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Relative weights of the observational constraints on the determination of stellar parameters

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Abstract

We study the effect of using different observed quantities (oscillation frequencies, binarity, interferometric radius) and the impact of their accuracy on constraining the uncertainities of global free stellar parameters (i.e. the mass, the age etc.). We use the Singular Value Decomposition (SVD) formalism to analyse the behavior of the χ^2 fitting function around its minimum. This method relates the errors in observed quantities to the precision in the model parameters. We apply this tool to the α Cen A for which, seismic, binarity and interferometric properties are known with high accuracy. We determine how changes of the accuracy of the observable constraints affect the precision obtained on the global stellar parameters.
Individual Objects: α Cen A

The Method

Given a set of n measurements $y_{obs,i}$ (e.g. T_{eff}, L, $\Delta\nu$, etc.) with associated error bars and a set of m free parameters x_j (e.g. τ (age), α, M, etc.), we first determine the reference model (RM) which minimizes the χ^2 fitting function. The linear transformation of the model ($y_{the,i}$) around the minimum of the reference set of parameters (x_{j0}) produces a derivative matrix which is the so-called design matrix **D**. This matrix relates small changes in the parameters to corresponding changes in the observables. This minimisation problem is most conveniently solved by using the SVD method as **D** may be decomposed as $D_{n\times m} = U_{n\times m}W_{m\times m}V_{m\times m}$. **U** and **V** are orthonormal and **W** is a diagonal matrix. By studying the behaviour of χ^2 around its minimum using the SVD method, we analyse the sensitivity of the observables to the parameters. The behaviour of χ^2 around its minimum is expressed by an m dimensional ellipsoidal equation $\Delta\chi^2$.

$$\Delta\chi^2 = \frac{\|V^{(1)}\delta x\|^2}{W_1^{-2}} + ... + \frac{\|V^{(m)}\delta x\|^2}{W_m^{-2}}, \quad Cov(\delta x_j, \delta x_k) = \sum_i^N \frac{V_{ji}V_{ki}}{W_i^2} \qquad (1)$$

where $\delta\mathbf{x} = \mathbf{x_j} - \mathbf{x_{j0}}$. A major advantage of the SVD method is that the columns of **V** are precisely the principal axes of the error ellipsoid, while the corresponding values of $\mathbf{W}^{-1}$ are the lengths of these axes. The estimation of the variances-covariances matrix $Cov(\delta x_j, \delta x_k)$ of the free parameters due to the measurement errors on the n observables is expressed above. The solution of this minimisation problem by the SVD method was proposed in the case of solar-like oscillation by Brown et al. (1994) and applied also by Creevey et al. (2007). To study the relation between different observables and free stellar parameters (the mixing length parameter α, the mass M, and the metalicity Z/X), we analyse the error ellipsoids in the parameter space by the SVD method.

Application to Different Astrophysical Situation Based on the α Cen A

The α Cen A is used here as a template. Its nature, its proximity, and the detection of solar-like oscillation constraints provide a unique opportunity to test the SVD method.

The stellar models of the α Cen A are computed with the stellar evolution code CESAM (Morel 1997) starting from the ZAMS. The adopted physical description is the standard MLT for convection calculation (Böhm- Vitense 1958); the OPAL opacities (Iglesias & Roger 1996) completed at low temperatures with the opacities of Alexander & Ferguson (1994); the OPAL equation of state; and an Eddington atmosphere as the surface boundary condition. The adiabatic oscillation frequencies are calculated for ℓ = 0-3 and n = 15-25 using the adiabatic oscillation code Losc (Scuflaire et al. 2008). To construct the derivative matrix D, we vary each of the parameters x_j. Each derivative is computed from differences centered on the reference parameter values given in Table 1. The interval δx has to be sufficiently small such that the linear approximation is good, yet still large enough to avoid numerical problems. The increments used are 20 Myr for (τ) age, 0.05 for α, 0.005 for M(M$\odot$), 0.003 for initial helium abundance Y_0 and 0.0005 for initial metallicity Z/X_0. The constraints of α CenA and all characteristics of our RM are summarized in Table 1. The technique used in this study to estimate all charracteristics of our RM follows that of Miglio & Montalbán (2005).

Table 1: Observations of α Cen A and the properties of the RM

	T_{eff} [1]	$L/L_\odot$ [1]	Z/X [2]	$R/R_\odot$ [3]	$\Delta\nu$ [4]	$\delta\nu$ [4]	$M/M_\odot$ [5]
α CenA	5810	1.522	0.039	1.224	105.5	5.6	1.105
σ	50K	0.030	0.06	0.003	$0.1\mu Hz$	$0.7\mu Hz$	0.007
RM	5782	1.516	0.039	1.229	105.5	5.7	1.099
x_j of RM		$\tau(Gyr)$	α	M	Y_0	Z/X_0	
		5.65	1.6747	1.099	0.280	0.039	

Apart from the real α Cen system, we study several different cases to cover a large range of realistic situations. Case 1 describes α Cen itself ($d = 1.3$pc), while Case 2 corresponds to a system located ten times further away ($d = 13$pc). For distant objects where the binary and interferometric data are unavailable, the seismic data are the major source of information (Case 3). We discuss also the influence of the seismic data precision.

Table 2 shows the results from these different cases. In all these cases, the mass is the best constrained parameter with these observables. We therefore focus here on its uncertainity. In Case 1 for α Cen itself, we see that the mean large separation $\Delta\nu$ and the radius $R/R_\odot$ both give more or less the same precision on the mass parameter ($\epsilon = 2.09\%$, and 2.20% ,respectively). If these two constraints are considered together, the relative precision on the mass is much higher ($\epsilon = 0.75\%$). This comes from the fact that $M \propto \Delta\nu^2 R^3$. Using the mean $\Delta\nu$ or the individual $\Delta\nu_i$ large separation give about the same precision on the mass, as we are close to the asymptotic regime. Comparing Case 2 with Case 1 allows us to estimate the effect of increasing distance on determination of the precision of mass parameter for the same combination of observables. The observables depending on the distance (i.e, mass, radius, luminosity) become less effective to constrain the mass parameter. For example the uncertainity on the mass parameter increases from $\epsilon = 0.75\%$ to $\epsilon = 2.13\%$ when the radius and the seismic data are considered together. In Case 3, the seismic information alone gives $\epsilon = 2.19\%$. As the precision on the seismic data does not depend on distance, the ϵ does not change ($\epsilon = 2.09\%$ for d=1.3pc, $\epsilon = 2.19\%$ for d=13pc). Contrary to Case 1, in Case 3, even if the mass or radius information were available, they would not reduce this value significantly ($\epsilon = 2.13\%$ if the radius is available, $\epsilon = 2.06\%$ if the mass). We also reduce the precision of seismic data taking $\sigma_{\Delta\nu} = 2$ μHz as the worst case scenario. For this precision, the uncertainty on the mass parameter increases only slightly because of the flatness of the error ellipsoid as we increase the error on the oscillation frequencies by a factor of 20.

Table 2: The rms error on the mass ($\epsilon(M)$), taking into account different sets of observables for α Cen A and for the case if it were located at 13pc. $Q_i = (T_{eff}, L/L_\odot, Z/X_0)$ represent the set of classical observables and are included in all cases. The symbol ($\surd$) indicates that the observable is included, and the symbol ($\times$) indicates that the observable is not included in the SVD analyse.

Observables					d=1.3pc	d=13pc
Classic		Seismic (μHz)				
$R/R_\odot$	$M/M_\odot$	$(\Delta\nu, \delta\nu)$	$(\Delta\nu_i, \delta\nu_i)$	$\sigma_{\Delta\nu}$	$\epsilon(M)$ (%)	$\epsilon(M)$(%)
×	×	×	×	–	2.21	2.26
×	×	√	×	0.1	2.09	2.19
×	×	×	√	–	1.73	1.75
×	×	√	×	2	2.12	2.20
√	×	×	×	–	2.20	2.24
√	×	√	×	0.1	0.75	2.13
×	√	×	×	–	0.61	2.12
×	√	√	×	0.1	0.61	2.06

Conclusion

We approach the problem of the accuracy on the determination of the stellar parameters by computing the error ellipsoids in the parameter space. The impact of each constraint is obtained by performing the SVD analysis for different cases taking into account or not each observable. We apply this method to the real α Cen system and for different cases in changing the distance and the precision on the constraints.

We have shown that, except for α Cen itself for which all the seismic and classical constraints are available to high precision, the seismic constraints play a dominant role in determining stellar free parameters for the distant objects.

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