Strömberg's method in deriving the luminosity law has the advantage of giving everything that can be obtained from the proper motions and radial velocities. If, however, it is applied without proper weighing of the observed data there is a danger of obtaining details which have no inherent significance.

Columbia University:
1931 *November.*

ON THE BEHAVIOUR OF THE BANDS OF *CH* AND *CN* IN THE SPECTRUM OF δ CEPHEI.

*P. Swings.**

(Communicated by Dr. Otto Struve.)

1. Quantitative spectrophotometric data are almost entirely lacking in the case of the Cepheid variables, in spite of the fact that such information would be invaluable for a discussion of the pulsation hypothesis and of the conditions of temperature, pressure and ionisation as functions of phase in the light-curve. Although several spectrophotometric investigations have appeared within the last few years, most of these are of a more or less qualitative character, and some of them are based upon spectrograms taken with very small dispersion and with objective prisms. Lack of resolving power is especially dangerous in the case of molecular bands. In a recent note, P. Swings and O. Struve † have shown that it is necessary carefully to avoid blends with atomic lines if one wishes to study the bands of *CH* and *CN* in the spectra of stars of types F and G. In the same note the conclusion was reached that, contrary to the general idea, the bands of *CH* and *CN* disappear at very nearly the same place in the spectral sequence, viz. spectral class F8. The astronomical literature contains numerous errors which have been caused by blends of molecular and atomic lines; for example, the *CH* band was believed by some investigators to persist as far as F0 or even earlier. We shall enumerate some of these errors in the last section of this note.

On the other hand, many investigations have been made with the use of non-standardised plates, in which case the microphotometric deflections can give qualitative results only, and even these may be vitiated if the measured photographic blackenings do not fall in a good region of the Hurter and Driffield curve of the emulsion.

The best method of obtaining definite and useful results for the spectra of Cepheids consists in using a slit spectrograph of sufficient dispersion and in taking spectra on well-standardised plates of fine grain.

2. We have made a careful investigation of the bands of *CH* and *CN* in the spectrum of 27 δ Cephei. We used slow Eastman process plates in connection with the Bruce spectrograph of the Yerkes Observatory, the

* Advanced Fellow of the C.R.B. Educational Foundation.
† P. Swings and O. Struve, *Physical Review*, 1932 January.

dispersion being 25 A. per mm. On each plate we impressed a photometric scale of intensity marks. For the examination of the bands we used the regions indicated in the above-mentioned paper of P. Swings and O. Struve. The spectrograms were analysed with the registering microphotometer of the Yerkes Observatory, according to the method adopted by C. T. Elvey and O. Struve.

TABLE I

Plate	Date U.T. 1931 d h m	Phase	Observer	Sp. Type
1 R 9931	Oct. 22 0 12	0·09	σ	F6–F7
9933	22 6 00	0·13	σ, *S*	F7
9941	Nov. 2 3 48	0·16	σ, *O*, *S*	F7
9910	Oct. 18 0 54	0·35	σ, *Sw*	G3
9912	18 5 15	0·38	σ, *Sw*, *S*	G3
9917	19 1 08	0·53	*M*, *S*	G5
9945	Nov. 4 5 34	0·55	*Sw*, *S*	G5
9919	Oct. 19 7 01	0·58	σ, *M*, *S*	G5
9903	9 5 21	0·71	σ, *M*, *S*	G6
9954	Nov. 16 1 40	0·75	σ, *Sw*, *S*	G6
9955	16 3 17	0·76	σ, *Sw*, *S*	G6
9921	Oct. 21 0 14	0·90	*M*, *Sw*	F8–F9
9950	Nov. 6 2 51	0·90	*M*, *S*	F8–F9
9909	Oct. 10 4 27	0·90	*Sw*, *S*	F8–F9
9922	21 1 40	0·91	*M*, *Sw*, *S*	F8
9923	21 2 53	0·92	*Sw*, *S*	F8
9924	21 3 41	0·92	*Sw*, *S*	F8
9925	21 4 28	0·93	*Sw*, *S*	F7–F8
9926	21 5 24	0·94	*Sw*, *S*	F7–F8
9927	21 6 42	0·95	σ, *Sw*, *S*	F7
9928	21 8 10	0·96	σ, *S*	F7
9929	21 9 41	0·97	σ, *S*	F6
9937	Nov. 1 2 54	0·97	σ, *O*, *S*	F6
9938	1 4 48	0·98	σ, *O*, *S*	F4–F5

The names of the observers are indicated as follows: σ=O. Struve; *O*=K. Ogrodnikoff; *M*=W. W. Morgan; *Sw*=P. Swings; *S*=F. R. Sullivan.

Table I contains a list of the plates examined, giving the serial number in the Yerkes collection, the date U.T., and the spectral type as deduced from the diagram of variation of spectral type of δ Cephei by H. Shapley * after a correction of 0.4 spectral class (following Miss Payne †) had been applied.

3. In fig. 1 the full-drawn line shows the light-curve of δ Cephei as given by M. Luizet.‡ We have indicated on this curve the points corresponding to maximum, minimum and mean volumes by using the curve of radial velocities and also the pulsation diagram given by W. Baade. §

The dotted line shows the light-curve as it would be if the star retained

* H. Shapley, *Astrophysical Journal*, **44**, 290, 1916.

† C. H. Payne, *The Stars of High Luminosity*, p. 309.

‡ M. Luizet, *Annales de l'Université de Lyon*, Nouvelle série, fasc. 33, 1912.

§ W. Baade, *Astronomische Nachrichten*, **228**, 5468, p. 359, 1926.

always the same (mean) volume ; this curve must cross the full line in the points corresponding to mean volume. The corrections (without considering the sign) reach maximum when the star is of maximum or minimum volume ; the actual values of these corrections were determined by means of the formula given by P. ten Bruggencate * ; they are, of course, only approximate.

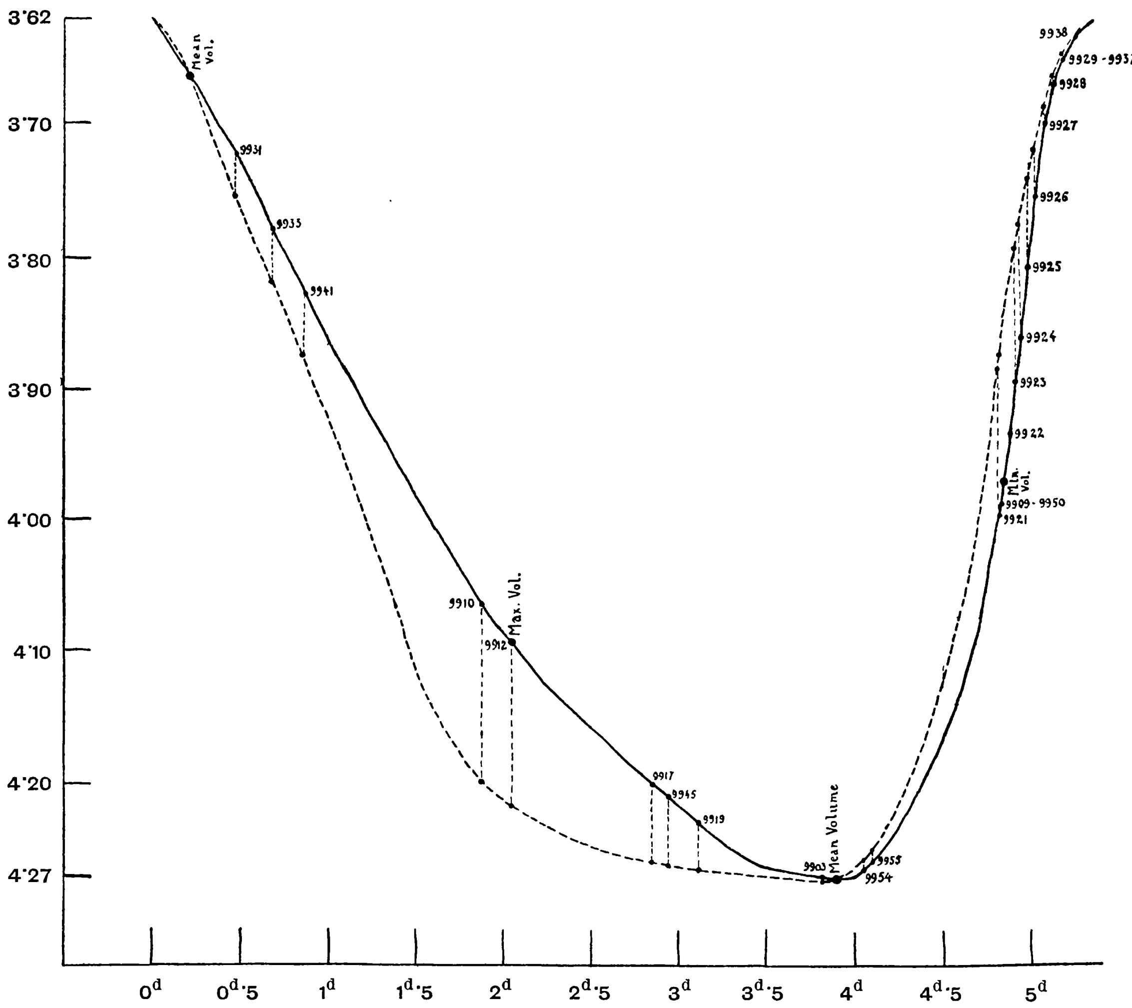

FIG. 1.—*Light-curve of δ Cephei*

4. Examination of the plates revealed immediately that the *CH* and *CN* bands undergo quite similar variations. Both are absent near maximum light ; they appear at about phase 0·16, or $0^{d}\cdot85$ after the maximum of light (plate 9941) ; their intensities, extremely faint on plate 9941, are gradually increasing and reach their maximum near phases 0·70–0·75 (plates 9903, 9954, 9955 ; light minimum). The intensities thereafter decrease, and the bands disappear before phase 0·90.

It is of especial interest to note that plate 9941, on the descending branch of the curve, shows a very faint *CH* absorption band, while plates 9924 and 9925, taken on the ascending branch when the star was of the same apparent

* P. ten Bruggencate, *Die veränderlichen Sterne*, pp. 34, 37.

magnitude, show no trace of *CH*. This seems to indicate that two states of the same apparent brightness do not necessarily have equal *CH* and *CN* absorptions; it agrees with the pulsation hypothesis, according to which each point on the descending curve would correspond to a lower temperature than the corresponding point of the same brightness on the ascending curve.

5. On four of the best plates we have measured the absorption of the *CH* molecules at λ 4311A (this being a region free from blends, as shown in the paper by P. Swings and O. Struve). The measured values are given in Table II.

TABLE II

Plate	Phase	Absorption Per cent.	N *	Sp. Class †	Temp. ‡
1 R 9912	0·38	12·5	39	G3	5100
9945	0·547	17	72	G5	4700
9903	0·71	20	100	G6	4600
9954	0·754	20	100	G6	4600

For comparison we have also measured the *CH* absorption for the same wave-length in the solar spectrum; we found 41 per cent. This indicates an "absolute magnitude effect" for the molecular bands, δ Cephei and the Sun being respectively a super-giant and a dwarf.

6. Knowing these absorptions, the question arises of determining for the different phases the ratios of the numbers of molecules of *CH* in the stellar atmosphere.§

Let $N_{j''}$ be the number of *CH* molecules per cubic centimetre, which are in the energy state j'' (j'' thus represents the ensemble of the electronic, vibrational and rotational energies), and let $B_{j''j'}$ be the probability of transition of the molecule from the j'' level to a j' level of higher energy. If λ_0 is the wave-length of the centre of the $j''j'$ absorption line, the absorption coefficient σ is a function ϕ of the difference $\lambda - \lambda_0$ between the centre of the line and the examined point; σ is also proportional to the product $N_{j''}B_{j''j'}$. Hence

$$\sigma(\lambda) = C\phi(\lambda - \lambda_0)N_{j''}B_{j''j'},$$

C being a constant depending upon λ_0 and the molecular properties. The residual intensity for each wave-length λ will be expressed by

$$I(\lambda) = I^*(\lambda)\,\psi(\sigma),$$

$I^*(\lambda)$ being the intensity for wave-length λ corresponding to the continuous spectrum and $\psi(\sigma)$ a function of σ. This function differs according to

* See end of § 6.

† H. Shapley, *loc. cit.*

‡ Russell, Dugan and Stewart, *Astronomy*, **2**, p. 753. The temperatures are expressed in absolute degrees.

§ This procedure is similar to that followed by O. Struve and C. T. Elvey for the comparison of the relative intensities of the components of a Si III triplet in stellar and laboratory spectra, *Astrophysical Journal*, **72**, 267, 1930.

whether we consider the case of pure absorption without any re-emission of the same wave-length or the case of a diffusing atmosphere. But, as was shown by Unsöld,* the difference between these two ψ functions is not great. We shall adopt here the formula

$$I(\lambda) = I^*(\lambda)\, e^{-\int \sigma dl},$$

l being the thickness of the layer ; this formula may be written

$$I(\lambda) = I^*(\lambda)\, e^{-\sigma l}$$

if σ does not depend upon the position in the reversing layer. Unsöld † has shown also that the expression for σ in the case of pure radiation damping is

$$\sigma = K \frac{N_{j''} B_{j''j'}}{(\lambda - \lambda_0)^2},$$

K being a constant.

The whole energy absorbed in the line is thus

$$E = I_0 \int_{-\infty}^{+\infty} \left\{ 1 - e^{-\frac{K N_{j''} B_{j''j'} l}{(\lambda - \lambda_0)^2}} \right\} d\lambda = K_{j''j'} \sqrt{N_{j''} B_{j''j} l},$$

I_0 being the intensity of the incident radiation, and $K_{j''j'}$ another constant.

We must pay attention to the fact that $N_{j''}$ is a certain proportion of the total number N of molecules CH in the atmosphere ; we shall write

$$N_{j''} = a_{j''} N.$$

Let us consider two different temperatures included between our limiting temperatures for the determinations of absorption, *i.e.* between 4600° and 5100° C. The ratio $\frac{E_1}{E_0}$ of the absorbed energies is

$$\frac{E_1}{E_2} = \sqrt{\frac{a_{1j''}}{a_{2j''}}} \sqrt{\frac{B_{1j''j'}}{B_{2j''j'}}} \sqrt{\frac{l_1}{l_2}} \sqrt{\frac{N_1}{N_2}}. \qquad (1)$$

We shall examine the importance of each of these factors.

(*a*) $\sqrt{\frac{a_{1j''}}{a_{2j''}}}$. An increase of temperature changes the value of the rotational quantum number m_0'' of the greatest number of molecules ; this change gives rise to a displacement of the maximum of intensity in the rotational lines of a partial band.

The Boltzmann formula gives

$$\mathcal{J} \frac{w_m^2}{2} = \tfrac{1}{2} kT,$$

where $\mathcal{J}$ is the moment of inertia of the molecule (with respect to the axis of

* A. Unsöld, *Zeitschrift für Physik*, **59**, 363, 1930 ; see also O. Struve and C. T. Elvey, *loc. cit.*

† *Ibid.*, **44**, 793, 1927 ; **46**, 768, 1928. See also R. Ladenburg, *ibid.*, **4**, 454, 1921.

rotation), w_m the frequency of rotation and k the Boltzmann constant. As a first approximation we shall use the old quantum theory, which gives us

$$J w_m = \frac{m_0'' h}{2\pi}.$$

Thus we get

$$m_0'' = \frac{2\pi}{h}\sqrt{kTJ}.$$

For two different temperatures, T_1 and T_2, we find

$$\frac{m_0''(T_1)}{m_0''(T_2)} = \sqrt{\frac{T_1}{T_2}}.$$

In our problem, the maximum value of this ratio is

$$\frac{m_0''(5100)}{m_0''(4600)} = 1{\cdot}05.$$

The influence of this factor is not worth considering, especially if the rotational lines which we consider are not too far from the band-head.

(*b*) $\sqrt{\frac{B_{1j''j'}}{B_{2j''j'}}}$. This ratio is almost certainly very near unity.

(*c*) $\sqrt{\frac{l_1}{l_2}}$. It is almost certain that the thickness of the absorbing layer does not change appreciably.*

(*d*) $\sqrt{\frac{N_1}{N_2}}$. This is certainly the most important factor in equation (1). We shall therefore assume that the intensity of absorption is proportional to $\sqrt{N}$.

Our reasoning implies that there is only one absorption line at the point of the spectrum where the absorption is measured. In reality this point is a blend of several *CH* lines; but this does not change the result. The absorbed energy is

$$E = \Sigma K_{j''j'}\sqrt{N_{j''}B_{j''j}\,l} = \sqrt{N}(\Sigma K_{j''j'}\sqrt{a_{j''}B_{j''j'}\,l}),$$

the Σ being applied to the different $(j''j')$ lines; $K_{j''j'}$ has very nearly the same value for all the blended lines. On the other hand, we have shown above that l, $a_{j''}$ and $B_{j''j'}$ do not change much when T varies between 5100 and 4600. Thus we may still consider E proportional to $\sqrt{N}$.

Accordingly we find for the relative numbers of molecules the values given in column 4 of Table II.

7. We shall now show that these values of the numbers of molecules

* P. ten Bruggencate, *loc. cit.*, p. 34.

are in close agreement with the values computed by means of the equation of dissociation-equilibrium of *CH*. This equation is *

$$\log K_p = -\frac{110,000}{4 \cdot 541 T} + 1 \cdot 5 \log T + \log\left(1 - e^{-\frac{4000}{T}}\right) - 0 \cdot 40,$$

K_p being the coefficient of equilibrium $= \frac{p_C p_H}{p_{CH}}$, where p_C and p_H are the partial pressures of *C* and *H* (assumed to be constant); *T* is the absolute temperature.

By applying this equation to various temperatures we find:

Table III

Sp. Class	T	$\log K_p$	N
G6	4600	9·6267	100
G5	4700	9·7461	76
	4800	9·8606	59
	4900	9·9708	45
G4	5000	0·0764	36
G4	5100	0·1781	28
	5200	0·2758	23
	5300	0·3700	18
	5400	0·4609	15

The measured numbers of molecules are 100, 72, 39, corresponding to the following temperatures: 4600°, 4720° and 4950° C., *i.e.* to spectral types G6, G5 and G3.

The agreement between theory and observation seems perfect.

8. We shall close by pointing out a few uncertain conclusions found in the astronomical literature concerning the *CH* and *CN* bands in Cepheids.

(*a*) In a recent paper † on three Cepheid variables (η Aquilæ, RT Aurigæ and α Ursæ Minoris), Dr. A. V. Douglas points out that the variation of the *CN* band is in phase with the periodic variation of many lines of ionised elements and of the Balmer lines, and that there is a great difference of phase between the variation of the *CH* and *CN* bands. She suggests, in order to explain this phenomenon, that the molecule *CN* is in the ionised state.

From the identical behaviour of the *CH* and *CN* bands in δ Cephei it seems extremely probable that this is not the case. Dr Douglas's plates were not standardised, so that her quantitative determinations are rather doubtful; furthermore, it is certain that not enough caution was used to eliminate blends.

(*b*) For the same reason we feel little confidence in the values of the intensity of the *CH* band and in the curve of variation of the ratio $\frac{\text{int. of G band}}{\text{int. of } Cr\ 4289}$ given by F. Henroteau and A. V. Douglas.‡

* P. Swings and O. Struve, *loc. cit.*

† A. V. Douglas, *M.N.*, **90**, 798, 1930.

‡ F. Henroteau and A. V. Douglas, *Publications of the Dominion Observatory, Ottawa*, **9**, No. 7, Table III, and fig. 9, 1929.

(*c*) It is also almost certain that the results obtained by G. Tiercy * for SU Cassiopeiæ are vitiated by blends in the case of the G band. † On account of the small dispersion used by him, it is certain that what he calls the " G band " is mostly made up of blends of atomic lines. This explains his observation that in a spectrum of class F5 the G band is more conspicuous than in the preceding spectra of classes F1–F2 and F1. We have shown formerly ‡ that the G band disappears at F8.

(*d*) Recent papers of P. ten Bruggencate § on the variations of the spectra of several Cepheids also include a discussion of the behaviour of the *CH* band. Some of his results are probably caused by the fact that his microphotometric deflections are not essentially due to the *CH* molecules.

(*e*) Miss C. H. Payne ‖ points out an increase in intensity of the *CH* band with decrease of pressure at constant temperature. This seems difficult to explain. It may perhaps be due to blends with atomic lines. Unfortunately our observations do not settle this question.

It is a great pleasure to record my gratitude to Professor O. Struve for helpful advice and criticism at all times.

Yerkes Observatory :
1931 *November* 25.

NOTES ON THE CONSTITUTION OF THE STARS : ADDENDUM.

H. N. Russell.

I am greatly indebted to Professor H. Vogt for calling my attention to the fact that he published in 1926 (*A.N.*, **226,** 301) the proposition that a series of stars of fixed composition must exhibit exact relations connecting mass, luminosity, etc. ; and certain other general theorems. Had I known of this, I would have eliminated in my recent paper (*M.N.*, **91,** 951, 1931) all reference to my publication of these propositions a year later in a text-book. The humorous form of this reference expressed my conviction that Professor Vogt was quite unaware of my publication, and that, in the circumstances, he was under no obligation to know of it.

* G. Tiercy, *Publicazioni Oss. Arcetri*, fasc. 44, 7, 1927.

† This is, of course, only a minor detail in Tiercy's papers, which contain numerous results of very great interest.

‡ P. Swings and O. Struve, *loc. cit.*

§ P. ten Bruggencate, *Annalen v. d. Bosscha Sterrenwacht, Lembang*, **5,** 1st part, 1931 ; *Die veränderlichen Sterne, Ergebnisse der exakten Naturwissenschaften*, **10,** 33–44, 1931.

‖ C. H. Payne, *The Stars of High Luminosity*, p. 214.