Masses of $[\mathbf{70}, \ell^+]$ Baryons in the $1/N_c$ Expansion

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Abstract

The masses of positive $[\mathbf{70}, 0^+]$ and $[\mathbf{70}, 2^+]$ nonstrange and strange baryons are calculated in the $1/N_c$ expansion. The approach is based on the separation of a system of N_c quarks into an excited core and an excited quark. The previous work for two flavor baryons is now extended to include strange baryons, to first order in SU(3)-flavor breaking. We show that the extension to $N_f = 3$ maintains the regularities previously observed in the behaviour of the linear term in N_c, of the spin-spin and of the spin-orbit terms. In particular the contribution of the spin-dependent terms decrease with the excitation energy, the dominant term remaining the spin-spin term.

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I. INTRODUCTION

The large N_c limit of QCD suggested by 't Hooft [1] and the power counting rules of Witten [2] lead to a consistent perturbative $1/N_c$ expansion method to study baryon spectroscopy, which allows to compute $1/N_c$ corrections in a systematic way. A perspective on the current research status can be found, for example, in Ref. [3]. The method is based on the result that baryons satisfy a contracted spin-flavor algebra in the large N_c limit of QCD [4], which reduces to SU($2N_f$) for ground state baryons, where N_f is the number of flavors. For $N_c \to \infty$ the baryon masses are degenerate. At large N_c, the mass splitting starts at order $1/N_c$ for the ground state baryons (N = 0 band). They belong to the **56** representation of SU(6), and have been described with remarkable success [4, 5, 6, 7, 8, 9, 10]. The applicability of the approach to excited states is a subject of current investigation. Although the SU(6) symmetry is broken for excited states, the experimental facts suggest a small breaking, which then implies that the $1/N_c$ expansion can still be applied. In this case the splitting starts at order N_c^0, as we shall see below.

The excited states belonging to the $[\mathbf{70}, 1^-]$ multiplet (N = 1 band) have been studied extensively in SU(4) ($N_f = 2$) [11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. The approach has been extended to $N_f = 3$ in Ref. [21] and it included first order in SU(3) symmetry breaking. There are also a few studies of the physically important multiplets belonging to the N = 2 band. These are related to $[\mathbf{56}', 0^+]$ in SU(4) [22], to $[\mathbf{56}, 2^+]$ in SU(6) [23] and to $[\mathbf{70}, \ell^+]$ in SU(4) [24]. The method had also been applied to highly excited nonstrange and strange baryons [25] belonging to the $[\mathbf{56}, 4^+]$ multiplet (N = 4 band). So far, configuration mixing has been neglected in the N = 2 band. It would involve new parameters under the form of mixing angles which, to be well determined, would require, generally, much more than the existing data. However the power counting for configuration mixing is quite well established [26].

The 35 SU(6) generators are

$$S^i = \frac{\sigma^i}{2} \otimes \mathbb{1}; \; T^a = \mathbb{1} \otimes \frac{\tau^a}{2}; \; G^{ia} = \frac{\sigma^i}{2} \otimes \frac{\tau^a}{2}, \tag{1}$$

where $i = 1, 2, 3$ and $a = 1, 2, \ldots, 8$. For excited states the mass operator is a linear combination of SU($2N_f$) and SO(3) scalars with coefficients to be determined from a fit. They incorporate the dynamics of quarks and it is important to understand their behaviour. Operators which break SU($2N_f$), but are rotational invariant, can also be added to the mass operator. They embed the SU(3)-flavor breaking, due to the difference in the mass of the strange and nonstrange quarks. The general form of an SU(6) $\times$ SO(3) scalar is

$$O^n = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{2}$$

where $O_\ell^{(k)}$ is a k-rank tensor in SO(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2), but scalar in SU(3)-flavor. This implies that O^n is a combination of SO(3) generators ℓ_i and of SU(6) generators (see below). In calculating the mass spectrum, the general procedure is to split the baryon into an excited quark and a core. The latter is in its ground state for the N = 1 band but generally carries some excitation for N > 1 (for example the $[\mathbf{70}, \ell^+]$ multiplet [24]). The excitation is implemented into the orbital part of the wave function. The advantage of this method is that the problem is reduced to the known case of the ground state, because the spin-flavor part of the core wave function remains always symmetric. But a disadvantage is that one introduces a large number of operators of type (2). Let us denote the excited quark operators by ℓ_q^i, s^i, t^a and g^{ia} and the corresponding core operators by ℓ_c^i, S_c^i, T_c^a and G_c^{ia}. Then, for example, for the $[\mathbf{70}, 1^-]$ multiplet with $N_f = 2$ one has 12 linearly independent operators up to $1/N_c$ power included [15].

In this practice the matrix elements of the excited quark are straightforward, as being single-particle operators. The matrix elements of the core operators S_c^i and T_c^a are also simple to calculate, while those of G_c^{ia} are more involved. Analytic group theory formulas for the matrix elements of all SU(4) generators have been derived in Ref. [27]. They are factorized according to a generalized Wigner-Eckart theorem into a reduced matrix element and an SU(4) Clebsch-Gordan coefficient. They have been used in nuclear physics, which is governed by the SU(4) symmetry, but can be straightforwardly be applied to a system of arbitrary N_c quarks containing the isodoublet u, d. Recently we have extended the approach of Ref. [27] to SU(6) [28] and obtained matrix elements of all SU(6) generators between symmetric $[N_c]$ states. These matrix elements are used below. The matrix elements of G_c^{ia} with nonzero strangeness presented in Ref. [29] are particular cases of the results of Ref. [28].

We should keep in mind that the excited states are resonances and have a finite width. Generic large N_c counting rules give widths of order N_c^0 [11, 13, 14, 16, 20, 30]. According to Ref. [11] the narrowness of the excited states is an artifact of simple quark model assumptions. Here, as in constituent quark models, we do ignore the finite width and treat the resonances as bound states.

The paper is organized as follows. In the next section we recall the orbital structure of the wave functions of the $[\mathbf{70}, \ell^+]$ baryon multiplet. Section 3 is devoted to the formalism of the mass operator. In Sec. 4 we present results for the masses of 47 nonstrange and strange baryons, most of which are predictions. The last section contains our conclusions. Appendix A is devoted to the operators O_3, O_4 and to the isospin operator O_6 which are of order $\mathcal{O}(N_c^0)$. The first two are operators for which the matrix elements change the analytic form as a function of N_c, when going from $N_f = 2$ to $N_f = 3$. Appendix B gives the general formula for the matrix elements of SU(3)-flavor breaking operators needed to construct B_1, B_2 and B_4. Appendix C gives the matrix elements of the spin-orbit operator O_2.

II. THE WAVE FUNCTIONS OF $[\mathbf{70}, \ell^+]$ EXCITED STATES

For the time being, we adopt the usual practice and divide the system of N_c quarks into an excited quark and a core, which can be excited or not. Below we use the notations given in our previous work [24]. We introduce the quark model indices ρ and λ to distinguish between the two independent orbital wave functions of the multiplet $[\mathbf{70}, \ell^+]$. The first is associated with states which are antisymmetric under the permutation of the first two particles while the second implies symmetry under the same permutation. Then, for $\ell = 0$ the orbital wave function is

$$|\mathbf{N_c - 1, 1}, 0^+\rangle_{\rho,\lambda} = \sqrt{\frac{1}{3}}|[N_c - 1, 1]_{\rho,\lambda}(0s)^{N_c-1}(1s)\rangle + \sqrt{\frac{2}{3}}|[N_c - 1, 1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2\rangle. \quad (3)$$

In the first term $1s$ is the first (single particle) radially excited state with $n = 1$, $\ell = 0$ ($N = 2n + \ell$). In the second term the two quarks are excited to the p-shell to get $N = 2$. They are coupled to $\ell = 0$. By analogy, for $\ell = 2$ one has

$$|\mathbf{N_c - 1, 1}, 2^+\rangle_{\rho,\lambda} = \sqrt{\frac{1}{3}}|[N_c - 1, 1]_{\rho,\lambda}(0s)^{N_c-1}(0d)\rangle + \sqrt{\frac{2}{3}}|[N_c - 1, 1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2\rangle, \quad (4)$$

where the two quarks in the p-shell are coupled to $\ell = 2$. One can see that the coefficients of the linear combinations (3) and (4) are independent of N_c so that both terms have to be considered in the large N_c limit. In Eqs. (3) and (4) the first term can be treated as in the $[\mathbf{70}, 1^-]$ sector, *i.e.* as an excited quark coupled to a ground state core [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The second term will be treated here as an excited quark coupled to an excited core. To see this, we rewrite it by using the fractional parentage technique to get

$$
\begin{aligned}
|[N_c-1,1]_{\rho,\lambda}(0s)^{N_c-2}(0p)^2,\ell^+\rangle &= \sqrt{\frac{N_c-2}{N_c}}\Psi_{[N_c-1]}((0s)^{N_c-2}(0p))\phi_{[1]}(0p) \\
&\quad -\sqrt{\frac{2}{N_c}}\Psi_{[N_c-1]}((0s)^{N_c-3}(0p)^2)\phi_{[1]}(0s), \qquad (5)
\end{aligned}
$$

both for $\ell=0$ and 2. Here all states are normalized. The first factor in each term in the right-hand side is a symmetric (N_c-1)-particle wave function and $\phi_{[1]}$ is a one particle wave function associated to the N_c-th particle. One can see that for large N_c the coefficient of the first term is $\mathcal{O}(1)$ and of the second $\mathcal{O}(N_c^{-1/2})$. Then, in the large N_c limit, one can neglect the second term and take into account only the first term, where both the core and N_c-th particle have an $\ell=1$ excitation.

Each of the above configurations $(0s)^{N_c-1}(1s)$, $(0s)^{N_c-1}(0d)$ or $(0s)^{N_c-2}(0p)^2$ represent orbital parts of a given total wave function. We denote by ℓ_q and ℓ_c the angular momenta of the excited quark and of the excited core respectively. They are coupled to a total angular momentum ℓ. Then in SU(6) $\times$ SO(3) the most general form of the wave function is

$$
\begin{aligned}
|\ell S; JJ_3; (\lambda\mu)YII_3\rangle &= \sum_{m_c,m_q,m_\ell,S_3} \left(\begin{array}{cc|c} \ell_c & \ell_q & \ell \\ m_c & m_q & m_\ell \end{array}\right) \left(\begin{array}{cc|c} \ell & S & J \\ m_\ell & S_3 & J_3 \end{array}\right) \\
&\times \sum_{pp'} c^{[N_c-1,1]}_{pp'}(S)|SS_3;p\rangle|(\lambda\mu)YII_3;p'\rangle|\ell_q m_q\rangle|\ell_c m_c\rangle, \qquad (6)
\end{aligned}
$$

where

$$
|SS_3;p\rangle = \sum_{m_1,m_2} \left(\begin{array}{cc|c} S_c & \frac{1}{2} & S \\ m_1 & m_2 & S_3 \end{array}\right) |S_c m_1\rangle|1/2 m_2\rangle, \qquad (7)
$$

with $p=1$ if $S_c=S-1/2$ and $p=2$ if $S_c=S+1/2$ and

$$
|(\lambda\mu)YII_3;p'\rangle = \sum_{Y_c,I_c,I_{c3},y,i,i_3} \left(\begin{array}{cc|c} (\lambda_c\mu_c) & (10) & (\lambda\mu) \\ Y_cI_cI_{c3} & yii_3 & YII_3 \end{array}\right) |(\lambda_c\mu_c)Y_cI_cI_{c3}\rangle|(10)yii_3\rangle, \qquad (8)
$$

where $p'=1$ if $(\lambda_c\mu_c)=(\lambda-1,\mu)$, $p'=2$ if $(\lambda_c\mu_c)=(\lambda+1,\mu-1)$ and $p'=3$ if $(\lambda_c\mu_c)=(\lambda,\mu+1)$. The spin-flavor part of the wave function (6) of symmetry $[N_c-1,1]$ results from the inner product of the spin and flavor wave functions. The indices p and p' represent the row where the last particle (the excited quark) is located in the Young diagram of SU(2)-spin and SU(3)-flavor states respectively. Thus the coefficients $c^{[N_c-1,1]}_{pp'}(S)$ are isoscalar factors [31, 32] of the permutation group of N_c particles, the expressions of which are [28]

$$
\begin{aligned}
c_{11}^{[N_c-1,1]}(S) &= -\sqrt{\frac{(S+1)(N_c-2S)}{N_c(2S+1)}}, \\
c_{22}^{[N_c-1,1]}(S) &= \sqrt{\frac{S[N_c+2(S+1)]}{N_c(2S+1)}}, \\
c_{12}^{[N_c-1,1]}(S) &= c_{21}^{[N_c-1,1]}(S) = 1, \\
c_{13}^{[N_c-1,1]}(S) &= 1.
\end{aligned}
\tag{9}
$$

In Eqs. (10)-(13) below, we illustrate their application for $N_c = 7$. In each inner product the first Young diagram corresponds to spin and the second to flavor. Accordingly, one can see that Eq. (10) stands for $^2 10$, Eq. (11) for $^4 8$, Eq. (12) for $^2 8$ and Eq. (13) for $^2 1$. Each inner product contains the corresponding isoscalar factors and the position of the last particle is marked with a cross. In the right hand side, from the location of the cross one can read off the values of p and of p'. The equations are

$$
\begin{ytableau} \ & \ & \ & \ & \ & \ \\ \times \end{ytableau} = c_{21}^{[6,1]} \begin{ytableau} \ & \ & \ & \ \\ \ & \ & \times \end{ytableau} \times \begin{ytableau} \ & \ & \ & \ & \times \\ \ & \ \end{ytableau}, \tag{10}
$$

$$
\begin{ytableau} \ & \ & \ & \ & \ & \ \\ \times \end{ytableau} = c_{12}^{[6,1]} \begin{ytableau} \ & \ & \ & \ & \times \\ \ & \ \end{ytableau} \times \begin{ytableau} \ & \ & \ & \ \\ \ & \ & \times \end{ytableau}, \tag{11}
$$

$$
\begin{aligned}
\begin{ytableau} \ & \ & \ & \ & \ & \ \\ \times \end{ytableau} &= c_{11}^{[6,1]} \begin{ytableau} \ & \ & \ & \times \\ \ & \ & \ \end{ytableau} \times \begin{ytableau} \ & \ & \ & \times \\ \ & \ & \ \end{ytableau} \\
&+ c_{22}^{[6,1]} \begin{ytableau} \ & \ & \ & \ \\ \ & \ & \times \end{ytableau} \times \begin{ytableau} \ & \ & \ & \ \\ \ & \ & \times \end{ytableau},
\end{aligned}
\tag{12}
$$

$$
\begin{ytableau} \ & \ & \ & \ & \ & \ \\ \times \end{ytableau} = c_{13}^{[6,1]} \begin{ytableau} \ & \ & \ & \times \\ \ & \ & \ \end{ytableau} \times \begin{ytableau} \ & \ & \ \\ \ & \ & \ \\ \times \end{ytableau}. \tag{13}
$$

For the configurations $(0s)^{N_c-1}(1s)$ and $(0s)^{N_c-1}(0d)$ the expression (6) slightly simplifies because $\ell_c = 0$. Only for the configuration $(0s)^{N_c-2}(0p)^2$ the core is excited with $\ell_c = 1$, in agreement with the discussion following Eq. (5).

III. THE MASS OPERATOR

For the $[\mathbf{70}, \ell^+]$ multiplet the mass operator can be written as the linear combination

$$
M_{[\mathbf{70},\ell^+]} = \sum_{i=1}^{6} c_i O_i + d_1 B_1 + d_2 B_2 + d_4 B_4, \tag{14}
$$

where the operators O_i are of type (2) and B_i are SU(6) breaking operators defined below. The values of the coefficients c_i and d_i which encode the QCD dynamics, are given in Table I. They were found by a numerical fit described in the next section.

The building blocks of O_i and B_i are the excited core operators ℓ^i_c, S^i_c, T^a_c and G^{ia}_c and the excited quark operators ℓ^i_q, s^i, t^a and g^{ia}. We also introduce the rank $k = 2$ tensor operator [39]

$$\ell^{(2),ij}_{ab} = \frac{1}{2}\left\{\ell^i_a, \ell^j_b\right\} - \frac{1}{3}\delta_{i,-j}\vec{\ell}_a \cdot \vec{\ell}_b, \tag{15}$$

with $a = c$, $b = q$ or vice versa or $a = b = c$ or $a = b = q$. For simplicity when $a = b$, we use a single index c, for the core, or q for the excited quark so that the tensor operators become $\ell^{(2),ij}_c$ and $\ell^{(2),ij}_q$ respectively. The latter case represents the tensor operator used in the analysis of the $[\mathbf{70}, 1^-]$ multiplet (see *e.g.* Ref. [15]).

There are many linearly independent operators O_i and B_i which can be constructed from the excited quark and the core operators. Here, due to lack of data, we have considered a restricted list containing the most dominant operators in the mass formula. The selection was determined from the previous experience of Refs. [15] and [24] for $N_f = 2$ and of Ref. [21] for $N_f = 3$. The operators O_i entering Eq. (14) are listed in Table I. O_1 is linear in N_c and is the most dominant in the mass formula. At $N_c \to \infty$ is the only one which survives. O_2 is the dominant part of the spin-orbit operator. It acts on the excited quark and is of order N^0_c. The operator O_3 is a composite two-body operator. It contains the tensor operator (15) which acts on the excited quark and the generators g^{ia} and G^{ja}_c acting on the the excited quark and on the core respectively. The contribution of G^{ja}_c sums coherently, thus it introduces an extra power in N_c, which implies that the matrix elements O_3 are of order N^0_c. For the same reason the matrix elements of O_4 are also of order N^0_c. As explained in the next section, we could not obtain its coefficient c_4, because of scarcity of data for the $[\mathbf{70}, \ell^+]$ multiplet. The spin-spin operator O_5 is of order $1/N_c$, but its contribution dominates over all the other terms of the mass operator containing spin.

Here we take into account the isospin-isospin operator, denoted by O_6, having matrix elements of order N^0_c due to the presence of T_c which sums coherently. Up to a subtracting constant, it is one of the four independent operators of order N^0_c, which, together with O_1, are needed to describe the submultiplet structure of $[\mathbf{70}, 1^-]$ [33]. Incidently, this operator has been omitted in the analysis of Ref. [21]. Its coefficent c_6 is indicated in Table I.

In Tables II, III and IV we show the diagonal matrix elements of the operators O_i for octet, decuplet and flavor singlet states respectively. From these tables one can obtain the large N_c mentioned above. Details about O_3 are given in Appendix A. Its matrix elements change the analytic dependence on N_c in going from SU(2) to SU(3). This happens for octet resonances which can be seen by comparing the column 3 of Table II with the corresponding result from Ref. [24]. The change is that the factor $N_c + 1$ in SU(2) becomes $N_c + 1/3$ in SU(3). The same change takes place for all operators O_i containing G_c^{ja} as for example the operator O_4 also presented in Appendix A.

The SU(6) breaking operators, B_1 and B_2 and B_4 in the notation of Ref. [21], expected to contribute to the mass are listed in Table I. The operators B_1, B_2 are the standard breaking operators while B_4 is directly related to the spin-orbit splitting. They break the SU(3)-flavor symmetry to first order in $\epsilon \simeq 0.3$ where ϵ is proportional to the mass difference between the strange and u, d quarks. Table V gives the matrix elements of the excited quark operator t_8 and of the core operator T_8^c which are necessary to construct the matrix elements of B_1 and B_2. These expressions have been obtained as indicated in Appendix B. It is interesting to note that they are somewhat different from those of Ref. [21]. However for all cases with physical quantum numbers but any N_c, our values are identical to those of Ref. [21], so that for $N_c = 3$ there is no difference.

For completeness, Table VI gives the matrix elements of $3\ell^i g^{i8}$ needed to construct B_4. They were obtained from the formula (B3) derived in Appendix B. As above, they are different from these of Ref. [21] except for physical quantum numbers. Unfortunately none of the presently known resonances has nonvanishing matrix elements for B_4. By definition all B_i have zero matrix elements for nonstrange resonances. In addition, the matrix elements of B_4 for $\ell = 0$ resonances also cancel and for the two remaining experimentally known strange resonances they also cancel out. For this reason the coefficient d_4 could not be determined.

IV. RESULTS

Comparing Table I with our previous results Ref. [24] for nonstrange baryons, one can see that the addition of strange baryons in the fit have not much changed the values of the coefficients c_1 and c_5 (previously c_4). The spin-orbit coefficient c_2 had changed sign but in absolute value remains small. The resonance $F_{05}(2100)$ is mostly responsible for this change. But actually the crucial experimental input for the spin-orbit contribution should come from Λ's, as in the case of the $[\mathbf{70}, 1^-]$ multiplet [21]. Unfortunately data for the two flavor singlets with $\ell \neq 0$, ${}^2\Lambda'[\mathbf{70}, 2^+]5/2$ and ${}^2\Lambda'[\mathbf{70}, 2^+]3/2$, which are spin-orbit partners are missing (see Table VII). If observed, they will help to fix the strength and sign of the spin-orbit terms unambiguously inasmuch as O_3, O_4 and O_5 do not contribute to their mass.

Presently, due to the large uncertainty obtained from the fit of c_2, there is still some overlap with the value obtained from nonstrange resonances. The coefficient c_3 is about twice smaller in absolute value now. Interestingly, the present values of the coefficients c_1, c_2 and c_5 follow the trend discussed in Ref. [24], namely the spin-spin and the spin-orbit contributions decrease with the excitation energy, the dominant part remaining the spin-spin term, similar to constituent quark model results with a hyperfine interaction.

Regarding the SU(3) breaking terms, the coefficient d_1 is has opposite sign as compared to that of Ref. [21] and is about four times larger in absolute value. The coefficient d_2 has the same sign and about the same order of magnitude. One can conclude that the SU(3)-flavor breaking is roughly similar in the $[\mathbf{70}, 1^-]$ and the $[\mathbf{70}, \ell^+]$ multiplets.

The resonances belonging to the $[\mathbf{70}, \ell^+]$ together with their calculated masses are presented in Table VII. The angular momentum coupling allows for 8 octets, with J ranging from 7/2 to 1/2, three decuplets with J from 5/2 to 1/2 and three flavor singlets with $J =$ 5/2, 3/2 or 1/2. Ignoring isospin breaking, there are in all 47 resonances from which 12 are fitted and 35 are predictions. The best fit gave $\chi^2_{\rm dof} \simeq 1$. Among the presently 12 resonances only five are new, the strange resonances. This reflects the fact that the experimental situation is still rather poor in this energy range. The known resonances are three-, two- and one-star.

For all masses the main contribution comes from the operator O_1. In the context of a constituent quark model this corresponds to the contribution of the spin-independent part of the Hamiltonian, namely the free mass term plus the kinetic and the confinement energy. A difference is that, this contribution is constant for all resonances here, while in quark models the mass difference between the strange and the u, d quarks is taken into account explicitly in the free mass term. Here this difference is embedded into the flavor breaking terms B_i .

The spin-orbit operator O_2 naturally contributes to states with $\ell \neq 0$ only. The operator O_3 contributes to states with $S = 3/2$ only. For $S = 1/2$ states it gives no contribution either due to the cancellation of a $6j$ coefficient or when the wave function has $S_c = 0$, as for example for flavor singlet states.

We have analyzed the role of the operator O_4 described in Appendix B. This is an operator of order N_c^0, like O_2, O_3 and O_6. As in Refs. [15] and [21], the combination $O_2 + O_4$ is of order $1/N_c$ for octets and decuplets, but this is no longer valid for flavor singlets. It means that the operators O_2 and O_4 are independent in SU(3) and both have to be included in the fit. However, the inclusion of O_4 considerably deteriorated the fit, by abnormally increasing the spin-orbit contribution with one order of magnitude. Therefore the contribution of O_4 cannot be constrained with the present data and we have to wait until more data will be available, especially on strange resonances.

To estimate the role of the isospin-isospin operator O_6 we have made a fit without the contribution of this operator. This fit gave $\chi^2_{\rm dof} \simeq 0.9$ and about the same values for c_i and d_i as that with O_6 included. This means that the presence of O_6 is not essential at the present stage.

The fitted value of the $N(1990)F_{17}$ resonance slightly deteriorates with respect to the SU(4) case. The reason is the negative contribution of the spin-orbit term. Further analysis, based on more data, is needed in the future, to clarify the change of sign in the spin-orbit term.

Of special interest is the fact that the resonance $\Lambda(1810)P_{01}$ gives the best fit when interpreted as a flavor singlet. Such an interpretation is in agreement with that of Ref. [34] where the baryon spectra were derived from a flavor-spin hyperfine interaction, rooted in pseudo-scalar meson (Goldstone boson) exchange. Thus the flavor-spin symmetry is common to both calculations. Moreover, the dynamical origin of the operator O_3, which does not directly contribute to $\Lambda(1810)P_{01}$, but plays an important role in the total fit, is thought to be related to pseudo-scalar meson exchange [15]. Hopefully, this study may help in shedding some light on the QCD dynamics hidden in the coefficients c_i.

V. CONCLUSIONS

The present results confirm the behaviour of some of the coefficients c_i of the mass formula at large excitation energy, observed previously [24]. This shows that the importance of spin-dependent terms of the mass operators vanish with the excitation energy. At any energy, these terms are dominated by the spin-spin contribution, like in constituent quark model studies. Thus the $1/N_c$ expansion can provide a deeper understanding of the successes of the quark models.

We have also found that the SU(3) breaking corrections are comparable in size with the $1/N_c$ corrections, as for the $[\mathbf{70}, 1^-]$ multiplet [21] which successfully explained the $\Lambda(1520) - \Lambda(1405)$ splitting.

The analysis of the $[\mathbf{70}, \ell^+]$ remains an open problem. It depends on future experimental data which may help to clarify the role of various terms contributing to the mass operator and in particular of O_2 and O_4. The present approach provides the theoretical framework to pursue this study.

APPENDIX A: THE OPERATORS O_3 AND O_4

Here we derive analytic expressions for the matrix elements of the operators O_3 and O_4 in SU(6). The compact form of O_3 is given in Table I

$$O_3 = \frac{3}{N_c} \ell_q^{(2),ij} g^{ia} G_c^{ja}. \tag{A1}$$

Writing the scalar products in an explicit form we have

$$O_3 = \frac{3}{N_c} \sum_{ij} (-1)^{i+j} \ell_q^{(2),-i,-j} \sum_{Y^a I_3^a} (-1)^{I_3^a + Y^a/2} g^{ia} G_c^{ja}, \tag{A2}$$

with $i, j = 1, 2, 3$ and $a = 1, 2, ..., 8$ and where $\ell_q^{(2)ij}$ is defined in Eq. (15). The matrix elements of the tensor operator are

$$\langle \ell' m' | \ell^{(2),ij} | \ell m \rangle = \delta_{\ell\ell'} \left[\frac{\ell(\ell+1)(2\ell-1)(2\ell+3)}{6} \right]^{1/2} \sum_\mu \left(\begin{array}{cc|c} 1 & 1 & 2 \\ i & j & \mu \end{array} \right) \left(\begin{array}{cc|c} \ell & 2 & \ell' \\ m & \mu & m' \end{array} \right). \tag{A3}$$

The other basic ingredients are the matrix elements of the operators g^{ia} and G_c^{ja}. As explained in Ref [28], we have

$$\langle \frac{1}{2} m_2; (10) y' i' i_3' | g^{ia} | \frac{1}{2} m_2; (10) y i i_3 \rangle = \left(\begin{array}{cc|c} \frac{1}{2} & 1 & \frac{1}{2} \\ m_2 & i & m_2' \end{array} \right) \left(\begin{array}{cc|c} (10) & (11) & (10) \\ y i i_3 & y^a i^a i_3^a & y' i' i_3' \end{array} \right), \tag{A4}$$

and

$$\langle [N_c - 1] S_c' m_1'; (\lambda_c' \mu_c') Y_c' I_c' I_{c3}' | G_c^{ja} | [N_c - 1] S_c m_1; (\lambda_c \mu_c) Y_c I_c I_{c3} \rangle =$$

$$\frac{1}{\sqrt{2}}\sqrt{\frac{5}{12}(N_c-1)(N_c+5)}\left(\begin{array}{cc|c} S_c & 1 & S'_c \\ m_1 & j & m'_1 \end{array}\right)\left(\begin{array}{cc|c} I_c & I^a & I'_c \\ I_{c3} & I^a_3 & I'_{c3} \end{array}\right)$$
$$\times\sum_{\rho=1,2}\left(\begin{array}{cc||c} (\lambda_c\mu_c) & (11) & (\lambda'_c\mu'_c) \\ Y_cI_c & Y^aI^a & Y'_cI'_c \end{array}\right)_\rho\left(\begin{array}{cc||c} [N_c-1] & [21^4] & [N_c-1] \\ (\lambda_c\mu_c)S_c & (11)1 & (\lambda'_c\mu'_c)S'_c \end{array}\right)_\rho, \qquad \text{(A5)}$$

where the SU(3) isoscalar factors are from Ref. [35] and the SU(6) isoscalar factors can be found in Table 1 of Ref. [28]. The final formula for the matrix elements of O_3 between states of mixed orbital symmetry $[N_c-1,1]$ is

$$\langle \ell S'; JJ_3; (\lambda\mu)YII_3|O_3|\ell S; JJ_3; (\lambda\mu)YII_3\rangle =$$
$$(-1)^{\ell_q+\ell_c+S'+J+1}\frac{5}{4}(2\ell+1)\sqrt{\ell_q(\ell_q+1)(2\ell_q-1)(2\ell_q+1)(2\ell_q+3)}\left\{\begin{array}{ccc} \ell & 2 & \ell \\ \ell_q & \ell_c & \ell_q \end{array}\right\}$$
$$\times\sqrt{2(N_c-1)(N_c+5)(2S+1)(2S'+1)}\left\{\begin{array}{ccc} S & 2 & S' \\ \ell & J & \ell \end{array}\right\}$$
$$\times\sum_{p,p',q,q'} c^{[N_c-1,1]}_{pp'}(S)c^{[N_c-1,1]}_{qq'}(S')\sqrt{2S'_c+1}\left\{\begin{array}{ccc} S'_c & S' & 1/2 \\ 1 & 2 & 1 \\ S_c & S & 1/2 \end{array}\right\}$$
$$\times\sum_{\rho=1,2} U((\lambda_c\mu_c)(11)(\lambda\mu)(10);(\lambda'_c\mu'_c)(10))_\rho\left(\begin{array}{cc||c} [N_c-1] & [21^4] & [N_c-1] \\ (\lambda_c\mu_c)S_c & (11)1 & (\lambda'_c\mu'_c)S'_c \end{array}\right)_\rho, \qquad \text{(A6)}$$

where the coefficients $c^{[N_c-1,1]}_{pp'}(S)$ are given by Eqs. (9). We recall that $S_c = S-1/2$ for $p=1$ and $S_c = S+1/2$ for $p=2$ and by analogy $S'_c = S'-1/2$ for $q=1$ and $S'_c = S'+1/2$ for $q=2$. Also $(\lambda_c\mu_c) = (\lambda-1,\mu)$ for $p'=1$, $(\lambda_c\mu_c) = (\lambda+1,\mu-1)$ for $p'=2$ and $(\lambda_c\mu_c) = (\lambda,\mu+1)$ for $p'=3$ and an analogous situation for $(\lambda'_c\mu'_c) = (\lambda-1,\mu)$ if $q'=1$, $(\lambda'_c\mu'_c) = (\lambda+1,\mu-1)$ if $q'=2$ and $(\lambda'_c\mu'_c) = (\lambda,\mu+1)$ if $q'=3$.

When applied on the excited quark the operator O_4 reads [21]

$$O_4 = \frac{4}{N_c+1}\ell^i_q t^a G^{ia}_c. \qquad \text{(A7)}$$

Writing the scalar products explicitly we have

$$O_4 = \frac{4}{N_c+1}\sum_i(-1)^i\ell^i_q\sum_{Y^aI^a_3}(-1)^{I^a_3+Y^a/2}t^{-a}G^{-ia}_c\ . \qquad \text{(A8)}$$

The matrix elements of t^a are [28]

$$\langle(10)y'i'i'_3|t^{-a}|(10)yii_3\rangle = \sqrt{\frac{4}{3}}\left(\begin{array}{cc|c} i & I^a & i' \\ i_3 & -I^a_3 & i'_3 \end{array}\right)\left(\begin{array}{cc||c} (10) & (11) & (10) \\ yi & -Y^aI^a & y'i' \end{array}\right) \qquad \text{(A9)}$$

Inserting the above expression and the matrix elements of G_c^{ia} , Eq. (A5), into (A7) one obtains

$$\langle \ell S'; JJ_3; (\lambda\mu)YII_3|O_4|\ell S; JJ_3; (\lambda\mu)YII_3\rangle = (-1)^{\ell_q+\ell_c+S'-S+J+1/2}\frac{4}{N_c+1}$$
$$\times(2\ell+1)\sqrt{\ell_q(\ell_q+1)(2\ell_q+1)}\begin{Bmatrix} \ell & 1 & \ell \\ \ell_q & \ell_c & \ell_q \end{Bmatrix}$$
$$\times\sqrt{\frac{5}{18}(N_c-1)(N_c+5)(2S+1)(2S'+1)}\begin{Bmatrix} J & \ell & S \\ 1 & S' & \ell \end{Bmatrix}$$
$$\times\sum_{p,p',q,q'} c_{pp'}^{[N_c-1,1]}(S)c_{qq'}^{[N_c-1,1]}(S')\sqrt{2S'_c+1}(-1)^{-S'_c}\begin{Bmatrix} S' & 1/2 & S'_c \\ S_c & 1 & S \end{Bmatrix}$$
$$\times\sum_{\rho=1,2} U((\lambda_c\mu_c)(11)(\lambda\mu)(10);(\lambda'_c\mu'_c)(10))_\rho \left(\begin{array}{cc} [N_c-1] & [21^4] \\ (\lambda_c\mu_c)S_c & (11)1 \end{array}\middle\| \begin{array}{c} [N_c-1] \\ (\lambda'_c\mu'_c)S'_c \end{array}\right)_\rho . \quad \text{(A10)}$$

The unitary Racah coefficients U, defined according to Ref. [35], which are needed to calculate (A6) and (A10) have been obtained as in Ref. [28]. Their explicit forms are

$$\begin{aligned}
U((\lambda-1,\mu)(11)(\lambda\mu)(10);(\lambda+1,\mu-1)(10)) &= -\frac{1}{2}\sqrt{\frac{3(\lambda+2)\mu}{2(\lambda+1)(\mu+1)}}, \\
U((\lambda+1,\mu-1)(11)(\lambda\mu)(10);(\lambda-1,\mu)(10)) &= \frac{1}{2}\sqrt{\frac{3\lambda(\lambda+\mu+1)}{2(\lambda+1)(\lambda+\mu+2)}}, \\
U((\lambda,\mu+1)(11)(\lambda\mu)(10);(\lambda,\mu+1)(10))_{\rho=1} &= \frac{\lambda+2\mu+8}{4\sqrt{g_{\lambda,\mu+1}}}, \\
U((\lambda,\mu+1)(11)(\lambda\mu)(10);(\lambda,\mu+1)(10))_{\rho=2} &= \frac{1}{4}\sqrt{\frac{3\lambda(\lambda+2)(\mu+3)(\lambda+\mu+4)}{(\mu+1)(\lambda+\mu+2)g_{\lambda,\mu+1}}}, \\
U((\lambda-1,\mu)(11)(\lambda\mu)(10);(\lambda-1,\mu)(10))_{\rho=1} &= -\frac{2\lambda+\mu-2}{4\sqrt{g_{\lambda-1,\mu}}}, \\
U((\lambda-1,\mu)(11)(\lambda\mu)(10);(\lambda-1,\mu)(10))_{\rho=2} &= \frac{1}{4}\sqrt{\frac{3(\lambda+\mu)(\lambda-1)\mu(\mu+2)}{(\lambda+1)(\lambda+\mu+2)g_{\lambda-1,\mu}}}, \\
U((\lambda+1,\mu-1)(11)(\lambda\mu)(10);(\lambda+1,\mu-1)(10))_{\rho=1} &= \frac{\lambda-\mu+5}{4\sqrt{g_{\lambda+1,\mu-1}}}, \\
U((\lambda+1,\mu-1)(11)(\lambda\mu)(10);(\lambda+1,\mu-1)(10))_{\rho=2} &= \\
-\frac{1}{4}\sqrt{\frac{3(\lambda+\mu+1)(\lambda+\mu+3)(\lambda+3)(\mu-1)}{(\lambda+1)(\mu+1)g_{\lambda+1,\mu-1}}}, & \qquad \text{(A11)}
\end{aligned}$$

where

$$g_{\lambda\mu} = \lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu. \quad \text{(A12)}$$

All U coefficients, but the 4th one, are of order $\mathcal{O}(N_c^0)$ which can be seen by inserting $\lambda = 2S$ and $\mu = N_c/2 - S$. This helps in finding the order of the matrix elements of O_4.

The matrix elements of O_6 are given by the product of $1/N_c$ and

$$\langle \ell S J J_3; (\lambda'\mu')Y'I'I'_3 | t^a T^a_c | \ell S J J_3; (\lambda\mu)YII_3\rangle = \delta_{\lambda\lambda'}\delta\mu\mu'\delta_{YY'}\delta_{II'}\delta_{I_3I'_3}$$
$$\times(-1)\sum_{pp'}\left[c^{[N_c-1,1]}_{pp'}(S)\right]^2 \frac{2\sqrt{g_{\lambda_c\mu_c}}}{3} U((\lambda_c\mu_c)(11)(\lambda\mu)(10);(\lambda_c\mu_c)(10))_1 \quad \text{(A13)}$$

where the (-1) sign results from a phase entering the symmetry property of SU(3) Clebsch-Gordan coefficients [36]. This is

$$\left(\begin{array}{cc|c} (10) & (11) & (10) \\ YI & Y^aI^a & Y'I' \end{array}\right) = \xi_1(-1)^{I+I^a-I'} \left(\begin{array}{cc|c} (11) & (10) & (10) \\ Y^aI^a & YI & Y'I' \end{array}\right). \quad \text{(A14)}$$

where $\xi_1 = -1$ in this case. The same property has also been used in the calculation of the matrix elements of O_3 and O_4. A simpler alternative is to calculate the matrix elements of O_6 by using the identity

$$t \cdot T_c = \frac{1}{2}(T^2 - T_c^2 - t^2)$$

which gives

$$\langle \ell S J J_3; (\lambda\mu)YII_3 | t^a T^a_c | \ell S J J_3; (\lambda\mu)YII_3\rangle = \frac{1}{6}\left\{ g_{\lambda\mu} - \sum_{pp'}\left[c^{[N_c-1,1]}_{pp'}(S)\right]^2 g_{\lambda_c\mu_c} - 4\right\} \quad \text{(A15)}$$

The formulas (A13) and (A15) give identical results.

APPENDIX B

Here we reproduce the general formulas [28] of the matrix elements of the flavor breaking operators t_8, T^c_8 and $\ell^i_q g^{i8}$ which have been used to generate Table V and VI. These are

$$\langle \ell S J J_3; (\lambda'\mu')Y'I'I'_3 | T^8_c | \ell S J J_3; (\lambda\mu)YII_3\rangle = \delta_{YY'}\delta_{II'}\delta_{I_3I'_3} \sum_{p,p',p''} c_{pp'}(S) c_{pp''}(S)$$
$$\times \sum_{Y_c,I_c,y,i} \frac{3Y_c}{2\sqrt{3}} \left(\begin{array}{cc|c} (\lambda_c\mu_c) & (10) & (\lambda\mu) \\ Y_cI_c & yi & YI \end{array}\right)\left(\begin{array}{cc|c} (\lambda_c\mu_c) & (10) & (\lambda'\mu') \\ Y_cI_c & yi & YI \end{array}\right), \quad \text{(B1)}$$

$$\langle \ell S J J_3; (\lambda'\mu')Y'I'I'_3 | t^8 | \ell S J J_3(\lambda\mu)YII_3\rangle = \delta_{YY'}\delta_{II'}\delta_{I_3I'_3} \sum_{p,p',p''} c_{pp'}(S) c_{pp''}(S)$$

$$\times \sum_{Y_c,I_c,y,i} \frac{3y}{2\sqrt{3}} \left(\begin{array}{cc||c} (\lambda_c\mu_c) & (10) & (\lambda\mu) \\ Y_cI_c & yi & YI \end{array} \right) \left(\begin{array}{cc||c} (\lambda_c\mu_c) & (10) & (\lambda'\mu') \\ Y_cI_c & yi & YI \end{array} \right) \tag{B2}$$

and

$$\begin{aligned}
\langle \ell S' J J_3; (\lambda'\mu') Y' I' I'_3 | \ell_q^i g^{i8} | \ell S J J_3 (\lambda\mu) Y I I_3 \rangle &= \delta_{YY'} \delta_{II'} \delta_{I_3 I'_3} (-1)^{J+\ell_q+\ell_c-1/2} \\
\times (2\ell+1) \sqrt{\ell_q(\ell_q+1)(2\ell_q+1)} \sqrt{(2S+1)(2S'+1)} & \left\{ \begin{array}{ccc} \ell & 1 & \ell \\ \ell_q & \ell_c & \ell_q \end{array} \right\} \left\{ \begin{array}{ccc} J & \ell & S \\ 1 & S' & \ell \end{array} \right\} \\
\times \sum_{p,p',q,q'} (-1)^{S_c} c^{[N_c-1,1]}_{pp'}(S) c^{[N_c-1,1]}_{qq'}(S') & \left\{ \begin{array}{ccc} S' & 1 & S \\ 1/2 & S_c & 1/2 \end{array} \right\} \\
\times \sum_{Y_c,I_c,y,i} \frac{3y}{2\sqrt{2}} \left(\begin{array}{cc||c} (\lambda_c\mu_c) & (10) & (\lambda\mu) \\ Y_cI_c & yi & YI \end{array} \right) & \left(\begin{array}{cc||c} (\lambda_c\mu_c) & (10) & (\lambda'\mu') \\ Y_cI_c & yi & YI \end{array} \right).
\end{aligned} \tag{B3}$$

To obtain Table V and VI we have used the Eqs. (9) for the coefficients $c^{[N_c-1,1]}_{pp'}$ and Table IV of Ref. [37] for the isoscalar factors of SU(3). From their expressions one can find that all these coefficients and isoscalar factors are of order N_c^0. Then it follows that for states with spin and strangeness of order N_c^0, the matrix elements of T^c_8 are of order N_c because $Y_c = Y - y$, $Y = N_c/3 + \mathcal{S}$ so that $Y_c \sim N_c$.

APPENDIX C

For completeness here we give the matrix elements of the spin-orbit operator O_2 . They are a generalization from SU(4) [15] to SU(6) and refer to an excited core with $\ell_c \neq 0$.

$$\begin{aligned}
\langle \ell_q s \rangle = \delta_{J'J} \delta_{J'_3 J_3} \delta_{\lambda'\lambda} \delta_{\mu'\mu} \delta_{Y'Y} \delta_{I'I} \delta_{I'_3 I_3} (-1)^{J-1/2+\ell_q+\ell_c} & \sqrt{\frac{3}{2}(2S+1)(2S'+1)} \\
\times \sqrt{(2\ell+1)(2\ell'+1)\ell_q(\ell_q+1)(2\ell_q+1)} & \left\{ \begin{array}{ccc} \ell & 1 & \ell' \\ \ell_q & \ell_c & \ell_q \end{array} \right\} \left\{ \begin{array}{ccc} 1 & \ell & \ell' \\ J & S' & S \end{array} \right\} \\
\times \sum_{pp'p''} (-1)^{-S_c} c_{p'p}(S) c_{p''p}(S') & \left\{ \begin{array}{ccc} S & 1 & S' \\ \frac{1}{2} & S_c & \frac{1}{2} \end{array} \right\}
\end{aligned} \tag{C1}$$

where $S_c = S - 1/2$ for $p' = 1$ and $S_c = S + 1/2$ for $p' = 2$ and similarly $S_c = S' - 1/2$ for $p'' = 1$ and $S_c = S' + 1/2$ for $p'' = 2$.

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TABLE I: List of operators and the coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 1.0$.

Operator	Fitted coef. (MeV)
$O_1 = N_c \mathbb{1}$	$c_1 =$ 556 $\pm$ 11
$O_2 = \ell_q^i s^i$	$c_2 =$ -43 $\pm$ 47
$O_3 = \frac{3}{N_c} \ell_q^{(2)ij} g^{ia} G_c^{ja}$	$c_3 =$ -85 $\pm$ 72
$O_4 = \frac{4}{N_c+1} \ell^i t^a G_c^{ia}$	
$O_5 = \frac{1}{N_c}(S_c^i S_c^i + s^i S_c^i)$	$c_5 =$ 253 $\pm$ 57
$O_6 = \frac{1}{N_c} t^a T_c^a$	$c_6 =$ -25 $\pm$ 86
$B_1 = t^8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 =$ 365 $\pm$ 169
$B_2 = T_c^8 - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 =$ -293 $\pm$ 54
$B_4 = 3\ell_q^i g^{i8} - \frac{\sqrt{3}}{2} O_2$	

TABLE II: Matrix elements for octet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$^4 8[\mathbf{70}, 2^+]\frac{7}{2}$	N_c	$\frac{2}{3}$	$-\frac{3N_c+1}{18N_c}$	$-\frac{2(3N_c+1)}{9(N_c+1)}$	$\frac{5}{2N_c}$	$\frac{N_c-13}{12N_c}$
$^2 8[\mathbf{70}, 2^+]\frac{5}{2}$	N_c	$\frac{2(2N_c-3)}{9N_c}$	0	$-\frac{4(N_c+3)(3N_c-2)}{27N_c(N_c+1)}$	$\frac{N_c+3}{4N_c^2}$	$\frac{N_c^2-4N_c-9}{12N_c^2}$
$^4 8[\mathbf{70}, 2^+]\frac{5}{2}$	N_c	$-\frac{1}{9}$	$\frac{5(3N_c+1)}{36N_c}$	$\frac{3N_c+1}{27(N_c+1)}$	$\frac{5}{2N_c}$	$\frac{N_c-13}{12N_c}$
$^4 8[\mathbf{70}, 0^+]\frac{3}{2}$	N_c	0	0	0	$\frac{5}{2N_c}$	$\frac{N_c-13}{12N_c}$
$^2 8[\mathbf{70}, 2^+]\frac{3}{2}$	N_c	$-\frac{2N_c-3}{3N_c}$	0	$\frac{2(N_c+3)(3N_c-2)}{9N_c(N_c+1)}$	$\frac{N_c+3}{4N_c^2}$	$\frac{N_c^2-4N_c-9}{12N_c^2}$
$^4 8[\mathbf{70}, 2^+]\frac{3}{2}$	N_c	$-\frac{2}{3}$	0	$\frac{2(3N_c+1)}{9(N_c+1)}$	$\frac{5}{2N_c}$	$\frac{N_c-13}{12N_c}$
$^2 8[\mathbf{70}, 0^+]\frac{1}{2}$	N_c	0	0	0	$\frac{N_c+3}{4N_c^2}$	$\frac{N_c^2-4N_c-9}{12N_c^2}$
$^4 8[\mathbf{70}, 2^+]\frac{1}{2}$	N_c	-1	$-\frac{7(3N_c+1)}{36N_c}$	$\frac{3N_c+1}{3(N_c+1)}$	$\frac{5}{2N_c}$	$\frac{N_c-13}{12N_c}$

TABLE III: Matrix elements for decuplet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$^2 10[\mathbf{70}, 2^+]\frac{5}{2}$	N_c	$-\frac{2}{9}$	0	$\frac{2(3N_c+7)}{27(N_c+1)}$	$\frac{1}{N_c}$	$\frac{N_c+5}{12N_c}$
$^2 10[\mathbf{70}, 2^+]\frac{3}{2}$	N_c	$\frac{1}{3}$	0	$-\frac{3N_c+7}{9(N_c+1)}$	$\frac{1}{N_c}$	$\frac{N_c+5}{12N_c}$
$^2 10[\mathbf{70}, 0^+]\frac{1}{2}$	N_c	0	0	0	$\frac{1}{N_c}$	$\frac{N_c+5}{12N_c}$

TABLE IV: Matrix elements for singlet resonances.

	O_1	O_2	O_3	O_4	O_5	O_6
$^2 1[\mathbf{70}, 2^+]\frac{5}{2}$	N_c	$\frac{2}{3}$	0	0	0	$-\frac{N_c+5}{6N_c}$
$^2 1[\mathbf{70}, 2^+]\frac{3}{2}$	N_c	-1	0	0	0	$-\frac{N_c+5}{6N_c}$
$^2 1[\mathbf{70}, 0^+]\frac{1}{2}$	N_c	0	0	0	0	$-\frac{N_c+5}{6N_c}$

TABLE V: Matrix elements of t_8 and T_8^c as a function of N_c, the isospin I and the strangeness $\mathcal{S}$. The off-diagonal matrix elements have $(\mathcal{S}=-1, I=1)$ or $(\mathcal{S}=-2, I=1/2)$ for ${}^28_J - {}^210_J$ and $(\mathcal{S}=0, I=0)$ for ${}^28_J - {}^21_J$.

	t_8	T_8^c
${}^28_J$	$\frac{N_c^3+[\mathcal{S}(5-\mathcal{S})+4I(I+1)-1]N_c^2-3[\mathcal{S}(2-\mathcal{S})+4I(I+1)-2]N_c+9\mathcal{S}}{2\sqrt{3}N_c(N_c-1)(N_c+3)}$	$\frac{N_c^4+(3\mathcal{S}+1)N_c^3+[(\mathcal{S}(\mathcal{S}+1)-4I(I+1)-2]N_c^2-3[\mathcal{S}(\mathcal{S}+1)-4I(I+1)+2]N_c-9\mathcal{S}}{2\sqrt{3}N_c(N_c-1)(N_c+3)}$
${}^48_J$	$\frac{2N_c-4I(I+1)+\mathcal{S}(\mathcal{S}+4)+1}{4\sqrt{3}(N_c-1)}$	$\frac{2N_c^2+2(3\mathcal{S}-2)N_c+4I(I+1)-\mathcal{S}(\mathcal{S}+10)-1}{4\sqrt{3}(N_c-1)}$
${}^210_J$	$\frac{2N_c+4I(I+1)-\mathcal{S}(\mathcal{S}-8)-5}{4\sqrt{3}(N_c+5)}$	$\frac{2N_c^2+2(3\mathcal{S}+4)N_c-4I(I+1)+\mathcal{S}(\mathcal{S}+22)+5}{4\sqrt{3}(N_c+5)}$
${}^21_J$	$\frac{-2N_c^2-2(3\mathcal{S}+1)N_c+12I(I+1)-3\mathcal{S}(\mathcal{S}+2)+3}{2\sqrt{3}(N_c+1)(N_c+3)}$	$\frac{N_c^3+3(\mathcal{S}+2)N_c^2+(18\mathcal{S}+5)N_c-12I(I+1)+3\mathcal{S}(\mathcal{S}+5)-3}{2\sqrt{3}(N_c+1)(N_c+3)}$
${}^28_J-{}^210_J$	$\sqrt{\frac{2}{3}}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$	$-\sqrt{\frac{2}{3}}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$
${}^28_J-{}^21_J$	$\frac{3(N_c-1)}{2\sqrt{N_c}(N_c+3)}$	$-\frac{3(N_c-1)}{2\sqrt{N_c}(N_c+3)}$

TABLE VI: Matrix elements of the term $3\ell^i g^{i8}$ of B_4.

	$3\ell^i g^{i8}$
$^4 8[\mathbf{70}, 2^+]\frac{7}{2}$	$\frac{2N_c - 4I(I+1) + \mathcal{S}(\mathcal{S}+4) + 1}{2\sqrt{3}(N_c-1)}$
$^2 8[\mathbf{70}, 2^+]\frac{5}{2}$	$\frac{4N_c^3 + 4I(I+1)(9+N_c(7N_c-12)) - 9(\mathcal{S}-1)^2 - N_c^2(\mathcal{S}-1)(7\mathcal{S}-19) + 12N_c(\mathcal{S}(\mathcal{S}-5)+1)}{6\sqrt{3}N_c(N_c-1)(N_c+3)}$
$^4 8[\mathbf{70}, 2^+]\frac{5}{2}$	$-\frac{2N_c - 4I(I+1) + \mathcal{S}(\mathcal{S}+4) + 1}{12\sqrt{3}(N_c-1)}$
$^4 8[\mathbf{70}, 0^+]\frac{3}{2}$	0
$^2 8[\mathbf{70}, 2^+]\frac{3}{2}$	$-\frac{4N_c^3 + 4I(I+1)(9+N_c(7N_c-12)) - 9(\mathcal{S}-1)^2 - N_c^2(\mathcal{S}-1)(7\mathcal{S}-19) + 12N_c(\mathcal{S}(\mathcal{S}-5)+1)}{4\sqrt{3}N_c(N_c-1)(N_c+3)}$
$^4 8[\mathbf{70}, 2^+]\frac{3}{2}$	$-\frac{2N_c - 4I(I+1) + \mathcal{S}(\mathcal{S}+4) + 1}{2\sqrt{3}(N_c-1)}$
$^2 8[\mathbf{70}, 0^+]\frac{1}{2}$	0
$^4 8[\mathbf{70}, 2^+]\frac{1}{2}$	$-\frac{\sqrt{3}(2N_c - 4I(I+1) + \mathcal{S}(\mathcal{S}+4) + 1)}{4(N_c-1)}$
$^2 10[\mathbf{70}, 2^+]\frac{5}{2}$	$-\frac{2N_c + 4I(I+1) - \mathcal{S}(\mathcal{S}-8) - 5}{6\sqrt{3}(N_c+5)}$
$^2 10[\mathbf{70}, 2^+]\frac{3}{2}$	$\frac{2N_c + 4I(I+1) - \mathcal{S}(\mathcal{S}-8) - 5}{6\sqrt{3}(N_c+5)}$
$^2 10[\mathbf{70}, 0^+]\frac{1}{2}$	0
$^2 1[\mathbf{70}, 2^+]\frac{5}{2}$	$\frac{3 + 12I(I+1) - 2N_c(N_c+1) - 3\mathcal{S}(\mathcal{S}+2N_c+2)}{\sqrt{3}(N_c+1)(N_c+3)}$
$^2 1[\mathbf{70}, 2^+]\frac{3}{2}$	$-\frac{\sqrt{3}(3 + 12I(I+1) - 2N_c(N_c+1) - 3\mathcal{S}(\mathcal{S}+2N_c+2))}{2(N_c+1)(N_c+3)}$
$^2 1[\mathbf{70}, 0^+]\frac{1}{2}$	0

TABLE VII: The partial contribution and the total mass (MeV) predicted by the $1/N_c$ expansion. The last two columns give the empirically known masses.

	Part. contrib. (MeV)							Total (MeV)	Exp. (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_5O_5	c_6O_6	d_1B_1	d_2B_2			
$^4N[\mathbf{70},2^+]\frac{7}{2}$	1667	-29	16	211	7	0	0	1872 ± 46	2016 ± 104	$F_{17}(1990)$**
$^4\Lambda[\mathbf{70},2^+]\frac{7}{2}$						0	254	2125 ± 72	2094 ± 78	$F_{07}(2020)$*
$^4\Sigma[\mathbf{70},2^+]\frac{7}{2}$						-211	85	1745 ± 95		
$^4\Xi[\mathbf{70},2^+]\frac{7}{2}$						-105	423	2189 ± 81		
$^2N[\mathbf{70},2^+]\frac{5}{2}$	1667	-10	0	42	3	0	0	1703 ± 29		
$^2\Lambda[\mathbf{70},2^+]\frac{5}{2}$						-105	169	1766 ± 26		
$^2\Sigma[\mathbf{70},2^+]\frac{5}{2}$						-105	169	1766 ± 26		
$^2\Xi[\mathbf{70},2^+]\frac{5}{2}$						-211	338	1830 ± 58		
$^4N[\mathbf{70},2^+]\frac{5}{2}$	1667	5	-39	211	7	0	0	1850 ± 44	1981 ± 200	$F_{15}(2000)$**
$^4\Lambda[\mathbf{70},2^+]\frac{5}{2}$						0	254	2104 ± 39	2112 ± 40	$F_{05}(2110)$***
$^4\Sigma[\mathbf{70},2^+]\frac{5}{2}$						-211	85	1724 ± 111		
$^4\Xi[\mathbf{70},2^+]\frac{5}{2}$						-105	423	2167 ± 54		
$^4N[\mathbf{70},0^+]\frac{3}{2}$	1667	0	0	211	7	0	0	1885 ± 17	1879 ± 17	$P_{13}(1900)$**
$^4\Lambda[\mathbf{70},0^+]\frac{3}{2}$						0	254	2138 ± 42		
$^4\Sigma[\mathbf{70},0^+]\frac{3}{2}$						-211	85	1758 ± 100		
$^4\Xi[\mathbf{70},0^+]\frac{3}{2}$						-105	423	2202 ± 56		
$^2N[\mathbf{70},2^+]\frac{3}{2}$	1667	14	0	42	3	0	0	1727 ± 31		
$^2\Lambda[\mathbf{70},2^+]\frac{3}{2}$						-105	169	1790 ± 29		
$^2\Sigma[\mathbf{70},2^+]\frac{3}{2}$						-105	169	1790 ± 29		
$^2\Xi[\mathbf{70},2^+]\frac{3}{2}$						-211	338	1854 ± 59		
$^4N[\mathbf{70},2^+]\frac{3}{2}$	1667	29	0	211	7	0	0	1914 ± 33		
$^4\Lambda[\mathbf{70},2^+]\frac{3}{2}$						0	254	2167 ± 41		
$^4\Sigma[\mathbf{70},2^+]\frac{3}{2}$						-211	85	1787 ± 103		
$^4\Xi[\mathbf{70},2^+]\frac{3}{2}$						-105	423	2231 ± 56		

	Part. contrib. (MeV)							Total (MeV)	Exp. (MeV)	Name, status
	c_1O_1	c_2O_2	c_3O_3	c_5O_5	c_6O_6	d_1B_1	d_2B_2			
$^2N[\mathbf{70},0^+]\frac{1}{2}$	1667	0	0	42	3	0	0	1712 ± 27	1710 ± 30	$P_{11}(1710)$***
$^2\Lambda[\mathbf{70},0^+]\frac{1}{2}$						-105	169	1776 ± 24		
$^2\Sigma[\mathbf{70},0^+]\frac{1}{2}$						-105	169	1776 ± 24	1760 ± 27	$P_{11}(1770)$*
$^2\Xi[\mathbf{70},0^+]\frac{1}{2}$						-211	338	1839 ± 57		
$^4N[\mathbf{70},2^+]\frac{1}{2}$	1667	43	55	211	7	0	0	1983 ± 26	1986 ± 26	$P_{11}(2100)$*
$^4\Lambda[\mathbf{70},2^+]\frac{1}{2}$						0	254	2237 ± 57		
$^4\Sigma[\mathbf{70},2^+]\frac{1}{2}$						-211	85	1857 ± 90		
$^4\Xi[\mathbf{70},2^+]\frac{1}{2}$						-105	423	2301 ± 68		
$^2\Delta[\mathbf{70},2^+]\frac{5}{2}$	1667	10	0	84	-6	0	0	1756 ± 32	1976 ± 237	$F_{35}(2000)$**
$^2\Sigma'[\mathbf{70},2^+]\frac{5}{2}$						-105	169	1819 ± 46		
$^2\Xi'[\mathbf{70},2^+]\frac{5}{2}$						-211	338	1883 ± 77		
$^2\Omega[\mathbf{70},2^+]\frac{5}{2}$						-316	507	1946 ± 113		
$^2\Delta[\mathbf{70},2^+]\frac{3}{2}$	1667	-14	0	84	-6	0	0	1731 ± 35		
$^2\Sigma'[\mathbf{70},2^+]\frac{3}{2}$						-105	169	1795 ± 48		
$^2\Xi'[\mathbf{70},2^+]\frac{3}{2}$						-211	338	1859 ± 78		
$^2\Omega[\mathbf{70},2^+]\frac{3}{2}$						-316	507	1922 ± 113		
$^2\Delta[\mathbf{70},0^+]\frac{1}{2}$	1667	0	0	84	-6	0	0	1746 ± 31	1744 ± 36	$P_{31}(1750)$*
$^2\Sigma'[\mathbf{70},0^+]\frac{1}{2}$						-105	169	1810 ± 45	1896 ± 95	$P_{11}(1880)$**
$^2\Xi'[\mathbf{70},0^+]\frac{1}{2}$						211	338	1873 ± 77		
$^2\Omega[\mathbf{70},0^+]\frac{1}{2}$						316	507	1937 ± 112		
$^2\Lambda'[\mathbf{70},2^+]\frac{5}{2}$	1667	-29	0	0	11	-105	169	1713 ± 51		
$^2\Lambda'[\mathbf{70},2^+]\frac{3}{2}$	1667	43	0	0	11	-105	169	1785 ± 62		
$^2\Lambda'[\mathbf{70},0^+]\frac{1}{2}$	1667	0	0	0	11	-105	169	1742 ± 40	1791 ± 64	$P_{01}(1810)$***