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Domain of validity of a 1D second order perturbative approach for the effects of rotation on stellar oscillations

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Abstract

At rotational velocities such as that of upper Main Sequence stars – between 50 to 200 km/s – the effects of rotation on oscillation frequencies must be included. Considering the accuracy reached by available ground-based and space observations, the aim of this study is to determine the limits – in terms of rotational velocity – of a perturbative approach to model the effects of rotation on oscillation frequencies. We thus compare the oscillation frequencies computed by 1D second order perturbative methods to the ones obtained in Reese et al. (2006) – direct integration of a 2D eigenvalue system. To do so, we use polytropic models (N=3) in uniform rotation, and we discuss the results for a β Cephei star (8.2 $M_\odot$, 5.04 $R_\odot$).

Equilibrium state of a 1D uniformly rotating polytrope

The 1D polytropic model is computed by solving the Lame-Emden equation in which the spherical part of the centrifugal acceleration is included in an effective gravity:

$$\frac{1}{x^2}\frac{d}{dx}(x^2\frac{dy}{dx}) + y^N = \Omega'^2 \quad (1)$$

Where y is defined by $\tilde{\rho} = \rho_c y^N$ and $\tilde{p} = p_c y^{N+1}$, x is the radius normalised to the critical radius of the polytrope, and $\Omega'^2 = \Omega^2/2\pi G\rho_c$, - Ω the rotational angular velocity. The resolution of the Lame-Emden equation yields the spherically symmetric equilibrium quantities – $\tilde{\rho}(r)$, $\tilde{p}(r)$, and $\tilde{\Phi}(r)$. Their second order non-spherically symmetric parts are computed as perturbations of the spherical case to the second order, such that for a quantity f: $f(r,\theta) = \tilde{f}(r) + f_{22}(r)P_2(cos\theta)$, P_2 being the second order Legender polynomial.
In the nonperturbative case, structure variables are expanded on around 50 spherical harmonics using a code developed by M. Rieutord.

Computation of oscillation frequencies

In Reese et al. (2006) the nonperturbative oscillation frequencies are calculated by direct integration of a 2D eigenvalue system. In the perturbative approach, we apply a perturbation about an axisymmetric steady configuration. After ignoring the resonant interaction due to near degeneracy, we derive the perturbed oscillation frequency by obtaining an additional correction of the order of $O(\Omega^2)$ to the frequency without rotation.

Implicit assumptions made in the perturbative method are no longer valid when two or three modes are close to each other in terms of frequencies. Those modes are then coupled according to selection rules, and the frequencies are then modified (Dziembowski & Goode 1992, Soufi et al. 1998, Suarez et al. 2006, and references therein).

Results for a β Cephei star (8.2 $M_\odot$, 5.04 $R_\odot$)

We treated the 2D nonperturbative frequencies like observation results, and calculated the discrepancies due to the perturbative approach. We present in Figure 1 the discrepancies on the splittings between the two models, where the error is defined as:

$$\delta = \frac{1}{2\ell}[(\omega_{n,\ell,-m} - \omega_{n,\ell,m})_{Npert} - (\omega_{n,\ell,-m} - \omega_{n,\ell,m})_{Pert}] \qquad (2)$$

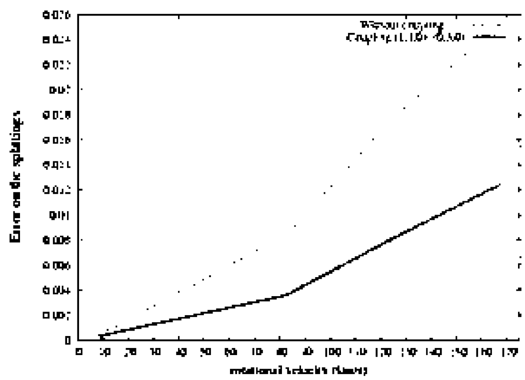

Figure 1: Comparison of the splitting computed by the perturbative and nonperturbative approach for the pressure mode P_1, $\ell = 1$, $m = \pm 1$.

Coupling due to near degeneracy clearly improves the accuracy of the perturbative method: if one accepts an observational relative error of 1.310^{-3}, which corresponds to 20 days of observation, the discrepancies are above this limit around 15 km s^{-1}. When coupling is included, this limit is pushed up to about 20 km s^{-1}. Considering the splittings, the discrepancy is much lower, and is under the error bars until around 40 km/s. Moreover, we studied the impact of these discrepancies on, for example, the measurement of a rotational angular velocity. We found that if we accept an accuracy of 1%, on the splittings, the error made by the perturbative approach on the rotational velocity is around 0.5%.

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