

Highly Excited Baryons in Large N_c QCD

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Abstract. We use the $1/N_c$ expansion of QCD to analyse the spectrum of positive parity resonances with strangeness $S = 0, -1, -2$ and -3 in the 2–3 GeV mass region, supposed to belong to the $[56, 4^+]$ multiplet. The mass operator is similar to that of $[56, 2^+]$, previously studied in the literature. The analysis of the latter is revisited. In the $[56, 4^+]$ multiplet we find that the spin-spin term brings the dominant contribution and that the spin-orbit term is entirely negligible in the hyperfine interaction, in agreement with the constituent quark model practice, where this interaction is usually neglected. More data are strongly desirable, especially in the strange sector in order to fully exploit the power of this approach. We discuss possibilities of extending the calculations to other excited baryons belonging to the $N = 2$ or the $N = 4$ band.

1. INTRODUCTION

The $1/N_c$ expansion of QCD suggested by 't Hooft [1] about 30 years ago and analysed in a greater detail by Witten [2] has lead to a powerful algebraic method to study baryon spectroscopy. This method is based on the result that the SU(6) spin-flavor symmetry is exact in the large N_c limit [3]. It has been applied with a great success to the ground state baryons which correspond to the SU(6) **56** multiplet [4, 5, 6, 7, 8, 9] as well as to some excited states, as for example the $[70, 1^-]$ negative parity states [10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

The applicability of the large N_c QCD to the description of excited baryon states is a problem of active investigation. Here we analyse the applicability of the method to the $[56, 4^+]$ multiplet and develop considerations for the treatment of multiplet as $[70, 0^+]$ or $[70, 2^+]$.

2. THE MASS OPERATOR

The study of the $[56, 4^+]$ multiplet is similar to that of $[56, 2^+]$ as analysed in Ref. [18], where the mass spectrum is calculated in the $1/N_c$ expansion up to and including $\mathcal{O}(1/N_c)$ effects. The SU(2) isospin symmetry is supposed to be exact. The SU(3) symmetry breaking is implemented to $\mathcal{O}(\varepsilon)$, where $\varepsilon \sim 0.3$ gives a measure of this breaking by the strange quark mass. As the **56** is a symmetric

representation of SU(6), it is not necessary to distinguish between excited and core quarks for the construction of a basis of mass operators, as explained in Ref. [18]. Then the mass operator of the SU(3) multiplets has the following structure

$$M = \sum_i c_i O_i + \sum_i b_i \bar{B}_i \quad (1)$$

given in terms of the linearly independent operators O_i and \bar{B}_i defined in Table 1. Here O_i ($i = 1, 2, 3$) are rotational invariants and SU(3) flavor singlets [10], \bar{B}_1 is the strangeness quark number operator with negative sign, and the operators \bar{B}_i ($i = 2, 3$) are also rotational invariants but contain the SU(6) spin-flavor generators G_{i8} as well. The operators \bar{B}_i ($i = 1, 2, 3$) provide SU(3) breaking and are defined to have vanishing matrix elements for nonstrange baryons. The relation (1) contains the effective coefficients c_i and b_i as parameters. They represent reduced matrix elements that encode the QCD dynamics. The values of the corresponding coefficients which we obtained from fitting the experimentally known masses are given in Table 1 both for the $[56, 4^+]$ and the presently revisited $[56, 2^+]$.

3. THE $[56, 2^+]$ MULTIPLY REVISITED

As mentioned above, the study of the $[56, 4^+]$ multiplet is similar to that of $[56, 2^+]$. Here we first revisit the $[56, 2^+]$ multiplet for two purposes: 1) to get a consistency test of our procedure of calculating matrix elements of the operators in Table 1 and 2) to analyse a new assignment of the $\Delta_{5/2^+}$ resonances.

The matrix elements of O_1 , O_2 , O_3 and \bar{B}_1 are trivial to calculate for both multiplets under study. For the $[56, 2^+]$ one can find them in Table 2 of Ref. [18] and for the $[56, 4^+]$ they are given in the next section.

To calculate the diagonal and off-diagonal matrix elements of \bar{B}_2 we use the definition

$$G_{i8} = G^{i8} = \frac{1}{2\sqrt{3}} (S^i - 3S_s^i), \quad (2)$$

where S^i and S_s^i are components of the total spin and of the total strange-quark spin respectively [8]. Using (2) we can rewrite \bar{B}_2 from Table 1 as

$$\bar{B}_2 = -\frac{\sqrt{3}}{2N_c} \vec{l} \cdot \vec{S}_s \quad (3)$$

with the decomposition

$$\vec{l} \cdot \vec{S}_s = l_0 S_{s0} + \frac{1}{2} (l_+ S_{s-} + l_- S_{s+}). \quad (4)$$

We calculated the matrix elements from the wave functions used in constituent quark model studies, where the center of mass coordinate has been removed and only the internal Jacobi coordinates appear (see, for example, Ref. [20]). The expressions we found for the matrix elements of \bar{B}_2 were identical with those of

Ref. [18], based on Hartree wave functions, exact in the $N_c \rightarrow \infty$ limit only. This proves that in the Hartree approach no center of mass corrections are necessary for the $[\mathbf{56}, 2^+]$ multiplet. We expect the same conclusion to stand for any $[\mathbf{56}, \ell^+]$. For mixed representations the situation is more intricate [12], see section 5.

For \bar{B}_3 , we used the following relation [14]

$$S_i G_{i8} = \frac{1}{4\sqrt{2}} \left[3I(I+1) - S(S+1) - \frac{3}{4} N_s(N_s+2) \right] \quad (5)$$

in agreement with [8]. Here I is the isospin, S is the total spin and N_s the number of strange quarks. As for the matrix elements of \bar{B}_2 , we found identical results to those of Ref. [18]. Note that only \bar{B}_2 has non-vanishing off-diagonal matrix elements. Their role is very important in the state mixing in particular in the octet-decuplet mixing. We found that the diagonal matrix elements of O_2 , O_3 , \bar{B}_2 and \bar{B}_3 of strange baryons satisfy the following relation

$$\frac{\bar{B}_2}{\bar{B}_3} = \frac{O_2}{O_3}, \quad (6)$$

for any state, irrespective of the value of J in both the octet and the decuplet. Such a relation also holds for the multiplet $[\mathbf{56}, 4^+]$ studied in the next section and might possibly be a feature of all $[\mathbf{56}, \ell^+]$ multiplets. It can be used as a check of the analytic expressions in Table 3. In spite of the relation (6) which holds for the diagonal matrix elements, the operators O_i and \bar{B}_i are linearly independent, as it can be easily proved.

The other issue for the $[\mathbf{56}, 2^+]$ multiplet is that the analysis performed in Ref. [18] is based on the standard identification of resonances due to the pioneering work of Isgur and Karl [21]. In that work the spectrum of positive parity resonances was calculated from a Hamiltonian containing a harmonic oscillator confinement and a hyperfine interaction of one-gluon exchange type. The mixing angles in the $\Delta_{5/2^+}$ sector turned out to be

State	Mass (MeV)	Mixing angles
${}^4\Delta[\mathbf{56}, 2^+]_{\frac{5}{2}}^+$	1940	$\begin{bmatrix} 0.94 & 0.38 \\ -0.38 & 0.94 \end{bmatrix}$
${}^4\Delta[\mathbf{70}, 2^+]_{\frac{5}{2}}^+$	1975	

which shows that the lowest resonance at 1940 MeV is dominantly a $[\mathbf{56}, 2^+]$ state. As a consequence, the lowest observed F_{35} $\Delta(1905)$ resonance was interpreted as a member of the $[\mathbf{56}, 2^+]$ multiplet.

In a more realistic description, based on a linear confinement [22], the structure of the $\Delta_{5/2^+}$ sector appeared to be different. The result was

State	Mass (MeV)	Mixing angles
${}^4\Delta[\mathbf{56}, 2^+]_{\frac{5}{2}}^+$	1962	$\begin{bmatrix} 0.408 & 0.913 \\ 0.913 & -0.408 \end{bmatrix}$
${}^4\Delta[\mathbf{70}, 2^+]_{\frac{5}{2}}^+$	1985	

which means that in this case the higher resonance, of mass 1985 MeV, is dominantly $[\mathbf{56}, 2^+]$. Accordingly, here we interpret the higher experimentally observed resonance $F_{35} \Delta(2000)$ as belonging to the $[\mathbf{56}, 2^+]$ multiplet instead of the lower one. Thus we take as experimental input the mass 1976 ± 237 MeV, determined from the full listings of the PDG [23] in the same manner as for the one- and two-star resonances of the $[\mathbf{56}, 4^+]$ multiplet (see below). The fit for $[\mathbf{56}, 2^+]$ multiplet based on this assignment is shown in Table 1. The χ_{dof}^2 obtained is 0.58, as compared to $\chi_{\text{dof}}^2 = 0.7$ of Ref. [18]. The contribution of the spin-orbit operator O_2 is slightly smaller here than in Ref. [18]. Although $\Delta(2000)$ is a two-star resonance only, the incentive of making the above choice was that the calculated pion decay widths of the $\Delta_{5/2^+}$ sector were better reproduced [24] with the mixing angles of the model [22] than with those of the standard model of Ref. [21]. It is well known that decay widths are useful to test mixing angles. Moreover, it would be more natural that the resonances $\Delta_{1/2}$ and $\Delta_{5/2}$ would have different masses, contrary to the assumption of Ref. [18] where these masses were taken to be identical.

TABLE 1. Operators of Eq. (1) and coefficients resulting from the fit with $\chi_{\text{dof}}^2 \simeq 0.58$ for $[\mathbf{56}, 2^+]$ and $\chi_{\text{dof}}^2 \simeq 0.26$ for $[\mathbf{56}, 4^+]$.

Operator	Fitted coef. (MeV)	
	$[\mathbf{56}, 2^+]$	$[\mathbf{56}, 4^+]$
$O_1 = N_c \mathbb{1}$	$c_1 = 540 \pm 3$	$c_1 = 736 \pm 30$
$O_2 = \frac{1}{N_c} l_i S_i$	$c_2 = 14 \pm 9$	$c_2 = 4 \pm 40$
$O_3 = \frac{1}{N_c} S_i S_i$	$c_3 = 247 \pm 10$	$c_3 = 135 \pm 90$
$\bar{B}_1 = -\mathcal{S}$	$b_1 = 213 \pm 15$	$b_1 = 110 \pm 67$
$\bar{B}_2 = \frac{1}{N_c} l_i G_{i8} - \frac{1}{2\sqrt{3}} O_2$	$b_2 = 83 \pm 40$	
$\bar{B}_3 = \frac{1}{N_c} S_i G_{i8} - \frac{1}{2\sqrt{3}} O_3$	$b_3 = 266 \pm 65$	

4. THE $[56, 4^+]$ MULTIPLY

Tables 2 and 3 give all matrix elements needed for the octets and decuplets belonging to the $[56, 4^+]$ multiplet. They are calculated following the prescription of the previous section. This means that the matrix elements of O_1 , O_2 , O_3 and \bar{B}_1 are straightforward and for \bar{B}_3 we use the formula (5). The matrix elements of \bar{B}_2 are calculated from the wave functions given explicitly in Ref. [25], firstly derived and employed in constituent quark model calculations [24]. One can see that the relation (6) holds for this multiplet as well.

As mentioned above, only the operator \bar{B}_2 has non-vanishing off-diagonal matrix elements, so \bar{B}_2 is the only one which induces mixing between the octet and decuplet states of $[56, 4^+]$ with the same quantum numbers, as a consequence of the SU(3) flavor breaking. Thus this mixing affects the octet and the decuplet Σ and Ξ states. As there are four off-diagonal matrix elements (Table 3), there are also four mixing angles, namely, θ_J^Σ and θ_J^Ξ , each with $J = 7/2$ and $9/2$. In terms of these mixing angles, the physical Σ_J and Σ'_J states are defined by the following basis states

$$|\Sigma_J\rangle = |\Sigma_J^{(8)}\rangle \cos\theta_J^\Sigma + |\Sigma_J^{(10)}\rangle \sin\theta_J^\Sigma, \quad (7)$$

$$|\Sigma'_J\rangle = -|\Sigma_J^{(8)}\rangle \sin\theta_J^\Sigma + |\Sigma_J^{(10)}\rangle \cos\theta_J^\Sigma, \quad (8)$$

and similar relations hold for Ξ . The masses of the physical states become

$$M(\Sigma_J) = M(\Sigma_J^{(8)}) + b_2 \langle \Sigma_J^{(8)} | \bar{B}_2 | \Sigma_J^{(10)} \rangle \tan\theta_J^\Sigma, \quad (9)$$

$$M(\Sigma'_J) = M(\Sigma_J^{(10)}) - b_2 \langle \Sigma_J^{(8)} | \bar{B}_2 | \Sigma_J^{(10)} \rangle \tan\theta_J^\Sigma, \quad (10)$$

where $M(\Sigma_J^{(8)})$ and $M(\Sigma_J^{(10)})$ are the diagonal matrix of the mass operator (1), here equal to $c_1 O_1 + c_2 O_2 + c_3 O_3 + b_1 \bar{B}_1 + b_2 \bar{B}_2 + b_3 \bar{B}_3$, for Σ states and similarly for Ξ states (see Table 4). If replaced in the mass operator (1), the relations (9) and (10) and their counterparts for Ξ , introduce four new parameters which should be included in the fit. Actually the procedure of Ref. [18] was simplified to fit the coefficients c_i and b_i directly to the physical masses and then to calculate the mixing angle from

$$\theta_J = \frac{1}{2} \arcsin \left(2 \frac{b_2 \langle \Sigma_J^{(8)} | \bar{B}_2 | \Sigma_J^{(10)} \rangle}{M(\Sigma_J) - M(\Sigma'_J)} \right). \quad (11)$$

for Σ_J states and analogously for Ξ states.

Due to the scarcity of data in the 2–3 GeV mass region, even such a simplified procedure is not possible at present in the $[56, 4^+]$ multiplet.

The fit of the masses derived from Eq. (1) and the available empirical values used in the fit, together with the corresponding resonance status in the Particle Data Group [23] are listed in Table 4. The values of the coefficients c_i and b_1 obtained from the fit are presented in Table 1, as already mentioned. For the four and three-star resonances we used the empirical masses given in the summary table. For the others, namely the one-star resonance $\Delta(2390)$ and the two-star resonance $\Delta(2300)$ we adopted the following procedure. We considered as “experimental”

TABLE 2. Matrix elements of SU(3) singlet operators.

	O_1	O_2	O_3
$^2 8_{7/2}$	N_c	$-\frac{5}{2N_c}$	$\frac{3}{4N_c}$
$^2 8_{9/2}$	N_c	$\frac{2}{N_c}$	$\frac{3}{4N_c}$
$^4 10_{5/2}$	N_c	$-\frac{15}{2N_c}$	$\frac{15}{4N_c}$
$^4 10_{7/2}$	N_c	$-\frac{4}{N_c}$	$\frac{15}{4N_c}$
$^4 10_{9/2}$	N_c	$\frac{1}{2N_c}$	$\frac{15}{4N_c}$
$^4 10_{11/2}$	N_c	$\frac{6}{N_c}$	$\frac{15}{4N_c}$

mass the average of all masses quoted in the full listings. The experimental error to the mass was defined as the quadrature of two uncorrelated errors, one being the average error obtained from the same references in the full listings and the other was the difference between the average mass relative to the farthest off observed mass. The masses and errors thus obtained are indicated in the before last column of Table 4.

Due to the lack of experimental data in the strange sector it was not possible to include all the operators \bar{B}_i in the fit in order to obtain some reliable predictions. As the breaking of SU(3) is dominated by \bar{B}_1 we included only this operator in Eq. (1) and neglected the contribution of the operators \bar{B}_2 and \bar{B}_3 . At a later stage, when more data will hopefully be available, all analytical work performed here could be used to improve the fit. That is why Table 1 contains results for c_i ($i = 1, 2$ and 3) and b_1 only. The χ^2_{dof} of the fit is 0.26, where the number of degrees of freedom (dof) is equal to one (five data and four coefficients).

The first column of Table 4 contains the 56 states (each state having a $2I + 1$ multiplicity from assuming an exact SU(2) isospin symmetry¹). The columns two to five show the partial contribution of each operator included in the fit, multiplied by the corresponding coefficient c_i or b_1 . The column six gives the total mass according to Eq. (1). The errors shown in the predictions result from the errors on the coefficients c_i and b_1 given in Table 1. As there are only five experimental data available, nineteen of these masses are predictions. The breaking of SU(3) flavor due to the operator \bar{B}_1 is 110 MeV as compared to 200 MeV produced in the $[56, 2^+]$ multiplet.

¹ Note that the notation Σ_J , Σ'_J is consistent with the relations (9), (10) inasmuch as the contribution of \bar{B}_2 is neglected (same remark for Ξ_J , Ξ'_J and corresponding relations).

TABLE 3. Matrix elements of SU(3) breaking operators. Here, $a_J = 5/2, -2$ for $J = 7/2, 9/2$, respectively and $b_J = 5/2, 4/3, -1/6, -2$ for $J = 5/2, 7/2, 9/2, 11/2$, respectively.

	\bar{B}_1	\bar{B}_2	\bar{B}_3
N_J	0	0	0
Λ_J	1	$\frac{\sqrt{3} a_J}{2N_c}$	$-\frac{3\sqrt{3}}{8N_c}$
Σ_J	1	$-\frac{\sqrt{3} a_J}{6N_c}$	$\frac{\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{2\sqrt{3} a_J}{3N_c}$	$-\frac{\sqrt{3}}{2N_c}$
Δ_J	0	0	0
Σ_J	1	$\frac{\sqrt{3} b_J}{2N_c}$	$-\frac{5\sqrt{3}}{8N_c}$
Ξ_J	2	$\frac{\sqrt{3} b_J}{N_c}$	$-\frac{5\sqrt{3}}{4N_c}$
Ω_J	3	$\frac{3\sqrt{3} b_J}{2N_c}$	$-\frac{15\sqrt{3}}{8N_c}$
$\Sigma_{7/2}^8 - \Sigma_{7/2}^{10}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Sigma_{9/2}^8 - \Sigma_{9/2}^{10}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0
$\Xi_{7/2}^8 - \Xi_{7/2}^{10}$	0	$-\frac{\sqrt{35}}{2\sqrt{3}N_c}$	0
$\Xi_{9/2}^8 - \Xi_{9/2}^{10}$	0	$-\frac{\sqrt{11}}{\sqrt{3}N_c}$	0

Our results can be summarized as follows:

- The main part of the mass is provided by the spin-flavor singlet operator O_1 , which is $\mathcal{O}(N_c)$.
- The spin-orbit contribution given by $c_2 O_2$ is small. This fact reinforces the practice used in constituent quark models where the spin-orbit contribution is usually neglected in order to obtain a good fit. It is also consistent with the intuitive picture of Ref. [26] where the spin-orbit interaction vanishes at high excitation energy.
- The breaking of the SU(6) symmetry keeping the flavor symmetry exact is mainly due to the spin-spin operator O_3 . This hyperfine interaction produces a splitting between octet and decuplet states of approximately 130 MeV which is smaller than that obtained in the $[56, 2^+]$ case [18], which gives 240 MeV.
- The contribution of \bar{B}_1 per unit of strangeness, 110 MeV, is also smaller here than in the $[56, 2^+]$ multiplet [18], where it takes a value of about 200 MeV. That may be quite natural, as one expects a shrinking of the spectrum with the excitation energy.
- As it was not possible to include the contribution of \bar{B}_2 and \bar{B}_3 in our fit, a degeneracy appears between Λ and Σ .

TABLE 4. Masses (in MeV) of the $[\mathbf{56}, 4^+]$ multiplet predicted by the $1/N_c$ expansion as compared with the empirically known masses. The partial contribution of each operator is indicated for all masses. Those partial contributions in blank are equal to the one above in the same column.

1/ N_c expansion results					Total (MeV)	Empirical (MeV)	Name, status
Partial contribution (MeV)							
$c_1 O_1$	$c_2 O_2$	$c_3 O_3$	$b_1 \bar{B}_1$				
$N_{7/2}$	2209	-3	34	0	2240 ± 97		
$\Lambda_{7/2}$				110	2350 ± 118		
$\Sigma_{7/2}$				110	2350 ± 118		
$\Xi_{7/2}$				220	2460 ± 166		
$N_{9/2}$	2209	2	34	0	2245 ± 95	2245 ± 65	N(2220)****
$\Lambda_{9/2}$				110	2355 ± 116	2355 ± 15	$\Lambda(2350)$ ***
$\Sigma_{9/2}$				110	2355 ± 116		
$\Xi_{9/2}$				220	2465 ± 164		
$\Delta_{5/2}$	2209	-9	168	0	2368 ± 175		
$\Sigma_{5/2}$				110	2478 ± 187		
$\Xi_{5/2}$				220	2588 ± 220		
$\Omega_{5/2}$				330	2698 ± 266		
$\Delta_{7/2}$	2209	-5	168	0	2372 ± 153	2387 ± 88	$\Delta(2390)^*$
$\Sigma'_{7/2}$				110	2482 ± 167		
$\Xi'_{7/2}$				220	2592 ± 203		
$\Omega_{7/2}$				330	2702 ± 252		
$\Delta_{9/2}$	2209	1	168	0	2378 ± 144	2318 ± 132	$\Delta(2300)^{**}$
$\Sigma'_{9/2}$				110	2488 ± 159		
$\Xi'_{9/2}$				220	2598 ± 197		
$\Omega_{9/2}$				330	2708 ± 247		
$\Delta_{11/2}$	2209	7	168	0	2385 ± 164	2400 ± 100	$\Delta(2420)$ ****
$\Sigma_{11/2}$				110	2495 ± 177		
$\Xi_{11/2}$				220	2605 ± 212		
$\Omega_{11/2}$				330	2715 ± 260		

In conclusion we have studied the spectrum of highly excited resonances in the 2–3 GeV mass region by describing them as belonging to the $[\mathbf{56}, 4^+]$ multiplet. This is the first study of such excited states based on the $1/N_c$ expansion of QCD. A better description should include multiplet mixing, following the lines developed,

for example, in Ref. [27].

We support previous assertions that better experimental values for highly excited non-strange baryons as well as more data for the Σ^* and Ξ^* baryons are needed in order to understand the role of the operator \bar{B}_2 within a multiplet and for the octet-decuplet mixing. With better data the analytic work performed here will help to make reliable predictions in the large N_c limit formalism.

5. PERSPECTIVES

As clearly stated in Ref. [18] the study of excited baryons is not free of difficulties. The use of the spin-flavor symmetry at zero-th order can be justified only from practical point of view, due to the smallness of the spin-orbit effects and of configuration mixings. More fundamentally, the excited baryons are unstable states for which the consistency condition is more difficult to ensure in large N_c QCD [28].

Moreover the analysis of the multiplet $[56, \ell^+]$ is simplified by the fact that a distinction between excited and core quarks is not necessary. This is not the case for mixed representations.

Our next objective is to analyse the $[70, 0^+]$ and $[70, 2^+]$ multiplets. The simplicity of the $[56, \ell^+]$ does not hold anymore. In addition, one can not assume that the excitation is associated to a single quark which can be decoupled from a core free of excitations, as in the case of $[70, 1^-]$.

This can be illustrated by writing, for example, the two independent wave functions associated to $[70, 2^+]$ in the form where the center of mass motion has been removed. For $N_c = 3$ we have

$$|70, 2^+\rangle_{\rho, \lambda} = \sqrt{\frac{1}{3}}|[21]_{\rho, \lambda}(0s)^2(0d)\rangle + \sqrt{\frac{2}{3}}|[21]_{\rho, \lambda}(0s)(0p)^2\rangle. \quad (12)$$

The coefficients of the linear combination above are independent of N_c (we could prove this assertion for $N_c = 4, 5$ and 6). This implies that the two terms contribute to the same order and we have to consider both of them in the calculations. The first is common to $[56, 2^+]$ and will be treated accordingly. For the second we have to include excitations both in the core and the decoupled quark. We shall analyse the role of an excited core in a future publication.

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