the DPD evolution equations at different virtualities. In Sec. IV we guess the evolution equations for DPD at next-to-leading order accuracy. Finally we summarise our results in Sec. V.

II. PRELIMINARIES

The double-parton distributions (DPD) $D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2)$ are interpreted as the two-particle inclusive probability of finding in a target hadron a couple of partons of flavour j_1 and j_2 , fractional momenta x_1 and x_2 and virtualities up to Q_1^2 and Q_2^2 , respectively. The special case in which $Q_1^2 = Q_2^2 = Q^2$ has been considered in detail in Ref. [15]. According to Jet Calculus [18], the distributions at the final scales, Q_1^2 and Q_2^2 , are constructed through the parton-to-parton functions, E , which themselves obey DGLAP-type [14] evolution equations:

$$Q^2 \frac{\partial}{\partial Q^2} E_i^j(x, Q_0^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} P_k^i(u) E_i^k(x/u, Q_0^2, Q^2), \quad (1)$$

where $P_k^i(u)$ are the Altarelli-Parisi splitting functions. Inserting the initial condition $E_i^j(x, Q_0^2, Q^2) = \delta_i^j \delta(1-x)$ eq. (1) can iteratively be solved to give

$$E_i^j(x, Q_0^2, Q^2) = \delta_i^j \delta(1-x) + \frac{\alpha_s}{2\pi} P_i^j(x) \ln \frac{Q^2}{Q_0^2} + \mathcal{O}(\alpha_s^2). \quad (2)$$

Therefore the functions E provide the resummation of collinear logarithms up to the accuracy with which the $P_k^i(u)$ are specified. We may therefore express, by Jet Calculus rules [18], the double-parton distributions $D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2)$ as

$$\begin{aligned} D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2) = & \quad (3) \\ & \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j_1', j_2'}(z_1, Q_0^2, z_2, Q_0^2) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, Q_0^2, Q_1^2\right) E_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, Q_0^2, Q_2^2\right) + \\ & \int_{Q_0^2}^{\text{Min}(Q_1^2, Q_2^2)} d\mu_s^2 \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_{h, \text{corr}}^{j_1', j_2'}(z_1, z_2, \mu_s^2) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, \mu_s^2, Q_1^2\right) E_{j_2'}^{j_2}\left(\frac{x_2}{z_2}, \mu_s^2, Q_2^2\right). \end{aligned}$$

The first term on r.h.s., usually addressed as the homogeneous term, takes into account the uncorrelated evolution of the active partons found at a scale Q_0^2 in $D_h^{j_1', j_2'}$ up to Q_1^2 and Q_2^2 , respectively. The second term, the inhomogeneous one, takes into account the probability

to find the active partons at Q_1^2 and Q_2^2 as a result of a splitting at a scale μ_s^2 , integrated over all the intermediate scale at which such splitting may occur. The distribution $D_{h,corr}^{j'_1,j'_2}$ is

$$D_{h,corr}^{j'_1,j'_2}(z_1, z_2, \mu_s^2) = \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \frac{F_h^{j'}(z_1 + z_2, \mu_s^2)}{z_1 + z_2} \widehat{P}_{j'}^{j'_1,j'_2}\left(\frac{z_1}{z_1 + z_2}\right). \quad (4)$$

The distributions $F_h^{j'}$ in eq. (4) are the single parton distributions and the $\widehat{P}_{j'}^{j'_1,j'_2}$ are the real Altarelli-Parisi splitting functions [18]. Both terms in eq. (3) are shown in Fig. (1).

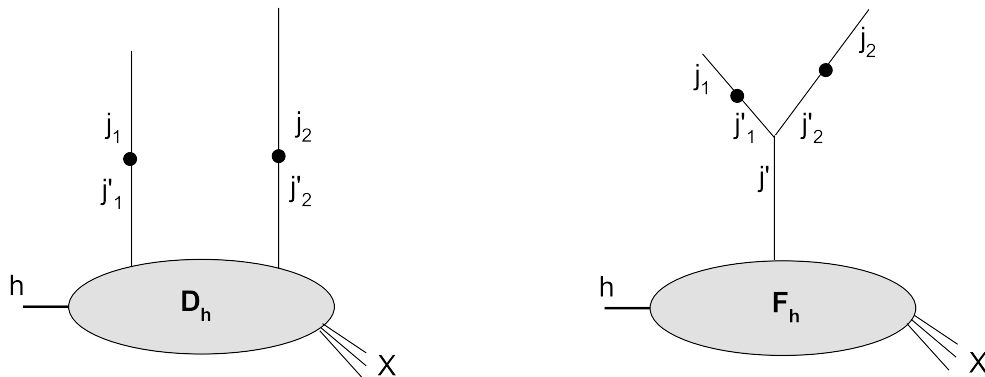


FIG. 1: Pictorial representation of both terms on right hand side of eq. (3). Black dots symbolize the parton-to-parton evolution function, E .

Due to strong ordering in parton virtualities, the maximum scale in the μ_s^2 integral is set to $\text{Min}(Q_1^2, Q_2^2)$. The scale Q_0^2 is in general the (low) scale at which DPD are usually modelled, in complete analogy with the single-parton distributions case. In the present context it also acts as the factorization scale for the correlated term, since all unresolved splittings for which $\mu_s^2 < Q_0^2$ are effectively taken into account in the definition of $D_h^{j'_1,j'_2}(z_1, Q_0^2, z_2, Q_0^2)$. The limits on convolutions integrals in eq. (4) are fixed by momentum conservation,

$$z_1 \geq x_1, \quad z_2 \geq x_2, \quad z_1 + z_2 \leq 1, \quad (5)$$

where z_1 and z_2 are intermediate partons fractional momenta and the last condition guarantees that their sum never exceeds the incoming hadron fractional momentum. The, lowest-order, real Altarelli-Parisi splitting functions $\widehat{P}_q^{qg}(u)$ and $\widehat{P}_g^{gg}(u)$ both contain an infrared singularity at the endpoint, $u = 1$. It is however easy to show that such a singularity is always outside the triangle defined by eq. (5) in the $[z_1, z_2]$ plane, provided that the trivial condition $x_1, x_2 > 0$ holds. In the “equal scales” case, $Q_1^2 = Q_2^2 = Q^2$, we may take

the logarithmic derivative with respect to Q^2 in eq. (3) and recover the result presented in Ref.[15]:

$$Q^2 \frac{\partial D_h^{j_1, j_2}(x_1, x_2, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x_1}{1-x_2}}^1 \frac{du}{u} P_k^{j_1}(u) D_h^{j_2, k}(x_1/u, x_2, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x_2}{1-x_1}}^1 \frac{du}{u} P_k^{j_2}(u) D_h^{j_1, k}(x_1, x_2/u, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \frac{F_h^{j'}(x_1 + x_2, Q^2)}{x_1 + x_2} \widehat{P}_{j'}^{j_1, j_2}\left(\frac{x_1}{x_1 + x_2}\right). \quad (6)$$

The first and second terms on the right-hand side are obtained through the Q^2 dependence contained in the E functions, while the last is obtained from the Q^2 dependent limit in the μ_s^2 integration in the correlated term. The evolution equations therefore resum large contributions of the type $\alpha_s \ln(Q^2/Q_0^2)$ and $\alpha_s \ln(Q^2/\mu_s^2)$ appearing in the uncorrelated and correlated term of eq. (3), respectively.

III. EVOLUTION EQUATIONS FOR DIFFERENT VIRTUALITIES

Let us now consider the general case in which the partons initiating the two separate hard scatterings have different virtualities, Q_1^2 and Q_2^2 , respectively with $Q_1^2 < Q_2^2$. The evolution equations for the higher scale is obtained by taking the logarithmic derivative of eq. (3) with respect to Q_2^2

$$Q_2^2 \frac{\partial D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2)}{\partial Q_2^2} = \left[\int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} D_h^{j_1', j_2'}(z_1, Q_0^2, z_2, Q_0^2) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, Q_0^2, Q_1^2\right) + \int_{Q_0^2}^{Q_1^2} d\mu_s^2 \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \frac{F_h^{j'}(z_1 + z_2, \mu_s^2)}{z_1 + z_2} \widehat{P}_{j'}^{j_1', j_2'}\left(\frac{z_1}{z_1 + z_2}\right) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, \mu_s^2, Q_1^2\right) \right] \cdot \frac{\alpha_s(Q_2^2)}{2\pi} \int_{\frac{x_2}{z_2}}^1 \frac{du}{u} P_k^{j_2}(u) E_{j_2'}^k\left(\frac{x_2}{z_2 u}, \mu_s^2, Q_2^2\right), \quad (7)$$

and using eq. (1). Reordering the integrals, we get

$$Q_2^2 \frac{\partial D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2)}{\partial Q_2^2} = \frac{\alpha_s(Q_2^2)}{2\pi} \int_{\frac{x_2}{1-x_1}}^1 \frac{du}{u} P_k^{j_2}(u) \left[\int_{x_1}^{1-\frac{x_2}{u}} \frac{dz_1}{z_1} \int_{\frac{x_2}{u}}^{1-z_1} \frac{dz_2}{z_2} D_h^{j_1', j_2'}(z_1, Q_0^2, z_2, Q_0^2) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, Q_0^2, Q_1^2\right) E_{j_2'}^k\left(\frac{x_2}{z_2 u}, Q_0^2, Q_2^2\right) + \int_{Q_0^2}^{Q_1^2} d\mu_s^2 \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \int_{x_1}^{1-\frac{x_2}{u}} \frac{dz_1}{z_1} \int_{\frac{x_2}{u}}^{1-z_1} \frac{dz_2}{z_2} \frac{F_h^{j'}(z_1 + z_2, \mu_s^2)}{z_1 + z_2} \widehat{P}_{j'}^{j_1', j_2'}\left(\frac{z_1}{z_1 + z_2}\right) E_{j_1'}^{j_1}\left(\frac{x_1}{z_1}, \mu_s^2, Q_1^2\right) \cdot E_{j_2'}^k\left(\frac{x_2}{z_2 u}, \mu_s^2, Q_2^2\right) \right]. \quad (8)$$

It is now easy to recognize, through direct comparison with eq.(3), that the term in square brackets is the double parton distribution $D_h^{j_1,k}(x_1, Q_1^2, x_2/u, Q_2^2)$. The desired evolution equations then becomes

$$Q_2^2 \frac{\partial D_h^{j_1,j_2}(x_1, Q_1^2, x_2, Q_2^2)}{\partial Q_2^2} = \frac{\alpha_s(Q_2^2)}{2\pi} \int_{\frac{x_2}{1-x_1}}^1 \frac{du}{u} P_k^{j_2}(u) D_h^{j_1,k}(x_1, Q_1^2, x_2/u, Q_2^2). \quad (9)$$

We could obtain the same result in a rather different way. We can in fact exploit the following property of the E function

$$E_i^j(x, Q_0^2, Q_2^2) = \int_x^1 \frac{du}{u} E_i^k\left(\frac{x}{u}, Q_0^2, Q_1^2\right) E_k^j(u, Q_1^2, Q_2^2). \quad (10)$$

The latter can be checked, for example, by expanding the E functions in power of α_s as given in eq. (2). By using eq. (10), eq. (3) can be recast in the much compact form

$$D_h^{j_1,j_2}(x_1, Q_1^2, x_2, Q_2^2) = \int_{x_2}^{1-x_1} \frac{dw_2}{w_2} D_h^{j_1,k}(x_1, Q_1^2, w_2, Q_1^2) E_k^{j_2}\left(\frac{x_2}{w_2}, Q_1^2, Q_2^2\right). \quad (11)$$

By direct substitution it can be checked that eq. (11) is indeed a solution of eq. (9). With respect to “equal scale” DPD evolution equations we notice the disappearance of the inhomogeneous term. This is due to the fact that the correlations up to a scale Q_1^2 given by the inhomogeneous term are taken into account by the “equal scales” evolution equations and properly built into $D_h^{j_1,k}(x_1, Q_1^2, w_2, Q_1^2)$. The evolution of the second parton from Q_1^2 to Q_2^2 is uncorrelated due to strong ordering in virtualities assumed in the leading logarithmic approximation. From the numerical point of view therefore DPD at different virtualities can be obtained evolving $D_h^{j_1,k}(x_1, Q_1^2, w_2, Q_1^2)$ with the “equal scale” evolution equations up to Q_1^2 , eq. (6), and then using the latter output as initial condition in eq. (9), for $Q_2^2 > Q_1^2$. We have therefore proven the conjecture put forward in Ref. [16] and actually implemented numerically [26]. For completeness we have also considered the DPD evolution equations in Q_1^2 . Provided that $Q_1^2 < Q_2^2$ and using the same techniques through which we have derived eqs. (6) and (9) we get

$$Q_1^2 \frac{\partial D_h^{j_1,j_2}(x_1, Q_1^2, x_2, Q_2^2)}{\partial Q_1^2} = \frac{\alpha_s(Q_1^2)}{2\pi} \int_{\frac{x_1}{1-x_2}}^1 \frac{du}{u} P_k^{j_1}(u) D_h^{k,j_2}(x_1/u, Q_1^2, x_2, Q_2^2) + \frac{\alpha_s(Q_1^2)}{2\pi} \int_{x_2}^{1-x_1} \frac{dz_2}{z_2} \frac{F_h^j(x_1+z_2, Q_1^2)}{x_1+z_2} \hat{P}_j^{j_1,j_2}\left(\frac{x_1}{x_1+z_2}\right) E_{j_2}^{j_2}\left(\frac{x_2}{z_2}, Q_1^2, Q_2^2\right). \quad (12)$$

In this case the evolution equations contain an inhomogeneous term which arises due to the explicit Q_1^2 dependence on the μ_s^2 integral in eq. (3). Since the factorization scale are kept

different, the latter does contain explicitly the function $E(Q_1^2, Q_2^2)$, which cannot be further simplified. To avoid a direct calculations of the E function, the double-parton distributions for unequal final scales should be obtained therefore via the two step procedure mentioned above.

IV. EVOLUTION EQUATIONS TO NLLA

In this section we address the problem of deriving the structure of DPS evolution equations at next-to-leading logarithmic accuracy. The aim therefore is to provide some guidance for an eventual *ab initio* calculation. At present, in fact, such an accuracy is not required since, given the scarce experimental information available, we do not even have sufficient data to test whether the scale dependence predicted by DPD evolution is supported. Jet Calculus techniques has been succesfully extended up next-to-leading logarithmic accuracy to improve the perturbative description of time-like parton cascades [20]. For space-like parton cascades instead, which is the case we are actually interested in, the formalism has not been extended beyond leading-logarithmic accuracy. However a couple of calculations have been performed in the context of semi-inclusive Deep Inelastic Scattering. In particular the one-particle inclusive cross sections up to order $\mathcal{O}(\alpha_s^2)$ have been calculated in Refs. [21, 22]. Such calculations carefully consider hadron production collinear to the hadron remnant where the introduction of fracture functions [23] is shown to be necessary to factorize additional collinear singularities appearing in the calculations in that phase-space region. The fixed order calculations at $\mathcal{O}(\alpha_s^2)$ allows the authors to derive the fracture functions evolution equations to next-to-leading logarithmic accuracy, as well as the two-loop, unknown, real splitting functions, $\widehat{P}^{(1)}$. Fracture functions evolution equations can be calculated, as DPD, within the Jet Calculus formalism [24, 25] and they do contain an inhomogenous term as well. While in the fracture functions case the partons emitted by the active one hadronizes through a fragmentation function, in the DPD one, the emitted parton is allowed to further evolve and eventually initiate a second hard scattering.

When evaluating the evolutions equations at next-to-leading logarithmic accuracy the evolution equations for the parton-to-parton functions E must be properly modified to

$$Q^2 \frac{\partial}{\partial Q^2} E_i^j(x, Q_0^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{du}{u} \left[P_k^{(0),i}(u) + \frac{\alpha_s(Q^2)}{2\pi} P_k^{(1),i}(u) \right] E_i^k(x/u, Q_0^2, Q^2), \quad (13)$$

where $P^{(0)}(u)$ and $P^{(1)}(u)$ are the one- and two-loops [19] Altarelli-Parisi splitting functions, respectively. This in turn implies that the two homogenous terms in DPD evolution equations in eq. (6) are modified by adding the two-loop splitting functions contributions. On the contrary, the derivation of the inhomogenous term to next-to-leading logarithmic accuracy is not trivial so, in the following, we will construct it explicitly in the ‘‘equal scales’’ case. The correlated term can be written therefore as

$$D_{h,corr}^{j'_1,j'_2}(x_1, x_2, Q_0^2, Q^2) = \int_{Q_0^2}^{Q^2} \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \int_{x_1+x_2}^1 dw \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \int dr_1 dr_2 du_1 du_2 \cdot F_h^{j'}(w, \mu_s^2) \left[\widehat{P}_{j'}^{(0),j'_1,j'_2}(u_1) \delta(1-u_1-u_2) + \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \widehat{P}_{j'}^{(1),j'_1,j'_2}(u_1, u_2) \right] \cdot E_{j'_1}^{j_1}(r_1, \mu_s^2, Q^2) E_{j'_2}^{j_2}(r_2, \mu_s^2, Q^2) \delta(x_1 - r_1 z_1) \delta(x_2 - r_2 z_2) \delta(z_1 - u_1 w) \delta(z_2 - u_2 w). \quad (14)$$

In the above equations $\widehat{P}_{j'}^{(1),j'_1,j'_2}(u_1, u_2)$ gives the probability that a parton j' splits to three partons, where the first, j'_1 , and a second, j'_2 , have respectively a fraction u_1 and u_2 of the incoming parton momentum j' and the third is integrated over. Integrating the δ -functions, which implements longitudinal momentum conservation, one gets

$$D_{h,corr}^{j'_1,j'_2}(x_1, x_2, Q_0^2, Q^2) = \int_{Q_0^2}^{Q^2} \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \int_{x_1+x_2}^1 \frac{dw}{w^2} F_h^{j'}(w, \mu_s^2) \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2} \left[\widehat{P}_{j'}^{(0),j'_1,j'_2}\left(\frac{z_1}{w}\right) \delta\left(1 - \frac{z_1}{w} - \frac{z_2}{w}\right) + \frac{\alpha_s(\mu_s^2)}{2\pi\mu_s^2} \widehat{P}_{j'}^{(1),j'_1,j'_2}\left(\frac{z_1}{w}, \frac{z_2}{w}\right) \right] E_{j'_1}^{j_1}\left(\frac{x_1}{z_1}, \mu_s^2, Q^2\right) E_{j'_2}^{j_2}\left(\frac{x_2}{z_2}, \mu_s^2, Q^2\right). \quad (15)$$

As already noted, the inhomogenous term in DPD evolution equations is due to the explicit Q^2 dependence in the upper limit of μ_s^2 integration. In order to obtain it we set $\mu_s^2 = Q^2$ in eq.(15), multiply by Q^2 , and use initial condition on E , $E_i^j(x, Q^2, Q^2) = \delta_i^j \delta(1-x)$. Adding the homogeneous contributions, the final result reads

$$Q^2 \frac{\partial D_h^{j_1,j_2}(x_1, x_2, Q^2)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x_1}{1-x_2}}^1 \frac{du}{u} \left[P_k^{(0),j_1}(u) + \frac{\alpha_s(Q^2)}{2\pi} P_k^{(1),j_1}(u) \right] D_h^{k,j_2}(x_1/u, x_2, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{x_2}{1-x_1}}^1 \frac{du}{u} \left[P_k^{(0),j_2}(u) + \frac{\alpha_s(Q^2)}{2\pi} P_k^{(1),j_2}(u) \right] D_h^{j_1,k}(x_1, x_2/u, Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \int_{x_1+x_2}^1 \frac{dw}{w^2} F_h^{j'}(w, Q^2) \left[w \widehat{P}_{j'}^{(0),j_1,j_2}\left(\frac{x_1}{w}\right) \delta(w-x_1-x_2) + \frac{\alpha_s(Q^2)}{2\pi} \widehat{P}_{j'}^{(1),j_1,j_2}\left(\frac{x_1}{w}, \frac{x_2}{w}\right) \right]. \quad (16)$$

It should be noted however that the kernels $\widehat{P}_{j'}^{(1),j'_1,j'_2}(u,v)$ reported in Refs. [21, 22] do express the probability that a parton j' splits into a parton j'_1 with a momentum fraction u of the incoming parton, into a parton j'_2 with a momentum fraction v of j'_1 , the third being integrated over. Therefore they are related to the ones appearing in eq. (16) by the following mapping

$$\widehat{P}_{j'}^{(1),j'_1,j'_2}(u_1, u_2) = \frac{1}{u_1} \widehat{P}_{j'}^{(1),j'_1,j'_2}\left(u_1, \frac{u_2}{u_1}\right). \quad (17)$$

The additional integral in the inhomogeneous term does appear since the momentum is not anymore constrained in the $1 \rightarrow 2$ splitting. The DPS evolution equations to next-to-leading logarithmic accuracy for different scales can be obtained by the same arguments given in Sec. III. We just quote the final result which reads

$$Q_2^2 \frac{\partial D_h^{j_1, j_2}(x_1, Q_1^2, x_2, Q_2^2)}{\partial Q_2^2} = \frac{\alpha_s(Q_2^2)}{2\pi} \int_{\frac{x_2}{1-x_1}}^1 \frac{du}{u} \left[P_k^{(0), j_2}(u) + \frac{\alpha_s(Q_2^2)}{2\pi} P_k^{(1), j_2}(u) \right] \cdot D_h^{j_1, k}(x_1, Q_1^2, x_2/u, Q_2^2), \quad (18)$$

provided that $Q_1^2 < Q_2^2$.

V. SUMMARY

We have considered double parton distributions in the general case in which the two factorization scales are kept different and derived the corresponding evolution equations. The results of the present calculation support the guess put forward in Ref. [16] and recently implemented numerically [26] widening the range of possible phenomenological investigations on double-parton scatterings. We have also derived the general structure of the DPD evolution equations at next-to-leading logarithmic accuracy and indicated how to transform the two-loops real splitting functions present in the literature in order to be used in the present context. Both results should be confirmed by performing an *ab initio* calculation.

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