# Teaching versus research: A multi-tasking approach to multi-department universities 

Axel Gautier*, Xavier Wauthy<br>CEREC FUSL and CORE, UCL, Bd du jardin botanique 43, 1000 Brussels, Belgium<br>Received 21 January 2005; accepted 11 February 2006<br>Available online 18 April 2006


#### Abstract

We study the possible implications of incentive schemes as a tool to promote efficiency in the management of universities. In this paper, we show that by designing internal financial rules which create yardstick competition for research funds, a multi-department university may induce better teaching quality and research, as compared to the performance of independent departments.


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## 1. Introduction

In European countries, higher education institutions are mostly funded with public money. While universities are essentially organized along nonmarket-based structure, the recent evolution of recruitment on the one hand and of public budgets on the other hand put pressures on the management of those universities. Nowadays, much emphasis is placed on the efficient use of the universities' resources and university management is making increasing use of techniques inherited from the private sector. For instance, Ball and Butler (2004) show that the UK Research Assessment Exercise induced a marked tendency within universities to adopt business-like methods to improve their quality

[^0]ratings and reporting strategies. Quite remarkably, a flourishing literature is emerging in which methodologies inherited from management techniques are developed in order to ensure an efficient assignment of budgets within and across universities. The splitting of funds between teaching and research is particularly at stake here (see for instance Abbott and Doucouliagos, 2003; Caballero et al., 2001, 2004; Fandel and Gal, 2001; Parkan, 2004). These researches focus on the design of performance measures and their use in the funding allocation process. However, very little attention is paid on the incentive mechanisms that are induced by such instruments. In the present paper, we precisely address that point. Moreover, in particular, we show that when properly designed, incentive schemes based on performance indicators may take advantage of the multidepartment structure of universities in order to enhance both teaching quality and research quality within each department.

Because universities count teaching and research as part of their core social goals, evaluating the performance of any university system calls for answering the following questions. Do universities manage to combine high quality teaching and high quality research? And, if they do, how? Obviously, one would like to see any university to excel in both dimensions but beyond wishful thinkings, very little is known about how to effectively realize this ideal. On the one hand, evidences from UK (see Shattock, 2002) suggest that universities that perform very well in research also perform well in teaching. On the other hand, it is hard to see what happens exactly in those universities which are not in the top 10. The situation is even more opaque in many continental systems where university assessment is in its very infancy. As argued by Neary et al. (2003), it is widely accepted that "...poor governance structures and inappropriate incentives...still characterize so many European Universities" (p. 1240). While recent empirical studies tend to become more and more sophisticated when evaluating the quality of research (see for instance the December 2003 issue of the Journal of the European Economic Association), very little effort has been devoted to the development of formal models of university governance, in particular regarding the organization of teaching and research within universities.

Fortunately enough, combining high quality teaching and high quality research is actually desirable for the universities themselves. This is in particular true in a system where universities are mostly financed on a per student basis and where students' choice depend on (1) teaching quality and (2) university's prestige (which is related to research quality). However, combining high quality teaching and high quality research is often viewed as a challenge or a source of conflicts within universities taken as an aggregate.

This is partly due to the individual incentives faced by academics. For them, research and teaching activities are most often substitutes. Moreover, in many education systems, academics benefit from a large discretion in the allocation of their working time. Tavernier and Wilkin (2001) show that academics do use this discretion to a large extent, resulting in very different occupation profiles for academics. Accordingly the actual splitting of their time among the various tasks they are assigned to is largely a matter of taste and incentives. There are a priori many ways by which a university could reconcile the individual conflict between teaching and research faced by its academics and the vital interest of performing well in both dimensions in the aggregate. Promoting specialization, with some academics being teaching professors and others full time researchers is a possible solution. Designing incentive schemes that value both aspects simultaneously is another one. However, there are also constraints limiting what can actually be
implemented. Suppose for instance that at the individual level high quality research and high quality teaching are complements, i.e. in order to be a good teacher, one must also be a good researcher. Then the "specialization" route must be abandoned. Suppose instead that research and teaching quality are not equally easy to assess. Then, the actual implementation of targeted schemes is hampered. In the context of an emergent market for academics, a similar argument could be made whenever research and teaching abilities are not equally easy to signal to alternative employers.

As a matter of fact, the tensions between teaching and research have not been widely studied in the economic literature. A few exceptions are worth being pointed out. Del Rey (2001) models competition between universities who decide on the allocation of funds between teaching and research activities. In her model, teaching achievements and research records enter the university objective function and funding is positively related to the number of students. She studies the balance between research and teaching efforts as a function of the funding rules. However, a key feature of her analysis is that academics and the university authority share the same objective. De Fraja and Iossa (2002) point out that increased students' mobility favors the emergence of "elite" institutions, i.e. a limited number of high research records universities co-existing with other universities focusing on teaching activities. Beath et al. (2003) focus on the tensions between pure and applied research under binding budget constraints. However, the teaching side of the academics' job is not considered in their paper.

In this paper, we start from the agency relationship that links academics to their authority (rectorate, deans,...) and we focus on the links between the multi-tasking nature of a professor-university relationship and the multi-unit nature of these universities. The paper is organized around two simple ideas. First, universities are active in multiple fields: science, economics, law, ... . Each field is organized within one department, with more or less autonomy. To a large extent, research and teaching are discipline-specific and the decisions made regarding some discipline are largely independent from those taken in other disciplines. ${ }^{1}$ In addition, universities are headed by a central authority which in particular has the final decision on the allocation of funds. Most often, the budget is centralized and the resource constraint applies at the university level. It means that the allocation of resources is done at the university level too. Thus, universities rely on an internal financing system which is very similar to the internal capital market of a conglomerate firm (see Coupé, 2001). Such practices are widely documented and to a certain extent can be viewed as socially desirable. University completeness is sometimes believed to be part of a university's mission (and this argument might be sufficient to justify a-possibly inefficient-form of redistribution). However, the ensuing solidarity between "cash-cow" departments and smaller units does not go without tensions.

The second building block of our analysis is the relationship between academics and authorities. We view it as a multi-task agency problem. The university authority wants to provide incentives on the two dimensions of teaching and research and we take it as an assumption that academics must perform teaching and research. While teaching and research require some effort, we assume that research is more valuable to the academic than teaching. This might simply reflect the tastes of the academic but there are more fundamental reasons for that. In particular, the emergence of a market for academics induces more severe career concerns. As a matter of fact, while the quality of individual

[^1]research output is reasonably easily assessed, teaching quality is most often evaluated at the level of a whole program, rather than at an individual level. Therefore, an academic is likely to put more effort on research than teaching because research outputs are more easily appropriable than teaching efforts. ${ }^{2}$ All in all, inducing effort on the teaching task might be more difficult to achieve than on the research task.

Consider now the issue of funding. If a university's funding mostly depends on the number of students and if these students are (at least partially) responsive to the reputation of a university program, it is especially crucial to ensure high teaching quality. Because more funds allow for a better research environment, good teaching performance makes high research records less costly. Thus, even if academics dislike teaching, they may exert significant efforts on improving teaching quality because, by attracting students, they will obtain funds that will make high research records less costly. Think of an extreme case where each academic is totally independent: She teaches the students who choose to attend her courses and finances her research with their enrolment fees. Clearly, whatever strong her distaste for teaching might be, the academic has to teach if she wants her research to be funded. The funding of research creates a complementarity between the two tasks.

How does the multi-department nature of a university affect the previous argument? Obviously, the problem comes from the possibility of reallocating funds dedicated to research between the different departments. The presence of such an internal market for research funds makes it more costly to induce teaching effort. First, because of an insurance effect (even if one does not raise any fund it will benefit from research funding). Second, because the marginal value of effort is smaller (because only part of the funds raised through teaching effort will be appropriated ex post). In a multi-department university (MDU), teaching efforts can be viewed as private resources spent at contributing to the constitution of a common resource. Self-interested academics are therefore very likely to free-ride on such efforts. On the other hand, because it counts several departments, the university is able to trigger yardstick competition between academics for the allocation of research funds. It is for instance the case when the authority decides to allocate funds to the most valuable research projects. When such a yardstick competition is at work, high quality research by an academic is likely to induce high effort by the others. Because it induces both free-riding and yardstick competition, a multi-unit organization for the university might a priori be thought of as a "good" with respect to research quality and a "bad" with respect to teaching efforts.

However, we show hereafter that the conflict between research and teaching can be resolved within the multi-unit institution. The multi-department structure preserves the complementarity between teaching efforts and research efforts which counteracts the substitution effect possibly present in the academics' utility function. Building on this, we show that the conglomerate nature of universities may actually be instrumental in promoting the quality of the teaching-research bundle as compared to a collection of single departments. However there is a limit to the virtue of the conglomerate structure: When departments are too numerous, part of the efficiency associated with redistribution must be sacrificed by leaving more budgetary autonomy to the departments.

[^2]Our work is related to the literature on multi-task agency problems. Two main issues are dealt with in this literature. First, it derives contracting mechanisms that can align the interests of the principal and the agent, particularly when the tasks are complements for the principal and substitutes for the agent (Sinclair-Desgagné, 1999; MacDonald and Marx, 2001). Sinclair-Desgagné (1999) for example proposes an audit mechanism where the performance on the task which is harder to assess (teaching) is audited whenever the agent shows a high performance on the other task (research). The payment to the agent is higher when an audit occurs unless the audit yields a bad assessment of performance on teaching, where in this case, the agent incurs a monetary penalty. Clearly with such a mechanism, performing well on research is useless if the academic does not perform well on teaching too. Hence, the agent views the two tasks as complements and this may counterbalance the fact that the two tasks are substitutes in his cost function. In this article, we show that the multi-department structure of a university can achieve the same goal: It is a tool that can make the efforts complements rather than substitutes in the academics' utility function. However, in our framework, we do not use an explicit contracting process. A second issue of interest in multi-tasking models is the problem of job design (Holmström and Milgrom, 1991), that is, given their characteristics, how the various tasks should be clustered together and delegated to the agent(s). We do not deal with this problem within universities since we assume that the academics must perform both teaching and research.

Notice that, by focusing on the tensions between teaching and research, we abstract from other important problems. For instance, we neglect the implications of the now standard distinction between pure and applied research (see Jensen and Thursby, 2001) on performance assessment. We also overlook the third and fourth basic tasks an academic is asked to perform, namely service to the society and administrative duties. (Tavernier and Wilkin (2001) show that these activities may indeed crowd out a significant share of an academic's working time.)

The paper is organized as follows. First, we develop our stylized model. This is done in Section 2. Sections 3 and 4 characterize optimal contracts and their implications for the relation between teaching and research. Comparative static results are also dealt with in these sections. Section 5 discusses the limitations of our analysis as well as possible extensions. Section 6 concludes.

## 2. The model

We consider a university with $N \geqslant 2$ departments. Each department is personified by a unique professor, the so-called "academic". There are $N+1$ players: $N$ academics and one university dean. Thus, we only consider a two layer hierarchy: University dean and professors. ${ }^{3}$

Academics allocate their time between two activities: Teaching and Research. The vector $a=(t, r)$ is the vector of actions where $t$ refers to teaching effort and $r$ refers to research effort. Performing a level of action $a$ costs the professor $C(a)$. We assume that the cost

[^3]function is increasing and convex in both arguments: For $l=t, r, \partial C(a) / \partial l>0$, $\partial^{2} C(a) / \partial l^{2}>0$.

The sign of $\partial^{2} C(a) / \partial t \partial r$ is obviously critical for the analysis to follow. On the one hand, one could argue that a negative sign for the cross derivative makes sense for relatively low levels of research output. However, when time constraint becomes really binding, the two efforts are likely to become substitutes in the academic's cost function. Casual observation suggests indeed that those academics who are really active in research are prompt to call for teaching load reductions from their department. ${ }^{4}$ Notice that Dundar and Lewis (1995) actually find empirical support for a negative sign. They identify economies of scope between teaching and research at the department level. However their results hold only when graduate teaching is considered. Indeed, graduate students might contribute to an academic's output as research assistants. A similar result is not likely to hold at undergraduate teaching level. On the other hand, assuming that the cross derivative is positive better fits the received literature on multi-tasking. It is indeed under the assumption that efforts are substitutes that aligning the academics' incentives in the two tasks with those of the institution is most problematic. In view of these mixed arguments, we shall not impose a priori any restriction on the sign of $\partial^{2} C(a) / \partial t \partial r$. However, large negative synergies between teaching and research would obviously lead to the specialization of academics in either teaching or research.

Each academic is endowed with a vector of "talent". This vector represents the professor's ability to do research and teaching. Talents are denoted by a two-dimensional vector $\eta_{i}=\left(\tau_{i}, \rho_{i}\right)$, where the first element represents the professor's teaching talent and the second, the professor's research talent.

A variable $\theta_{i}$ identifies a proxy for the quality of the research projects undertaken by professor $i$. It depends on the effort in research activity and on the professor's research talent:

$$
\theta_{i}=f\left(r_{i}, \rho_{i}\right)
$$

with $\partial f / \partial r_{i}>0, \partial f / \partial \rho_{i}>0$. Research effort and talent are not perfect substitutes: $\partial^{2} f / \partial r_{i} \partial \rho_{i}>0$. A research project of quality $\theta_{i}$ leads to a research output only if it is combined with financial resources.

Students' choice is not explicitly modeled. However, we assume that $n_{i}$, the number of students in discipline $i$, depends on teaching quality in field $i$. Quality itself depends on the combination of teaching effort and teaching talent of professor $i$. Hence we assume

$$
n_{i}=g\left(t_{i}, \tau_{i}\right)
$$

with $\partial g / \partial t_{i}>0, \partial g / \partial \tau_{i}>0$. We assume that talent and effort in teaching activities are not perfect substitutes: $\partial^{2} g / \partial t_{i} \partial \tau_{i}>0$. Teaching efforts contribute to the constitution of the general budget of the university through enrolment fees.

Academics receive a fixed wage $z$ from the university. For simplicity, this fixed pay is normalized to zero. In addition, there is a reward $w$ proportional to the research output. Denoting research output by the variable $R_{i}$, we assume the utility of professor $i$ is given by

$$
U_{i}=w R_{i}-C(a) .
$$

[^4]$w R_{i}$ could be interpreted either as a private benefit from research or as a future job opportunity, i.e. the professor's value on the academic market. In the first interpretation, $w R_{i}$ represents the private benefits an academic enjoys from his research achievements. Private benefits of research could be notoriety, job opportunities, consultancy contracts, tenure position, ... . Clearly, in all these examples, the benefit is tied to the academic's research output. What is specific in this model is the linear specification of the private benefit.

Alternatively, $w R_{i}$ can be interpreted as the academic's market value. Professorships exhibit nowadays high mobility and high turnovers ${ }^{5}$ with the consequence that there exists a true market for academics (see Siow, 1995 on the organization of the market for professors). The value of an academic on this market is largely influenced by his research performance. Hence, a high research output $R_{i}$ translates into better job opportunities and a larger pay. Under this interpretation, our model assumes that the market for academics values research at a per unit price of $w$. The professor's value $w R_{i}$ can be interpreted as his future reservation wage, either inside his institution or elsewhere. ${ }^{6}$

Notice that the teaching activity does not enter positively in the professor's utility function. This rather extreme assumption is meant to capture the idea that teaching is valuable inside a given university but it has little value outside. For instance, it might be difficult to signal to the job market high teaching quality. By contrast, research has a high visibility outside university, and can be used as a signal of quality (talent) on the market. Therefore, we assume there is a private benefit associated to the research output and no private benefit associated with teaching. Clearly, this assumption makes the worst case for teaching efforts.

Regarding Universities, we assume the following. The budget of the university is noted by $B$. The university receives a fixed transfer $F$ from the government and a tuition fee $s$ per student. In state owned systems, the tuition is partially paid by the government. $B$ is then equal to

$$
B=F+s \sum_{k=1}^{N} n_{k} .
$$

In the remaining of the paper, we consider the budget $B$ as the total amount of resources available for funding research projects. $B$ is thus the university's resources net of the academic's wages (normalized to zero) and all the other spending of a university (which could account for a large amount). This specification of the budget constraint implicitly assumes the existence of scale economies in the teaching activity. When the number of students increases, it is likely that the teaching staff increases as well. However, it is sufficient for our purpose that it increases less than proportionally, so that the residual budget available for research increases.

The research output of professor $i$ is denoted by $R_{i}$. It depends on (i) the project's quality $\theta_{i}$ and (ii) the budget $y_{i}$ allocated to research in department $i$. The research budget includes labs, research assistants, sabbatical year, .... The production function for research output

[^5]is the following:
$$
R_{i}=\theta_{i} v\left(y_{i}\right) \quad \forall i=1, \ldots, N
$$

The function $v($.$) is increasing and concave. The concavity of the v($.$) function implies that$ the allocation of the research budget to the departments will not be of the form "winner takes all". Specifically, we assume that $v(y)=y^{1-h}$ with $h \in(0,1)$. Note that to simplify the analysis, we consider that the production function is not department specific. However, this does not mean that all academics are identical with respect to the research activity. Heterogeneity could indeed be incorporated in the academic's research talent $\rho_{i}$. Talent could then be interpreted either as a specific academic talent or as a field specific talent (or both). The same is true for teaching.

This specification for the budget takes as constant other forms of funding which are to a large extent accessible to a university (typically institutional research fundings by federal or private agencies). Notice also that we assume that departments do not compete among themselves for students. In other words, we assume that students' preferences determine their field of studies while teaching efforts may attract them to the particular university we study. Thus, teaching effort affects the choice of the university, not the field chosen. This is why teaching efforts increase the university budget.

Within this framework, we may now specify the internal financing rules of the university and the timing of the events.

1. Academics simultaneously choose the actions $a_{i}$.
2. Students choose their university and the values of $n_{i}$ and $\theta_{i}$ are observed.
3. Given the qualities of the research projects and the total budget, the university allocates the research budget to academics.

The following assumption underlines the game structure: The university cannot commit ex ante (that is, before the professors choose the actions) to a particular sharing of the university budget. Thus, the budget will be allocated ex post (once $n_{i}$ and $\theta_{i}$ are realized). Under this allocation rule, whether the university knows academics' talents or not makes no difference. We assume that ex post, the university allocates funds to research projects in order to maximize the aggregate research output $\sum_{k=1}^{N} R_{k}$. Actually, ex ante commitment to a particular distribution rule would be highly demanding: The university should indeed be able to specify the allocation of resources given all possible realizations of $\theta_{k}$ and $n_{k}$, $k=1, \ldots, N$. Such a rule would be a mapping from the $N^{2}$-dimensional space of project quality and students' number to the $N$-dimensional space of investments. It is reasonable to assume that the costs of writing such an allocation rule are prohibitive. ${ }^{7}$ Moreover, ex post, the university would still have an incentive to re-negotiate such an arrangement to allocate its scare funds to the more valuable projects i.e. to maximize the aggregate research output given the budget. ${ }^{8}$

[^6]Lack of commitment implies that we do not need to specify the objective of the university beyond maximization of the aggregate research output given the budget. The allocation of resources by the university will then be similar to the winner-picking contest of Stein (1997). ${ }^{9}$ The analogies between our model of MDUs and models analyzing conglomerate-for profit-firms will be discussed later on. The specificity of our analysis is to integrate the multi-tasking nature of the incentive problem, which we view as inherent to the academic job. Be it for teaching or research, the quality of the output essentially results from the academic's effort and an academic should perform both tasks.

## 3. The trade-off between research and teaching efforts

### 3.1. Optimal efforts

At the last stage of the game, given the budget $B$ and the value of research projects $\left(\theta_{1}, \ldots, \theta_{i}, \ldots, \theta_{N}\right)$, the university allocates $B$ in order to maximize the research output

$$
\begin{align*}
& \max _{y_{1}, \ldots, y_{N}} \sum_{k=1}^{N} R_{k} \\
& \text { subject to : } \sum_{k=1}^{N} y_{k}=B \tag{P.1}
\end{align*}
$$

We may then specify the optimal allocation rule for the university budget.
Proposition 3.1. The optimal allocation of the budget is: For all $i \in N$

$$
y_{i}=\alpha_{i} B
$$

where $\alpha_{i}=\theta_{i}^{1 / h} / \sum_{k=1}^{N} \theta_{k}^{1 / h}$.
Proof. The first-order conditions for problem (P.1) are

$$
\theta_{i}(1-h) y_{i}^{-h}=\lambda .
$$

Solving for $y_{i}$ we obtain

$$
y_{i}=\frac{\lambda^{-1 / h}}{(1-h)^{-1 / h}} \theta_{i}^{1 / h}
$$

Summing over all departments we obtain

$$
\sum_{k=1}^{N} y_{k}=\frac{\lambda^{-1 / h}}{(1-h)^{-1 / h}} \sum_{k=1}^{N} \theta_{k}^{1 / h}
$$

Given that $\sum_{k=1}^{N} y_{k}=B$, we have

$$
\frac{B}{\sum_{k=1}^{N} \theta_{k}^{1 / h}}=\frac{\lambda^{-1 / h}}{(1-h)^{-1 / h}}
$$

Replacing $\lambda^{-1 / h} /(1-h)^{-1 / h}$ by $y_{i} / \theta_{i}^{1 / h}$, we have finally $y_{i}=\theta_{i}^{1 / h} / \sum_{k=1}^{N} \theta_{k}^{1 / h} B$.

[^7]Our assumptions on $v($.$) imply that the shares of the budget \alpha$ are independent of the size of the budget. From Proposition 3.1 the following comparative static results are immediate:

Corollary 3.1. The optimal allocation of the budget satisfies the following:

$$
\begin{align*}
& \frac{\partial \alpha_{i}}{\partial \theta_{i}}>0,  \tag{3.1}\\
& \frac{\partial \alpha_{i}}{\partial \theta_{j}}<0 . \tag{3.2}
\end{align*}
$$

In other words, the resource allocation process is based on the relative quality of research projects, an effect which is typical of a winner-picking contest. ${ }^{10}$ This means that in a MDU, the quality of project $i$ alone cannot explain the budget allocated to professor $i$. In a MDU, budget allocation depends on the quality of all projects. As we will see, this allocation scheme, which is specific to a MDU, creates yardstick competition between academics and as such, is an important part of the incentive package.

University behavior in the last stage is perfectly anticipated by academics. Using Proposition 3.1, we may now analyze the first stage of the game. Integrating the optimal budget allocation scheme, the professor $i$ 's utility function is

$$
U_{i}=w\left[\theta_{i} v\left(\alpha_{i} B\right)\right]-C\left(a_{i}\right)
$$

Let us denote:

- $C_{l_{i}}$ is the partial derivative of $C\left(a_{i}\right)$ with respect to $l_{i}, l=t, r$,
- $\alpha_{i}^{\prime}$ is the partial derivative of $\alpha_{i}$ with respect to $\theta_{i}$,
where indices $i$ refer to professor $i$. First-order conditions read as follows:

$$
\begin{align*}
C_{t_{i}} & =w\left[\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}\right] \frac{\partial B}{\partial t_{i}}  \tag{3.3}\\
C_{r_{i}} & =w\left[v\left(\alpha_{i} B\right)+\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}^{\prime} B\right] \frac{\partial \theta_{i}}{\partial r_{i}} . \tag{3.4}
\end{align*}
$$

The RHS of (3.3) and (3.4) are, respectively, the marginal benefit of teaching effort and research effort. We assume that there exists a unique interior solution to this system. ${ }^{11} \mathrm{We}$ denote this solution $t_{i}^{*}$ and $r_{i}^{*}$. Obviously, an increase in the marginal benefit of task $l$ leads to an increase of $l_{i}^{*}, l=t, r$. We now state:

Lemma 3.1. The marginal benefit of teaching (resp. research) effort increases with the level of research effort (resp. teaching effort).

[^8]Proof. Take the derivative of the RHS of (3.3) with respect to $r_{i}$. The sign of the expression is given by

$$
\begin{equation*}
v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}+\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}^{\prime}+\theta_{i} v^{\prime \prime}\left(\alpha_{i} B\right) \alpha_{i} \alpha_{i}^{\prime} B . \tag{3.5}
\end{equation*}
$$

Only the last term in this expression is negative (since $v($.$) is concave and \alpha_{i}^{\prime}>0$ ). A sufficient condition for a positive sign of (3.5) is $v^{\prime}\left(\alpha_{i} B\right)>-v^{\prime \prime}\left(\alpha_{i} B\right) \alpha_{i} B$, which is always true given the assumption on $v($.$) . Note that we could reach the same result by taking the derivative of$ (3.3) with respect to $t_{i}$; the sign of this expression is obviously given by (3.5).

The MDU creates a complementarity between teaching and research efforts. The previous lemma implies the following: (1) If teaching and research are complements in the academics' cost function ( $\left.\partial^{2} C(a) / \partial t \partial r<0\right)$, the multi-unit structure of the university reinforces this complementarity. (2) If teaching and research are substitutes in the academics' cost function ( $\left.\partial^{2} C(a) / \partial t \partial r>0\right)$, the two tasks are complements in a MDU if

$$
\begin{equation*}
v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}+\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}^{\prime}+\theta_{i} v^{\prime \prime}\left(\alpha_{i} B\right) \alpha_{i} \alpha_{i}^{\prime} B-\frac{\partial^{2} C\left(a_{i}\right)}{\partial t_{i} \partial r_{i}} \geqslant 0 . \tag{3.6}
\end{equation*}
$$

Eq. (3.5) measures the complementarity created by the MDU structure. If it is strong enough, it can countervail the potential negative synergies coming from the cost side. We now show that this complementarity decreases with the university overall budget $B$, that is, a larger university creates less complementarity than a smaller one.

Lemma 3.2. The complementarity between teaching effort and research effort created by the MDU structure decreases with the university's budget $B$.

Proof. The derivative of (3.5) with respect to $B$ is

$$
\begin{equation*}
v^{\prime \prime}\left(\alpha_{i} B\right)\left(\alpha_{i}^{2}+2 \alpha_{i} \alpha_{i}^{\prime} \theta_{i}\right)+\alpha_{i}^{2} \alpha_{i}^{\prime} \theta_{i} B v^{\prime \prime \prime}\left(\alpha_{i} B\right) \tag{3.7}
\end{equation*}
$$

Given the specific functional form we impose on $v($.$) , we have \alpha_{i} B v^{\prime \prime \prime}\left(\alpha_{i} B\right)=-(1-h)$ $v^{\prime \prime}\left(\alpha_{i} B\right)$. Hence, (3.7) is clearly negative.

We are now in a position to address our key issue, i.e. to assess the efficiency of a conglomerate form for the university. Since under our assumptions there exist no synergies between departments outside those resulting from the budget sharing, any difference between a single department university (SDU) and a MDU is explained by the organizational form of the university.

## 3.2. $S D U$ and $M D U$

Consider a SDU, i.e. a university made of a unique academic endowed with a vector of talent $(\tau, \rho)$. In a SDU, there is no budget allocation scheme, since there is only one research project of quality $\theta=f(r, \rho)$ available; therefore, $y=B$. The professor's utility is then

$$
U=w[\theta v(B)]-C(a) .
$$

Take the first-order condition to derive the optimal teaching and research levels:

$$
\begin{align*}
& C_{t}=w\left[\theta v^{\prime}(B)\right] \frac{\partial B}{\partial t},  \tag{3.8}\\
& C_{r}=w[v(B)] \frac{\partial \theta}{\partial r} \tag{3.9}
\end{align*}
$$

Compare first the marginal benefit of teaching in a MDU vs a SDU (Eqs. (3.3) and (3.8)). Everything else being equal (the research budget $B$ in a SDU equals $\alpha_{i} B$ in a MDU; and the value of the research project), the marginal benefit of teaching in a MDU is $\alpha_{i}<1$ times the marginal benefit in a single department. Hence, a MDU provides the academics with less incentives to do teaching.

The reason is that the budget is a pure private good in a SDU while it tends to be a common resource in a MDU. If in a SDU any additional resource created by attracting more students is invested in the department's research project, in a MDU, any additional resource goes to the common pool of resources from which department $i$ gets only a fraction $\alpha_{i}<1$. Hence an academic can only appropriate a fraction $\alpha_{i}$ of the additional budget.

An insurance effect is also at play in our framework. Indeed, the availability of funds within the university affects the marginal benefit of teaching through the term $v^{\prime}\left(\alpha_{i} B\right)$. By concavity of $v($.$) , the larger the budget, the lower the marginal benefit of teaching. An$ academic would have lower incentives to teach should its university be endowed with a large budget $B$ and, conversely, a low budget stimulates teaching effort. Lemma 3.3 will illustrate this effect more clearly.

Lemma 3.3. In a MDU, teaching efforts $t_{i}$ and $t_{j}$ are strategic substitutes:

$$
\frac{\partial t_{i}^{*}}{\partial t_{j}}<0
$$

Proof. An increase in $t_{j}$ leads to an increase of the university budget $B$. To measure the effect of a change in $t_{i}$, take the derivative of (3.3) with respect to $t_{j}$ :

$$
\begin{equation*}
w\left[\theta_{i} v^{\prime \prime}\left(\alpha_{i} B\right) \alpha_{i}^{2}\right] \frac{\partial B}{\partial t_{i}} \frac{\partial B}{\partial t_{j}}<0 . \tag{3.10}
\end{equation*}
$$

Clearly, the marginal benefit of teaching for professor $i$ decreases when the budget increases i.e. when $t_{j}$ increases.

Thus teaching efforts are unambiguously strategic substitutes. This illustrates the insurance effect we mentioned. When there are more resources, the individual incentives to create additional resources diminish.

Turning to the comparison of marginal benefits of research in a MDU and a SDU (Eqs. (3.4) and (3.9)), we note the additional term $\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}^{\prime} B\left(\partial \theta_{i} / \partial r_{i}\right)$ in (3.4). This term is positive. Hence, everything else being equal (the research budget $B$ in a SDU equals $\alpha_{i} B$ in a MDU), the marginal benefit of a research effort is larger in a MDU than in a SDU.

The additional term in (3.4) measures the competitive effect of having a MDU. In a MDU, professors compete for the research budget. Given that the budget allocation scheme is based on the relative quality of projects, the professors have to produce higher quality research to grab the resources from the university. Increasing research effort increases the share of the budget captured by the academic. The competition for research funding, as induced by the MDU structure, leads to an increase of research quality.

Summing up, integrating departments in a MDU changes academics' incentives: It creates free-riding on teaching and yardstick competition on research, i.e. a MDU provides more incentives to do research and less incentives for teaching than a SDU.

Does it mean that MDU has a better research and a lower quality teaching than SDU? The answer is no because in those circumstances where tasks are complements (i.e. Eq. (3.6) holds) higher effort in one task induces higher effort in the other task. Hence, it is possible that a MDU offers a better quality teaching and performs better research than a $S D U$. The reverse could also be true. Last, it is also possible that MDU performs better on one dimension only. However, the next proposition shows that a MDU cannot perform better than a SDU on the teaching side only when tasks are complements. To establish this result, we compare a MDU with a collection of SDU replicating the MDU's departments.

Proposition 3.2. If condition (3.6) holds and for all $i=1, \ldots, N, t_{i}^{\mathrm{MDU}}>t^{\mathrm{SDU}}$ then $r_{i}^{\mathrm{MDU}}>r^{\mathrm{SDU}}$.

Proof. Suppose $t_{i}^{\mathrm{MDU}}>t^{\mathrm{SDU}}$ and $r_{i}^{\mathrm{MDU}}<r^{\mathrm{SDU}}$. Because $r_{i}^{\mathrm{MDU}}<r^{\mathrm{SDU}}$, the marginal benefit of teaching efforts is smaller in the MDU than in the SDU. In order for this to be compatible with higher teaching efforts in the MDU, we need that the marginal cost of teaching effort is smaller in the MDU than in the SDU when research effort is smaller. This cannot be the case if $\partial^{2} C(a) / \partial t \partial r \leqslant 0$. If $\partial^{2} C(a) / \partial t \partial r>0$, the decrease in the marginal cost of teaching must be large enough to compensate for the decrease in marginal benefit. However, this requires that teaching and research efforts are substitutes, i.e. condition (3.6) is not satisfied.

Proposition 3.2 shows that a sufficient condition for having more research effort in all departments is to have more teaching in all departments. However, depending on the distribution of talents, it is possible that in some department, a MDU has a better performance, while in other department a MDU does worse. We propose hereafter an example which allows to discuss the basic intuitions underlying the general trade-off between teaching and research incentives.

### 3.3. An example

Each academic is characterized by a vector $\eta_{i}=\left(\tau_{i}, \rho_{i}\right)$ of talent. The research output in department $i$ is $R_{i}\left(\theta_{i}, y_{i}\right)=\theta_{i} y_{i}^{1-h}$, with $h<1$ where $\theta_{i}=r_{i} \rho_{i}$. We assume that all the academics are identical with respect to talent. This means that $\eta_{i}=(\tau, \rho), \forall i$.

The university is financed exclusively by a per student fee $s$. The number of students in department $i$ is $n_{i}\left(t_{i}, \tau_{i}\right)=t_{i} \tau$. Hence the total budget is $B=s \tau \sum_{k=1}^{N} t_{k}$.

The costs of teaching and research efforts are separable. Specifically, we assume $C\left(a_{i}\right)=t_{i}^{2} / 2+r_{i}^{2} / 2$.

In a SDU, the optimal behavior of the academic is obtained by solving the following program:

$$
\max _{t, r} w\left[\theta B^{1-h}\right]-C(a)
$$

The first-order conditions are

$$
\begin{align*}
& w\left[\theta(1-h)(s \tau t)^{-h}\right](s \tau)=t  \tag{3.11}\\
& w\left[(s \tau t)^{1-h}\right] \rho=r \tag{3.12}
\end{align*}
$$

Solving them for the efforts we obtain

$$
\begin{align*}
& t^{\mathrm{SDU}}=(1-h)^{1 / 2 h} w^{1 / h} \rho^{1 / h}(s \tau)^{(1-h) / h}  \tag{3.13}\\
& r^{\mathrm{SDU}}=(1-h)^{(1-h) / 2 h} w^{1 / h} \rho^{1 / h}(s \tau)^{(1-h) / h} \tag{3.14}
\end{align*}
$$

In a MDU each academic $i$ solves

$$
\max _{t_{i}, r_{i}} w\left[\theta_{i}\left(\alpha_{i} B\right)^{1-h}\right]-C\left(a_{i}\right)
$$

where $B=s \tau \sum_{k=1}^{N} t_{k}, \alpha_{i}$ is given in Proposition 3.1: $\alpha_{i}=\theta_{i}^{1 / h} / \sum_{k=1}^{N} \theta_{k}^{1 / h}$ and $\alpha_{i}^{\prime}$ is given by

$$
\begin{equation*}
\alpha_{i}^{\prime}=\frac{\partial \alpha_{i}}{\partial \theta_{i}}=\frac{1}{h} \theta_{i}^{1 / h} \theta_{i}^{-1} \frac{\sum_{k=1, k \neq i}^{N} \theta_{k}^{1 / h}}{\left(\sum_{k=1}^{N} \theta_{k}^{1 / h}\right)^{2}}=\frac{1}{h \theta_{i}} \alpha_{i} \sum_{k=1, k \neq i}^{N} \alpha_{k}>0 \tag{3.15}
\end{equation*}
$$

The first-order conditions are

$$
\begin{align*}
& w\left[\theta_{i}(1-h)\left(s \tau \alpha_{i} \sum_{k=1}^{N} t_{k}\right)^{-h}\right]\left(s \tau \alpha_{i}\right)=t_{i}  \tag{3.16}\\
& w\left[\left(s \tau \alpha_{i} \sum_{k=1}^{N} t_{k}\right)^{1-h}+\theta_{i}(1-h)\left(s \tau \alpha_{i} \sum_{k=1}^{N} t_{k}\right)^{-h}\left(s \tau \sum_{k=1}^{N} t_{k}\right) \alpha_{i}^{\prime}\right] \rho=r_{i} . \tag{3.17}
\end{align*}
$$

Using the fact that all academics are identical, we can replace $\sum_{k=1}^{N} t_{k}$ by $N t_{i}, \alpha_{i}$ by $1 / N$ and $\alpha_{i}^{\prime}$ by $\left(1 / h \theta_{i}\right) \alpha_{i}\left(1-\alpha_{i}\right)=\left(1 / h \theta_{i}\right)\left((N-1) / N^{2}\right)$.

Simplifying and solving the system for the efforts $t_{i}$ and $r_{i}$, we obtain an explicit relation between optimal values in the single division university and a multi-division one:

$$
\begin{align*}
& t_{i}^{\mathrm{MDU}}=(1-h)^{1 / 2 h} w^{1 / h} \rho^{1 / h}(s \tau)^{(1-h) / h} \frac{1}{N^{1 / h}}\left(\frac{N+h-1}{h}\right)^{1 / 2 h}=t^{\mathrm{SDU}} g_{1}(N)  \tag{3.18}\\
& r_{i}^{\mathrm{MDU}}=(1-h)^{(1-h) / 2 h} w^{1 / h} \rho^{1 / h}(s \tau)^{(1-h) / h} \frac{1}{N^{1 / h}}\left(\frac{N+h-1}{h}\right)^{(1+h) / 2 h}=r^{\mathrm{SDU}} g_{2}(N) \tag{3.19}
\end{align*}
$$

It follows that

$$
\begin{aligned}
& t_{i}^{\mathrm{MDU}}>t^{\mathrm{SDU}} \Leftrightarrow g_{1}(N)>1, \\
& r_{i}^{\mathrm{MDU}}>r^{\mathrm{SDU}} \Leftrightarrow g_{2}(N)>1,
\end{aligned}
$$

with $g_{1}(N)=\left(1 / N^{1 / h}\right)((N+h-1) / h)^{1 / 2 h} \quad$ and $\quad g_{2}(N)=\left(1 / N^{1 / h}\right)((N+h-1) / h)^{(1+h) / 2 h}$. Moreover, direct computations yield

$$
R_{i}^{\mathrm{MDU}}>R^{\mathrm{SDU}} \Leftrightarrow g_{2}(N) g_{1}(N)^{1-h}>1
$$

Proposition 3.3 summarizes our findings:
Proposition 3.3. For all $h<1$, there exist real numbers $N_{1}<N_{r}<N_{2}$, with $N_{r}>1$ and $N_{1} \geqslant 1$ if $h \leqslant \frac{1}{2}$ such that

- $\forall N \in\left[1, \operatorname{Max}\left[1, N_{1}\right]\right]$, we have $t_{i}^{\mathrm{MDU}} \geqslant t^{\mathrm{SDU}}, r_{i}^{\mathrm{MDU}} \geqslant r^{\mathrm{SDU}}$, and $R_{i}^{\mathrm{MDU}} \geqslant R^{\mathrm{SDU}}$.
- $\forall N \in\left[\operatorname{Max}\left[1, N_{1}\right], N_{r}\right]$, we have $t_{i}^{\mathrm{MDU}} \leqslant t^{\mathrm{SDU}}, r_{i}^{\mathrm{MDU}} \geqslant r^{\mathrm{SDU}}$, and $R_{i}^{\mathrm{MDU}} \geqslant R^{\mathrm{SDU}}$.
- $\forall N \in\left[N_{r}, N_{2}\right]$, we have $t_{i}^{\mathrm{MDU}} \leqslant t^{\text {sDU }}, r_{i}^{\mathrm{MDU}} \geqslant r^{i \mathrm{DU}}$, and $R_{i}^{\mathrm{MDU}} \leqslant R^{\mathrm{SDU}}$.
- $\forall N \geqslant N_{2}$, we have $t_{i}^{\mathrm{MDU}} \leqslant t^{\mathrm{SDU}}, r_{i}^{\mathrm{MDU}} \leqslant r^{\mathrm{SDU}}$, and $R_{i}^{\mathrm{MDU}} \leqslant R^{\mathrm{SDU}}$.

Proof. The equation $g_{1}(N)=1$ is equivalent to $-h N^{2}+N+h-1=0$. The roots of this second-degree equation are: $N=1$ and $N=N_{1}=(1-h) / h . g_{1}(N)$ is larger than 1 when $N$ lies between the two roots. When $h>\frac{1}{2}, N_{1}<1$ and $\forall N>1$, we have, $g_{1}(N)<1$ and $t_{i}^{\mathrm{MDU}}<t^{\mathrm{SDU}}$. When $h \leqslant \frac{1}{2}, N_{1} \geqslant 1$ and $\forall N \in\left[1, N_{1}\right]$, we have $g_{1}(N) \geqslant 1$ and $t_{i}^{\mathrm{MDU}} \geqslant t^{\mathrm{SDU}}$. For all $N>N_{1} \geqslant 1$, we have $g_{1}(N)<1$ and $t_{i}^{\mathrm{MDU}}<t^{\mathrm{SDU}}$.

The equation $g_{2}(N)=1$ is equivalent to $-h N^{2 /(1+h)}+N+h-1=0$. We first show that if $h<1$, this equation has two roots in the interval $\left[1,+\infty\left[: N=1\right.\right.$ and $N_{2}>1$. First, $N=1$ is clearly a root of this equation. Second, $-h N^{2 /(1+h)}+N+h$ is increasing up to $N=$ $((1+h) / 2 h)^{(1-h) /(1+h)}>1$ and decreasing afterwards. Hence, there is another root $N_{2}>1$. $g_{2}(N)$ is larger than 1 when $N$ lies in between the two roots 1 and $N_{2}$. Hence for all $N \in\left[1, N_{2}\right], g_{2}(N) \geqslant 1$ and $r_{i}^{\mathrm{MDU}} \geqslant r^{\mathrm{SDU}}$, and for all $N>N_{2}, g_{2}(N)<1$ and $r_{i}^{\mathrm{MDU}}<r^{\mathrm{SDU}}$.

The root $N_{2}$ is larger than $N_{1}$. Indeed, $g_{2}(N)=g_{1}(N)((N+h-1) / h)^{1 / 2}$. Clearly $g_{2}(N)>g_{1}(N)$ for all $N>1$. Hence, when $g_{1}\left(N_{1}\right)=1, g_{2}\left(N_{1}\right)$ is still larger than 1 .

The equation $g_{1}(N)^{1-h} g_{2}(N)=1$ is equivalent to $-h N^{2-h}+N+h-1=0$. For $h<1$, this equation has two roots in the interval $\left[1,+\infty\left[: N=1\right.\right.$ and $N_{r}>1$. The analysis is similar to the equation $g_{2}(N)=1:-h N^{2-h}+N+h-1=0$ increases up to $N=(1 /(h(2-$ $h)))^{1 /(1-h)}>1$ and decreases after. Hence for all $N \in\left[1, N_{r}\right], R_{i}^{\mathrm{MDU}} \geqslant R^{\mathrm{SDU}}$, and for all $N>N_{r}, R_{i}^{\mathrm{MDU}}<R^{\mathrm{SDU}}$.

Last, the root $N^{\mathrm{r}}$ lies in between $N_{1}$ and $N_{2}$. Indeed, when $N=N_{1}, g_{1}\left(N_{1}\right)^{1-h} g_{2}\left(N^{1}\right)$ is larger than 1 since the first term equals 1 and the second is $>1$. When $N=N_{2}$, $g_{1}\left(N_{2}\right)^{1-h} g_{2}\left(N_{2}\right)$ is lower than 1 since the second term equals 1 and the first is $<1$.

Figs. 1 and 2 illustrate Proposition 3.3 for $h=\frac{1}{4}$ and $\frac{1}{2}$.
$N_{1}$ then defines the critical "size" beyond which the free-riding effect of the MDU more than compensates the initial positive effect. $N_{2}$ defines the critical level beyond which the MDU structure leads to less research efforts. When there are too many departments, competition is too fierce, and this induces less effort. As predicted by Proposition 3.2, we


Fig. 1. $h=\frac{1}{4}$.


Fig. 2. $h=\frac{1}{2}$.
observe that $N_{1}<N_{2}$. This ordering is explained by the following argument: When the number of department increases, the free-riding effect comes into play as well as the competition effect. However, when the number is small enough the introduction of competition for research funding increases efforts in the research dimension which increases effort in the teaching dimension as well, because of the complementarity effect. In other words, because teaching and research activities remain complements, a bit of competition for research fundings overcomes the free-riding effect. We identify by $N^{\mathrm{r}}$ the critical number of departments for which the research outputs are identical in the SDU and in the MDU. Notice then that this research output is achieved with a lower budget in the MDU, which is then compensated by a larger research effort.

The numerical approximations for our critical values for $N$ are $\left\{N_{1}, N_{r}, N_{2}\right\}=$ $\{3,5.15,8.66\}$ for $h=\frac{1}{4}$ and $\left\{N_{1}, N_{r}, N_{2}\right\}=\{1,2.62,6.22\}$ for $h=\frac{1}{2}$. Although we have not been able to prove it formally, our computations indicate that all of these threshold values are decreasing in $h$. Recall that in this example a larger $h$ means that $v($.$) is more$ concave, so that the marginal contribution of funds to research output is decreasing quickly. By contrast, when $h=1$ the marginal contribution is constant. In other words, a lower $h$ means that redistribution possibilities become more valuable. It is therefore not surprising that a MDU remains more efficient than a collection of SDU for a larger number of departments.

The following corollary is an immediate consequence of Proposition 3.3:
Corollary 3.2. There exists a unique $N^{*}>1$ such that the aggregate research output is maximal.

Proof. $N^{*}$ is the maximum of $g_{1}(N)^{1-h} g_{2}(N)$. The derivative of $g_{1}(N)^{1-h} g_{2}(N)$ with respect to $N$ is (after simplification)

$$
\left(\frac{N+h-1}{h}\right)^{1 / h} \frac{1}{h N^{2 / h}}\left(\frac{1}{N(N+h-1)}-(2-h)\right)
$$

$N^{*}$ is the solution of $1 /(N(N+h-1))=(2-h)$. Solving the equation, if $h<1$, there is a unique positive root $N^{*}=\frac{1}{2}\left[(1-h)+\sqrt{(1-h)^{2}+4 /(2-h)}\right]$. Clearly, $N^{*}>1$.

This last result shows that in order to maximize the research output, the university should have some level of diversification. Notice however that $N^{*}$ is actually less than 2 , whatever the value of $h$. Accordingly, when we take the restriction $N \geqslant 2$ into account, the desirability of a multi-unit university cannot be evaluated through $N^{*}$. The relevant comparison is between $N^{\mathrm{r}}$ and 2 . Condition $N^{\mathrm{r}}>2$ is not satisfied for all $h \in[0,1]$ but our
numerical computations show that $N^{\mathrm{r}}$ is decreasing in $h$. Actually, unless $h$ is large, there always exists a feasible MDU structure which exhibits a better research output than the corresponding collection of single-unit divisions. The fact that $h$ cannot be too large is intuitive: Suppose $h$ is arbitrarily close to 1 , then research output (almost) does not depend on research funding. In this case, very few is to be gained through redistribution opportunities, while free-riding already undermines teaching efforts. On the other hand, if $h$ is smaller, the competition for funds is fiercer. Accordingly, its positive effect overcomes the negative free-riding effect for a larger number of divisions.

## 4. Mixed research funding

So far, we assumed that the allocation of the university's budget to research projects is a pure winner-picking contest, i.e. the financing of an academic's research only depends on the relative quality of its project. This allocation rule has a positive effect on research incentives due to yardstick competition but a negative effect on teaching incentives due to free-riding. In fact, the absence of reward for the teaching effort is the most important problem associated with winner-picking, especially when the number of academics is large.

Obviously, the university could alleviate this problem by departing from winner-picking and allocating its resources not only according to the relative quality of research projects but also depending on the teaching's quality, measured for example by the number of students in field $i .^{12}$ In fact, in most universities, the number of students per department matters for deciding on the allocation of research funds.

Limiting the scope of winner-picking and integrating the number of students as a determinant of the budget sharing rule, together with the project's relative quality, would have a positive effect on teaching incentives since it would make the teaching effort more appropriable by the academic. However, this kind of sharing rule proves hard to use because the university should ex ante, that is, before the academics choose the efforts, design (and commit to) a sophisticated sharing rule. As discussed in Section 2, full commitment would imply that the university decides ex ante the way the budget will be allocated given all possible realizations of $n_{k}$ and $\theta_{k}, k=1, \ldots, N$. Commitment to such a rule would require that both the number of students and the projects' quality are verifiable. ${ }^{13}$ If it does not seem to be a problem for the students' number, it is much more demanding in terms of information for the projects' quality. ${ }^{14}$ Outsiders, such as a court would need a lot of information and a great expertise to verify the qualities of the research projects.

With nonverifiable $\theta$, the university can implement ex ante two kind of sharing rules: Winner-picking and rules based on the number of students only. Despite the nonverifiability of $\theta$, winner-picking is implementable since it corresponds to the optimal allocation of resources ex post (once they are created). Hence, to implement winnerpicking, the university simply decides to postpone the definition of the sharing rule until the budget is realized. Rules based on the number of students create strong incentives for

[^9]teaching because they reduce the free-riding, but they also reduce the yardstick competition effect. It is particularly clear in the rule that replicates the stand alone university: $y_{i}=s n_{i}$, where there is no free-riding but no yardstick competition. Rules based on the number of students (or more generally on teaching quality) thus reduce the benefits of the conglomerate structure of a university.

To see how things are going with full commitment and a verifiable project's quality, let us consider a particular rule that mixes winner-picking with an allocation based on the number of students. Suppose that the university commits to split the budget $B$ into two parts: A fraction $\gamma$ of the budget $B$ will be allocated according to the relative quality of the projects (winner-picking), while a fraction $1-\gamma$ will be allocated according to the relative number of students enroled in the departments. With this rule, the research budget of academic $i$ is

$$
\begin{equation*}
y_{i}=\gamma \alpha_{i} B+(1-\gamma) \beta_{i} B \tag{4.20}
\end{equation*}
$$

where $\alpha_{i}$ is given by Proposition 3.1 and $\beta_{i}=n_{i} / \sum_{k=1}^{N} n_{k}$.
If the university applies the sharing rule given by (4.20), funds could be inefficiently allocated ex post: Once the $n_{k}$ and $\theta_{k}$ are realized, there is room for a redistribution that increases the aggregate research output. Strong commitment by the university is then necessary to apply this rule.

When the research budget is given by (4.20), the first-order conditions read as follows:

$$
\begin{align*}
C_{t_{i}} & =w\left[\theta_{i} v^{\prime}\left(y_{i}\right)\right]\left[\left(\gamma \alpha_{i}+(1-\gamma) \beta_{i}\right) \frac{\partial B}{\partial t_{i}}+(1-\gamma) B \frac{\partial \beta_{i}}{\partial t_{i}}\right]  \tag{4.21}\\
C_{r_{i}} & =w\left[v\left(y_{i}\right)+\theta_{i} v^{\prime}\left(y_{i}\right) \gamma \alpha_{i}^{\prime} B\right] \frac{\partial \theta_{i}}{\partial r_{i}} \tag{4.22}
\end{align*}
$$

Consider first the incentives to perform research effort. Yardstick competition is still present but the incentive effect is reduced because the academics compete only for a fraction $\gamma$ of the budget. Moreover, for those academics who benefit from a large research financing because a lot of students attend their field, the benefit of competing for the university budget is lower. Consider next the teaching effort. There is still free-riding because only a fraction of the incremental budget created by academic $i$ will be invested in his research project. However, there are more incentives to teach because the share of the budget for project $i$ is increasing with teaching effort by academic $i$. Hence, the new sharing rule increases the incentives for teaching and decreases the incentives for research. But because of the complementarity between the two tasks, the global effect is ambiguous. We may rely on the example developed in the above section to shed some light on the various effects at work.

As for the case of mixed-funding, our example reveals the following. Suppose $\gamma<1$. Given that all academics are identical, in the first-order conditions (4.21) and (4.22), $\alpha_{i}$ and $\beta_{i}$ can both be replaced by $1 / N$. Solving for the effort levels we have

$$
\begin{align*}
& t_{i}^{\mathrm{MDU}}=t^{\mathrm{SDU}} g_{3}(N, \gamma),  \tag{4.23}\\
& r_{i}^{\mathrm{MDU}}=r^{\mathrm{SDU}} g_{4}(N, \gamma), \tag{4.24}
\end{align*}
$$

with $g_{4}(N, \gamma)=\left(1 /\left(N^{1 / h}\right)\right)((h N+(1-h)(n-1) \gamma) / h)^{(1+h) / 2 h}(1+(1-\gamma)(N-1))^{(1-h) / 2 h}$ and $g_{3}(N, \gamma)=\left[g_{4}(N, \gamma)(1+(1-\gamma)(N-1))\right]^{1 /(1+h)}$. Notice that $g_{3}(N, 1)=g_{1}(N) \quad$ and $g_{4}(N, 1)=g_{2}(N)$.

The aggregate research output is

$$
\sum_{k=1}^{N} R_{k}=N r_{i} \rho\left(s t_{i} \tau\right)^{1-h}=N \rho r^{\mathrm{SDU}}(s \tau)^{1-h}\left(t^{\mathrm{SDU}}\right)^{1-h} g_{3}(N, \gamma)^{1-h} g_{4}(N, \gamma)
$$

If the university sets $\gamma$ in order to maximize the aggregate research output, we have:
Proposition 4.1. If $N \geqslant 2$, it is efficient to set $\gamma=\gamma^{*}=((1-h) N) /((N-1)(2-h))$ with $0<\gamma^{*}<1$.

Proof. $\gamma^{*} \equiv \max _{\gamma} g_{3}(N, \gamma)^{1-h} g_{4}(N, \gamma)$.
With identical academics, there is no distortion in the allocation of resources to the academics since $\alpha_{i}=\beta_{i}$. Hence, there is no loss due to a misallocation of resources ex post. This would no longer be true if the academics were different. Selecting $\gamma^{*}$ just reflects the balance between incentives to teach and to do research. The fact that the university optimally sets $\gamma^{*}<1$ means that it achieves a larger aggregate research output with the redistribution rule (4.20). Hence, the benefits of conglomerate organization for a university increases when it can make research budgets contingent on both the research's and the teaching's quality.

As an immediate corollary of the above proposition we note also that $\partial \gamma^{*} / \partial N<0$. Accordingly, the more departments there are, the more budgetary autonomy these departments should be left with.

## 5. Comments

### 5.1. Robustness

Sections 3 and 4 have been devoted to disentangling the nature of the trade-off between teaching and research efforts in a MDU. Roughly speaking, our analysis suggests that a MDU may actually achieve some form of redistribution in research funding among departments without giving away efficiency, i.e. inducing more efforts on teaching quality and research quality. This is especially true in cases where research outputs are heavily dependent on funding levels ( $h$ small). Since our model is quite specific, we now question its robustness to alternative assumptions.

Pay related to performance scheme: In the analysis, we assumed that there is no direct pay related to performance for the academics and, in particular, there is no reward to teaching. Indeed, for academics, a higher teaching quality is valuable only because it increases the total research budget. This assumption can be justified by the fact that teaching quality, unlike research quality, is difficult to assess. Moreover, if research quality is comparable across academics in the same field, measures of teaching quality are often institution specific, hence less comparable. Suppose however that the university designs a measure of teaching quality. Typically this measure could result from students' evaluations. Denote by $q_{i}$ a proxy for the teaching quality by professor $i$. The relevant measure of teaching quality should be correlated with teaching effort and teaching talent: $q_{i}\left(t_{i}, \tau_{i}\right)$ with $\partial q_{i} / \partial t_{i}>0$ and $\partial q_{i} / \partial \tau_{i}>0$.

With observable teaching quality $q_{i}$, the market value of a professor $i$ is now: $w\left[R_{i}+\delta q_{i}\right]$ where $\delta$ is the weight given to teaching. Integrating this pay structure in the academic's
utility function, the first-order conditions of the optimization problem are

$$
\begin{equation*}
C_{t_{i}}=w\left[\theta_{i} v^{\prime}\left(\alpha_{i} B\right) \alpha_{i}\right] \frac{\partial B}{\partial t_{i}}+w \delta \frac{\partial q_{i}}{\partial t_{i}} \tag{5.25}
\end{equation*}
$$

and (3.4) which remains unchanged.
Obviously, the effect on optimal teaching effort is positive. Moreover, the more precise is the quality measure, the more positive the impact. Given Lemma 3.1 we may conclude that research efforts will increase as well.

Alternative financing sources: A key feature of our model is that departments rely exclusively on the university central budget to finance research. Real life departments have also access to alternative source of funding. We will not address here the possible funding related to consultancy, and more generally private funding related to applied research. Obviously, the incentives to rely on such sources are larger when basic research efforts are less appropriable. Aside from consultancy, academics may rely on institutional funds to finance their basic research. Academics also face yardstick competition for these funds. Moreover, their competitors are in general working in a similar discipline, so that competition is likely to be tougher. It therefore seems reasonable to consider that research effort is likely to be larger there. However, the key implication of these fundings is that they are unrelated to the result of teaching efforts. ${ }^{15}$ Accordingly, wider access to these funds breaks the complementarity between teaching and research efforts. Notice here that the argument equally applies to SDUs and MDUs. However, because the MDU already faces a free-riding problem in the teaching activity, the negative impact of these external fundings on teaching efforts could actually be stronger.

### 5.2. Career concerns

There is no direct incentive or performance related to pay in our model. Rather, the model relies on implicit incentives where current actions influence future opportunities. To put it differently, implicit incentives mean that the university cannot control the per unit reward $w$.

This incentive structure shares features with the career concern model of Holmström (1982) and Dewatripont et al. (1999a,b). We should stress however that our framework differs from career concern ones in one fundamental respect. In the career concern model, the agent's pay reflects the market's expectation of the unknown agent's talent given the observables. Applied to our framework, this means that $w R_{i}$ is the expected wage of a professor on the academic market, given observables i.e. given research records. In this case, the market would pay academics according to their perceived talent. Clearly, the market will use the research output to assess the talent of an academic, and the academic's pay will increase with the research output. But market assessment of research talent would also take into account the amount available for the research of professor $i$. And to make correct assessment about the research budget $y_{i}$, the market needs to infer the value of the total budget $B$ and the redistribution rule that applies within the university. If it is common knowledge that the university allocates its budget in order to maximize the aggregate

[^10]research output (an assumption we make in this model) the market expectation of the budget $y_{i}$ will depend on the expectation of the unknown talents $\rho_{j}$ of all professors $j=1, \ldots, N$ within the same university since redistribution will be based not only on the project's quality $\theta_{i}$ but also on the qualities of the other projects $\theta_{j}$ as all professors compete for the same budget. Moreover, the market should evaluate the total budget size, which depends not only on the expectation of the unknown talents $\tau_{j}, j=1, \ldots, N$. Hence, the market value of professor $i$ depends not only on his observed research output but also on the observed research output of his colleague in other fields. This would make the model extremely difficult to solve. We then take another route and assume the benefit of research is simply proportional to the professor's research output. Accordingly, our model cannot be viewed as a career concern model. The market values identically a research record resulting from a high effort and/or high talent than a research record resulting from a poor talent but a large research budget. Hence, we consider a myopic market, where the professor's value depends only on his observable: His research achievement $R_{i}$.

## 6. Conclusion

Universities are most often organized as multi-unit departments headed by a single central authority. They also obtain a very significant share of their funds from enrolment fees and/or per student subsidies. Universities are asked to perform well in teaching and research activities. In this respect, a multi-unit organization allows for a redistribution of research funds which may be relatively independent of the origin of funds. However, it is often argued that such a redistribution weakens academics' incentives to perform well in their teaching duties. This is especially intuitive if the bulk of an academic pay and prestige depends on research records rather than teaching performance.

In this paper, we show that the multi-unit organization of universities is not incompatible with improved performance in both teaching and research. In other words, the lack of direct incentives towards teaching activities can be overcome. Central to this result is the organization of yardstick competition between departments which combined with the strategic complementarity between teaching and research may promote academics' efforts in both dimensions. However, the number of departments cannot be too large.

Our analysis is related to the literature on incentives in conglomerate firms. This literature considers that divisions are in charge of only one task. Brusco and Panunzi (2005) and Gautier and Heider (2002) consider the case where divisional managers exert efforts to raise funds for investment while the quality of the investment projects is given. They show that the redistribution of funds reduces incentives to exert effort in the fund raising activity. In Inderst and Laux (2005), resources are given and managers exert effort to develop valuable projects. The allocation of resources to the most valuable projects by the corporate headquarter increases incentives. In the present paper, we combine the two dimensions in a multi-tasking framework where the resources and the projects' value are endogenous and result from the performance of a manager (professor). In this respect, our analysis suggests that the benefits of a conglomerate structure essentially depend on the number of divisions, i.e. the value of diversification has to be linked with the level (or degree) of diversification. However, it does not address the question of which type of departments to merge in a university. Moreover, in particular, if the university dean has the choice between different academic profiles, what is an efficient combination of such
profiles? Similarly, which type of divisions should be combined in order to maximize the value of a conglomerate firm? Notice also that an interesting extension of the present model would consider the presence of effective competition for students either within some given universities or among different universities. These questions are left for future research.

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[^0]:    *Corresponding author. Tel.: + 3222117938 .
    E-mail addresses: gautier@fusl.ac.be (A. Gautier), xwauthy@fusl.ac.be (X. Wauthy).

[^1]:    ${ }^{1}$ This is in particular true if students choose first a discipline and second a university where they attend.

[^2]:    ${ }^{2}$ Moreover, the evaluation of research quality through publication scores or patent holdings is much easy to establish from outside the university and to transmit than the quality of teaching, which requires internal access to the institution.

[^3]:    ${ }^{3}$ Obviously, reality is more complex. Universities display multiple layer hierarchy. However, the present simplification allows us to combine the multi-tasking issues faced by academics with the redistribution problem in a tractable model.

[^4]:    ${ }^{4}$ For instance by buying these reductions with the research grant they receive.

[^5]:    ${ }^{5}$ See in particular the recent contribution of Ehrenberg (2003).
    ${ }^{6}$ Note that this part of the professor's reward is delayed. It represents the future pay prospect of a professor, hence, it does not need to be paid immediately out of the university's budget. Clearly, this interpretation bears some resemblance with career concern models. This is discussed in Section 5.

[^6]:    ${ }^{7}$ For references on the cost of writing contracts, see Tirole (1999).
    ${ }^{8}$ In Section 5 we discuss the case where the university can fully commit ex ante to simple redistribution rules where a fraction $\gamma$ of the budget is allocated according to the projects' relative quality and a fraction $1-\gamma$ of the budget is allocated according to the number of students. This rule may improve the global performance of the university. In this sense our present assumption leads to a model whose outcome defines a lower bound on the possible achievements of the university structure.

[^7]:    ${ }^{9}$ 'Simply put, individual projects must compete for the scarce funds, and the headquarters' job is to pick the winners and the losers in this competition.' (Stein, 1997, p. 111).

[^8]:    ${ }^{10}$ Specifically, the extent to which any given project gets funded in an internal capital market will depend not only on that project's own absolute merits, but also on its merits relative to other projects in the company's overall portfolio' (Stein, 1997, p. 112).
    ${ }^{11}$ Since $v($.$) is strictly concave and C($.$) is convex, a sufficient condition for the program to be globally concave is$ that the cross derivative of the cost function is not too large.

[^9]:    ${ }^{12}$ Notice that such a rule is likely to conflict with equity considerations since it introduces a bias in favor of those disciplines which are lucky enough to attract large cohorts of students simply because of labor market conditions.
    ${ }^{13}$ Recall indeed that only verifiable information could be included in a contract.
    ${ }^{14}$ Remember that the project quality $\theta_{i}$ is only a part of the final research record $R_{i}$, the latest being perfectly verifiable.

[^10]:    ${ }^{15}$ The research assessment exercise currently undertaken in the UK provides a very clear example of such practices. Approximately 1 million $£$ per year of public funds are distributed according to the quality ratings obtained through this exercise, without any references to teaching quality or enrolment.

