CONVECTION-OSCILLATIONS INTERACTION IN F-G TYPE MAIN SEQUENCE STARS

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ABSTRACT

The derivation of pulsation models taking the non-adiabatic interaction with convection into account is very important in helio- and asteroseismology. The mode excitation and damping as well as the amplitudes and phases can be computed by these models and then confronted to observations. We have implemented in our non-radial non-adiabatic pulsation code the perturbation of the convective flux, the turbulent pressure and the turbulent kinetic energy dissipation, according to the theory of Gabriel (1974, 1996, 1998). In the case of solar-like oscillations, a major problem is to avoid the occurrence of unphysical spatial oscillations of the eigenfunctions in the efficient part of the convective envelope. We present here a new local solution to this problem. Finally, application to the Sun, δ Sct and γ Dor stars are presented.

Key words: Stars: oscillations; Convection; Stars: interiors.

1. INTRODUCTION

In this paper, we consider the problem of the interaction between convection and pulsation, following a linear perturbative approach. Such approach requires an a-priori distinction between the convective motions supposed to have short wavelengths and the acoustic motions supposed to have longer wavelengths. On the one hand, we consider the convective *fluctuations* corresponding to the difference between the physical conditions in a convective cell and in the average medium, and on the other hand we consider the *perturbation* of the mean structure corresponding to the oscillations.

The problem of the interaction between convection and pulsation has been studied by many authors, following different approaches. In particular, two different approaches of the mixing-length theory (MLT) have been proposed, which lead to the same equations at equilibrium but differ when we consider their perturbations: the theory of Gough (1965, 1977) and the theory of Unno (1967). Gabriel et al. (1974, 1975) generalized the theory of Unno (1967) to the case of non-radial modes. Also, some terms neglected by Unno are not neglected in the theory of Gabriel (1987, 1996, 1998). We first recall in this paper the basic equations and approximations and the last improvements of the Gabriel's theory. Then, we present the applications to solar-type, δ Sct and γ Dor stars.

2. EQUATIONS FOR THE CONVECTIVE FLUCTUATIONS

As usually in the study of turbulence, we split the variables in two parts, describing respectively the average model and the convection. Therefore, we write $y = \overline{y} + \Delta y$ and $\vec{v} = \vec{u} + \vec{V}$, where y is any of the variables ρ, p, T, etc. $\vec{v}$ is the velocity vector. $\overline{y}$ and $\vec{u}$ are the average values, while Δy and $\vec{V}$ describe the convection. We do not recall here the general hydrodynamic equation and the equations for the average medium and begin directly with the equations for convection obtained by taking the difference between the two. In our treatment, we use the Boussinesq approximation, which gives for the continuity equation:

$$\nabla \cdot \vec{V} = 0 \quad \text{or} \quad \nabla \cdot (\rho \vec{V}) = 0. \tag{1}$$

Concerning the equation of motion for convection, some simplifications are necessary in order to recover the MLT. Hence, our treatment neglects a large number of characteristics of the convection, including the cascade of the energy bound to the coupling of the convective motions at different scales. Following Unno (1967), we assume:

$$\frac{\Delta\rho}{\overline{\rho}}\nabla\cdot(\overline{\beta}_g+\overline{\beta}_R+\overline{\beta}_t)-\nabla\cdot(\Delta\beta_g+\Delta\beta_R+\Delta\beta_t) = \Lambda\frac{\overline{\rho}\vec{V}}{\tau_c}, \tag{2}$$

where β_i are the differences between the gas (g), radiation (R) and Reynolds (t) stress tensors and the corresponding gas, radiation and turbulent pressure. The equa-

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tion of motion for convection is then given by:

$$\overline{\rho}\frac{\mathrm{d}\vec{V}}{\mathrm{d}t} = \frac{\Delta\rho}{\overline{\rho}}\nabla\overline{p} - \nabla\Delta p - \rho\vec{V}\cdot\nabla\vec{u} - \Lambda\frac{\overline{\rho}\vec{V}}{\tau_c}, \quad (3)$$

where $p = p_g + p_R + p_t$. Λ is a dimensionless constant. In our case we take $\Lambda = 8/3$ in order to have the compatibility with our equilibrium MLT models. τ_c is the lifetime of the convective elements. It is related to the mixing-length $l = -\alpha(\mathrm{d}\ln p/\mathrm{d}r)^{-1}$ and the mean turbulent velocity by:

$$\tau_c = l/V_r. \quad (4)$$

We neglect the pressure fluctuation everywhere but in the equation of motion (Boussinesq). Also, we keep only the first order terms in the fluctuations. For similar reasons as for the derivation of Eq. (2), we assume for the closure of the energy equation (Unno 1967):

$$\begin{aligned} \overline{\rho}\overline{T}\frac{\Delta s}{\tau_c} &= -\overline{\rho}\overline{T}\vec{V}\cdot\nabla\overline{s} - \rho\epsilon_2 + \overline{\rho\epsilon_2} \\ &+ (\rho T\nabla s)\cdot\vec{V} - \overline{(\rho T\nabla s)\cdot\vec{V}}. \end{aligned} \quad (5)$$

The energy equation for convection is then given by:

$$\frac{\Delta(\rho T)}{\overline{\rho T}}\frac{\mathrm{d}\overline{s}}{\mathrm{d}t} + \frac{\mathrm{d}\Delta s}{\mathrm{d}t} + \vec{V}\cdot\nabla\overline{s} = -\frac{\omega_R\tau_c + 1}{\tau_c}\Delta s. \quad (6)$$

$$\text{with} \quad \omega_R = \frac{1}{\tau_R} = \frac{4ac}{3}\frac{\overline{T}^3}{c_p\overline{\kappa\rho}^2\mathcal{L}^2}. \quad (7)$$

τ_R is the cooling characteristic time of turbulent eddies due to radiative losses. $\mathcal{L}$ is the characteristic length of the eddies. It is related to the mixing-length l by $\mathcal{L}^2 = (2/9)l^2$ in order to recover the MLT used in our equilibrium stellar models. Finally, the total flux of energy transported by convection is:

$$\vec{F}_c = \overline{\rho}\overline{T}\,\overline{\Delta s\vec{V}}. \quad (8)$$

3. PERTURBATION OF THE MEAN STRUCTURE

In this section, we perturb the equations of the mean structure, which gives the linear non-radial non-adiabatic pulsation equations. The Lagrangian variation of any quantity y is denoted, for a given spheroidal mode by $\delta y(\vec{r},t) = \delta y(r)\exp(i\sigma t)\,Y_\ell^m(\theta,\varphi)$. In order to be able to distinguish global perturbations from convective motions, we must consider ℓ values small enough so that $r/\ell \gg l$. In what follow, we omit the overlining of the mean quantities when no risk of confusion is present.

The perturbed equation of mass conservation is:

$$\frac{\delta\rho}{\rho} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\xi_r\right) = \ell(\ell+1)\frac{\xi_h}{r}, \quad (9)$$

where we used the notation of Unno et al. (1989) for the displacement vector $\vec{\xi}$.

For the perturbation of the divergence of the Reynolds tensor, we use the following notation:

$$\begin{aligned} \delta(\nabla\cdot\beta_t) &= -\Xi_r(r)\,Y_\ell^m(\theta,\varphi)\,\vec{e}_r \\ &\quad -\Xi_h(r)\left(r\,\vec{\nabla}_h Y_l^m(\theta,\phi)\right), \end{aligned}$$

where $\vec{\nabla}_h$ is the transversal component of the gradient. We define $A = 0.5\,V_r^2/V_\theta^2$ (for an isotropic turbulence, $A = 1/2$). The radial component of the perturbed equation of motion takes then the following form:

$$\begin{aligned} \sigma^2\xi_r &= \frac{\mathrm{d}\delta\Phi}{\mathrm{d}r} + \frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}r}\left(\delta p_{g+R} + \delta p_t\right) \\ &\quad + g\frac{\delta\rho}{\rho} + \frac{2A-1}{A}\frac{p_t}{r\rho}\frac{\mathrm{d}\xi_r}{\mathrm{d}r} + \frac{\Xi_r}{\rho}, \end{aligned} \quad (10)$$

where $p_{g+R} = p_g + p_R$. The transversal component of the perturbed equation of motion is:

$$\begin{aligned} \sigma^2 r\,\xi_h &= \delta\Phi + \frac{r\,\Xi_h}{\rho} + \frac{\delta p_{g+R} + \delta p_t}{\rho} \\ &\quad + \frac{2A-1}{A}\frac{p_t}{\rho}\left(\frac{\xi_r}{r} - \frac{\xi_h}{r}\right). \end{aligned} \quad (11)$$

For the perturbed energy equation, we find:

$$\begin{aligned} i\sigma T\delta s = &-\frac{\mathrm{d}\,\delta(L_R + L_c)}{\mathrm{d}m} + \left[\frac{\delta\epsilon}{\epsilon} + \ell(\ell+1)\frac{\xi_h}{r}\right]\epsilon \\ &+ \frac{\ell(\ell+1)}{4\pi r^3\rho}\left[L_R\left(\frac{\delta T}{r\,(\mathrm{d}T/\mathrm{d}r)} - \frac{\xi_r}{r}\right) - L_c\frac{\xi_h}{r}\right] \\ &+ \frac{\ell(\ell+1)}{\rho r}\delta F_{c,h} + \delta\left[\epsilon_2 + \vec{V}\cdot\frac{\nabla(p_g + p_R)}{\overline{\rho}}\right], \end{aligned} \quad (12)$$

where we used the following notation for the perturbation of the convective flux:

$$\begin{aligned} \delta\vec{F}_c &= \delta F_{c,r}(r)\;Y_\ell^m(\theta,\phi)\;\vec{e}_r \\ &\quad + \delta F_{c,h}(r)\;\left(r\nabla_h Y_\ell^m(\theta,\phi)\right). \end{aligned} \quad (13)$$

The last term of Eq. (12) is the perturbed rate of dissipation of turbulent kinetic energy into heat. It is also present in the perturbed equation of turbulent kinetic energy conservation:

$$i\sigma\rho\,\delta\left[\frac{\rho\vec{V}^2}{2\rho}\right] = -\delta\left[\overline{\rho\epsilon_2 + \vec{V}\cdot\nabla p_{g+R}}\right] - i\sigma\overline{\rho\vec{V}\vec{V}}\otimes\nabla\vec{\xi}. \quad (14)$$

4. PERTURBATION OF THE CONVECTION

Stationary solutions of the equations for the convective fluctuations (Eqs. (1), (3) and (6)) lead to the classical MLT treatment adopted in our equilibrium models

(Gabriel et al. 1974). In order to determine the perturbation of the terms linked to convection we proceed as follow. We perturb Eqs. (1), (3) and (6). Then we search for solutions of the form $\delta\left(\Delta X\right)=\delta\left(\Delta X\right)_{\vec{k}}\, e^{i\,\vec{k}\cdot\vec{r}}\, e^{i\sigma\, t}$, assuming constant coefficients. Then we integrate these particular solutions over all values of k_θ and k_ϕ such that $k_\theta^2+k_\phi^2=A\,k_r^2$, assuming A constant and that every direction of the horizontal component of $\vec{k}$ have the same probability. We have to introduce this distribution of $\vec{k}$ values in order to obtain an expression for the perturbation of the Reynolds tensor which allows the proper separation of the variables in the equation of motion (Gabriel 1987). Finally, horizontal averages are computed on a scale larger than the eddies size but smaller than the horizontal wave-length of the non-radial oscillations. The perturbation of Eq. (1) gives: $\vec{k}\cdot\delta\vec{V}=0$. The perturbation of Eq. (6) gives:

$$\left(\frac{\Delta\rho}{\overline{\rho}}+\frac{\Delta T}{\overline{T}}\right)\frac{\mathrm{d}\delta\overline{s}}{\mathrm{d}t}+\frac{\mathrm{d}\left(\delta\Delta s\right)}{\mathrm{d}t}$$
$$+\,\delta\vec{V}\cdot\nabla\overline{s}+\vec{V}\cdot\delta\left(\nabla\overline{s}\right)$$
$$=-\omega_{\mathrm{R}}\,\delta\Delta s-\delta\omega_{\mathrm{R}}\Delta s-\delta\left(\frac{\Delta s}{\tau_{\mathrm{c}}}\right). \tag{15}$$

We recall that the term $\Delta s/\tau_{\mathrm{c}}$ corresponds to the closure approximation adopted in our MLT treatment for the energy equation (Eq. (5)). When $\sigma\,\tau_{\mathrm{c}}\ll 1$, convection instantaneously adapts to the changes due to oscillations and we can assume:

$$\delta\left(\frac{\Delta s}{\tau_{\mathrm{c}}}\right)=\frac{\Delta s}{\tau_{\mathrm{c}}}\left(\frac{\delta\Delta s}{\Delta s}-\frac{\delta\tau_{\mathrm{c}}}{\tau_{\mathrm{c}}}\right). \tag{16}$$

This is the treatment adopted in Gabriel (1996). In Sect. 5, we will propose another way to perturb $\Delta s/\tau_{\mathrm{c}}$. The perturbation of Eq. (3) gives:

$$i\sigma\overline{\rho}\,\delta\vec{V}=\delta\left(\frac{\Delta\rho}{\overline{\rho}}\right)\nabla\overline{p}+\frac{\Delta\rho}{\overline{\rho}}\delta\left(\nabla\overline{p}\right)-\delta\left(\nabla\Delta p\right)$$
$$-\overline{\rho}\vec{V}\cdot\delta\nabla\vec{u}-\frac{\Lambda\overline{\rho}\vec{V}}{\tau_{\mathrm{c}}}\left(\frac{\delta\overline{\rho}}{\overline{\rho}}-\frac{\delta\tau_{\mathrm{c}}}{\tau_{\mathrm{c}}}\right)-\frac{\Lambda\overline{\rho}\delta\vec{V}}{\tau_{\mathrm{c}}}. \tag{17}$$

In what follows, we use the notations: $D=(i\sigma\tau_{\mathrm{c}}+\omega_{\mathrm{R}}\tau_{\mathrm{c}}+1)^{-1}$, $B=(i\sigma\tau_{\mathrm{c}}+\Lambda)/\Lambda$, $C=D(\omega_{\mathrm{R}}\tau_{\mathrm{c}}+1)$. Taking the divergence of Eq. (17) enables to determine $\delta\Delta p$. We define $Q=\partial\ln T/\partial\ln\rho|_p$ and neglect Δp in the relation between $\Delta\rho$ and Δs. $\delta\tau_{\mathrm{c}}$ and $\delta\omega_{\mathrm{R}}$ are related to δl by perturbing Eqs. (4) and (7). From Eqs. (15) and (17) and following the procedure described at the beginning of this section, we can then compute the different perturbed correlation terms: $\overline{\delta V_r/V_r}$, $\overline{\delta\Delta s/\Delta s}$, $\overline{V_\theta\delta V_r/V_r^2}$, $\overline{\delta V_\theta/V_r}$ and $\overline{V_\theta\,\delta V_\theta/V_r^2}$. We give here only $\overline{\delta V_r/V_r}$ and refer to Gabriel (1996) for the other correlation terms:

$$\frac{\overline{\delta V_r}}{V_r}=\frac{1}{B+(i\sigma\tau_{\mathrm{c}}+1)D}$$
$$\cdot\left\{-\frac{\delta c_{\mathrm{p}}}{c_{\mathrm{p}}}-\frac{\delta Q}{Q}-\frac{\delta\rho}{\rho}+\frac{\mathrm{d}\delta p}{\mathrm{d}p}-\frac{\mathrm{d}\xi_r}{\mathrm{d}r}\right.$$
$$-\;i\sigma\tau_{\mathrm{c}}D\frac{(Q+1)}{Q}\frac{\delta s}{c_{\mathrm{p}}}+C\left[\frac{\mathrm{d}\delta s}{\mathrm{d}s}-\frac{\mathrm{d}\xi_r}{\mathrm{d}r}\right]$$
$$-\;\frac{A}{A+1}\frac{i\sigma\tau_{\mathrm{c}}}{\Lambda}\left(\frac{\mathrm{d}\xi_r}{\mathrm{d}r}+\frac{1}{A}\frac{\xi_r}{r}-\frac{\ell(\ell+1)}{2A}\frac{\xi_h}{r}\right)$$
$$-\;\omega_{\mathrm{R}}\tau_{\mathrm{c}}D\left(3\frac{\delta T}{T}-\frac{\delta c_{\mathrm{p}}}{c_{\mathrm{p}}}-\frac{\delta\kappa}{\kappa}-2\frac{\delta\rho}{\rho}\right)$$
$$\left.+\;(i\sigma\tau_{\mathrm{c}}+3\omega_{\mathrm{R}}\tau_{\mathrm{c}}+2)D\,\frac{\delta l}{l}\right\}. \tag{18}$$

From the perturbation of Eq. (8), we obtain then the perturbed convective flux. The radial component is given by:

$$\frac{\delta F_{\mathrm{c},r}}{F_{\mathrm{c},r}}=\frac{\delta\rho}{\rho}+\frac{\delta T}{T}-i\sigma\tau_{\mathrm{c}}D\frac{(Q+1)}{Q}\frac{\delta s}{c_{\mathrm{p}}}$$
$$+C\left[\frac{\mathrm{d}\delta s}{\mathrm{d}s}-\frac{\mathrm{d}\xi_r}{\mathrm{d}r}\right]+(2\omega_{\mathrm{R}}\tau_{\mathrm{c}}+1)D\,\frac{\delta l}{l}$$
$$-\omega_{\mathrm{R}}\tau_{\mathrm{c}}D\left(3\frac{\delta T}{T}-\frac{\delta c_{\mathrm{p}}}{c_{\mathrm{p}}}-\frac{\delta\kappa}{\kappa}-2\frac{\delta\rho}{\rho}\right)$$
$$+(i\sigma\tau_{\mathrm{c}}+2\omega_{\mathrm{R}}\tau_{\mathrm{c}}+1)D\frac{\overline{\delta V_r}}{V_r}, \tag{19}$$

and the transversal component is given by:

$$\frac{\delta F_{\mathrm{c},h}}{F_{\mathrm{c},r}}=\frac{C\,(B+1)}{2A\,(B-C)}\frac{\delta s}{\mathrm{d}s/\mathrm{d}\ln r}$$
$$+\,\frac{1}{2AB}\left[\frac{C\,(B+1)}{B-C}+A+2\right]\frac{\delta p}{\mathrm{d}p/\mathrm{d}\ln r}$$
$$+\left[\frac{C\,(B+1)\,(2BA+B+1)}{2BA\,(A+1)\,(B-C)}\right.$$
$$\left.+\,\frac{B-1}{2B\,(A+1)}+\frac{A+2}{2AB}\right]\left(\frac{\xi_h}{r}-\frac{\xi_r}{r}\right)$$
$$-\,\frac{B-1}{2B\,(A+1)}\left[\frac{C\,(B+1)}{B-C}+A+2\right]\frac{\mathrm{d}\xi_h}{\mathrm{d}r} \tag{20}$$

The perturbed turbulent pressure is simply given by:

$$\frac{\delta p_{\mathrm{t}}}{p_{\mathrm{t}}}=\frac{\delta\rho}{\rho}+2\frac{\overline{\delta V_r}}{V_r}. \tag{21}$$

Since a term proportional to $\mathrm{d}\delta s/\mathrm{d}s$ is present in Eq. (18), the differential system is one order larger when the perturbation of the turbulent pressure is taken into account in the equation of motion (Eq. (10)).

We consider now the perturbation of the rate of dissipation of turbulent kinetic energy into heat (last term of Eq. (12)). From Eq. (14), we find after some algebra:

$$\delta\left(\overline{\rho\epsilon_2+\vec{V}\cdot\nabla p_{\mathrm{g+R}}}\right)=-i\sigma p_{\mathrm{t}}\left[\frac{A+1}{2A}\left(\frac{\delta p_{\mathrm{t}}}{p_{\mathrm{t}}}-\frac{\delta\rho}{\rho}\right)\right.$$
$$\left.+\,\frac{\mathrm{d}\xi_r}{\mathrm{d}r}+\frac{1}{2A}\left(2\frac{\xi_r}{r}-\ell\,(\ell+1)\,\frac{\xi_h}{r}\right)\right]. \tag{22}$$

A source of uncertainty in any mixing-length perturbative theory of convection comes from the expression which is adopted for the perturbation of the mixing-length l. In our non-adiabatic pulsation code, we can adopt optionally: $\delta l/l = \delta H_{\rm p}/H_{\rm p}$ or $\delta l/l = (1 + (\sigma\tau_{\rm c})^2)^{-1}\delta H_{\rm p}/H_{\rm p}$.

5. A NEW PERTURBATION OF THE CLOSURE EQUATIONS

A well known problem of this treatment is the occurrence of spatial oscillations of the thermal eigenfunctions with a wave-length much shorter than the mixing-length in the part of the convective envelope where $\sigma\,\tau_{\rm c} \gg 1$, which is in contradiction with the basic assumptions of the MLT (Gonczi & Osaki 1980). The same problem also arises in the local mixing-length perturbative theory of Gough (Baker & Gough 1979). The explanation of this phenomenon is the following. Let us consider the conservation of energy equation for a radial mode when most of the energy is transported by convection:

$$i\sigma T\,\delta s = -\frac{{\rm d}\delta L_{\rm c}}{{\rm d}m}. \quad (23)$$

Isolating ${\rm d}\delta s/{\rm d}s$ in Eq. (19) and considering the case $\sigma\,\tau_{\rm c} \gg 1 \gg \omega_{\rm R}\tau_{\rm c}$, we can write:

$$\frac{\delta L_{\rm c}}{L_{\rm c}} \simeq \left(\frac{\delta L_{\rm c}}{L_{\rm c}}\right)_1 + \frac{1}{i\sigma\tau_{\rm c}}\,\frac{{\rm d}\delta s/{\rm d}r}{{\rm d}s/{\rm d}r}. \quad (24)$$

Combining Eqs. (23), (24) and the equilibrium relations of the MLT, we find after some algebra:

$$\frac{\tau_{\rm c}}{T}\left[\frac{{\rm d}\,(\delta L_{\rm c})_1}{{\rm d}m} + \frac{2i\pi}{\sigma}\frac{{\rm d}(\rho r^2 T V_r^2)}{{\rm d}m}\frac{{\rm d}\delta s}{{\rm d}r}\right] - \frac{1}{2}\,\frac{l^2}{i\sigma\tau_{\rm c}}\,\frac{{\rm d}^2\delta s}{{\rm d}r^2} + i\sigma\tau_{\rm c}\delta s = 0. \quad (25)$$

This is the equation of an oscillator whose solutions have a wavelength of: $\sqrt{2}l/(\sigma\tau_{\rm c})$. In order to avoid these non-physical oscillations, non-local treatments have been proposed (Gonczi 1986; Balmforth 1992; Xiong et al. 1997).

In the method presented above, we have adopted Eq. (16) for the perturbation of the energy closure equation. A lot of complex physical process, including all the cascade of energy are extremely simplified in this approach. Therefore, it is clear that a lot of uncertainty is associated to the perturbation of this term. A point to emphasize is that the occurrence of the non-physical spatial oscillations is directly linked to the perturbation of this closure term. When these oscillations occur ($\sigma\,\tau_{\rm c} \gg 1$), the radial derivatives of $\delta\overline{s}$ and $\delta\Delta s$ are of the order of $(\sigma\tau_{\rm c}/l)\delta\overline{s}$ and $(\sigma\tau_{\rm c}/l)\delta\Delta s$ respectively. Therefore, if we take Eq. (16), we see that the order of magnitude of the perturbation of the right hand side member of Eq. (5) is $\sigma\tau_{\rm c}$ times larger than the left hand side member. To have the same order of magnitude, the perturbation of the left hand side member should rather now be given by:

$$\delta\left(\frac{\Delta s}{\tau_{\rm c}}\right) = \beta\,\sigma\,\delta\Delta s - \Delta s\frac{\delta\tau_{\rm c}}{\tau_{\rm c}^2}, \quad (26)$$

where β is a coefficient of the order of unity. In order to get a formula which switch continuously from Eq. (16) to Eq. (26), we propose to adopt the following expression:

$$\delta\left(\frac{\Delta s}{\tau_{\rm c}}\right) = \frac{\Delta s}{\tau_{\rm c}}\left((1+\beta\sigma\tau_{\rm c})\frac{\delta\Delta s}{\Delta s} - \frac{\delta\tau_{\rm c}}{\tau_{\rm c}}\right). \quad (27)$$

With this expression for the perturbation of the closure term, the coefficient $D = (i\sigma\tau_{\rm c} + \omega_{\rm R}\tau_{\rm c} + 1)^{-1}$ is replaced by $D = ((i+\beta)\sigma\tau_{\rm c} + \omega_{\rm R}\tau_{\rm c} + 1)^{-1}$. Therefore, in the case $\sigma\,\tau_{\rm c} \gg 1$, the coefficient of ${\rm d}^2\delta\overline{s}/{\rm d}r^2$ in Eq. (25) is approximately $-1/2\; l^2/((i+\beta)\sigma\tau_{\rm c})$ instead of $-1/2\; l^2/(i\sigma\tau_{\rm c})$ and thanks to the real part of the complex parameter β, the non-physical spatial oscillations of the eigenfunctions are no longer present in the solution, as we will show in Sect. 9.

6. INTEGRAL EXPRESSIONS

A useful quantity in the analysis of the driving and damping mechanisms in pulsating stars is the work integral: $-\int_0^M \Im\{\delta\rho^*\delta p/\rho^2\}{\rm d}m$. We can isolate the contribution of the different physical terms in this integral.

$$W_{\rm f_i} = -\int_0^m (\Gamma_3 - 1)\,\Re\left\{\frac{1}{\sigma}\frac{\delta\rho^*}{\rho}\frac{{\rm d}\,\delta L_i}{{\rm d}m}\right\}{\rm d}m \quad (28)$$

is the work due to the radiative and convective flux radial component perturbation ($i = {\rm c}$ for convection and $i = {\rm R}$ for radiation);

$$W_{{\rm f}_{i,h}} = \ell(\ell+1)\int_0^m (\Gamma_3 - 1)\Re\left\{\frac{\delta\rho^* F_i}{r\rho^2\sigma}\left[\frac{\delta F_{i,h}}{F_i} - \frac{\xi_h}{r}\right]\right\}{\rm d}m \quad (29)$$

is the work due to the flux transversal components perturbation.

$$W_{\rm p_t} = -\int_0^m \Im\left\{\frac{\delta\rho^*}{\rho}\frac{\delta p_{\rm t}}{\rho}\right\}{\rm d}m \quad (30)$$

is the work due to the perturbation of the turbulent pressure; and finally for a radial mode and isotropic turbulence ($A = 1/2$):

$$W_{\epsilon_2} = \int_0^m \Re\left\{\frac{1}{\sigma}\frac{\delta\rho^*}{\rho}\,\delta\left(\epsilon_2 + \vec{V}\cdot\frac{\nabla p_{\rm g+R}}{\rho}\right)\right\}{\rm d}m = \int_0^m \frac{3}{2}(\Gamma_3 - 1)\Im\left\{\frac{\delta\rho^*}{\rho}\frac{\delta p_{\rm t}}{\rho}\right\}{\rm d}m \quad (31)$$

is the work due to turbulent kinetic energy dissipation variations. In agreement with Ledoux & Walraven (1958), we see from this equation that the perturbation of turbulent pressure and the perturbation of dissipation rate of turbulent kinetic energy have an opposite effect on the work integral and thus on the excitation and damping of the modes. Therefore, it is important to take both into account.

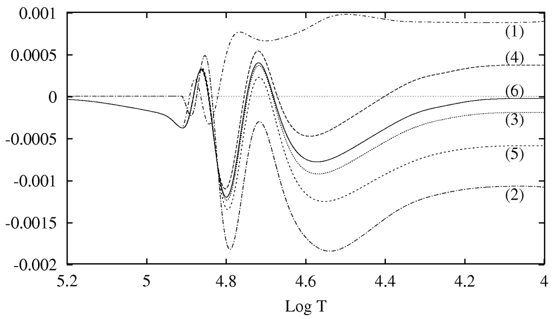

Figure 1. Different physical components of the work integral obtained with our TDC treatment for the radial p_3 mode of a model with $M = 1.8 M_\odot$, $T_{\rm eff} = 6681$ K and $\alpha = 1.8$. (1) is $W_{\rm f_R}$, (2) is $W_{\rm f_c}$, (3) is $W_{\rm f_R} + W_{\rm f_c}$, (4) is $W_{\rm f_R} + W_{\rm f_c} + W_{\rm p_t}$, (5) is $W_{\rm f_R} + W_{\rm f_c} + W_{\epsilon_2}$ and (6) is $W = W_{\rm f_R} + W_{\rm f_c} + W_{\rm p_t} + W_{\epsilon_2}$.

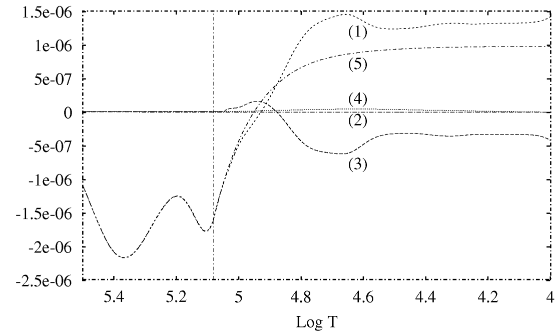

Figure 2. Different physical components of the work integral obtained with our TDC treatment for the mode $\ell = 1$, g_{50}. (1) is $W_{\rm f_R}$, (2) is $W_{\rm f_{Rh}}$, (3) is $W_{\rm f_c}$, (4) is $W_{\rm f_{ch}}$ and (5) is $W_{\rm f_R} + W_{\rm f_{Rh}} + W_{\rm f_c} + W_{\rm f_{ch}}$. Model with $M = 1.6 M_\odot$, $T_{\rm eff} = 6935$ K and $\alpha = 2$.

7. APPLICATION TO δ SCT STARS

δ Sct are a well known class of A-F type near main sequence variable stars whose periods (0.5 to 6 hours) correspond to low order p and g-modes. As shown in Dupret et al. (2004a, 2004b) and Grigahcène et al. (2004), our time-dependent convection treatment (TDC) succeeds in explainining the stabilization of the δ Sct radial and non-radial modes at the red edge of the instability strip. In Dupret et al. (these proceedings), we present theoretical instability strips, amplitude ratios and phase-lags obtained with our TDC treatment for these stars.

In Fig. 1, we give the work integral obtained with our TDC treatment including $\delta \vec{F_c}$, δp_t and $\delta \epsilon_2$. We give also the contribution of each of these 3 terms on the total work, according to Eqs. (28), (30) and (31). We see that $W_{\rm f_R}$ (curve 1) has a driving effect at the CE base. It is due to a flux blocking mechanism. However, for δ Sct stars, $W_{\rm f_c}$ (curve 2 of Fig. 1) is significant in all CE and compensates the driving effect of $W_{\rm f_R}$ (as shown in curve 3 $= W_{\rm f_R} + W_{\rm f_c}$), so that the stabilization of the modes at the red edge of the instability strip is obtained with TDC models. $W_{\rm p_t}$ (difference between curve 4 and curve 3) has a driving effect in this model. But it is nearly compensated by W_{ϵ_2} (difference between curve 5 and curve 3) which has a damping effect on the oscillations. Therefore $W_{\rm f_R} + W_{\rm f_c}$ (3) and W (6) are not very different.

8. APPLICATION TO γ DOR STARS

γ Dor stars are a recently discovered class of F-type main sequence variable stars whose long periods (between 0.4 and 3 days) correspond to pulsations in non-radial high order gravity modes. As shown in Dupret et al. (2004a, 2004b) and Grigahcène et al. (2004), our TDC treatment succeeds in explainining the driving of the γ Dor high order gravity modes. In Dupret et al. (these proceedings), we present theoretical instability strips, amplitude ratios and phase-lags obtained with our TDC treatment for these stars.

Flux blocking mechanism has been proposed by Guzik et al. (2000) as the driving mechanism of the γ Dor g-modes. However, they assumed frozen convection in their non-adiabatic models. In Fig. 2, we give the work integral obtained with our TDC treatment including $\delta \vec{F_c}$. In order to test the validity of the flux blocking mechanism, we give also the contribution of the radial and transversal components of the radiative and convective flux perturbations on the work integral. Fig. 2 shows that a significant driving occurs at the CE base for the mode $\ell = 1$, g_{50} (curve 5). The decomposition in radiative and convective flux contributions shows that the convective flux variations do not play a significant role at the CE base (curve 3). The main driving comes from the radiative flux variations (curve 1). This supports the flux blocking mechanism proposed by Guzik et al. (2000). We see also in this figure that the transversal components of radiative and convective flux variations (curves 2 and 4) do not play a significant role in the work integral.

9. APPLICATION TO THE SUN

It is now admitted that the p-modes of the Sun and solar-like stars are stochastically excited. For these stars, strong constraints on the TDC models are given by two types of observables. First, the linewidths in the power spectrum give a direct measurement of the damping rates of the modes, which can be confronted to the theoretical ones computed by a non-adiabatic pulsation code. Secondly, the observed amplitudes can be confronted to the theoretical ones: $V = |\vec{\xi}(r_s)| \sqrt{P/(2\eta I)}$ (Samadi et al. 2001). V is the velocity, $\vec{\xi}(r_s)$ is the displacement at the layer seen by the instrument, P is the power injected in the acoustic modes and can be computed by stochastic models (Samadi & Goupil 2001), $\eta = \Im(\sigma)$ is the angular damping rate and I is the inertia of the mode.

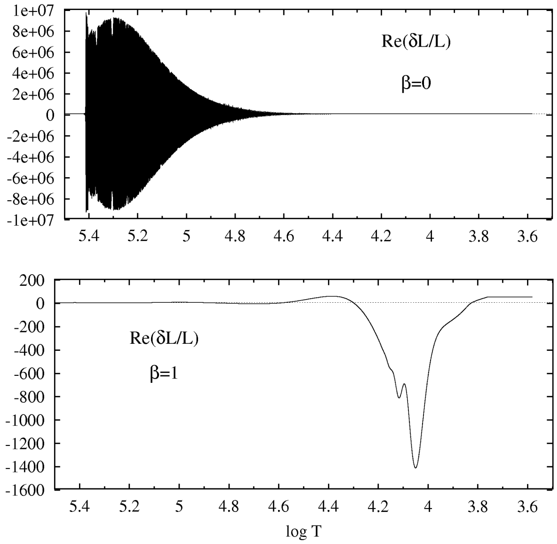

Figure 3. $\Re(\delta L/L)$ obtained for the radial p_{22} mode of a solar model with our TDC treatment. In the top panel $\beta = 0$ and in the bottom panel $\beta = 1$.

Solar-type stars have a large convective envelope, in which $\sigma\,\tau_c \gg 1$ for their typical p-modes frequencies. As we have shown in Sect. 5, this leads to spatial oscillations of the thermal eigenfunctions with a wave-length much shorter than the mixing-length. A striking illustration of these oscillations is given in the top panel of Fig. 3, where $\Re(\delta L/L)$ is given for the radial p_{22} mode of a typical solar model. We proposed in Sect. 5 to introduce a free complex parameter β in the energy closure equation in order to avoid these oscillations. In the bottom panel of Fig. 3, $\Re(\delta L/L)$ obtained with $\beta = 1$ is given. As can be seen, the short wave-length spatial oscillations disappear completely in this case.

In Fig. 4, the theoretical damping ($\Im(\sigma)/(2\pi)$) (radial modes) obtained with our TDC treatment including $\delta\vec{F_c}$, δp_t and $\delta\epsilon_2$ and assuming $\beta = -2.5{+}0.5i$ are confronted to the observations by BiSON (Chaplin et al. 2002, radial modes) and by GOLF (Baudin et al. 2004, $\ell = 1$ modes). We see that our TDC models succeed to reproduce the plateau around 3000 μHz.

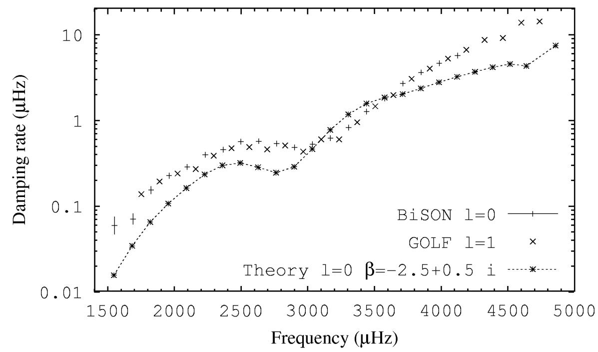

Figure 4. Half linewidths (μHz) observed by BiSON $(+)$ and GOLF $(\times)$ and damping rates $(\Im(\sigma)/(2\pi))$ obtained with our TDC models, assuming $\beta = -2.5 + 0.5\,i$.

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