Excited $[\mathbf{70}, \ell^+]$ baryon resonances in large N_c QCD

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We summarize results obtained in the $1/N_c$ expansion method for the masses of baryon resonances belonging to the $[\mathbf{70}, \ell^+]$ multiplet. They represent an extension of our previous studies from two to three flavors. A better approach to mixed symmetric states of any angular momentum and parity is also outlined.

1. Introduction

QCD is not a perturbative theory at low energy regime. Consequently, one does not know how to systematically study the structure of the ground and excited baryon states except, in principle, for lattice calculations. Large N_c QCD [1,2] introduces a new perturbative series where the parameter is the inverse of the number of colors N_c. The resulting $1/N_c$ expansion approach has proven to be an interesting way to study baryon spectroscopy.

The success is due to the discovery in 1984, by Gervais and Sakita [3] and independently, during the ninetieths, by Dashen and Manohar [4], from the study of baryon-meson scattering process in the $N_c \to \infty$ limit, that ground state baryons satisfy a contracted SU$(2N_f)_c$ spin-flavor algebra where N_f is the number of flavors. This means that when $N_c \to \infty$, ground state baryons form an infinite tower of degenerate states. At $N_c \to \infty$, the SU$(2N_f)$ algebra used in the constituent quark model becomes the SU$(2N_f)_c$ algebra. One can therefore use SU$(2N_f)$ to classify large N_c baryons and calculate matrix elements of various static observables. If SU$(2N_f)$ is broken the degenerate baryon states split at order $1/N_c$.

The first studies of excited baryons in the large N_c limit suggested that constituent quark baryons can be reduced to a symmetric core composed of $N_c - 1$ ground state quarks and one excited quark. The advantage is that one can treat the core in the same way as a ground state baryon. However, with this approach, the SU$(2N_f)$ symmetry is broken at order $\mathcal{O}(N_c^0)$ instead of $\mathcal{O}(1/N_c)$ as for the ground state [5]. This generates the conceptual problem that in the $N_c \to \infty$ limit, excited states do not form anymore an infinite tower of degenerate states. Furthermore, excited states are resonances and have widths of order N_c^0 [6]. Nevertheless, baryons belonging to various SU(6) excited multiplets have been studied with success during the last ten years [7,8,9,10,11,12,13,14,15,16,17,18]. More or less, all these studies consider excited baryons as bound states and split the SU$(2N_f)$ generators into two parts, one acting on the symmetric core and the other on the excited quark. Accordingly, the number of invariant operators needed in the description of observables becomes exceedingly large and the splitting starts at order $\mathcal{O}(N_c^0)$. Fortunately, these studies show that the N_c^0 breaking is small.

Recently, a new approach of the $[\mathbf{70}, 1^-]$ multiplet solved this conceptual problem by removing the splitting of generators and using orbital-flavor-spin wave functions respecting the permutation symmetry [19]. Still, the excited baryons as considered as bound states.

Below we shall analyze the $[\mathbf{70}, \ell^+]$ multiplets with $\ell = 0$ or 2 in the standard approach with splitting.

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2. The mass operator

The constituent quark model suggests that $[\mathbf{70},\ell^+]$ baryons are composed of one or two excited quarks and $\mathcal{O}(N_c)$ ground state quarks. The standard procedure, used in Ref. [18], for calculating the mass spectrum is to reduce the wave function to that of a product of a symmetric orbital and a symmetric flavor-spin wave function for the core of $N_c - 1$ quarks times the wave function of one excited quark. This implies that the total orbital-flavor-spin wave function is truncated to a single term, described by the product of two Young tableaux, each with the excited quark in the second row. Many others terms, related to Young tableaux with the excited quark in the first row, are neglected [19]. At the same time each SU(6) and SO(3) generator is splitted into two terms, one acting on the core and the other on the excited quark. We assume that baryons are bound states.

The mass operator must be rotationally invariant, parity and time reversal even. For the $[\mathbf{70},\ell^+]$ multiplet it has the following structure:

$$M_{[\mathbf{70},\ell^+]} = \sum_{i=1}^{6} c_i O_i + d_1 B_1 + d_2 B_2 + d_4 B_4, \quad (1)$$

where O_i are rotational invariants and SU(3)-flavor scalars, the operators B_i provide SU(3) breaking and are defined to have non-vanishing matrix elements for strange baryons only and the coefficients c_i and d_i, which encode the QCD dynamics, are unknown. One can obtain these coefficients from a fit to experimental data. As the number of data is limited, we have to make a selection among all the possible operators.

The first column of Table 1 shows the list of operators chosen for this study as thought to be the most dominant one. The choice is based on previous studies [9,12,17]. They are composed of SU(6)×SO(3) generators. Generators S_c^i, T_c^a, G_c^{ia} refer to the core part of the wave function and s^i, t^a, g^{ia} and ℓ_q^i act on the excited quark. Besides the SO(3) generators which are of rank $k=1$ we also need to introduce the rank $k=2$ tensor operator

$$\ell_q^{(2)ij} = \frac{1}{2}\left\{\ell^i,\ell^j\right\} - \frac{1}{3}\delta_{i,-j}\vec{\ell}\cdot\vec{\ell}. \quad (2)$$

For deriving the matrix elements of the operators O_i and B_i, we first needed to calculate the matrix elements of the core generators, the problem being trivial for the excited quark (single particle operators). It is not always an obvious task. To obtain the matrix elements, we have used a generalized Wigner-Eckart theorem and then derived SU(6) isoscalar factors. We have obtained analytic expressions for these isoscalar factors for SU(6) symmetric wave functions $|[N_c]\rangle$ containing any number of quarks [20]. The mixed symmetric case needs considerable group theory work. To our knowledge, isoscalar factors for mixed symmetric states $|[N_c-1,1]\rangle$ which can be applied to baryons composed of N_c quarks [19] for $N_f=3$ are yet inexistent. That is why, presently, it is not possible to treat the $[\mathbf{70},\ell^+]$ multiplet composed of strange baryons without simplifying the baryon wave function and splitting the SU(6) generators.

In Table 1, O_1 is the SU(6) scalar operator linear in N_c. O_2 and O_5 are the dominant part of the spin-orbit and spin-spin operators respectively. The first, which acts only on the excited quark, is of order N_c^0 but the two-body spin-spin operator is of order N_c^{-1}. The operators O_3 and O_4 are of order N_c^0 due to the presence of the SU(6) generator G_c^{ia} which sums coherently. O_6 represents the isospin-isospin operator, having matrix elements of order N_c^0 due to T_c^a which sums coherently too.

As already mentioned, the operators B_i break the SU(3)-flavor symmetry. The operators B_1 and B_2 are the standard breaking operators while B_4 is directly related to the spin-orbit splitting. They break the SU(3)-flavor symmetry to first order.

3. Results

The second column of Table 1 gives the values of the coefficients c_i and d_i resulting form the fit. The $\chi^2_{\rm dof}$ obtained is 1.0. Details concerning the calculations and data used in the fit are presented elsewhere [18]. One can see that the first order operator O_1 and the spin-spin operator O_5 are the most dominant ones, *i.e.* c_1 and c_5 are large. The spin-orbit coefficient is negative, at variance with previous studies [16] but remains small in absolute value. The coefficient c_3 is twice

Table 1
List of operators and the coefficients resulting from the fit with $\chi^2_{\rm dof} \simeq 1.0$ for the $[\mathbf{70},\ell^+]$ multiplets ($\ell = 0$ and 2).

Operator	Fitted coef. (MeV)			
$O_1 = N_c \mathbb{1}$	$c_1 =$	556	$\pm$	11
$O_2 = \ell^i_q s^i$	$c_2 =$	-43	$\pm$	47
$O_3 = \frac{3}{N_c}\ell_q^{(2)ij} g^{ia} G_c^{ja}$	$c_3 =$	-85	$\pm$	72
$O_4 = \frac{4}{N_c+1}\ell^i t^a G_c^{ia}$				
$O_5 = \frac{1}{N_c}(S_c^i S_c^i + s^i S_c^i)$	$c_5 =$	253	$\pm$	57
$O_6 = \frac{1}{N_c} t^a T_c^a$	$c_6 =$	-25	$\pm$	86
$B_1 = t^8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 =$	365	$\pm$	169
$B_2 = T_c^8 - \frac{N_c - 1}{2\sqrt{3}N_c} O_1$	$d_2 =$	-293	$\pm$	54
$B_4 = 3\ell_q^i g^{i8} - \frac{\sqrt{3}}{2} O_2$				

smaller in absolute value as compared to that of Ref. [16]. We had to exclude the operator O_4 from the fit because it considerably deteriorated the $\chi^2_{\rm dof}$. This study suggests that the isospin-isospin operator does not play an important role, the coefficient c_6 being small. Possibly, this is a consequence of the splitting of generators and of the truncation of the baryon wave function. In the improved approach of Ref. [19], applied to the analysis of the $[\mathbf{70},1^-]$ multiplet, the isospin term appears to have the same importance as the spin term.

The SU(3)-flavor breaking operators play an important dynamical role as it can be seen from the values of the coefficients d_1 and d_2. As all the matrix elements of B_4 cancel out for the available resonances, it was not possible for us to obtain an estimation of d_4.

Figure 1 shows the evolution of the spin-spin dynamical coefficient c_5 with the excitation energy. Here we have collected the presently known values with error bars for the orbitally excited states studied so far in the large N_c expansion: $N = 1$, Ref. [12], $N = 2$ (lower value [15], upper value [18]) and $N = 4$, Ref. [17]. This figure suggests that at large excitations, the spin-spin contribution vanishes.

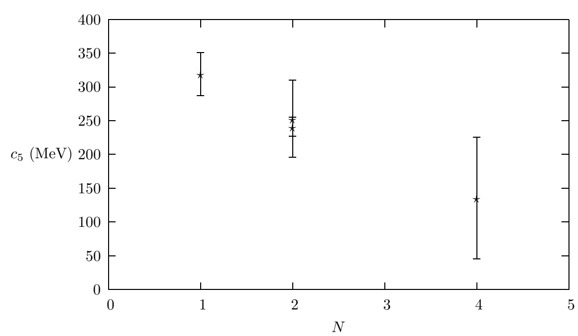

Figure 1. Evolution of the coefficient c_5 with the excitation energy corresponding to $N = 1, 2$ and 4 bands in a harmonic oscillator notation.

4. Conclusions

We have shown results obtained for the masses of resonances which we identified as belonging to the $[\mathbf{70},0^+]$ and $[\mathbf{70},2^+]$ multiplets. The present results confirm the dependence of the coefficients c_1, c_2 and c_5 as a function of excitation energy, namely that the contributions of the spin-dependent terms decrease with energy and eventually vanish at very large excitations [16]. The analysis of $[\mathbf{70}, \ell^+]$ remains open. More and better experimental data are needed to clarify the role of various terms contributing to the mass operator of the $[\mathbf{70}, \ell^+]$ multiplet. A new study should be made without truncating the baryon wave function.

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