EXCITED BARYONS IN THE $1/N_C$ EXPANSION

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We review results for the mass spectrum of orbitally excited baryons obtained in the $1/N_c$ expansion. We show the dependence of various contributions to the mass operator as a function of the excitation energy.

1. Introduction

At low energies, typical for baryon spectroscopy, QCD does not admit a perturbative expansion in the strong coupling constant. About 30 years ago, 't Hooft suggested an alternative approach based on an $1/N_c$ expansion where N_c is the number of colors[1]. Witten described the counting rules for such an expansion[2]. The method works very well for the ground state baryons inasmuch as they display an SU($2N_f$) exact symmetry, where N_f is the number of flavors[3]. Although for excited states this symmetry is broken, in the last few years it has been realized that an $1/N_c$ expansion can as well be used to describe states belonging to various SU(6) multiplets. A particular attention has been paid to the $[\mathbf{70}, 1^-]$ multiplet[4−10].

Here we review recent work on the mass spectrum of baryons in the $N = 2$ and $N = 4$ bands. We are especially concerned with orbitally excited baryons belonging to the $[\mathbf{56}, 2^+]$, Ref. [14], $[\mathbf{70}, \ell^+]$ ($\ell = 0, 2$), Ref. [15] and $[\mathbf{56}, 4^+]$, Ref. [16], multiplets .

2. Mass operator

The mass operator up to $\mathcal{O}(1/N_c)$ has the general form

$$M = \sum c_i O_i + \sum b_i \bar{B}_i \tag{1}$$

where the operators O_i are SU(3)-flavor singlets and the operators $\bar{B}_i$, which are defined to have vanishing expectation values for non-strange states, break the flavor symmetry. The coefficients c_i and b_i that encode QCD dynamics are evaluated by a numerical fit to the available data. The operators O_i and $\bar{B}_i$ can be expressed as positive parity and rotationally invariant products of generators of SU(6) $\otimes$ O(3).

The analysis of symmetric $[\mathbf{56}, \ell^+]$ states is rather simple[14,16]. The total wave functions are obtained by coupling an orbital part $\sim Y_{\ell m}$ to spin-flavor symmetric states. But for mixed-symmetric representations, it is necessary to split the wave function into two parts : a symmetric core composed of $N_c - 1$ quarks and a excited quark. Generally, in the case of the mixed symmetric representations, one has both core generators ℓ^i_c, S^i_c, T^a_i and G^{ia}_c and excited quark generators ℓ^i_q, s^i, t^a and g^{ia}. The multiplet $[\mathbf{70}, 1^-]$ is a particular case with $\ell_c = 0$.

As an illustration, Table 1 gives the list of operators chosen for the study of the $[\mathbf{70}, \ell^+]$ multiplets[15]. O_1 is the SU(6) scalar operator of order N_c, O_2 and O_4 are the dominant parts of the spin-orbit and spin-spin operators respectively. Note that O_2 is $\mathcal{O}(N_c^0)$ for mixed-symmetric states, in contrast to the symmetric states case where it is $\mathcal{O}(N_c^{-1})$, see Refs. [15, 17]. $O_3 \sim \mathcal{O}(N_c^0)$ due to G^{ja}_c. Strange baryons are not included in this study. Table 1 contains the values of the coefficients c_i obtained by fitting the available experimental data. We present in Table 2 the masses of the resonances which we have interpreted as belonging to the $[\mathbf{70}, 0^+]$ or $[\mathbf{70}, 2^+]$ multiplets.

Table 1. List of operators and the coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 0.83$ for the $[\mathbf{70}, \ell^+]$ multiplets.

Operator	Fitted coef. (MeV)			
$O_1 = N_c \mathbb{1}$	$c_1 =$	555	$\pm$	11
$O_2 = \ell^i_q s^i$	$c_2 =$	47	$\pm$	100
$O_3 = \frac{3}{N_c} \ell^{(2)ij}_q g^{ia} G^{ja}_c$	$c_3 =$	-191	$\pm$	132
$O_4 = \frac{1}{N_c}(S^i_c S^i_c + s^i S^i_c)$	$c_4 =$	261	$\pm$	47

3. The dependence of the coefficients c_i on the excitation energy

It is interesting to see the change of c_i with the excitation energy. In Figure 1, we collect the presently know values of c_i $(i = 1, 2, 4)$ with error bars for the orbitally excited states studied so far in the large N_c expansion: $N = 1$,

Table 2. The partial contribution and the total mass (MeV) predicted for the $[\mathbf{70}, \ell^+]$ by the $1/N_c$ expansion as compared with the empirically known masses.

	$1/N_c$ expansion results						
	Partial contribution				Total	Empirical	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_4O_4			
$^4N[\mathbf{70}, 2^+]\frac{7}{2}^+$	1665	31	42	217	1956 ± 95	2016 ± 104	F_{17}(1990)**
$^2N[\mathbf{70}, 2^+]\frac{5}{2}^+$	1665	10	0	43	1719 ± 34		
$^4N[\mathbf{70}, 2^+]\frac{5}{2}^+$	1665	-5	-106	217	1771 ± 88	1981 ± 200	F_{15}(2000)**
$^4N[\mathbf{70}, 0^+]\frac{3}{2}^+$	1665	0	0	217	1883 ± 17	1879 ± 17	P_{13}(1900)**
$^2N[\mathbf{70}, 2^+]\frac{3}{2}^+$	1665	-16	0	43	1693 ± 42		
$^4N[\mathbf{70}, 2^+]\frac{3}{2}^+$	1665	-31	0	217	1851 ± 69		
$^2N[\mathbf{70}, 0^+]\frac{1}{2}^+$	1665	0	0	43	1709 ± 25	1710 ± 30	P_{11}(1710)***
$^4N[\mathbf{70}, 2^+]\frac{1}{2}^+$	1665	-47	149	217	1985 ± 26	1986 ± 26	P_{11}(2100)*
$^2\Delta[\mathbf{70}, 2^+]\frac{5}{2}^+$	1665	-10	0	87	1742 ± 29	1976 ± 237	P_{35}(2000)**
$^2\Delta[\mathbf{70}, 2^+]\frac{3}{2}^+$	1665	16	0	87	1768 ± 38		
$^2\Delta[\mathbf{70}, 0^+]\frac{1}{2}^+$	1665	0	0	87	1752 ± 19	1744 ± 36	P_{31}(1750)*

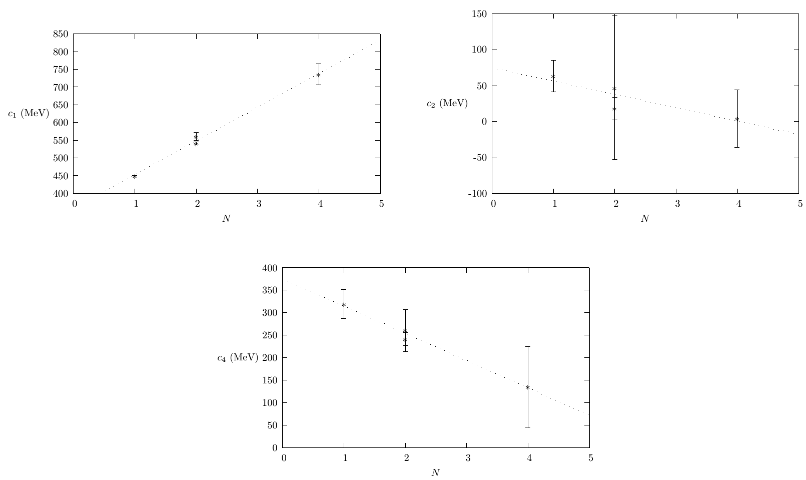

Figure 1. Evolution of the coefficients c_i with the excitation energy corresponding to $N = 1, 2$ and 4. The straight lines are to guide the eye.

Ref. [10], $N = 2$ (lower values[14], upper values[15]) and $N = 4$, Ref. [16]. This behavior shows that at large energies the dominant contribution comes from c_1 and the contributions of the spin-dependent terms vanish. These results are consistent with the quark model picture where the linear term in N_c contains the free mass term, the kinetic and the confinement energy. An intuitive model based on the chiral symmetry restoration has already predicted that the spin dependent interactions vanish at high energies[18].

4. Conclusions

Our work is based on the assumption that there is no multiplet mixing and is restricted to non-strange baryons. Future work is devoted to strange baryons. To better understand the applications of the $1/N_c$ expansion more and better data is desirable.

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