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Anomalous perturbative transport in tokamaks due to drift-wave turbulence

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A new method for calculating the anomalous transport in tokamak plasmas is presented. The renormalized nonlinear plasma response function is derived using the direct-interaction approximation (DIA). A complete calculation for the case of electrostatic drift-wave turbulence is presented. Explicit expressions for all coefficients of the anomalous transport matrix relating particle and heat fluxes to density and temperature gradients in the plasma are obtained. The anomalous transport matrix calculated using the DIA does not have the Onsager symmetry. As an example of application, the parameters of the Texas Experimental Tokamak (TEXT) [Nucl. Technol. Fusion **1**, 479 (1981)] are used to evaluate all transport coefficients numerically, as well as the spectrum modulation. The relation between the theoretical results and the experimental data is discussed. Although this paper focuses on electron transport for simplicity, the method can also be used to calculate anomalous transport due to ion instabilities, such as the ion-temperature-gradient instability.

I. INTRODUCTION

To sustain thermonuclear reactions in a tokamak, the plasma must be confined well enough to overcome heat losses. The confinement of the energy for a long enough time is a major difficulty in magnetic fusion.

It is well known that classical calculations of heat diffusion based on kinetic theory (using, for example, the Fokker–Planck or Balescu–Lenard collision operators; see, e.g., Braginskii[1] or Hinton[2]) yield transport coefficients which are in complete disagreement with observations. So-called "neoclassical" transport theories are still based on classical collisional processes, but take into account the toroidal geometry of a tokamak.[3,4] According to neoclassical theories, the electron heat conductivity is smaller than the ion heat conductivity by a factor $\sqrt{m_e/m_i}$. However, measurements (in Ohmic discharges) indicate that, although the ion heat transport appears to be well described by neoclassical results,[5–7] the electron heat transport is sometimes as much as two orders of magnitude *larger*.[5,8] The energy transport is therefore completely dominated by the electrons in those cases. This so-called "anomalous transport" is the subject of this work.

Today, the design of fusion reactors relies heavily on empirical scaling laws deduced from extrapolation of existing experimental data.[9–11] Unfortunately, these extrapolations are unreliable, since the parameters of future reactors are often very different from those of existing machines. A more fundamental understanding of transport in tokamak plasmas is therefore critical to the design of future generations of reactors.[12] Recent reviews on the present status of experiments and theoretical models can be found in Boozer *et al.*,[13] Wootton *et al.*,[14] Burrel *et al.*,[15] Houlberg *et al.*,[16] and Kaye *et al.*[17]

It is now widely accepted that the anomalous transport in tokamaks results very likely from the presence of a saturated spectrum of fluctuations in the plasma, produced by microscopic turbulence (see, e.g., the reviews by Tang,[8] Horton,[18] and Liewer[19]). For example, in the presence of fluctuating electric fields, transport of particles across the magnetic flux surfaces can occur through the $\mathbf{E}\times\mathbf{B}$ drift. In the presence of fluctuating magnetic fields, the magnetic flux surfaces are destroyed and electrons can flow outward along the perturbed field lines. Possible sources of turbulent fluctuations include drift modes,[20–23] dissipative trapped electron (DTE) modes,[24] ion-temperature-gradient modes,[25,26] electron-temperature-gradient modes,[27] and microtearing modes.[28]

The relatively low frequency ($\omega \lesssim 0.5$ MHz$\ll\Omega_i$, the ion Larmor frequency) of small-scale ($k_\perp \rho_i \lesssim 1$) density and potential fluctuations observed in some tokamak experiments[29–32] suggests that drift waves and DTE modes may be responsible for the anomalous transport. Drift waves are unstable, low-frequency waves produced in any plasma having spatial inhomogeneities and a magnetic field. They propagate mainly in the direction perpendicular to both the density gradient and the magnetic field, but they have a finite parallel wave number, which allows electrons to flow freely along the field lines. Their simplest dispersion relation is of the form $\omega=\mathbf{k}\cdot\mathbf{v}_*$, where $\mathbf{v}_*=-(cT/eB)\mathbf{b}_0\times(\nabla n_0/n_0)$ is the diamagnetic drift velocity (see, e.g., Nicholson[33]). Recently, it has become evident that the ion dynamics can also play a crucial role in the anomalous transport because of the ion temperature gradient (η_i) instability.[25,26] Indeed, it has been observed that the Ohmic confinement time saturates at high density an order of magnitude below the predictions of neoclassical

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theory, indicating anomalous ion losses.[34] In addition, it was observed that pellet injection improves the ion energy confinement, while leaving the anomalous electron thermal losses unchanged.[35] Far-infrared laser scattering experiments on the Texas Experimental Tokamak (TEXT)[36] showed that the onset of fluctuations propagating in the ion direction coincides with the saturation of the energy confinement time with density,[37] which strongly suggests an association between the anomalous ion losses and the saturation of the energy confinement time.

Many calculations of anomalous transport have been performed[38–41] in the so-called "quasilinear" approximation of weak-turbulence theory (see, e.g., Kadomtsev[42] and Sagdeev and Galeev[43]). In these calculations, the level of turbulence is assumed to be low enough so that the nonlinearities can be treated perturbatively. To obtain closure, the weak-turbulence expansion must be truncated to some order. The "quasilinear" theory corresponds to truncating this expansion to first order.[42,43] The transport can be calculated from correlations of fluctuations, obtained from the linearized equations. The important effect considered in quasilinear theory is the wave–particle interaction, but the mode–mode coupling terms are neglected. However, experiments suggest that mode–mode coupling effects are, in fact, very important, and that the weak-turbulence condition is usually not satisfied.[30]

In a strongly turbulent plasma, the nonlinearities are essential and the perturbative approach of weak-turbulence theory fails. The goal of renormalized plasma turbulence theories is to simplify the nonlinear problem by attempting to describe only the macroscopic statistical averages of the fluctuations. Examples of such renormalization techniques include the "resonance broadening"[44] and "clump"[45] calculations, as well as methods based on the direct interaction approximation.[42,46–49] Renormalization prescriptions are generally untested closure assumptions, which should be treated with caution. However, the direct interaction approximation (hereafter DIA) provides an exact description of certain stochastic models, such as the random-coupling model[50] and the Langevin equation.[51] Furthermore, the DIA method automatically satisfies the self-consistency constraints arising from the strong coupling between electromagnetic fields and particle densities through Maxwell's equations. These constraints are not satisfied in quasilinear calculations.

Since the DIA method is based on the evaluation of averaged infinitesimal response functions, it naturally provides the *incremental fluxes* resulting from small perturbations of the equilibrium profiles, rather than total fluxes accross the magnetic surfaces.[52] It is therefore particularly well suited to comparisons with the recent gas puffing, pellet injection, and heat pulse propagation experiments.[15,53–56] These experiments are performed either by using externally imposed modulated sources or by taking advantage of naturally occurring phenomena such as sawteeth oscillations.[56–59] In contrast to quasilinear calculations, we will show that the DIA formalism can also be used to study the modulation of the fluctuation spectrum caused by external modulated perturbations. Such spectrum modulation measurements have been performed recently in TEXT.

Traditionally, the transport of particles and heat in tokamaks have been studied as two independent problems, with the particle flux related to the density gradient by Fick's law, $\Gamma = -D\nabla n$, and the heat flux expressed in terms of the temperature gradient as $\mathbf{q} = -n\chi\nabla T$. In these simple laws, the diffusion coefficient D was assumed to be independent of the temperature gradient, and the heat conductivity χ was assumed to be independent of the density gradient. However, recent experimental and theoretical studies have shown that it is incorrect to assume no coupling between energy and particle transport.[59–61] If the transport is due to turbulence, the coefficients D and χ will be functions of the plasma parameters. Any theory of turbulent transport should therefore produce a full transport matrix, including off-diagonal coefficients, rather than just two coefficients D and χ (see Gentle *et al.*[61]).

In this work, we use the DIA technique to calculate the transport matrix (diagonal and off-diagonal coefficients) corresponding to perturbations in the density and temperature profiles. The theoretical expressions obtained for the transport coefficients are based on a specific set of equations describing the dynamics of the plasma and on the DIA, but do not require any further approximations.

Even though the current interpretation of transport experiments in tokamaks is that the ion dynamics plays a crucial role in the anomalous thermal transport, we concentrate, for simplicity, on the role played by the electrons, and give the ions a secondary role in this paper. Our method can, in principle, account for the ion dynamics as well by choosing an appropriate form for the ion nonlinear susceptibility (see Sec. II). Here, however, detailed numerical estimates are given only for a very simple ion susceptibility, which does not necessarily represent the complete physics of anomalous transport in tokamaks. This choice is discussed in more details in Secs. III and IV. In addition, we use the simplest possible model for the electrons themselves, ignoring the complications coming from the toroidal geometry, such as magnetic curvature and shear. With these simplifying assumptions, we are able to present a completely self-consistent, yet tractable calculation of anomalous transport in a strong-turbulence regime.

Our work is presented in two papers. In this paper we consider the transport due to electrostatic drift-wave turbulence. In a companion paper,[62] we use the method developed here to calculate the transport due to drift-Alfvén wave turbulence, i.e., including the effects of magnetic fluctuations.

This paper is organized as follows. In Sec. II, we give the basic assumptions and the model equations describing the dynamics of the plasma. We present a method based on the DIA to calculate the transport matrix coefficients. The results are given in a very general form, in terms of the potential fluctuation spectrum. In Sec. III, we give a brief review of fluctuations and transport experiments. As an example of application of our method, we give numerical estimates of the transport coefficients corresponding to the parameters of the TEXT tokamak. Finally, in Sec. IV, we give a summary of the principal results.

II. ANOMALOUS TRANSPORT FROM DRIFT-WAVE TURBULENCE

In this section, we develop a method to determine the transport coefficients associated with drift-wave turbulence in a tokamak plasma. In Sec. II A, we introduce the fundamental kinetic equations, which serve as a basis for the theory, and we specify the geometry of the problem. In Sec. II B, we exploit the separation of time scales and length scales in the problem to derive two separate systems of equations: one describing the transport, the other describing the turbulent fluctuations. Finally, in Sec. II C, we derive a statistical solution for these equations using the direct interaction approximation (DIA), and we give explicit expressions for the anomalous fluxes of particles and heat.

A. Basic equations

1. Electron dynamics

When considering plasma phenomena of low frequency compared to the electron gyrofrequency, such as drift waves, the electron dynamics can be described by the drift-kinetic equation (hereafter DKE). This equation describes the motion of the guiding center of the particles, and is obtained by averaging the Vlasov equation over the rapidly oscillating component of the motion (see, e.g., Kulsrud[63] and Landau and Pitaevskii[64]). It is given by

$$\frac{\partial f}{\partial t}+\nabla\cdot[(\mathbf{v}_E+\mathbf{v}_D)f]+\nabla\cdot(v_\parallel\,\mathbf{b}f)+\frac{\partial}{\partial v_\parallel}\times\left[\left(-\frac{e}{m_e}E_\parallel-\frac{\mu}{m_e}\mathbf{b}\cdot\nabla B\right)f\right]=C(f), \quad (1)$$

where $f(\mathbf{x},v_\parallel,\mu,t)$ is the electron distribution function, $\mu=m_e v_\perp^2/2B$ is the electron magnetic moment, $C(f)$ is the collision operator, and parallel $\parallel$ and perpendicular $\perp$ subscripts refer to the direction of the magnetic field. In this equation, the $\mathbf{E}\times\mathbf{B}$ drift velocity is given by $\mathbf{v}_E=(c/B)\mathbf{E}\times\mathbf{b}$, while the magnetic drift velocity $\mathbf{v}_D=(1/B\Omega_e)(v_\perp^2/2)\mathbf{b}\times\nabla B+(1/\Omega_e)v_\parallel^2(\mathbf{R}_B\times\mathbf{b})$, where $\mathbf{b}$ is a unit vector parallel to the magnetic field, Ω_e is the electron gyrofrequency, and $\mathbf{R}_B$ is vector curvature radius of the magnetic field lines.

In this section, we will consider the electrostatic limit ($\mathbf{E}=-\nabla\phi$, where ϕ is the electrostatic potential) of the DKE, which is valid when the plasma parameter $\beta\ll(m_e/m_i)$ (see, e.g., Miyamoto[65]). We consider the more general electromagnetic case $[\mathbf{E}=-\nabla\phi-(1/c)\partial\mathbf{A}/\partial t]$ in Paper II. We distinguish between the two different populations of electron in a tokamak: circulating and trapped (see, e.g., Miyamoto[65]). In this work, for simplicity, we ignore the magnetic shear[66] and curvature in the tokamak, i.e., we consider a uniform magnetic field $\mathbf{B}_0$. As a consequence, $\mathbf{v}_D=0$ and $\nabla B=0$. The presence of trapped particles is therefore the only ∇B effect that we consider.

The circulating electrons have $|v_\parallel/v_\perp|>(2\epsilon)^{1/2}$, where $\epsilon=r/R$ is the inverse aspect ratio of the tokamak (r and R are the minor and major radii of the tokamak). The orbits of circulating electrons have no turning points; hence they "circulate" around the torus. The distribution function $f^c(\mathbf{x},v_\parallel,v_\perp^2,t)$ of circulating electrons obeys the following DKE;

$$\frac{\partial f^c}{\partial t}+\mathbf{v}_E\cdot\nabla f^c+v_\parallel\,\nabla_\parallel f^c+\frac{e}{m_e}\nabla_\parallel\phi\,\frac{\partial f^c}{\partial v_\parallel}=C_c(f^c,f^t). \quad (2)$$

The collision operator C_c will be specified below [Eq. (25)].

The trapped electrons, instead, have $|v_\parallel/v_\perp|<(2\epsilon)^{1/2}$, implying the presence of turning points in their orbits ("banana orbits;" see, e.g., Miyamoto[65]). For these electrons, the DKE can be further simplified by "bounce averaging," i.e., averaging over the rapid oscillatory motion of the guiding center in the toroidal direction. The bounce-averaged DKE is valid for frequencies smaller than the bounce frequency, i.e., for $\omega<\omega_{be}=\sqrt{\epsilon}v_e/(Rq)$, where $v_e\equiv\sqrt{T_0/m_e}$ is the electron thermal velocity, T_0 is the electron temperature, m_e is the electron mass, and q is the tokamak safety factor (see, e.g., Gross[67]). The bounce-averaging procedure eliminates all the terms that are odd in $v_\parallel$ from the DKE. The distribution function $f^t(\mathbf{x}_\perp,v_\perp^2,t)$ for the trapped electrons therefore satisfies

$$\frac{\partial f^t}{\partial t}+\mathbf{v}_E\cdot\nabla f^t=C_t(f^c,f^t). \quad (3)$$

2. Ion dynamics

Our work will focus on the electron nonlinearities. For simplicity, the ion dynamics will be treated in the fluid limit. In this limit, the ion density n_i can be written quite generally as[68,69]

$$n_i/n_0=(e\phi/T_0)\chi_i(|\phi|^2), \quad (4)$$

where n_0 is the equilibrium ion density (we take $Z=1$), and $\chi_i(|\phi|^2)$ is the nonlinear ion susceptibility, which depends on the frequency and the power spectrum of the potential fluctuations. The results of this section are given in terms of an unspecified χ_i. In Sec. III we will introduce a specific form for χ_i, which will allow us to obtain numerical results.

3. Self-consistency constraint

It is important to maintain the self-consistency of the model, since the electromagnetic fields and particle densities are strongly coupled through Maxwell's equations. Theories that are not self-consistent, such as those based on the quasilinear approximation, usually violate the important property of ambipolarity of the particle fluxes.[40]

Here the system of equations (2)–(4) describing the particle and field dynamics is closed by the addition of the quasineutrality relation, valid for scales larger than λ_D (the Debye length),

$$n_i(\mathbf{x},t)=n_e(\mathbf{x},t), \quad (5)$$

where $n_e(\mathbf{x},t)=\int f_e\,d\mathbf{v}=\int(f^c+f^t)d\mathbf{v}$ is the total electron density.

4. Geometry

Since we assume that $\mathbf{B}_0$ is uniform and that the turbulent length scale is smaller than the macroscopic scales on which density and temperature vary (cf. Sec. II B), the problem can be studied locally. We can therefore use a local Cartesian geometry, in which $x \geqslant 0$ represents the radial coordinate, y represents the poloidal coordinate, and z is the coordinate along the field line. The origin is at the center of the minor radial section of the tokamak. The magnetic surfaces are therefore planes with $x=$const. Note that the plasma density and temperature are constant on magnetic surfaces (see, e.g., Gross[67]).

B. Separation of scales

1. Motivation

For our study of anomalous transport, the basic set of equations [(2)–(5)] can be rewritten as two separate systems of equations: one describing the turbulence, the other describing the transport. This separation can be done because turbulence and transport correspond to widely different time scales and length scales. The two systems of equations remain coupled through their nonlinear terms.

For drift-wave turbulence the typical frequencies and wave numbers are $\omega \sim \omega_*$, $k_\perp \sim \rho_s^{-1}$, and $k_\parallel \sim (qR)^{-1}$, where $\omega_* = -(cT_0/eB)\mathbf{k}\cdot\mathbf{b}_0 \times (\nabla n_0/n_0)$ is the electron diamagnetic frequency, ρ_s is the ion gyroradius at the electron temperature, and qR is the connection length (i.e., the length of the helical trajectory of a circulating particle; see, e.g., Gross[67]). In contrast, the transport occurs at the very low frequency ω_0, and very small wave number $\mathbf{k}_0$ of the external perturbations. For example, the typical frequency for modulated gas puff experiments in the Texas Experimental Tokamak (TEXT) is $\omega_0 \lesssim 240$ rad/s$\ll \omega_* \sim 10^6$ rad/s (cf. Table I). The poloidal and parallel wave numbers for the transport both vanish because the fluxes are averaged over the magnetic surfaces (perpendicular to the x axis). The radial wave number $k_{0x} \sim L_n^{-1} \ll \rho_s^{-1}$, where $L_n \equiv |n_0/\nabla n_0|$ is the (macroscopic) density gradient length scale. For example, in TEXT, $k_{0x} \simeq 0.04$ cm^{-1} and $\rho_s^{-1} \simeq 8$ cm^{-1} (cf. Table I).

2. Transport equations

In the following calculations, we will use an overcaret for quantities that vary on the transport scales, and an overtilde for turbulent quantities. Subscripts 0 will be used for the equilibrium values of the parameters.

In this work, we develop a method to calculate the anomalous fluxes resulting from small external perturbations. We study the modulation due to these perturbations on the transport scales. Therefore, we add external, infinitesimal, velocity-dependent source terms $\hat{\xi}^c$, $\hat{\xi}^t$, and $\hat{\xi}^i$ to Eqs. (2), (3), and (4) respectively. As described in Sec. II B, these sources have frequencies $\omega_0 \ll \omega_*$ and wave numbers $k_{0x} \ll \rho_s^{-1}$. The electron distribution functions $f^{c,t}$ are then the sum of an equilibrium part f_0, a fluctuating part $\tilde{f}^{c,t}$, and the response to the modulation $\hat{f}^{c,t}$:

$$f^{c,t}=f_0+\tilde{f}^{c,t}+\hat{f}^{c,t}, \tag{6}$$

where the equilibrium distribution function f_0 is a local Maxwellian,

$$f_0(\mathbf{x},v^2)=\frac{n_0(\mathbf{x})}{(2\pi)^{3/2}}\left(\frac{T_0(\mathbf{x})}{m_e}\right)^{-3/2}\exp\left(\frac{-m_e v^2}{2T_0(\mathbf{x})}\right), \tag{7}$$

n_0 and T_0 being the local electron density and temperature. We assume[70] $\phi_0=0$.

The equations describing the (infinitesimal) *response functions* $\hat{f}^t$, $\hat{f}^c$, $\hat{n}_i$, and $\hat{\phi}$ to the external perturbations are obtained by taking the low-frequency and long-wavelength limit of Eqs. (2)–(5), and averaging over the magnetic flux surfaces (i.e., $k_{\parallel 0}=0$). This gives

$$\frac{\partial \hat{f}^c}{\partial t}+\langle \widehat{\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}^c}\rangle=\langle \hat{C}_c\rangle+\hat{\xi}^c, \tag{8}$$

$$\frac{\partial \hat{\phi}^t}{\partial t}+\langle \widehat{\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}^t}\rangle=\langle \hat{C}_t\rangle+\hat{\xi}^t, \tag{9}$$

$$\hat{n}_i=n_0(e\hat{\phi}/T_0)\chi_i+\hat{\xi}^i, \tag{10}$$

$$\hat{n}_i=\hat{n}_e=\int(\hat{f}^c+\hat{f}^t)d\mathbf{v}, \tag{11}$$

where the angular brackets denotes an ensemble average over the turbulence. Equations (8) and (9) can also be written in terms of the incremental anomalous fluxes $\hat{\mathbf{\Gamma}}^{c,t}\equiv\langle\widehat{\tilde{\mathbf{v}}_E\tilde{f}^{c,t}}\rangle$ as[71]

$$\frac{\partial \hat{f}^{c,t}}{\partial t}+\nabla\cdot\hat{\mathbf{\Gamma}}^{c,t}=\langle \hat{C}_{c,t}\rangle+\hat{\xi}^{c,t}. \tag{12}$$

These phase-space fluxes $\hat{\mathbf{\Gamma}}^{c,t}$ are velocity dependent. However, since the collision frequency $\nu_{ei} \gg \omega_0$, the distributions $\hat{f}^c$ and $\hat{f}^t$ are nearly Maxwellian on transport scales. Therefore we can use the corresponding moment equations to calculate the fluxes of particles and energy. These are obtained by integrating (12) over velocity. We obtain

$$\frac{\partial \hat{n}}{\partial t}+\nabla\cdot\hat{\mathbf{\Gamma}}_n=\int\hat{\xi}\,d\mathbf{v}, \tag{13}$$

where the particle flux density

$$\hat{\mathbf{\Gamma}}_n\equiv\int(\hat{\mathbf{\Gamma}}^c+\hat{\mathbf{\Gamma}}^t)d\mathbf{v}, \tag{14}$$

and

$$\frac{\partial \hat{u}}{\partial t}+\nabla\cdot\hat{\mathbf{Q}}=T_0\int\hat{\xi}\,\frac{v^2}{2v_e^2}\,d\mathbf{v}, \tag{15}$$

where $\hat{u}=(\frac{3}{2})\widehat{nT}$ is the energy density and $\hat{\mathbf{Q}}\equiv T_0\int(\hat{\mathbf{\Gamma}}^c+\hat{\mathbf{\Gamma}}^t)(v^2/2v_e^2)d\mathbf{v}$ is the total energy flux density. Using the continuity equation (13), the energy equation (15) can be rewritten as an evolution equation for the temperature,

$$\frac{3}{2} n_0 \frac{\partial \hat{T}}{\partial t} + \nabla \cdot \hat{\mathbf{q}} + \frac{3}{2} \hat{\boldsymbol{\Gamma}}_n \cdot \nabla T_0 = T_0 \int \hat{\xi} \frac{v^2 - 3v_e^2}{2v_e^2} d\mathbf{v}, \quad (16)$$

where the net energy flux $\hat{\mathbf{q}}$ is obtained by substracting the energy convected by the particles from the total energy flux $\hat{\mathbf{Q}}$,

$$\hat{\mathbf{q}} \equiv \hat{\mathbf{Q}} - \frac{3}{2} T_0 \hat{\boldsymbol{\Gamma}}_n = T_0 \int (\hat{\boldsymbol{\Gamma}}^c + \hat{\boldsymbol{\Gamma}}^t) \frac{v^2 - 3v_e^2}{2v_e^2} d\mathbf{v}. \quad (17)$$

Note that we have omitted the terms representing the classical and neoclassical fluxes (i.e., those corresponding to the collision terms $\langle \hat{C}_{c,t} \rangle$) in Eqs. (15) and (16).

To evaluate $\hat{\boldsymbol{\Gamma}}_n$ and $\hat{\mathbf{q}}$, we consider perturbations of the equilibrium density and temperature profiles. The Maxwellian distribution depends on the temperature and density as $f_0 \propto n_0 T_0^{-3/2} \exp(-mv^2/2T_0)$. Therefore, if we perturb the density profile, f_0 is perturbed according to

$$\hat{n}_0 \frac{\partial f_0}{\partial n_0} = f_0 \frac{\hat{n}_0}{n_0},$$

and the corresponding sources are

$$\hat{\xi}^c = \hat{\xi}^t = -i\omega_0 f_0 \frac{\hat{n}_0}{n_0},$$
$$\hat{\xi}^i = \frac{1}{-i\omega_0} \int \hat{\xi}^e \, d\mathbf{v} = \hat{n}_0. \quad (18)$$

If we perturb the temperature profile, f_0 is perturbed according to

$$\hat{T}_0 \frac{\partial f_0}{\partial T_0} = f_0 \frac{v^2 - 3v_e^2}{2v_e^2} \frac{\hat{T}_0}{T_0},$$

and the sources are

$$\hat{\xi}^c = \hat{\xi}^t = (-i\omega_0) f_0 \frac{v^2 - 3v_e^2}{2v_e^2} \frac{\hat{T}_0}{T_0},$$
$$\hat{\xi}^i = \frac{1}{-i\omega_0} \int \hat{\xi}^e \, d\mathbf{v} = 0. \quad (19)$$

Note that in both cases we insist on injecting the same number of ions and electrons in order to preserve the quasineutrality of the plasma.

3. Turbulence

The anomalous fluxes $\hat{\boldsymbol{\Gamma}}^{c,t}$ depend on the turbulent spectrum of fluctuations. These fluctuations are described by the following set of nonlinear equations:

$$\left(\frac{\partial}{\partial t} + v_\parallel \nabla_\parallel \right) \tilde{f}^c + \tilde{\mathbf{v}}_E \cdot \nabla f_0 + \frac{e}{m_e} \nabla_\parallel \tilde{\phi} \frac{\partial f_0}{\partial v_\parallel} - \tilde{C}_c = -\tilde{\mathbf{v}}_E \cdot \nabla \tilde{f}^c, \quad (20)$$

$$\frac{\partial}{\partial t} \tilde{f}^t + \tilde{\mathbf{v}}_E \cdot \nabla f_0 - \tilde{C}_t = -\tilde{\mathbf{v}}_E \cdot \nabla \tilde{f}^t, \quad (21)$$

$$\frac{\tilde{n}_i}{n_0} = \frac{e\tilde{\phi}}{T_0} \chi_i, \quad (22)$$

$$\tilde{n}_i = \tilde{n}_e = \int (\tilde{f}^c + \tilde{f}^t) d\mathbf{v}, \quad (23)$$

obtained from Eqs. (2)–(5) by using the expansion (6). We have neglected the nonlinear term $-(e/m_e)\nabla_\parallel \tilde{\phi} \, \partial \tilde{f}/\partial v_\parallel$, since this term does not contribute directly to the spatial fluxes.

We now specify the form of the collision operators $\tilde{C}_c$ and $\tilde{C}_t$. On the time scale of drift-wave turbulence, the circulating electrons are collisionless, since $\omega_* \gg \nu_{ei}$. The trapped electrons, however, have an effective collision frequency $\nu_{\text{eff}} \sim \omega_* < \omega_b$. This "detrapping" frequency is velocity dependent, and is given by $\nu_{\text{eff}} = \nu_* (v_e/v)^3$, where $\nu_* = \nu_{ei}/\epsilon$. Some authors have used very simple collision operators, given by

$$\tilde{C}_t = -\nu_{\text{eff}} \tilde{f}^t,$$
$$\tilde{C}_c = 0. \quad (24)$$

However, these collision operators do not conserve the number of particles, since electrons detrapped by a collision do not reappear as circulating electrons. Therefore, the use of Eqs. (24) could introduce errors in the calculation of the transport. A better choice would be to adopt an energy-dependent linearized BGK operator (Bhatnagar, Gross, and Krook[72]) of the type

$$\tilde{C}_t = -\nu_{\text{eff}} \left[\tilde{f}^t - f_0 \left(\frac{\tilde{n}_e}{n_0} + \frac{\tilde{T}_e}{T_0} \frac{v^2 - 3v_e^2}{2v_e^2} \right) \right],$$
$$\tilde{C}_c = -\left(\frac{f_0}{n_{0c}} \right) \int_t \tilde{C}_t \, d\mathbf{v}, \quad (25)$$

where, $\tilde{n}_e = \tilde{n}_e^t + \tilde{n}_e^c$ is the total fluctuating electron density, and $n_{0c} = \int_c f_0 \, d\mathbf{v}$ is the equilibrium number density of circulating electrons. This collision operator not only conserves the total number of particles, but also vanishes when $\tilde{f}$ is a perturbed Maxwellian. Unfortunately, it couples the two electron equations [(20) and (21)] through their nonlinear terms. Although we could, in principle, solve the problem with this collision operator, it complicates the algebra substantially. For the sake of simplicity and clarity, we will replace $\tilde{n}_e$ and $\tilde{T}_e$ in Eq. (25) by their linear approximations $\tilde{n}_e^L$ and $\tilde{T}_e^L$, given by

$$\frac{\tilde{n}_e^L}{n_0} = \frac{1}{n_0} \int \tilde{f}^L \, d\mathbf{v} \equiv \frac{e\tilde{\phi}}{T_0} \chi_e^L \quad (26)$$

and

$$\frac{\tilde{T}_e^L}{T_0} = \frac{2}{3n_0} \int \tilde{f}^L \frac{v^2 - 3v_e^2}{2v_e^2} d\mathbf{v} \equiv \frac{2}{3} \frac{e\tilde{\phi}}{T_0} \chi_T^L, \quad (27)$$

where $\tilde{f}^L = (\tilde{f}^t + \tilde{f}^c)^L$ is determined from the linearized form of Eqs. (20)–(23). Explicit expressions for χ_e^L (the electron susceptibility) and χ_T^L are derived in Appendix B.

C. Statistical solution of the model equations

1. The direct interaction approximation

In well-developed turbulence, perturbed quantities can be treated as statistical variables. The equations for the moments of these statistical variables form an infinite set of differential equations. The direct interaction approximation (hereafter DIA) method provides an approximate way of closing this infinite set of coupled equations into a definite set, containing only moments below some finite order. It presents many advantages over alternative theories of turbulence. In particular, it conserves energy and the positivity of the energy spectrum, two important features that are not present in simpler theories such as those based on quasinormal approximations.[46]

In principle, the DIA method allows the determination of both the response functions and the turbulent spectrum at saturation. Here, however, we will use the method to determine the response functions in terms of the saturation spectrum, which we assume to be known experimentally (see Sec. III).

The DIA method is based on the evaluation of average infinitesimal response functions that naturally provide the *incremental fluxes* resulting from small perturbations of the equilibrium profiles, rather than total fluxes accross the magnetic surfaces.[52] This is particularly well suited to comparisons with the recent perturbative transport experiments.[15,61] Indeed, although anomalous transport has been studied traditionally in terms of total equilibrium fluxes, many experiments now study the transport of small perturbations, such as heat pulses. It has been observed experimentally[58] and can be shown analytically (see Appendix D) that *the incremental fluxes resulting from small perturbations are quite different from the equilibrium fluxes.*

For detailed descriptions of the DIA method and discussions about the validity of this procedure, the reader is referred to the review articles by Kadomtsev,[42] Leslie,[47] Krommes,[48] and Similon and Sudan.[49]

2. Anomalous fluxes

Let us denote the first-order fluctuating quantities in the small parameter expansion by $\tilde{f}^{c(1)}$, $\tilde{f}^{t(1)}$, $\tilde{n}_i^{(1)}$, and $\tilde{\phi}^{(1)}$. These "forced beat" fluctuations have frequencies $\omega'=\omega_0-\omega$ and wavelengths $\mathbf{k}'=\mathbf{k}_0-\mathbf{k}$. We rewrite the transport equations (8)–(11) as

$$\frac{\partial\hat{f}^c}{\partial t}+\langle\tilde{\mathbf{v}}_E^{(1)}\cdot\nabla\tilde{f}^c+\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}^{c(1)}\rangle=\langle\hat{C}_c\rangle+\hat{\xi}^c, \tag{28}$$

$$\frac{\partial\hat{f}^t}{\partial t}+\langle\tilde{\mathbf{v}}_E^{(1)}\cdot\nabla\tilde{f}^t+\tilde{\mathbf{v}}_E\cdot\nabla\tilde{f}^{t(1)}\rangle=\langle\hat{C}_t\rangle+\hat{\xi}^t, \tag{29}$$

$$\hat{n}_i=n_0\chi_i\frac{e\hat{\phi}}{T_0}+\hat{\xi}_i, \tag{30}$$

$$\hat{n}_i=\hat{n}_e=\int(\hat{f}^c+\hat{f}^t)d\mathbf{v}. \tag{31}$$

From Eqs. (28) and (29), we see that the anomalous fluxes $\hat{\mathbf{\Gamma}}^{c,t}$ are

$$\hat{\mathbf{\Gamma}}_{\rm DIA}^{c,t}=\langle\tilde{\mathbf{v}}_E^{(1)}\tilde{f}^{c,t}+\tilde{\mathbf{v}}_E\tilde{f}^{c,t(1)}\rangle. \tag{32}$$

On the other hand, the DIA equations for the forced beat fluctuations, obtained from the system of Eqs. (20)–(23), can be written as

$$\left(\frac{\partial}{\partial t}+v_\parallel\nabla_\parallel\right)\tilde{f}^{c(1)}+\tilde{\mathbf{v}}_E^{(1)}\cdot\nabla f_0-\tilde{C}_c^{(1)}$$
$$=-(\tilde{\mathbf{v}}_E\cdot\nabla\hat{f}^c+\hat{\mathbf{v}}_E\cdot\nabla\tilde{f}^c), \tag{33}$$

$$\frac{\partial}{\partial t}\tilde{f}^{t(1)}+\tilde{\mathbf{v}}_E^{(1)}\cdot\nabla f_0-\tilde{C}_t^{(1)}=-(\tilde{\mathbf{v}}_E\cdot\nabla\hat{f}^t+\hat{\mathbf{v}}_E\cdot\nabla\tilde{f}^t), \tag{34}$$

$$\tilde{n}_i^{(1)}=n_0(e\tilde{\phi}^{(1)}/T_0)\chi_i, \tag{35}$$

$$\tilde{n}_i^{(1)}=\tilde{n}_e^{(1)}=\int(\tilde{f}^{c(1)}+\tilde{f}^{t(1)})d\mathbf{v}. \tag{36}$$

Note that in Eq. (35) we have neglected the effects of the beat potential on the nonlinear ion susceptibility χ_i. This simplification is one of the main limitations of our treatment.

The set of Eqs. (33)–(36) is linear in the beat fluctuations. It can be solved explicitly for $\tilde{f}^{c(1)}$, $\tilde{f}^{t(1)}$, $\tilde{n}_i^{(1)}$, and $\tilde{\phi}^{(1)}$ (which vary on the turbulent scale) in terms of the response functions $\hat{f}^c$, $\hat{f}^t$, $\hat{n}_i$, $\hat{\phi}$, and the fluctuating quantities $\tilde{f}^c$, $\tilde{f}^t$, $\tilde{n}_i$, and $\tilde{\phi}$. These results are then used to calculate the velocity-dependent anomalous fluxes $\hat{\mathbf{\Gamma}}_{\rm DIA}^{c,t}$ in terms of the fluctuation spectrum. Here we only provide a sketch of the derivation. The reader is referred to Appendix C for more details.

The calculations are performed in Fourier space. The Fourier transforms (in space and time) of Eqs. (33)–(36) are

$$(g_{k'}^c)^{-1}\tilde{h}_{k'}^{c(1)}+b_{k'}^c f_0(e\tilde{\phi}_{k'}^{(1)}/T_0)$$
$$=-W_{k,k_0}(e/T_0)(\tilde{\phi}_k^*\hat{h}_{k_0}^c-\hat{\phi}_{k_0}\tilde{h}_k^{c*}), \tag{37}$$

$$(g_{k'}^t)^{-1}\tilde{h}_{k'}^{t(1)}+b_{k'}^t f_0(e\tilde{\phi}_{k'}^{(1)}/T_0)$$
$$=-W_{k,k_0}(e/T_0)(\tilde{\phi}_k^*\hat{h}_{k_0}^t-\hat{\phi}_{k_0}\tilde{h}_k^{t*}), \tag{38}$$

$$\tilde{n}_{ik'}^{(1)}=n_0(e\tilde{\phi}_{k'}^{(1)}/T_0)\chi_i, \tag{39}$$

$$\tilde{n}_{ik'}^{(1)}=\tilde{n}_{ek'}^{(1)}=n_0\left(\frac{e\tilde{\phi}_{k'}^{(1)}}{T_0}\right)+\int(\tilde{h}_{k'}^{c(1)}+\tilde{h}_{k'}^{t(1)})d\mathbf{v}. \tag{40}$$

Here $k_0\equiv(\mathbf{k}_0,\omega_0)$, $k\equiv(\mathbf{k},\omega)$, and $k'\equiv(\mathbf{k}',\omega')=k_0-k$ are the wave vectors of the external perturbations, the fluctuations, and the beat fluctuations, respectively. The electron distribution functions have been expanded into their adiabatic and nonadiabatic parts as $\tilde{f}^{c,t}=f_0e\tilde{\phi}/T_0+\tilde{h}^{c,t}$. The nonlinear coupling coefficient is given by

$$W_{k,k_0}=(cT_0/eB_0)\mathbf{b}_0\cdot(\mathbf{k}\times\mathbf{k}_0). \tag{41}$$

The propagators g_k^c and g_k^t for the circulating and trapped electrons are defined by

$$g_k^c=[-i(\omega-k_\parallel v_\parallel)]^{-1} \tag{42}$$

and

$$g_k^t=[-i\omega+\nu_{\rm eff}]^{-1}. \tag{43}$$

Finally, the coefficients b_k^c and b_k^t are

$$b_k^c = -i(\omega-\omega_*)+\sigma_k \tag{44}$$

and

$$b_k^t = -i(\omega-\omega_*)+\nu_{\rm eff}\left(1-\chi_e^L-\frac{2}{3}\chi_T^L\frac{v^2-3v_e^2}{2v_e^2}\right), \tag{45}$$

where $\sigma_k(v)$, given by Eq. (B10) of Appendix B, is a collision frequency defined by $C_k^c \equiv -\sigma_k f_0(e\phi_k/T_0)$, and $\omega_* = -(cT_0/eB_0)\mathbf{k}\cdot\mathbf{b}_0\times(\nabla n_0/n_0)$ is the diamagnetic frequency.

Solving the linear system of Eqs. (37)–(40) for $\tilde{h}_{k'}^{c(1)}$, $\tilde{h}_{k'}^{t(1)}$, $\tilde{n}_{ik'}^{(1)}$ and $\tilde{\phi}_{k'}^{(1)}$, we find

$$n_0\frac{e\tilde{\phi}_k^{(1)}}{T_0} = \frac{-1}{\chi_i-\chi_e^L}W_{k,k_0}\frac{e}{T_0}\int[\tilde{\phi}_k(g_k^c\hat{h}_{k_0}^c+g_k^t\hat{h}_{k_0}^t) - \hat{\phi}_{k_0}(g_k^c\tilde{h}_k^c+g_k^t\tilde{h}_k^t)]d\mathbf{v} \tag{46}$$

and

$$\tilde{h}_k^{c,t(1)} = -g_k^{c,t}b_k^{c,t}f_0n_0\frac{e\tilde{\phi}_k^{(1)}}{T_0} - W_{k,k_0}\frac{e}{T_0}(\tilde{\phi}_k\hat{h}_{k_0}^{c,t}-\hat{\phi}_{k_0}\tilde{h}_k^{c,t}). \tag{47}$$

The coefficients χ_e^L and χ_i have been defined in Eqs. (26) and (4). The quantity $\chi_i-\chi_e^L$ is proportional to the plasma dielectric function $\mathscr{E}(\omega,\mathbf{k}) = (\omega_{pe}/kv_e)^2(\chi_i-\chi_e^L)$. Equations (46) and (47) give the beat quantities in terms of the response functions $\hat{\phi}_{k_0}$ and $\hat{h}_{k_0}^{c,t}$, and the fluctuations. The DIA fluxes $\hat{\Gamma}_{\rm DIA}^{c,t}$ are then obtained in terms of the external perturbations as

$$\begin{aligned}\hat{\Gamma}_{\rm DIA}^{c,t} &= \langle\tilde{\mathbf{v}}_E^{(1)}\tilde{h}^{c,t}+\tilde{\mathbf{v}}_E\tilde{h}^{c,t(1)}\rangle \\ &= -i\left(\frac{cT_0}{eB}\right)\sum_k \mathbf{k}\times\mathbf{b}_0 W_{k,k_0}\frac{1}{-i\omega_0}\Bigg[g_k^{c,t}\hat{\xi}^{c,t}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle \\ &\quad -\frac{1}{\chi_i-\chi_e^L}\left(\int(g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t)d\mathbf{v}\right) \\ &\quad \times\left(\frac{1}{n_0}\left\langle\frac{e}{T_0}\tilde{\phi}_k\tilde{h}_k^{c,t*}\right\rangle + g_k^{c,t}b_k^{c,t}\frac{f_0}{n_0}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\right)\Bigg], \end{aligned}\tag{48}$$

where the response functions have been replaced by their lowest-order approximations (obtained by neglecting the nonlinearities),

$$\hat{h}_{k_0}^{c,t} = \frac{\hat{\xi}^{c,t}}{-i\omega_0} \quad \text{and} \quad \hat{\phi}_{k_0}=0. \tag{49}$$

Note that in Eqs. (46)–(48), we have used the separation of scale $|k_0| \ll |k|$, so that $k' = k_0 - k \simeq -k$.

Note that the expression (48) for the perturbed anomalous fluxes $\hat{\Gamma}_{\rm DIA}^{c,t}$ is independent of the details of the electron and ion equations, requiring only that they can be written in the form (37)–(39).

The test-particle result is recovered by letting $\tilde{\phi}_k^{(1)} \to 0$ in Eqs. (37)–(40) (This is equivalent to assuming that the electromagnetic fields are not directly affected by the perturbations.) This gives

$$\tilde{h}_{k'}^{c,t(1)} = -W_{k,k_0}(e/T_0)(\tilde{\phi}_k\hat{h}_{k_0}^{c,t}-\hat{\phi}_{k_0}\tilde{h}_k^{c,t}), \tag{50}$$

and the test-particle fluxes are

$$\hat{\Gamma}_{\rm TP}^{c,t} = -i\frac{cT_0}{eB_0}\sum_k \mathbf{k}\times\mathbf{b}_0 W_{k,k_0}\frac{1}{-i\omega_0}g_k^{c,t}\hat{\xi}^{c,t}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \tag{51}$$

We note that, as expected, expression (51) is independent of χ_i, i.e., $\hat{\Gamma}_{\rm TP}^{c,t}$ is independent of the ion dynamics. Indeed, since we have neglected classical collisions, electrons and ions can interact only through the electromagnetic fields. However, the test-particle approximation assumes that the fields are not affected by density perturbations, and therefore the electron incremental flux must be independent of the ion dynamics.

As an example of the test-particle limit, we can calculate the flux of circulating electrons for a purely resonant propagator, i.e., $g_k = \pi\delta(\omega - k_\parallel v_\parallel)$. Then

$$\begin{aligned}\hat{\Gamma}_{n,\rm TP} &= \int\hat{\Gamma}_{\rm TP}\,d\mathbf{v} \\ &= n_0\frac{cT_0}{eB_0}\sum_k(\mathbf{k}\times\mathbf{b}_0)\sqrt{\frac{\pi}{2}} \\ &\quad \times\frac{-(cT_0/eB_0)\mathbf{k}\cdot(\mathbf{b}_0\times i\mathbf{k}_0)(\hat{n}_0/n_0)}{|k_\parallel|\,v_e}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \end{aligned}\tag{52}$$

This result has the same form as the quasilinear result for the *equilibrium flux*, given by[18]

$$\Gamma_n = n_0\frac{cT_0}{eB_0}\sum_k k_y\sqrt{\frac{\pi}{2}}\frac{\omega_*-\omega}{|k_\parallel|\,v_e}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \tag{53}$$

The test-particle result (52) can be obtained from the quasilinear expression (53) by taking the perturbation of this expression, ignoring the perturbation of the spectrum and the normal frequency ω.

We now proceed with our DIA calculation. Using Eq. (51), the DIA fluxes can be rewritten as

$$\begin{aligned}\hat{\Gamma}_{\rm DIA}^{c,t} &= \hat{\Gamma}_{\rm TP}^{c,t} + i\frac{cT}{eB_0}\sum_k\mathbf{k}\times\mathbf{b}_0W_{k,k_0}\frac{1}{-i\omega_0}\frac{1}{\chi_i-\chi_e^L} \\ &\quad\times\left(\int(g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t)d\mathbf{v}\right)\left(\frac{1}{n_0}\left\langle\frac{e}{T_0}\tilde{\phi}_k\tilde{h}_k^{c,t*}\right\rangle\right. \\ &\quad\left. + g_k^{c,t}b_k^{c,t}\frac{f_0}{n_0}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\right), \end{aligned}\tag{54}$$

The additional term present in the DIA result is due to the quasineutrality constraint. This term is proportional to $1/(\chi_i-\chi_e^L)$, i.e., to the inverse of the plasma dielectric function $\mathscr{E}(\omega,\mathbf{k})$. Note, also, that the fluxes of trapped particles and circulating particles are coupled through $\mathscr{E}(\omega,\mathbf{k})$ in the DIA result.

We can now obtain the fluxes of particles and heat using Eqs. (14) and (17) for perturbations in the density and temperature profiles given by (18) and (19). If the wavelength of the perturbation is smaller than the density gradient length scale, i.e., if $k_{0x}L_n > 1$, the results can be written in the form

$$\hat{\Gamma}_n = -D_{nn}\nabla\hat{n}_0 - D_{nT}(n_0/T_0)\nabla\hat{T}_0 \tag{55}$$

and

$$\hat{\mathbf{q}} = -D_{Tn}T_0\nabla\hat{n}_0 - D_{TT}n_0\nabla\hat{T}_0, \tag{56}$$

where the four diffusion coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT} are given by

$$D_{nn} = \frac{-1}{n_0}\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right) \times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle \int (g_k^c+g_k^t)f_0\,d\mathbf{v}, \tag{57}$$

$$D_{nT} = \frac{-1}{n_0}\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right) \times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle \int (g_k^c+g_k^t)f_0\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}, \tag{58}$$

$$D_{Tn} = \frac{-1}{n_0}\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\Bigg[\int (g_k^c+g_k^t)f_0 \times\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v} - \frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right) \times\int (g_k^c+g_k^t)f_0\,d\mathbf{v}\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{59}$$

$$D_{TT} = \frac{-1}{n_0}\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\Bigg[\int (g_k^c+g_k^t)f_0 \times\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^2 d\mathbf{v} - \frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right) \times\int (g_k^c+g_k^t)f_0\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \tag{60}$$

Equations (55) and (56) can be combined using the anomalous transport matrix,

$$\begin{pmatrix}\hat{\Gamma}_n\\(1/T_0)\hat{\mathbf{q}}\end{pmatrix} = -\begin{pmatrix}D_{nn} & D_{nT}\\ D_{Tn} & D_{TT}\end{pmatrix}\begin{pmatrix}\nabla\hat{n}_0\\(n_0/T_0)\nabla\hat{T}_0\end{pmatrix}. \tag{61}$$

The four diffusion coefficients (57)–(60) will be evaluated in Sec. III for specific density and temperature profiles. We have chosen to write the transport matrix (61) in such a way that the flux of particles $\hat{\Gamma}_n$ and the net heat flux $\hat{\mathbf{q}}$ are given in terms of the density source $\hat{n}_0$ and the temperature source $\hat{T}_0$. The transport matrix is then symmetric in the test-particle limit, i.e., $D_{nT}^{\mathrm{TP}} = D_{Tn}^{\mathrm{TP}}$. It is important to realize that, although it is very simple in analytic calculations to consider pure density sources or pure temperature sources, this is not possible experimentally. Indeed, density and temperature are not eigenvalues of the coupled density and temperature evolution equations (13) and (16). Therefore, they will, in general, be perturbed simultaneously (cf. Sec. III B).

The integrals over velocity space in Eqs. (57)–(60) can be evaluated explicitly since f_0 is a Maxwellian distribution, and the propagators g_k^c and g_k^t are known. In terms of the "circulating integrals" H_n and the "trapped integrals" A_n defined in Appendix A, the diffusion coefficients can be written as

$$D_{nn} = -\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right) \times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\frac{1}{-i\omega}[(1-\tau)H_0+\tau A_0], \tag{62}$$

$$D_{nT} = -\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right) \times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\frac{1}{-i\omega}[(1-\tau)H_1+\tau A_1], \tag{63}$$

$$D_{Tn} = -\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\frac{1}{-i\omega}\Bigg[[(1-\tau)H_1 +\tau A_1] - \frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right) \times[(1-\tau)H_0+\tau A_0]\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{64}$$

$$D_{TT} = -\left(\frac{cT_0}{eB_0}\right)^2 \sum_k |\mathbf{k}\times\mathbf{b}_0|^2\frac{1}{-i\omega}\Bigg[[(1-\tau)H_2+\tau A_2] - \frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right) \times[(1-\tau)H_1+\tau A_1]\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{65}$$

where τ, calculated in Appendix A, is the fraction of trapped electrons.

The test-particle results are recovered by dropping the terms involving a $1/(\chi_i-\chi_e^L)$ in expressions (62)–(65). This is equivalent to letting the dielectric go to infinity, i.e., ignoring the electromagnetic field response.

Note that the expressions for the transport matrix coefficients are rather complicated, and cannot be expressed as simple scaling laws. Furthermore, there is no simple relation between the various transport coefficients. It has been recently argued that simple scaling laws cannot explain the experimentally observed features of the perturbative fluxes.[61]

3. Spectrum modulation

When the equilibrium density and temperature profiles are modified by external perturbations, the saturation spectrum is also perturbed as a result. In recent experiments in TEXT, the spectrum modulation resulting from the modulation of the gas feed has been measured. It is not possible to calculate the spectrum modulation in the quasilinear or test-particle approximations. Indeed, these methods neglect the effects of the beat potential, which is responsible for the spectrum modulation. However, as we show in this section, it is straightforward to obtain this information using the DIA method.

The spectrum at a wave number k is given by $I_k = \langle (e/T_0)^2 \tilde{\phi}_k \tilde{\phi}_k^* \rangle$. The modulation of the spectral component I_k at a frequency and wave number $k_0 \equiv (\mathbf{k}_0, \omega_0)$ is denoted by $\hat{I}_{k,k_0}$ and can be obtained using the DIA method. We obtain

$$\hat{I}_{k,k_0} = \langle (e/T_0)^2 \tilde{\phi}_k \tilde{\phi}^{(1)}_{k_0-k} \rangle + \langle (e/T_0)^2 \tilde{\phi}^{(1)}_k \tilde{\phi}_{k_0-k} \rangle$$
$$= 2\,\mathrm{Re} \langle (e/T_0)^2 \tilde{\phi}_k^* \tilde{\phi}_k^{(1)} \rangle, \tag{66}$$

where we have used the separation of scales $k' = k_0 - k \simeq -k$. In the test-particle limit, $\tilde{\phi}_k^{(1)} \to 0$ and $\hat{I}_{k,k_0} = 0$. Using the result (46) for $\tilde{\phi}_k^{(1)}$, we can rewrite (66) as

$$\hat{I}_{k,k_0} = \mathrm{Re}\left[\frac{-2}{\chi_i - \chi_e^L} W_{k,k_0} \frac{1}{n_0} \left(\frac{e}{T_0} \right)^2 \int \langle [\tilde{\phi}_k (g_k^c \hat{h}_{k_0}^c + g_k^t \hat{h}_{k_0}^t) \right.$$
$$\left. - \hat{\phi}_{k_0} (g_k^c \hat{h}_k^c + g_k^t \tilde{h}_k^t)] \tilde{\phi}_k^* \rangle d\mathbf{v} \right]. \tag{67}$$

Now, using Eqs. (18) and (19) for the perturbations of density and temperature, and Eq. (49) for the lowest-order response functions, we obtain the relation between the relative spectral modulation and the relative density and temperature modulations as

$$\frac{\hat{I}_{k,k_0}}{\langle | e\tilde{\phi}_k / T_0 |^2 \rangle} = I_n \frac{\nabla \hat{n}_0}{n_0} + I_T \frac{\nabla \hat{T}_0}{T_0}, \tag{68}$$

with the coefficients

$$I_n = \mathrm{Re}\left(\frac{2}{\chi_i - \chi_e^L} \frac{cT_0}{eB_0} k_\theta \frac{1}{-i\omega} [(1-\tau) H_0 + \tau A_0] \right) \tag{69}$$

and

$$I_T = \mathrm{Re}\left(\frac{2}{\chi_i - \chi_e^L} \frac{cT_0}{eB_0} k_\theta \frac{1}{-i\omega} [(1-\tau) H_1 + \tau A_1] \right). \tag{70}$$

In these equations, we have used the velocity integrals H_n and A_n defined by Eqs. (A3) and (A1) in Appendix A. Numerical estimates of these coefficients are given in Sec. III, and can be compared with those from modulation experiments.

III. THE ANOMALOUS TRANSPORT MATRIX

A. Summary of recent experiments

1. Measurements of the transport coefficients

Thermal and particle transport coefficients have been determined traditionally from the study of the steady-state power and particle balance. Recently, however, a new type of method has been developed, which involves perturbations of the steady-state profiles and the analysis of the subsequent relaxation processes. These perturbations can be imposed externally or can result from naturally occurring phenomena such as the sawtooth collapse.

The first direct measurements of transport coefficients from the propagation of heat pulses produced by the sawtooth collapse appeared in the late 1970s.[73,74] Since then, the method has been used in many experiments at the TFTR (Tokamak Fusion Test Reactor),[54,56] JET (Joint European Torus),[57] and TEXT (Texas Experimental Tokamak)[58] tokamaks (see Wesson's *Tokamaks*[75] for general information about these machines). Most recently, experiments have been performed where the injection of gas in the tokamak (gas feed) is modulated in time.[61] In these gas feed modulation experiments, it has been possible to measure the modulation of the density and temperature profiles as well as that of the turbulent spectrum.

Discrepancies have been reported between the transport coefficients measured from steady-state power balance analysis and those deduced from the analysis of pulse propagation. However, it is now understood that this discrepancy is perfectly natural, since the two methods actually measure different physical properties of the plasma.[76] Indeed, the diffusion coefficients associated with the evolution of perturbations of the profiles are obtained by *linearizing* the transport equations about an equilibrium state. We have shown in Sec. II that these coefficients are different from those determining the steady-state transport.

It is difficult to separate experimentally the effects due to density perturbations from those due to temperature perturbations. This is because changes in the density profile induce changes in the temperature profile, and vice versa. It was shown both experimentally[77] and theoretically[78] that the density and temperature are not eigenvectors of the transport problem. They are coupled through the off-diagonal terms of the transport matrix. As a result, the measured fluxes always involve several transport coefficients. Recently, a framework for the interpretation of coupled heat and particle transport has been developed,[76] and the experimental results expressed in terms of a transport matrix with nonzero off-diagonal terms.[59,76] In practice, modulated transport experiments allow the extraction of both diagonal and off-diagonal transport coefficients.

2. Measurements of the fluctuations spectrum

Another class of experiments has focused on the source of the anomalous transport, i.e., the plasma microturbulence.

In the late 1970s, a low-frequency, long-wavelength broadband turbulence spectrum propagating in the direction of the electron diamagnetic velocity was detected in TEXT using multichord FIR (far infrared radiation) interferometry.[31,79] Unfortunately, FIR scattering has a poor spatial resolution for small values of the wave number ($k_\perp \leqslant 4\ \mathrm{cm}^{-1}$).[80] This is a problem because the fluctuations spectrum is large in that range of wave numbers. The other method available to measure the spectrum in the confinement region, namely heavy ion beam probe (HIBP[81]) has a good spatial resolution, but is insensitive to large values of the wave number ($k_\perp \geqslant 4\ \mathrm{cm}^{-1}$). In particular, the two experimental methods give quite different measurements for the average poloidal wave numbers in TEXT

($\bar{k}_\theta \rho_s = 0.3$ from FIR results and $\bar{k}_\theta \rho_s = 0.1$ from HIBP results).[82]

Because of the complexity of the measurements and their interpretation, direct comparisons between theoretical and experimental results remain very difficult.

B. Numerical results

1. Choice of parameters

The theoretical expressions for the transport coefficients were obtained in Sec. II, with few assumptions besides the chosen plasma model and the DIA theory. However, in order to obtain numerical results that can be directly compared with experiments, we need to adopt a specific set of tokamak parameters, as well as an explicit form for the nonlinear ion susceptibility χ_i.

a. The TEXT tokamak. A fairly complete set of data for the fluctuation spectrum in the interior of a tokamak was obtained in the Texas Experimental Tokamak (TEXT).[31] We will therefore adopt the TEXT parameters for our numerical estimates. The major radius $R = 1$ m, the minor radius $a = 25$ cm, and the ratio $qR/a \sim 10$. For the electron temperature and magnetic field, we take $T_0 = 1$ keV and $B_0 = 26$ kG, which represent typical experimental values.[31] In Table I we list the values of the principle parameters in TEXT.

We adopt a parabolic radial profile for the equilibrium density, $n_0(r) = n_c(1 - r^2/a^2)$. For the equilibrium temperature profile we take $T_0(r) = T_c(1 - r^2/a^2)^2$, so that $\eta = d \ln T_0 / d \ln n_0 = 2$ is independent of radius. The coefficients n_c and T_c are the equilibrium values of the density and temperature at the center of the plasma, i.e., at $r = 0$.

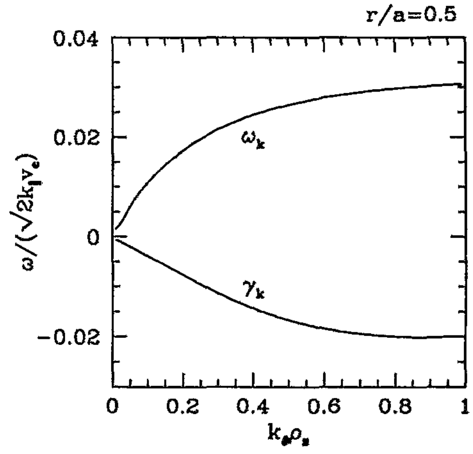

FIG. 1. Normal mode frequency ω_k and growth rate γ_k, normalized to $\sqrt{2}|k_\parallel|v_e$, as a function of the normalized poloidal wave number $k_\theta \rho_s$, in the midplane $r/a = 0.5$.

b. The ion susceptibility. The ions will be modeled by cold fluid equations with an eddy viscosity ν_i. This viscosity provides the necessary sink of free energy and overdamps the unstable electron waves. The ion continuity and momentum equations are

$$\frac{\partial \tilde{n}_i}{\partial t} + \nabla \cdot (n_0 \tilde{\mathbf{v}}_i) = 0 \tag{71}$$

TABLE I. TEXT tokamak parameters (cgs units).

Parameter	Symbol	Typical value
Major radius	R	1 m
Minor radius	a	25 cm
Safety factor	q	$qR/a \approx 10$
Central electron temperature	T_0	1 keV
Central electron density	n_0	4×10^{13} cm^{-3}
Toroidal magnetic field	B	26 kG
Plasma β parameter	$\beta = 4\pi n_0 T_0 / B^2$	0.001
Velocities		
Electron thermal velocity	$v_e = (T_0/m_e)^{1/2}$	1.325×10^9 cm/s
Ion sound velocity	$c_S = \sqrt{T_0/m_i}$	3.10×10^7 cm/s
Alfvén velocity	$v_A = B/(4\pi n_0 m_i)^{1/2}$	9×10^8 cm/s
Electron diamagnetic velocity	$v_* = 10^8\, T_0 |\nabla \ln n_0| / B$	1.5×10^5 cm/s
Frequencies		
Electron–ion collision frequency	$\nu_{ei} = 2.9 \times 10^{-6} \frac{n_e \ln \Lambda}{T_0^{3/2}}$	6×10^4 rad/s
Electron diamagnetic frequency	$\omega_* = k_\theta v_*$	10^6 rad/s
Ion cyclotron frequency	$\Omega_i = eB/mC$	2.49×10^8 rad/s
Electron bounce frequency	$\omega_b = v_e/qR$	5.30×10^8 rad/s
Electron plasma frequency	$\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$	3.57×10^{11} rad/s
Lengths		
Ion gyroradius	$\rho_s = C_S/\Omega_i$	0.124 cm
Debye length	$\lambda_D \cdot (T_0/4\pi n_0 e^2)^{1/2}$	3.7×10^{-3} cm

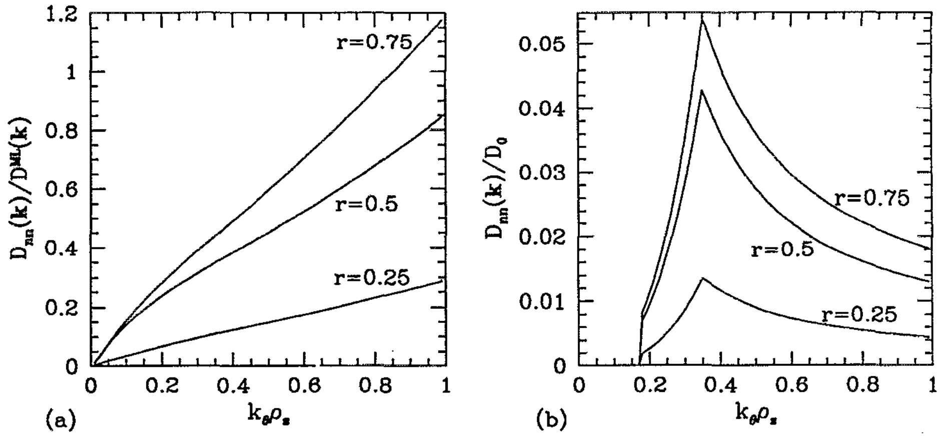

FIG. 2. (a) Diffusion coefficient $D_{nn}(k_\theta)$, normalized to the mixing-length estimate $D^{\rm ML}(k_\theta)$, as a function of the normalized poloidal wave number $k_\theta\rho_s$, for several values of the radius r/a. (b) Diffusion coefficient $D_{nn}(k_\theta)$, normalized to D_0, as a function of the normalized poloidal wave number $k_\theta\rho_s$, for several values of the radius r/a.

and

$$\frac{\partial \tilde{\mathbf{v}}_i}{\partial t}=\frac{e}{m_i}\left(-\nabla\tilde{\phi}+\frac{1}{c}\tilde{\mathbf{v}}_i\times\mathbf{B}_0\right)+\nu_i\nabla^2\tilde{\mathbf{v}}_i, \tag{72}$$

where $\tilde{\mathbf{v}}_i$ is the ion velocity and m_i is the ion mass. By taking the Fourier transforms of these equations in space and time and solving them perturbatively in the limit $\omega\ll\Omega_i$, we find

$$\frac{\tilde{n}_i}{n_0}=\frac{e\tilde{\phi}}{T_0}\chi_i,$$

with

$$\chi_i=\frac{\omega_*}{\omega}-k_\perp^2\rho_s^2\left(1+\frac{i\Gamma_i}{\omega}\right), \tag{73}$$

where $\Gamma_i\equiv k^2\nu_i$ is an effective eddy viscosity damping, which depends on the turbulent spectrum. Note that this very simple form of the ion susceptibility does not necessarily reflect any direct contribution of the ions themselves to the anomalous transport. Furthermore, we assume that the ions are cold, i.e., $T_i\ll T_e$, which is usually not true, especially in the larger tokamaks.

It is known from experiments[31] that the frequency spread $\Delta\omega_{\rm k}$ of the fluctuations spectrum remains comparable to the normal mode frequency $\omega_{\rm k}$ over the entire range of relevant wave numbers, i.e., for $0.1\leqslant k_\perp\rho_s\leqslant 1.0$. Since we assume that the ion nonlinearities are the only cause of spectrum broadening, the requirement that $\Delta\omega_{\rm k}\sim\omega_{\rm k}$ implies a scaling

$$\Gamma_i\simeq\omega_*/(2k_\perp^2\rho_s^2). \tag{74}$$

Since $\omega_*\propto k_\perp$, we have[83] $\Gamma_i\propto k_\perp^{-1}$.

The normal mode frequency and growth rate are found by solving the dispersion relation,

$$\mathscr{E}(\omega,\mathbf{k})\propto[\chi_i(\omega,\mathbf{k})-\chi_e^L(\omega,\mathbf{k})]=0,$$

where $\omega\equiv\omega_{\rm k}+i\gamma_{\rm k}$, χ_e^L is given by Eq. (B19) of Appendix B and χ_i is given by Eq. (73). The results for $\omega_{\rm k}$ and $\gamma_{\rm k}$ are plotted in Fig. 1 as a function of k_θ, in the tokamak midplane $r=0.5a$. Note that the damping rate $\gamma_{\rm k}$ is about half the mode frequency $\omega_{\rm k}$, in accordance with our choice for Γ_i, Eq. (74).

c. The fluctuation spectrum. We assume that the frequency dependence of the experimental potential fluctuation spectrum can be represented by a simple Lorentzian profile centered on the eigenfrequency, so that

$$\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle=\frac{2\gamma_{\rm k}}{(\omega-\omega_{\rm k})^2+\gamma_{\rm k}^2}I_{\rm k} \tag{75}$$

where $I_{\rm k}$, the frequency integrated spectrum, is defined by

$$I_{\rm k}\equiv\int\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\frac{d\omega}{2\pi}. \tag{76}$$

Note that $\gamma_{\rm k}$ gives the half-width at half-maximum.

The spectrum $I_{\rm k}$ has been measured in TEXT.[31] We model the data by

$$I_{\rm k}=0.41\times\begin{cases}0, & \text{if } k_\theta<1.41\ \text{cm}^{-1};\\ 1, & \text{if } 1.41\ \text{cm}^{-1}<k_\theta<2.83\ \text{cm}^{-1};\\ (k_\theta/2.83)^{-4}, & \text{if } k_\theta>2.83\ \text{cm}^{-1}.\end{cases} \tag{77}$$

Note that we introduce a cutoff at the small wave numbers, and the constant 0.41 is chosen such that $\int I_{\rm k}\,d\mathbf{k}/(2\pi)^3=1$. We have assumed isotropy in (k_r,k_θ), i.e., in the direction perpendicular to the magnetic field, as was also assumed in the interpretation of the experimental data.[31]

Finally, we use the "mixing-length" estimate of the potential fluctuation spectrum to model its radial dependence,

$$\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\sim\left(\frac{\rho_s}{L_n}\right)^2\sim\rho_s(0)\frac{4}{a^2}\left(\frac{r}{a}\right)^2. \tag{78}$$

2. The diffusion coefficients

In Sec. II, we have derived general expressions for the diffusion coefficients, given by Eqs. (62)–(65). We now

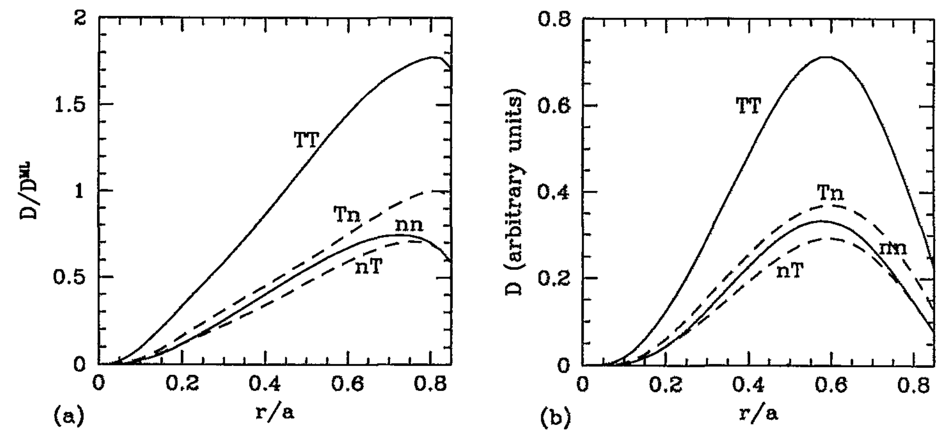

FIG. 3. (a) Spatial variation of the diffusion coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT}, normalized to the mixing-length estimate $D^{\rm ML}$. (b) Spatial variation of the diffusion coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT}, obtained assuming $D^{\rm ML}\propto(r/a)[1-(r/a)^2]^2$.

use the parameters described in the previous section to evaluate numerically the diffusion coefficients.

a. Wave number dependence of the diffusion coefficients. In Eqs. (62)–(65), the sum over the four-vector k, Σ_k, actually represents the integrals $\int d\mathbf{k}/(2\pi)^3\int d\omega/2\pi$. The four diffusion coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT} can therefore be written as

$$D_{ab}=\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{d\omega}{2\pi}D_{ab}(\mathbf{k},\omega),\tag{79}$$

where $D_{ab}(\mathbf{k},\omega)$ is a diffusion coefficient per unit frequency bandwidth and wave number bandwidth. If we use Eq. (75) for the frequency spectrum, the integrations over ω in (62)–(65) can be performed in the complex plane by the method of residues, closing the contour of integration in the upper-half plane. The pole of the plasma dielectric function $(\omega_{\mathbf{k}},\gamma_{\mathbf{k}})$ is in the lower-half complex plane, and does not contribute to the integral. We find

$$D_{nn}=\int\frac{d\mathbf{k}}{(2\pi)^3}D_{nn}(\mathbf{k}),\tag{80}$$

where

$$\begin{aligned}D_{nn}(\mathbf{k})&=\int\frac{d\omega}{2\pi}D_{nn}(\mathbf{k},\omega)\\&=\left(\frac{cT_0}{eB_0}\right)^2|\mathbf{k}\times\mathbf{b}_0|^2I_{\mathbf{k}}\left[\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right)\frac{1}{-i\omega}\right.\\&\quad\left.\times[(1-\tau)H_0+\tau A_0]\right]_{\omega=\omega_{\mathbf{k}}+i\gamma_{\mathbf{k}}},\end{aligned}\tag{81}$$

and similar expressions for the three other coefficients. In Eq. (81), $I_{\mathbf{k}}$ is the frequency-integrated spectrum, defined by (76). If we assume isotropy in the (k_r,k_θ) plane, the functions $D_{ab}(\mathbf{k})=D_{ab}(k_\theta)$. The mixing-length estimate of D_{nn} is given by

$$D^{\rm ML}\sim\left(\frac{c}{B}\right)^2|E|^2\tau,\tag{82}$$

where $\tau\sim\omega_*^{-1}$. Therefore,

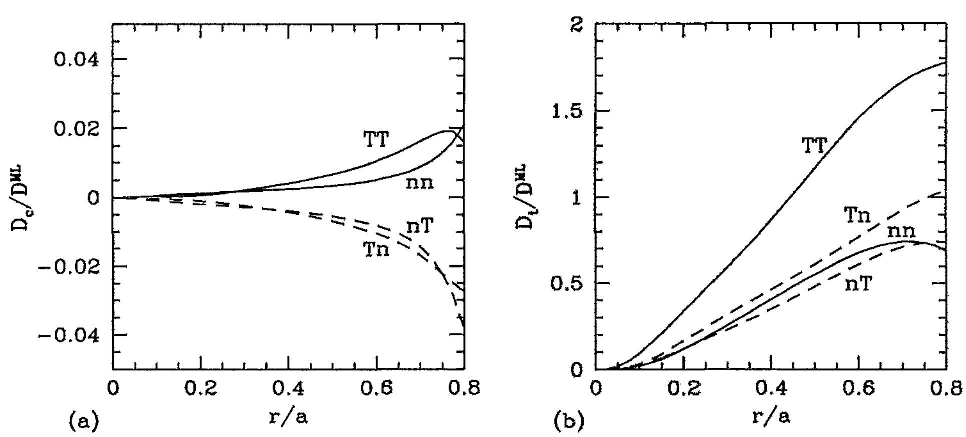

FIG. 4. (a) Contributions of the circulating electrons to the coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT}, normalized to the mixing-length estimate $D^{\rm ML}$. (b) Contributions of the trapped electrons to the coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT}, normalized to the mixing-length estimate $D^{\rm ML}$.

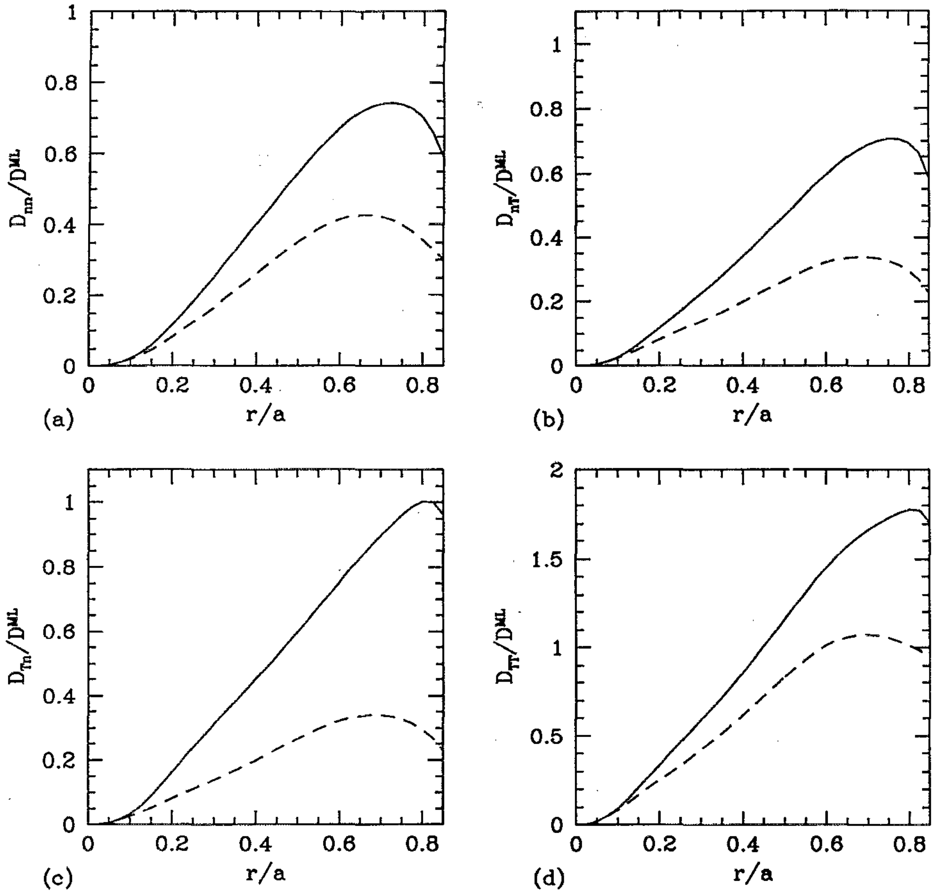

FIG. 5. (a)–(d) DIA (solid lines) and test-particle (dashed lines) results for the diffusion coefficients D_{nn}, D_{nT}, D_{Tn}, and D_{TT}, normalized to the mixing-length estimate D^{ML}.

$$D^{\mathrm{ML}}(\mathbf{k},\omega) \sim \left(\frac{cT}{eB}\right)^2 k^2 \left\langle \left|\frac{e\tilde{\phi}_k}{T_0}\right|^2 \right\rangle \omega_*^{-1} \quad (83)$$

and

$$D^{\mathrm{ML}}(\mathbf{k}) = \int \frac{d\omega}{2\pi} D^{\mathrm{ML}}(\mathbf{k},\omega) \sim \left(\frac{cT}{eB}\right) k^2 \omega_*^{-1} I_{\mathbf{k}}. \quad (84)$$

In Fig. 2(a), we show the diffusion coefficient $D_{nn}(k_\theta)$ normalized to the mixing-length value $D^{\mathrm{ML}}(k_\theta)$ for various values of r/a. These results are independent of our choice for the frequency-integrated spectrum $I_{\mathbf{k}}$. In Fig. 2(b), we show $D_{nn}(k_\theta)/D_0$, where $D_0 = 0.41(cT/eB)^2 \rho_s^{-2} \omega_*^{-1}$, for the particular choice of $I_{\mathbf{k}}$ given in Eq. (77). The three other diffusion coefficients, namely $D_{nT}(k_\theta)$, $D_{Tn}(k_\theta)$, and $D_{TT}(k_\theta)$ have k dependences very similar to that of $D_{nn}(k_\theta)$.

b. Spatial dependence of the diffusion coefficients. The transport coefficients D_{ab} are obtained by integrating the coefficients $D_{ab}(\mathbf{k})$ over k using (77) for $I_{\mathbf{k}}$,

$$D_{ab} = \int \frac{d\mathbf{k}}{(2\pi)^3} D_{ab}(\mathbf{k}).$$

In Fig. 3(a), we show the coefficients D_{ab}/D^{ML}, where $D^{\mathrm{ML}} = \int D^{\mathrm{ML}}(k) d\mathbf{k}/(2\pi)^3$, as a function of the normalized radius r/a. If we assume Eq. (78) for the radial dependence of the potential fluctuation spectrum, $D^{\mathrm{ML}} \propto 2(r/a)[1-(r/a)^2]^2$. In Fig. 3(b), we show the coefficients $(D_{ab}/D^{\mathrm{ML}}) \times 2(r/a)[1-(r/a)^2]^2$. All four coefficients have similar radial dependences. Experimentally, the heat conductivity D_{TT} is found to increase with r near the edge. Our result does not follow this prediction. However, all the calculations presented in this work are valid in the "interior" of the tokamak plasma, not near the edge, where other physical mechanisms prevail.

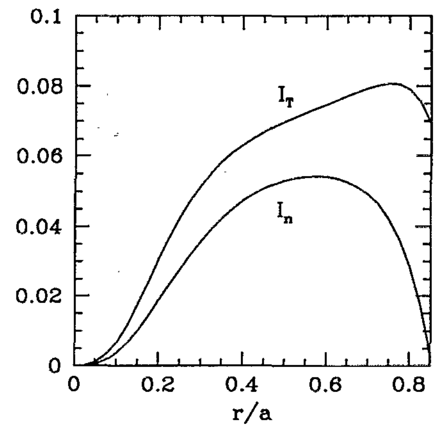

FIG. 6. Spectrum modulation coefficients I_n and I_T.

c. Relative contribution of trapped and circulating electrons. The diffusion coefficients D_{ab}, given by Eqs. (62)–(65) in Sec. II can be written as the sum of two components, representing the contributions of the trapped and circulating electrons. For example, D_{nn} can be written as $D_{nn}=D^c_{nn}+D^t_{nn}$ with

$$D^c_{nn}=-\left(\frac{cT_0}{eB_0}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\frac{1}{-i\omega}(1-\tau)H_0 \quad (85)$$

and

$$D^t_{nn}=-\left(\frac{cT_0}{eB_0}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\frac{1}{-i\omega}\tau A_0. \quad (86)$$

In Figs. 4(a) and 4(b), we show the contributions from the trapped and the circulating electrons to the coefficient D_{nn}/D^{ML}. The contribution from circulating electrons is very small compared to that of the trapped electrons, even though the numbers of circulating electrons and trapped electrons are comparable in the midplane of the tokamak ($\tau\sim\sqrt{2\epsilon}=\sqrt{2r/R}=0.5$; cf. Appendix A), and there are much more circulating electrons than trapped electrons at $r/a=0.1(1-\tau\sim0.8)$. Therefore the analysis of the problem could be simplified by considering adiabatic circulating electrons. This has been pointed out for quasilinear calculations as well.[84]

Note that all the diffusion coefficients are positive (cf. Fig. 3). The contribution from the circulating electrons to the off-diagonal diffusion coefficients D_{nT} and D_{Tn} is negative, whereas the contribution from the trapped electrons is positive. Since the trapped electrons dominate, the off-diagonal diffusion coefficients are positive. It has been suggested that the negative off-diagonal diffusion coefficients obtained in calculations involving only circulating electrons could explain the "inward" convective motion observed in modulation experiments. However, as we have shown here, the presence of even a very small population of trapped electrons is sufficient to guarantee the positivity of all four diffusion coefficients.

d. Comparison with test-particle results. The function D_{nn} is given by Eq. (62), obtained from our DIA calculations. The test-particle result is recovered by dropping the term proportional to $1/(\chi_i-\chi_e^L)$ in this expression. In Fig. 5 we compare the DIA and test-particle (TP) results for D_{ab}/D^{ML}.

The diffusion coefficients calculated by the DIA are larger by a factor 2–4 than those obtained in the TP approximation. The disagreement between the two methods is largest for D_{Tn}, breaking the Onsager symmetry in the DIA result.

e. Onsager relations. Systems that are out of equilibrium generally return to the equilibrium state through a variety of irreversible transport processes. Onsager[85,86] derived "reciprocal relations" connecting the corresponding transport coefficients. These relations reflect, on the macroscopic level, the time reversal invariance of the microscopic equations of motion. For example, a particle density gradient can cause a heat current, and a temperature gradient can cause a particle current. The transport coefficients for the two processes are equal. The entropy production (due to the irreversible processes occurring in the system) is a bilinear expression in the fluxes and thermodynamic forces. The calculation of the entropy production therefore provides a way of finding the proper "conjugate" irreversible fluxes and thermodynamic forces necessary for the establishment of the phenomenological equations whose coefficients obey the Onsager relations.

From Fig. 3, we note that the two off-diagonal coefficients of our transport matrix, D_{nT} and D_{Tn}, are not equal, i.e., the Onsager symmetry observed in quasilinear and test-particle results is not present here. This is not understood at present, but has been observed in other studies of anomalous transport.[87,88] One difficulty is to find the "correct" conjugate fluxes and forces. In particular, we have not been able to find an expression for the entropy production in the complex system studied here. It is possible that Onsager symmetry should not be expected in the case of anomalous transport, but this remains to be explained.

3. The spectrum modulation

In Sec. II, we have calculated the spectrum modulation produced by perturbations in density and temperature, and the result was written as

$$\frac{\hat{I}_{k,k_0}}{\langle|e\tilde{\phi}_k/T_0|^2\rangle}=I_n(\omega,k_\theta)\frac{\nabla\hat{n}_0}{n_0}+I_T(\omega,k_\theta)\frac{\nabla\hat{T}_0}{T_0}, \quad (87)$$

where $I_n(\omega,k_\theta)$ and $I_T(\omega,k_\theta)$ are given by (69) and (70). If we integrate these results over the frequency and wave number, we obtain

$$\frac{\int[d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\hat{I}_{k_0,k}}{(\sqrt{2}C_s/\omega_*)\int[d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\langle|e\tilde{\phi}_k/T_0|^2\rangle}=I_n\times\frac{\nabla\hat{n}_0}{n_0}+I_T\times\frac{\nabla\hat{T}_0}{T_0}, \quad (88)$$

where

$$I_n=\frac{\int[d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\langle|e\tilde{\phi}_k/T_0|^2\rangle\mathrm{Re}\{[(k_\theta\rho-s)/(\chi_i-\chi_e^L)](\omega_*/-i\omega)[(1-\tau)H_0+\tau A_0]\}}{\int[d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\langle|e\tilde{\phi}_k/T_0|^2\rangle} \quad (89)$$

and

$$I_T=\frac{\int [d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\langle|e\tilde{\phi}_k/T_0|^2\rangle\mathrm{Re}\{[(k_\theta\rho-s)/(\chi_i-\chi_e^L)](\omega_*/-i\omega)[(1-\tau)H_1+\tau A_1]\}}{\int [d\mathbf{k}/(2\pi)^3](d\omega/2\pi)\langle|e\tilde{\phi}_k/T_0|^2\rangle}. \tag{90}$$

We have plotted the coefficients I_n and I_T in Fig. 6. We find that I_n and I_T are of the order of 0.05, whereas $(\sqrt{2}C_s)/\omega_*\sim 44$. Therefore, the spectrum modulation is comparable in magnitude to the density modulation, as observed experimentally.[61]

We should note, however, that from Eq. (66), the coefficients I_n and I_T are real. Therefore, (68) implies a phase variation (along the radius r) for the spectrum modulation similar to the one for the density modulation. Experiments, on the other hand, show that the phase of the spectrum modulation is constant throughout the plasma, and does not change with the radius, whereas the phase of the density and temperature do vary with the radius.[89] This is not explained by our model.

IV. SUMMARY AND DISCUSSION

In this work, we have developed a new approach to the calculation of anomalous transport in tokamak plasmas. Our method is based on the direct-interaction approximation, a renormalized theory of turbulence that provides the response functions due to infinitesimal perturbations. The method is particularly well suited to comparisons with perturbative transport experiments. Moreover, important physical properties, such as the ambipolarity of the particle fluxes, are automatically satisfied.

The theoretical expressions obtained for the transport coefficients are based on a specific set of equations describing the dynamics of the plasma and on the DIA, but do not require any further approximations. We have not attempted to calculate the fluctuation spectrum from first principles. Rather, we have expressed our results in terms of the potential fluctuation spectrum, and we have used experimental data in all numerical evaluations.

We have studied the electron dynamics in detail, but we have used a simplified treatment of the ion dynamics, assuming the presence of an anomalous ion viscosity. The form of this anomalous viscosity was chosen in such a way that the frequency peak and bandwidth of the fluctuation spectrum agree with those observed in the Texas Experimental Tokamak (TEXT). It would be interesting to study what happens with different values of the ion viscosity, in particular, it would be interesting to try different power laws for its wave number dependence. According to some preliminary results, it seems that a high dissipation is needed at the long wavelengths. A simple power law with a positive exponent does not provide enough dissipation at the small wave numbers. It is possible that a more sophisticated model is needed, where different dissipation mechanisms prevail for different ranges of wave numbers. Dissipation at small scales can be provided by viscosity effects, whereas it is conceivable that dissipation at long wavelengths could result from geometry effects or shear. It is also possible that the nonlinear ion damping rate and the mode width are not related through each other locally in wave number space, as was assumed here, but instead through an integral relation.

We have concentrated on drift-wave turbulence, which appears to be an important source of anomalous transport in tokamaks. We have used a simplified slab geometry, neglecting the magnetic shear and curvature but retaining the distinction between trapped and circulating electrons. Magnetic shear and toroidicity have been shown to modify the stability of drift waves in the linear regime.[90,91] However, there are good reasons to believe that the effect of the shear in the strong turbulence regime might be less important than in the linear regime. Indeed, the decorrelation time of the mode[92] is very small in this regime ($\Delta\omega_\mathrm{k}\sim\omega_\mathrm{k}$), and is probably smaller than the characteristic time needed for the mode to be stabilized by shear. Nevertheless, it would be interesting to prove this particular point by calculating the transport coefficients, using the method developed here, with a model including the magnetic shear.

The theoretical expressions for the transport coefficients have been obtained with few assumptions besides the chosen plasma model and the DIA theory. However, to obtain numerical results that can be directly compared with experiments, we needed to adopt a specific set of tokamak parameters. We chose the TEXT tokamak, since a fairly complete set of data for the fluctuation spectrum in the interior of this tokamak was available.

All four diffusion coefficients (relating the fluxes of particles and heat to perturbations in the density and temperature gradients) were found to have similar radial dependences. Experimentally, the heat conductivity is found to increase with radius near the edge. Our result does not follow this prediction. However, all the calculations presented in this work are valid in the "interior" of the plasma, rather than near the edge, where other physical mechanisms might prevail. We found that the contribution from circulating electrons is very small compared to that of the trapped electrons. Therefore, the analysis of the problem could be simplified by considering only the transport due to the trapped electrons. It was found that all the diffusion coefficients are positive. The contribution from the circulating electrons to the off-diagonal diffusion coefficients is negative, whereas the contribution from the trapped electrons is positive. Since the trapped electrons dominate, the off-diagonal diffusion coefficients are positive. It has been suggested that the negative off-diagonal diffusion coefficients obtained in calculations involving only circulating electrons could explain the "inward" convective motion observed in modulation experiments. However, we showed that the presence of even a very small population of trapped electrons is sufficient to guarantee the positivity of all four diffusion coefficients. We have argued that this is not in disagreement with an observed effective inward convective velocity term in the expression for the particle flux. The diffusion coefficients are about two to four times larger than predicted by the test-particle approximation. The difference is largest for the coefficient relating the heat flux to a perturbation in the density profile, breaking the Onsager symmetry in the DIA result. This last result is very important, since Onsager symmetry is commonly assumed when analyzing experimental data. Our results show that it is incorrect to make that assumption when studying the anomalous transport matrix.

The modulation of the fluctuation spectrum that results from a modulation of the density or temperature profiles was calculated using the DIA method. This result cannot be obtained from simpler turbulence theories such as quasilinear theories. The spectrum modulation was found to be comparable in magnitude to the density modulation, as recently observed in TEXT.

It is now commonly accepted that perturbative and equilibrium fluxes differ if the diffusion coefficients depend on the plasma parameters, which is the case if they are produced by turbulence. Recent modulation experiments in TEXT showed that the perturbed fluxes are linearly proportional to the density modulation amplitude, but they are usually larger than the equilibrium fluxes. Because of the linear relation between the perturbative fluxes and the density perturbation, the incremental transport can be described by a transport matrix. On the other hand, if the equilibrium fluxes have a complicated dependence on the gradients, a transport matrix has very little meaning. We believe that much can be learned from perturbative transport experiments.

The results obtained here were expressed in a form such that they can be easily compared with experiments. Since we need experimental data for the frequency and wave number dependence of the fluctuation spectrum, new experiments, yielding more detailed results on the turbulence characteristics, would be welcome. In particular, it would be very useful to obtain the information on both the fluctuations and the perturbative transport during the same experiment. This would allow a direct comparison between theoretical predictions and measured data.

ACKNOWLEDGMENTS

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APPENDIX A: USEFUL INTEGRALS

The trapped-electron integrals are defined by

$$A_n=\frac{1}{n_{0t}}\int_t \frac{f_0}{1+i(\nu_{\text{eff}}/\omega)}\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^n d\mathbf{v}, \tag{A1}$$

where n_{0t} is the equilibrium density of trapped electrons. Since only v appears in the integrand, the integral can be rewritten as

$$\begin{aligned}A_n&=\frac{1}{\tau}\frac{1}{(2\pi v_e^2)^{3/2}}\int_0^\infty \frac{e^{-v^2/2v_e^2}}{1+i(\nu_*/\omega)(v/\sqrt{2}v_e)^{-3}}\\&\quad\times\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^n v^2\,dv\int_t d\Omega\\&=\frac{4}{\sqrt{\pi}}\int_0^\infty \frac{e^{-r^2}(r^2-\frac{3}{2})^n}{1+i(\nu_*/\omega)r^{-3}}\,r^2\,dr,\end{aligned} \tag{A2}$$

where $\tau=\sqrt{2\epsilon/(1+\epsilon)}$ is the fraction of trapped particles. These integrals can be calculated numerically.

The circulating-electron integrals are defined by

$$H_n=\frac{1}{n_{0c}}\int_c \frac{f_0}{1-k_\parallel v_\parallel/\omega}\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^n d\mathbf{v}, \tag{A3}$$

where n_{0c} is the equilibrium number density of circulating electrons. These integrals must be calculated by integrating over the proper range of velocities for the circulating particles:

$$\begin{aligned}H_n&=\frac{n_0}{n_{0c}}\frac{1}{(2\pi v_e^2)^{3/2}}\int_{-\infty}^{+\infty}dv_\parallel\int_0^{v_\parallel/\sqrt{r-1}}2\pi v_\perp\,dv_\perp\\&\quad\times\frac{e^{-(v_\parallel^2+v_\perp^2)/2v_e^2}}{1-k_\parallel v_\parallel/\omega}\left(\frac{v_\perp^2+v_\parallel^2-3v_e^2}{2v_e^2}\right)^n d\mathbf{v}.\end{aligned}$$

They can be performed analytically. For $n=0$, 1 and 2, we obtain

$$H_0=\frac{1}{1-\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\left[Z\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)-Z\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)\right], \tag{A4}$$

$$\begin{aligned}H_1&=\frac{1}{1-\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\left\{\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}+\left[\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2\right.\right.\\&\quad\left.-\frac{1}{2}\right]Z\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)-\frac{\omega}{\sqrt{2}k_\parallel v_e}-\left[\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2\right.\\&\quad\left.\left.-\frac{1}{2}\right]Z\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)\right\}\end{aligned} \tag{A5}$$

and

$$\begin{aligned}H_2&=\frac{1}{1-\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\left\{\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\left[\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2-\frac{1}{2}\right]\right.\\&\quad+\left[\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^4-\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2+\frac{5}{4}\right]Z\left(\frac{1}{\tau}\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)\\&\quad-\frac{\omega}{\sqrt{2}k_\parallel v_e}\left[\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2-\frac{1}{2}\right]-\left[\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^4\right.\\&\quad\left.\left.-\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)^2+\frac{5}{4}\right]Z\left(\frac{\omega}{\sqrt{2}k_\parallel v_e}\right)\right\}.\end{aligned} \tag{A6}$$

In these expressions, $Z(\zeta)$ is the plasma dielectric function. Note that in the limit $\tau\to 0$, we have $Z[(1/\tau)(\omega/\sqrt{2}k_\parallel v_e)]\to-[(1/\tau)(\omega/\sqrt{2}k_\parallel v_e)]^{-1}$, and all the terms involving $1/\tau$ disappear from the expressions for H_n.

APPENDIX B: LINEARIZED ELECTRON EQUATIONS

The linearized electron equations are obtained by letting the right-hand sides in Eqs. (20) and (21) go to zero.

The adiabatic part of the fluctuations can be eliminated by writing

$$\tilde{f}^{c,t}=f_0(e\tilde{\phi}/T_0)+\tilde{h}^{c,t}, \tag{B1}$$

where $\tilde{h}^c$ and $\tilde{h}^t$ are the nonadiabatic distribution fluctuations. If we Fourier transform in space and time, the linearized circulating and trapped electron equations then become

$$-i(\omega-k_\parallel v_\parallel)\tilde{h}_k^c-i(\omega-\omega_*)f_0(e\tilde{\phi}_k/T_0)-\tilde{C}_k^c(\tilde{f}_k)=0 \tag{B2}$$

and

$$-i\omega\tilde{h}_k^t-i(\omega-\omega_*)f_0(e\tilde{\phi}_k/T_0)-C_k^t(\tilde{f}_k)=0, \tag{B3}$$

where the velocity-dependent diamagnetic frequency ω_* is given by

$$\omega_*(v)=\omega_*^n+\omega_*^T\frac{v^2-3v_e^2}{2v_e^2}. \tag{B4}$$

The velocity-independent diamagnetic frequencies are given in terms of the density and temperature gradients by

$$\omega_*^n=-\frac{cT_0}{eB_0}\mathbf{k}\cdot\mathbf{b}_0\times\frac{\nabla n_0}{n_0} \tag{B5}$$

and

$$\omega_*^T=-\frac{cT_0}{eB_0}\mathbf{k}\cdot\mathbf{b}_0\times\frac{\nabla T_0}{T_0}=\eta\omega_*^n, \tag{B6}$$

where $\eta=|(\nabla T_0/T_0)/(\nabla n_0/n_0)|$. The collision operators, given by Eq. (25), can be written as

$$\tilde{C}_k^t=-\nu_{\text{eff}}\left[\tilde{h}_k^t+f_0\frac{e\tilde{\phi}_k}{T_0}\left(1-\chi_e^L-\frac{2}{3}\chi_T^L\frac{v^2-3v_e^2}{2v_e^2}\right)\right] \tag{B7}$$

and

$$\tilde{C}_k^c=-\frac{f_0}{n_{0c}}\int_t\tilde{C}_k^t\,d\mathbf{v}=-\frac{f_0}{n_{0c}}\frac{e\tilde{\phi}_k}{T_0}\int_t f_0\left[\frac{-i\nu_{\text{eff}}(\omega-\omega_*)}{-i\omega+\nu_{\text{eff}}}+\left(1-\chi_e^L-\frac{2}{3}\chi_T^L\frac{v^2-3v_e^2}{2v_e^2}\right)\frac{i\omega\nu_{\text{eff}}}{-i\omega+\nu_{\text{eff}}}\right]d\mathbf{v}, \tag{B8}$$

where we have used Eqs. (26) and (27), which define χ_e^L and χ_T^L. This result can be rewritten as

$$\tilde{C}_k^c=-\sigma f_0\frac{e\tilde{\phi}_k}{T_0}, \tag{B9}$$

with

$$\sigma=(n_{0t}/n_{0c})[-i(\omega-\omega_*^n)(1-A_0)-i\omega_*^n\eta A_1+i\omega(1-\chi_e^L)(1-A_0)+i\omega(\tfrac{2}{3})\chi_T^LA_1], \tag{B10}$$

where the trapped electrons integrals A_n have been defined in Appendix A by Eq. (A1). The linearized electron equations become

$$-i(\omega-k_\parallel v_\parallel)\tilde{h}_k^c+[-i(\omega-\omega_*)+\sigma]f_0(e\tilde{\phi}_k/T_0)=0 \tag{B11}$$

and

$$(-i\omega+\nu_{\text{eff}})\tilde{h}_k^t+\left[-i(\omega-\omega_*)+\nu_{\text{eff}}\left(1-\chi_e^L-\frac{2}{3}\chi_T^L\frac{v^2-3v_e^2}{2v_e^2}\right)\right]f_0\left(\frac{e\tilde{\phi}_k}{T}\right)=0. \tag{B12}$$

We define the propagators g_k^c and g_k^t by

$$(g_k^c)^{-1}=-i(\omega-k_\parallel v_\parallel), \tag{B13}$$

$$(g_k^t)^{-1}=-i\omega+\nu_{\text{eff}}, \tag{B14}$$

and we will also use the following notations:

$$b_k^c=-i(\omega-\omega_*)+\sigma, \tag{B15}$$

$$b_k^t=-i(\omega-\omega_*)+\nu_{\text{eff}}\left(1-\chi_e^L-\frac{2}{3}\chi_T^L\frac{v^2-3v_e^2}{2v_e^2}\right), \tag{B16}$$

so that the linearized electron equations can be rewritten as

$$(g_k^c)^{-1}\tilde{h}_k^c+b_k^cf_0(e\tilde{\phi}_k/T_0)=0, \tag{B17}$$

$$(g_k^t)^{-1}\tilde{h}_k^c+b_k^tf_0(e\tilde{\phi}_k/T_0)=0. \tag{B18}$$

The system of equations,

$$\chi_e^L=1-\frac{1}{n_0}\left(\int_c g_k^cb_k^cf_0\,d\mathbf{v}+\int_t g_k^tb_k^tf_0\,d\mathbf{v}\right) \tag{B19}$$

and

$$\chi_T^L=-\frac{1}{n_0}\left[\int_c g_k^cb_k^cf_0\left(\frac{v^2-3v_e^2}{2v_e^2}\right)d\mathbf{v}+\int_t g_k^tb_k^tf_0\left(\frac{v^2-3v_e^2}{2v_e^2}\right)d\mathbf{v}\right] \tag{B20}$$

is easily solved for χ_e^L and χ_T^L. More specifically, using the integrals A_n and H_n, defined in Appendix A by Eqs. (A1) and (A3), χ_e^L and χ_T^L are solutions of the following system of equations:

$$\chi_e^L[1-\tau(1-A_0)(1-H_0)]+(\tfrac{2}{3})\sqrt{2\epsilon}A_1(1-H_0)\chi_T^L=(1-\tau)[(1-H_0)+(\omega_*^n/\omega)(H_0+\eta H_1)]+\tau(\omega_*^n/\omega)[(A_0+\eta A_1)+(1-A_0-\eta A_1)H_0] \tag{B21}$$

and

$$\chi_e^L\tau[A_1+H_1(1-A_0)]+[(1-\tau)+(\tfrac{2}{3})\tau(A_2-A_1H_1)]\chi_T^L=\tau(\omega_*^n/\omega)[(A_1+\eta A_2)+(1-A_0-\eta A_1)H_1]+(1-\tau)[(\omega_*^n/\omega)(H_1+\eta H_2)-H_1]. \tag{B22}$$

If we use the simplest collision operator (24), the coefficients χ_e^L and χ_T^L are given by

$$\chi_e^L=(1-\tau)\left\{1+\left[\left(\frac{\omega_*^n}{\omega}\right)-1\right]H_0+\eta\left(\frac{\omega_*^n}{\omega}\right)H_1\right\}$$
$$+\tau\left(\frac{\omega_*^n}{\omega}\right)(A_0+\eta A_1) \quad \text{(B23)}$$

and

$$\chi_T^L=(1-\tau)\left\{\left[\left(\frac{\omega_*^n}{\omega}\right)-1\right]H_1+\eta\left(\frac{\omega_*^n}{\omega}\right)H_2\right\}$$
$$+\tau\left(\frac{\omega_*^n}{\omega}\right)(A_1+\eta A_2). \quad \text{(B24)}$$

APPENDIX C: DERIVATION OF THE TRANSPORT COEFFICIENTS

Working in Fourier space and using the notations introduced in Appendix B, the nonlinear equations for the nonadiabatic distribution fluctuations $\tilde{h}_k^c$ and $\tilde{h}_k^t$ are

$$g_k^{c-1}\tilde{h}_k^c+b_k^c f_0\left(\frac{e\tilde{\phi}_k}{T_0}\right)=\sum_{k'} W_{k',k-k'}\left(\frac{e\tilde{\phi}_{k'}}{T_0}\right)\tilde{h}_{k-k'}^c, \quad \text{(C1)}$$

$$g_k^{t-1}\tilde{h}_k^t+b_k^t f_0\left(\frac{e\tilde{\phi}_k}{T_0}\right)=\sum_{k'} W_{k',k-k'}\left(\frac{e\tilde{\phi}_{k'}}{T_0}\right)\tilde{h}_{k-k'}^t, \quad \text{(C2)}$$

where the nonlinear coupling coefficient $W_{k',k-k'}$ is given by $W_{k',k-k'}=(cT_0/eB)\mathbf{b}_0\cdot(\mathbf{k}'\times\mathbf{k})$ with $k\equiv(\mathbf{k},\omega)$. The equations for the forced beat fluctuations are therefore

$$g_k^{c-1}\tilde{h}_k^{c(1)}+b_k^c f_0(e\tilde{\phi}_k^{(1)}/T_0)$$
$$=-W_{k,k_0}(e/T_0)(\tilde{\phi}_k\hat{h}_{k_0}^c-\hat{\phi}_{k_0}\tilde{h}_k^c), \quad \text{(C3)}$$

$$g_k^{t-1}\tilde{h}_k^{t(1)}+b_k^t f_0(e\tilde{\phi}_k^{(1)}/T_0)$$
$$=-W_{k,k_0}(e/T)(\tilde{\phi}_k\hat{h}_{k_0}^t-\hat{\phi}_{k_0}\tilde{h}_k^t), \quad \text{(C4)}$$

$$\tilde{n}_{ik}^{(1)}=n_0\chi_i(e\tilde{\phi}_k^{(1)}/T_0), \quad \text{(C5)}$$

$$\tilde{n}_{ik}^{(1)}=\tilde{n}_{ek}^{(1)}$$
$$=\int(\tilde{f}_k^{c(1)}+\tilde{f}_k^{t(1)})d\mathbf{v}$$
$$=n_0\left(\frac{e\tilde{\phi}_k^{(1)}}{T_0}\right)+\int(\tilde{h}_k^{c(1)}+\tilde{h}_k^{t(1)})d\mathbf{v}, \quad \text{(C6)}$$

where $\mathbf{k}_0$ and ω_0 are the wave numbers and frequency of the perturbations. We have used the separation of scale $|k_0|\ll|k|$, so that $k-k_0\simeq k$.

We now solve this linear system for $\tilde{h}_k^{c(1)}$, $\tilde{h}_k^{t(1)}$, $\tilde{n}_{ik}^{(1)}$, and $\tilde{\phi}_k^{(1)}$. We find

$$n_0\frac{e\tilde{\phi}_k^{(1)}}{T_0}=\frac{-1}{\chi_i-\chi_e^L}W_{k,k_0}\left(\frac{e}{T_0}\right)\int[\tilde{\phi}_k(g_k^c\hat{h}_{k_0}^c+g_k^t\hat{h}_{k_0}^t)$$
$$-\hat{\phi}_{k_0}(g_k^c\tilde{h}_k^c+g_k^t\tilde{h}_k^t)]d\mathbf{v}, \quad \text{(C7)}$$

$$\tilde{h}_k^{c,t(1)}=g_k^{c,t}b_k^{c,t}f_0\frac{W_{k,k_0}}{\chi_i-\chi_e^L}\left(\frac{e}{T_0}\right)\int[\tilde{\phi}_k(g_k^c\hat{h}_{k_0}^c+g_k^t\hat{h}_{k_0}^t)$$
$$-\hat{\phi}_{k_0}(g_k^c\tilde{h}_k^c+g_k^t\tilde{h}_k^t)]d\mathbf{v}$$
$$-g_k^{c,t}W_{k,k_0}\left(\frac{e}{T_0}\right)(\tilde{\phi}_k\hat{h}_{k_0}^{c,t}-\hat{\phi}_{k_0}\tilde{h}_k^{c,t}). \quad \text{(C8)}$$

The anomalous fluxes, in the DIA framework, are therefore

$$\hat{\Gamma}_{\mathrm{DIA}}^{c,t}=\langle\tilde{\mathbf{v}}_E^{(1)}\tilde{h}^{c,t}+\tilde{\mathbf{v}}_E\tilde{h}^{c,t(1)}\rangle$$
$$=-i\left(\frac{cT_0}{eB}\right)\sum_k\mathbf{k}\times\mathbf{b}_0\left\langle\left(\frac{e\tilde{\phi}_k^{(1)}}{T_0}\right)\tilde{h}_k^{c,t*}-\left(\frac{e\hat{\phi}_k^*}{T_0}\right)\tilde{h}_k^{c,t(1)}\right\rangle$$
$$=-i\left(\frac{cT}{eB}\right)\sum_k\mathbf{k}\times\mathbf{b}_0\left\langle\left(\frac{e\tilde{\phi}_k^{(1)}}{T_0}\right)\right.$$
$$\times\left[\tilde{h}_k^{c,t*}+\left(\frac{e\tilde{\phi}_k^*}{T_0}\right)g_k^{c,t}b_k^{c,t}f_0\right]$$
$$\left.+\left(\frac{e\tilde{\phi}_k^*}{T_0}\right)g_k^{c,t}W_{k,k_0}\left(\frac{e}{T_0}\right)(\tilde{\phi}_k\hat{h}_{k_0}^{c,t}-\hat{\phi}_{k_0}\tilde{h}_k^{c,t})\right\rangle. \quad \text{(C9)}$$

Before we develop the calculation further, let us notice that the test-particle result, obtained by letting $\tilde{\phi}_k^{(1)}$ go to zero in the preceding formula, is given by

$$\hat{\Gamma}_{\mathrm{TP}}^{c,t}=-i\left(\frac{cT_0}{eB}\right)\sum_k\mathbf{k}\times\mathbf{b}_0\left(\frac{e}{T_0}\right)^2$$
$$\times W_{k,k_0}\langle\tilde{\phi}_k^*g_k^{c,t}(\tilde{\phi}_k\hat{h}_{k_0}^{c,t}-\hat{\phi}_{k_0}\tilde{h}_k^{c,t})\rangle. \quad \text{(C10)}$$

Using the lowest-order approximation for $\hat{h}_{k_0}^{c,t}$ and $\hat{\phi}_{k_0}$, we can express the fluxes in terms of the sources $\hat{\xi}$ as

$$\hat{\Gamma}_{\mathrm{TP}}^{c,t}=-i\left(\frac{cT_0}{eB}\right)\sum_k\mathbf{k}\times\mathbf{b}_0W_{k,k_0}g_k^{c,t}\frac{\hat{\xi}^{c,t}}{-i\omega_0}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \quad \text{(C11)}$$

Note that the fluxes of trapped particles and circulating particles are completely decoupled in this approximation.

The fluxes obtained with the DIA method can now be written as

$$\hat{\Gamma}_{\mathrm{DIA}}^{c,t}=\hat{\Gamma}_{\mathrm{TP}}^{c,t}+i\left(\frac{cT_0}{eB}\right)\left(\frac{1}{n_0}\right)\sum_k\mathbf{k}\times\mathbf{b}_0\left(\frac{e}{T_0}\right)$$
$$\times W_{k,k_0}\frac{1}{\chi_i-\chi_e^L}\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}d\mathbf{v}\right)$$
$$\times\left[\langle\tilde{\phi}_k\tilde{h}_k^{c,t*}\rangle+\left(\frac{e}{T_0}\right)g_k^{c,t}b_k^{c,t}f_0\langle\tilde{\phi}_k\tilde{\phi}_k^*\rangle\right]. \quad \text{(C12)}$$

We see here that the fluxes additional to those obtained in the quasilinear approximation couple the three species of particles through $\chi_i-\chi_e^L$, which is closely related to the plasma dielectric. The flux of electrons is obtained by integrating $\hat{\Gamma}_{\mathrm{DIA}}^{c,t}$ over the velocities,

$$\Gamma_{\rm DIA}=\int(\hat{\Gamma}^c_{\rm DIA}+\hat{\Gamma}^t_{\rm DIA})d\mathbf{v}=\Gamma_{\rm QL}-i\mathbf{k}_0\left(\frac{cT_0}{eB}\right)^2 \times\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}d\mathbf{v}\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle \tag{C13}$$

with

$$\Gamma_{\rm QL}=\int(\hat{\Gamma}^c_{\rm TP}+\hat{\Gamma}^t_{\rm TP})d\mathbf{v} =i\mathbf{k}_0\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}d\mathbf{v}\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C14}$$

and where we have used the ion equation (22) and the quasineutrality equation (23) to write

$$\left\langle\left(\frac{e\tilde{\phi}_k}{T_0}\right)\int(\tilde{h}_k^{c*}+\tilde{h}_k^{t*})d\mathbf{v}\right\rangle=(\chi_i^*-1)n_0\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \tag{C15}$$

The flux of heat is obtained from the second velocity moment of $\Gamma^{c,t}_{\rm anom}$,

$$\frac{1}{T_0}\mathbf{q}_{\rm DIA}=\int(\hat{\Gamma}^c_{\rm DIA}+\hat{\Gamma}^t_{\rm DIA})\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v} =\frac{1}{T_0}\mathbf{q}_{\rm QL}-i\mathbf{k}_0\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\frac{1}{\chi_i-\chi_e^L}\times\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}d\mathbf{v}\right)\left[\left\langle\tilde{\phi}_k\int(\tilde{h}_k^c+\tilde{h}_k^t)^*\times\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\right\rangle\left(\frac{e}{n_0T_0}\right)+\frac{1}{n_0}\int(g_k^cb_k^c+g_k^tb_k^t)\times f_0\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle\right]. \tag{C16}$$

Here, we still need to calculate the correlations,

$$\left\langle\tilde{\phi}_k\int(\tilde{h}_k^{c*}+\tilde{h}_k^{t*})\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\right\rangle.$$

These correlations can, in principle, be deduced from the spectral DIA equations. However, we will instead use a simpler procedure and *model* the velocity dependence of the distribution fluctuations $\tilde{h}_k^c$ and $\tilde{h}_k^t$. If we *assume* that the velocity dependence of $\tilde{h}_k^c$ and $\tilde{h}_k^t$ is the same as the velocity dependence of the *linearized* values of $\tilde{h}_k^c$ and $\tilde{h}_k^t$, i.e., that the correlations have the form

$$\langle\tilde{\phi}_k\tilde{h}_k^{c,t*}(\mathbf{v})\rangle=-g_k^{c,t*}b_k^{c,t*}f_0\langle(e\tilde{\psi}_k^*/T_0)\tilde{\phi}_k\rangle, \tag{C17}$$

with $\tilde{\psi}_k$ independent of velocity; then the quasineutrality relation determines $\langle\tilde{\psi}_k^*\tilde{\phi}_k\rangle$, and we find

$$\langle\tilde{\phi}_k\tilde{\psi}_k^*\rangle=\langle|\tilde{\phi}_k|^2\rangle\frac{\chi_i^*-1}{\chi_e^{L*}-1}.$$

Therefore, eq. (C17) simplifies to

$$\left\langle\tilde{\phi}_k\int(\tilde{h}_k^{c*}+\tilde{h}_k^{t*})\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\right\rangle =n_0\chi_T^{L*}\frac{\chi_i^*-1}{\chi_e^{L*}-1}\left\langle\frac{e\tilde{\phi}_k^*}{T_0}\tilde{\phi}_k\right\rangle, \tag{C18}$$

so that

$$\frac{1}{T_0}\mathbf{q}_{\rm DIA}=\frac{1}{T}\mathbf{q}_{\rm QL}-i\mathbf{k}_0\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\frac{1}{\chi_i-\chi_e^L}\times\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}d\mathbf{v}\right)\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C19}$$

with

$$\frac{1}{T_0}\mathbf{q}_{\rm QL}=\int(\Gamma^c_{\rm QL}+\Gamma^t_{\rm QL})\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}=i\mathbf{k}_0\left(\frac{cT_0}{eB}\right)^2\times\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\left(\int\frac{g_k^c\hat{\xi}^c+g_k^t\hat{\xi}^t}{-i\omega_0}\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\right)\times\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle. \tag{C20}$$

If the density profile is perturbed, the fluxes of particles and heat are

$$\Gamma_{\rm QL}=i\mathbf{k}_0\frac{\hat{n}_0}{n_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\times\int(g_k^c+g_k^t)f_0\,d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C21}$$

$$\Gamma_{\rm DIA}=i\mathbf{k}_0\frac{\hat{n}_0}{n_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right)\times\int(g_k^c+g_k^t)f_0\,d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C22}$$

$$\frac{1}{T_0}\mathbf{q}_{\rm QL}=i\mathbf{k}_0\frac{\hat{n}_0}{n_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\times\int(g_k^c+g_k^t)f_0\frac{v^2-3v_e^2}{2v_e^2}d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C23}$$

$$\frac{1}{T_0}\mathbf{q}_{\rm DIA}=i\mathbf{k}_0\frac{\hat{n}_0}{n_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\Bigg[\int(g_k^c+g_k^t)f_0\times\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}-\frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right)\times\int(g_k^c+g_k^t)f_0\,d\mathbf{v}\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C24}$$

where we have used Eq. (18) for $\hat{\xi}^c$ and $\hat{\xi}^t$. If the temperature profile is perturbed, the fluxes of particles and heat are

$$\Gamma_{\rm QL}=i\mathbf{k}_0\frac{\hat{T}_0}{T_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\times\int(g_k^c+g_k^t)f_0\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C25}$$

$$\Gamma_{\rm DIA}=i\mathbf{k}_0\frac{\hat{T}_0}{T_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times b_0|^2\int(g_k^c+g_k^t)\times f_0\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}\left(1-\frac{\chi_i^*-\chi_e^L}{\chi_i-\chi_e^L}\right)\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C26}$$

$$\frac{1}{T_0}\mathbf{q}_{\rm QL}=i\mathbf{k}_0\frac{\hat{T}_0}{T_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\times\int(g_k^c+g_k^t)f_0\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^2d\mathbf{v}\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C27}$$

$$\frac{1}{T_0}\mathbf{q}_{\rm DIA}=i\mathbf{k}_0\frac{\hat{T}_0}{T_0}\left(\frac{cT_0}{eB}\right)^2\sum_k|\mathbf{k}\times\mathbf{b}_0|^2\Bigg[\int(g_k^c+g_k^t)\times f_0\left(\frac{v^2-3v_e^2}{2v_e^2}\right)^2d\mathbf{v}-\frac{1}{\chi_i-\chi_e^L}\left(\frac{\chi_i^*-1}{\chi_e^{L*}-1}\chi_T^{L*}-\chi_T^L\right)\times\int(g_k^c+g_k^t)f_0\frac{v^2-3v_e^2}{2v_e^2}\,d\mathbf{v}\Bigg]\left\langle\left|\frac{e\tilde{\phi}_k}{T_0}\right|^2\right\rangle, \tag{C28}$$

where we have used Eq. (19) for $\hat{\xi}^c$ and $\hat{\xi}^t$.

APPENDIX D: INCREMENTAL VERSUS EQUILIBRIUM FLUXES

The anomalous transport has been conventionally studied analytically by calculating total fluxes across magnetic surfaces. However, many experiments measure the transport of *perturbations*, such as heat pulses. It has been observed experimentally as well as shown analytically that incremental fluxes resulting from small perturbations are quite different from equilibrium fluxes.

Let us assume that the nonlinear equilibrium electron flux Γ can be written under the following very general form:

$$\Gamma=-D_n\nabla n_0-D_T\nabla T_0+\mathbf{U}_n n_0+\mathbf{U}_T T_0, \tag{D1}$$

where n_0, T_0, ∇n_0, and ∇T_0 are the equilibrium values of the density, temperature, and their gradients, D_n and D_T are diffusion coefficients, and $\mathbf{U}_n$ and $\mathbf{U}_T$ are convection velocities. If the flux is produced by the turbulence, the parameters D_n, D_T, $\mathbf{U}_n$, and $\mathbf{U}_T$ are functions of the equilibrium plasma parameters. Now, let us perturb the plasma with an external infinitesimal source. The nonlinear flux Γ will be perturbed as well, and the linearized, or incremental flux can be written as

$$\delta\Gamma=\left(\frac{\partial\Gamma}{\partial n}\right)_0\delta n+\left(\frac{\partial\Gamma}{\partial\nabla n}\right)_0\delta\nabla n+\left(\frac{\partial\Gamma}{\partial T}\right)_0\delta T+\left(\frac{\partial\Gamma}{\partial\nabla T}\right)_0\delta\nabla T+\cdots, \tag{D2}$$

where δn, $\delta\nabla n$, δT, and $\delta\nabla T$ are the perturbations of the equilibrium plasma density, temperature, and their gradients, and the subscript "0" means that the quantities in parentheses must be evaluated at the equilibrium. Those derivatives can be evaluated from Eq. (D1) and written as

$$\left(\frac{\partial\Gamma}{\partial n}\right)_0=\left[-\left(\frac{\partial D_n}{\partial n}\right)_0\nabla n_0-\left(\frac{\partial D_T}{\partial n}\right)_0\nabla T_0+\left(\frac{\partial\mathbf{U}_n}{\partial n}\right)_0 n_0+\left(\frac{\partial\mathbf{U}_T}{\partial n}\right)_0 T_0+\mathbf{U}_n\right],$$

$$\left(\frac{\partial\Gamma}{\partial T}\right)_0=\left[-\left(\frac{\partial D_n}{\partial T}\right)_0\nabla n_0-\left(\frac{\partial D_T}{\partial T}\right)_0\nabla T_0+\left(\frac{\partial\mathbf{U}_n}{\partial T}\right)_0 n_0+\left(\frac{\partial\mathbf{U}T}{\partial T}\right)_0 T_0+\mathbf{U}_T\right],$$

$$\left(\frac{\partial\Gamma}{\partial\nabla n}\right)_0=\left[-\left(\frac{\partial D_n}{\partial\nabla n}\right)_0\nabla n_0-\left(\frac{\partial D_T}{\partial\nabla n}\right)_0\nabla T_0+\left(\frac{\partial\mathbf{U}_n}{\partial\nabla n}\right)_0 n_0+\left(\frac{\partial\mathbf{U}_T}{\partial\nabla n}\right)_0 T_0-D_n\right],$$

$$\left(\frac{\partial\Gamma}{\partial\nabla T}\right)_0=\left[-\left(\frac{\partial D_n}{\partial\nabla T}\right)_0\nabla n_0-\left(\frac{\partial D_T}{\partial\nabla T}\right)_0\nabla T_0+\left(\frac{\partial\mathbf{U}_n}{\partial\nabla T}\right)_0 n_0+\left(\frac{\partial\mathbf{U}_T}{\partial\nabla T}\right)_0 T_0-D_T\right].$$

Replacing these results in Eq. (D2), we can rewrite the linearized incremental flux as

$$\delta\Gamma=-D_n^{\rm incr}\delta\nabla n-D_T^{\rm incr}\delta\nabla T+\mathbf{U}_n^{\rm incr}\delta n+\mathbf{U}_T^{\rm incr}\delta T, \tag{D3}$$

with

$$D_n^{\text{incr}} = D_n + \left(\frac{\partial D_n}{\partial \nabla n}\right)_0 \nabla n_0 + \left(\frac{\partial D_T}{\partial \nabla n}\right)_0 \nabla T_0 - \left(\frac{\partial \mathbf{U}_n}{\partial \nabla n}\right)_0 n_0 - \left(\frac{\partial \mathbf{U}_T}{\partial \nabla n}\right)_0 T_0,$$

$$D_T^{\text{incr}} = D_T + \left(\frac{\partial D_n}{\partial \nabla T}\right)_0 \nabla n_0 + \left(\frac{\partial D_T}{\partial \nabla T}\right)_0 \nabla T_0 - \left(\frac{\partial \mathbf{U}_n}{\partial \nabla T}\right)_0 n_0 - \left(\frac{\partial \mathbf{U}_T}{\partial \nabla T}\right)_0 T_0,$$

$$\mathbf{U}_n^{\text{incr}} = \mathbf{U}_n - \left(\frac{\partial D_n}{\partial n}\right)_0 \nabla n_0 - \left(\frac{\partial D_T}{\partial n}\right)_0 \nabla T_0 + \left(\frac{\partial \mathbf{U}_n}{\partial n}\right)_0 n_0 + \left(\frac{\partial \mathbf{U}_T}{\partial n}\right)_0 T_0,$$

$$\mathbf{U}_T^{\text{incr}} = \mathbf{U}_T - \left(\frac{\partial D_n}{\partial T}\right)_0 \nabla n_0 - \left(\frac{\partial D_T}{\partial T}\right)_0 \nabla T_0 + \left(\frac{\partial \mathbf{U}_n}{\partial T}\right)_0 n_0 + \left(\frac{\partial \mathbf{U}_T}{\partial T}\right)_0 T_0.$$

Comparing the linearized incremental flux, we see that the incremental transport coefficients are very different from the equilibrium ones. The only case where these two sets of coefficients are identical is when the equilibrium flux is linear in the plasma parameters, which is not the case when this flux is produced by the turbulence.

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