A Multiphase Model for the Transport of Dispersed Phases in Environmental Flows: Theoretical Contribution

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Abstract

Civil and environmental engineers frequently rely on mathematical models to design hydraulic structures and examine a natural flow. In this respect, single-phase models fail to accurately describe the dispersed phase and to take into account its interaction with the water flow. Consequently, authors propose in this paper to develop multiphase models in order to address shortcomings of single-phase models, improve their accuracy and unify the mathematical description of transport phenomena. The paper covers the theoretical derivation of a two-phase model suitable for hydraulic structures and natural flows. In this respect, the drift-flux theory is proven to constitute an adequate alternative to single-phase models in order to derive mid- and large-scale free-surface multiphase models. In particular, an original one-dimensional drift-flux model for free-surface flows is derived to address the specific problems arising in civil and environmental engineering. Theoretically, it is shown that this approach succeeds in enhancing the mathematical accuracy of models for sediment, air and pollutant transport. By using the new set of equations, hydraulic software's may increase in accuracy and investigators may gain new insight into experimental data.

Keywords: Fluid Mechanics, Multiphase flows, Hydraulics, Free-surface flows.

1. Introduction

As a literature review easily shows, civil and environmental engineers make frequent use of mathematical models to handle environmental flow related problems as the simulation of river and channel flow, flood prediction, the assessment of sediment and pollutant transport, the evaluation of air-water interactions… In this respect, the need of consistent models has never been more pressing. In first approximation, such situations can be dealt with analyzing the flow as a pure water flowing in an immutable external environment [1-3]. However, mechanisms of interaction that could alter the flow behavior are multiple: flow interaction with natural bed [4], fluid-structure interactions [5], transport of pollutant and air-water interactions [6, 7]. Many models have been developed to date to deal with such dispersed phases. In the field of civil and environmental engineering, they usually consist of the Navier-Stokes equations coupled with a transport equation specific to the dispersed phase. Since 3D models remains expensive from a computational point of view, 2D shallow-water-like models [8] and 1D Saint-Venant-like models [1] are also constructed by coupling a pure water flow model with case specific transport equations [4]. None of these methods offers a unified framework of description. In parallel, a few attempts to use a two-phase description for sediment-transport problems has been published [9] but focus only on peculiar issues as vertical profiles or turbulence models.
The objective of this research is thus to rigorously develop a unified two-phase mathematical model to simulate a wide range of free-surface flows with transport mechanisms. Four conditions are sought in the development of the model. First, transport of most dispersed phase must be described by a unified mathematical model. Second, model must be derived through a multi-phase methodology. Third, the set of equations must handle correctly

the scale heterogeneities in time and space of practical applications and mechanisms encountered in free-surface hydraulics. Finally, computational effort to solve the model must be reduced by deriving a 1D model as well as a 2D model whose applicability is partially extended to 3D flows by enriching the vertical profile of the flow parameters (concentration, velocities and pressure).
In this paper, authors argue that the so-called drift-flux model, originally developed for chemical and mechanical engineering, achieve this objective in many respects. In section 2, 3D, 2D and 1D equations for the Drift-flux model are given and commented. In section 3, generality of the 1D model is exemplified by particularization to single-phase hydrodynamics, aerated flow dynamics and sedimentation engineering over mobile bed.

2. The Drift-flux theory

In principle, a rigorous two-phase flow solver should solve at the same time the local instant variables describing the behavior of each phase by means of the Local Navier-Stokes equations (holding for each phases separately) as well as jump conditions (expressing the law of conservations across interfaces) [10]:

$$\begin{cases} \nabla \mathbf{v}_k = 0 \qquad k=1,2 \\ \dfrac{\partial \mathbf{v}_k}{\partial t} + \nabla(\mathbf{v}_k\mathbf{v}_k) = -\dfrac{1}{\rho_k}\nabla p_k + \dfrac{1}{\rho_k}\nabla \boldsymbol{\tau}_k + \mathbf{g} \qquad k=1,2 \\ \sum_{k=1}^{2} \rho_k \mathbf{n}_k(\mathbf{v}_k-\mathbf{v}_i) = \sum_{k=1}^{2} \dot{\mathbf{m}}_k = 0 \\ \sum_{k=1}^{2} \left[\rho_k \mathbf{n}_k\cdot(\mathbf{v}_k - \mathbf{v}_i)\mathbf{v}_k - \mathbf{n}_k\cdot\tau_k + \mathbf{n}_k p_k\right] + \left(\mathbf{t}_\alpha A^{\alpha\beta}\sigma\right)_{,\beta} = 0 \end{cases} \qquad (1)$$

for the k[th] phase (k=1,2), ρ_k is the density, $\mathbf{v}_k$ is the velocity, p_k is the pressure, τ_k is the viscous stress and $\mathbf{g}$ is the acceleration of gravity. $\mathbf{v}_i$ is the velocity of the interface between the water and the dispersed phase and $\mathbf{n}_k$ is the normal vector to the interface. The whole term $\left(\mathbf{t}_\alpha A^{\alpha\beta}\sigma\right)_{,\beta}$ accounts for the surface flux contribution due to the surface tension [10, 11].

Obtaining a solution this way is however beyond the present computational capability for many engineering applications. This is why many simplified models are often derived. We aim here at proving that one of them, the drift-flux theory (summarized in Figure 1) is particularly suited to describe environmental flows with transport of dispersed phases [12, 13].

2.1 Three-dimensional Drift-flux Model

The 3D drift-flux model is obtained by Eulerian time-averaging the local instant formulation of Navier-Stokes equations (1) and by assuming that the multiphase flow may be described as a single-phase flow of mixture variables which refer to the motion of the centre of mass of the system. The motion of the dispersed phase is then treated in terms of diffusion through the mixture. The momentum equation for the dispersed phase is neglected in favor of a constitutive equation for the relative velocity between the centre of mass and each phase. The rigorous demonstration, presented in [10, 12], gives the following set of field equations:

$$\begin{cases} \dfrac{\partial \rho_m}{\partial t} + \nabla(\rho_m \mathbf{v}_m) = 0 \\ \dfrac{\partial \alpha_d \rho_d}{\partial t} + \nabla(\alpha_d\rho_d\mathbf{v}_m) + \nabla(\alpha_d\rho_d\mathbf{V}_{dm}) = \Gamma_d \\ \dfrac{\partial \rho_m \mathbf{v}_m}{\partial t} + \nabla(\rho_m\mathbf{v}_m\mathbf{v}_m) = -\nabla p_m + \nabla\left(\tau_m + \tau^T + \tau^D\right) \\ \qquad + \rho_m \mathbf{g} + \mathbf{M}_m \end{cases} \qquad (2)$$

where α_d is the concentration (or void fraction) of the dispersed phase, ρ_m is the mixture density and $\mathbf{v}_m$ is the velocity of the centre of mass of the system, which is different from both the water velocity and the dispersed phase velocity. The diffusion velocity $\mathbf{V}_{dm}$, defined as the relative velocity with respect to the mass center of the mixture, is given by a case-specific constitutive

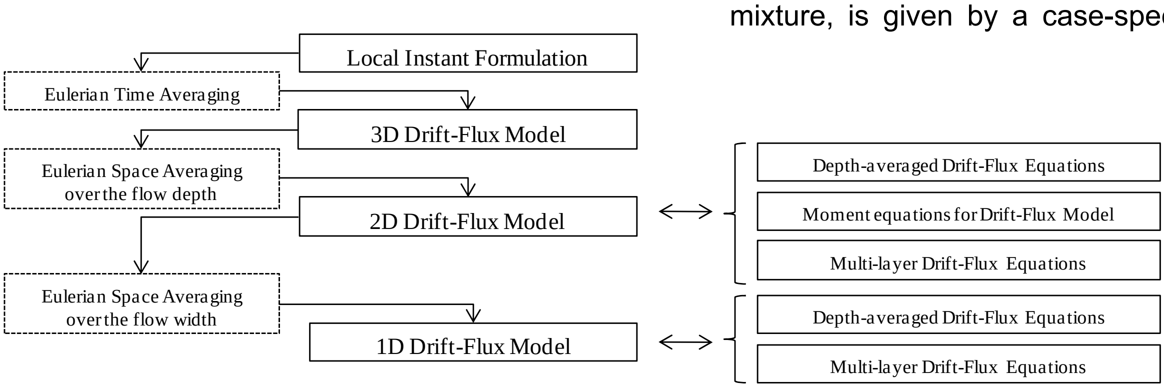

Figure 1: Drift-flux model - Description of the methodology and the results

equation [14, 15]. The mass source term Γ_d accounts for the exchange of mass between the water and the active dispersed phase and is usually given by a case-specific correlation. The mixture pressure p_m is a primitive unknown. The mixture momentum equation includes three kinds of stresses: the classical Newtonian viscous stresses τ_m, turbulent stresses τ^T and diffusion stresses τ^D due to the relative velocity between phases. All of them are given by constitutive equations and closure models [16, 17]. Finally, the mixture momentum source term $\mathbf{M}_m$ represents the effect of the surface tension on the mixture momentum. In conclusion, one can say that 3D drift-flux model for multiphase flows is analogous to the Navier-Stokes equation for pure water flows.

2.2 Two-dimensional Drift-flux Model

Computation of the three-dimensional Drift-flux Model requires a prohibitive computational effort in many cases, mainly because 3D meshes are complex to generate and include millions of cells. Furthermore, tracking the free-surface requires specific algorithm such as the Level Set method [18], the Volume of Fluid method [19], etc. In many practical applications in civil engineering, 2D models can be used as the vertical scale of the computational model is way smaller than the horizontal one. In this case, the 3D Drift-flux model (2) is integrated over the flow depth and the vertical momentum equation is cancelled thanks to a dimensional analysis [20]. Such a 2D shallow-water-like model offers a valuable alternative to the 3D model since various methods have been developed to accommodate depth-averaged models with vertical distribution of the parameters; namely the addition of Boussinesq/covariance coefficients [10], of moment equations [21] as well as multi-layer integration [22].

2D shallow-water-like drift-flux model is thoroughly presented and commented in the following book [12] to which we refer the interested reader. We prefer in this paper focusing on the 1D model which is the original contribution of this paper.

2.3 One-dimensional Drift-Flux Model

In particular applications, we can assume that the computational domain is essentially one-dimensional [1, 23]. It means both the flow depth and width are way smaller than the flow length (river, channel, pipe,…). In such cases, equations of momentum along two of the three axes may be simplified and the remaining equations of the 3D Drift-flux model are area-integrated over the flow cross-section (Figure 2). Again, definition of a multi-layer domain of integration and introduction of Boussinesq/covariance coefficients enable to enrich the vertical description of the flow parameters. The integration gives the following set of 3 partial differential equations for each layer:

$$\begin{cases} \dfrac{\partial \langle \rho_m \rangle_l \Omega_l}{\partial t} + \dfrac{\partial \langle \rho_m \rangle_l \tilde{u}_{m,l} \Omega_l}{\partial x} = q_{m,L,l} \\ \dfrac{\partial \langle \alpha_d \rangle_l \Omega_l}{\partial t} + \dfrac{\partial \langle \alpha_d \rangle_l \tilde{u}_{m,l} \Omega_l}{\partial x} + \dfrac{\partial}{\partial x}\left(\langle \alpha_d \rangle_l \dfrac{\rho_w}{\langle \rho_m \rangle_l} \tilde{U}_{dj,l} \Omega_l \right) = \\ \qquad \left\langle \Gamma_d / \rho_d \right\rangle_l \Omega_l + q_{\alpha,L,l} \\ \dfrac{\partial \langle \rho_m \rangle_l \tilde{u}_{m,l} \Omega_l}{\partial t} + \dfrac{\partial \beta_{x,x} \langle \rho_m \rangle_l \tilde{u}_{m,l} \tilde{u}_{m,l} \Omega_l + g P_\Omega}{\partial x} + \dfrac{\partial p_{s,l} \Omega_l}{\partial x} \\ + \dfrac{\partial}{\partial x}\left(\beta_{x,x} \dfrac{\langle \alpha_d \rangle_l}{1 - \langle \alpha_d \rangle_l} \dfrac{\rho_d \rho_w}{\langle \rho_m \rangle_l} \tilde{U}_{dj,l} \tilde{U}_{dj,l} \Omega_l \right) = \langle \rho_m \rangle_l g \left(i_l - J_l \right) \end{cases} \tag{3}$$

where the subscript l indicates the layer considered (Figure 2). $\langle \rho_m \rangle_l$ designates the mean density of the mixture in the l^{th} layer, Ω_l is the area of the l^{th} layer, $\tilde{u}_{m,l}$ is the mean mixture velocity (i.e. the mixture density weighted area-average of the mixture velocity), $q_{m,L,l}$ is the mixture mass lateral discharge, $\langle \alpha_d \rangle_l$ is the mean concentration in dispersed phase, ρ_w and ρ_d are

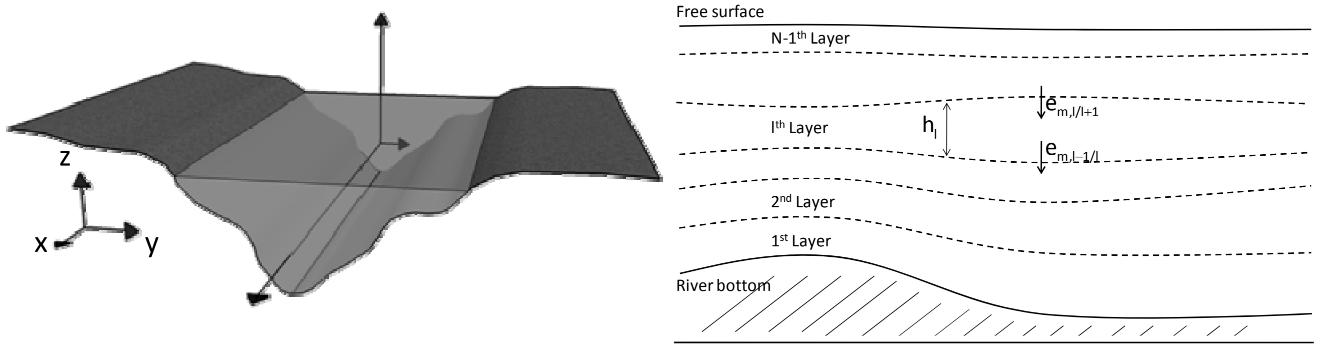

Figure 2: Integration domain for the multi-layer approach - 1D depth-averaged drift-flux model

the density of, respectively, the water phase and the dispersed phase. The area-averaged drift velocity $\tilde{U}_{dj,l}$ accounts for the relative velocity between both phases and is defined by a case specific constitutive equation [24]. $\langle \Gamma_d/\rho_d \rangle_l \Omega_l$ is the dispersed phase volume exchange term and $q_{\alpha,L,l}$ is the dispersed phase volumetric lateral discharge. $\beta_{x,x}$ is the Boussinesq coefficient which accounts for the non-uniformity of the mixture velocity over the l^{th} layer. gP_Ω is the hydrostatic pressure term. i_l is the topographic slope of the interface between the l^{th} and $(l-1)^{th}$ layer and J_l is the friction slope computed thanks to a friction correlation (Manning-Strickler, Colebrook, Martinelli-Lockhart,…). Finally, $p_{s,l}$ is the pressure exerted at the interface between l^{th} and $(l+1)^{th}$ layer.

3 PARTICULARIZATIONS

The 1D drift-flux model (3) constitutes a general framework for the simulation of transport phenomena. In this section, both its validity and its generality are exemplified by particularizing it to conventional models in free-surface hydraulics.

3.1 Single-Phase hydrodynamic

Equations (3) can be used to simulate pure water flows in rivers and pipes. If the number of layer is one and the concentration of the dispersed phase is set to zero, equations (3) reduces to the classical Saint-Venant equations [1]:

$$\begin{cases} \dfrac{\partial \Omega}{\partial t}+\dfrac{\partial u\Omega}{\partial x}=q_L \\ \dfrac{\partial u\Omega}{\partial t}+\dfrac{\partial \beta_{x,x} uu\Omega+gP_\Omega}{\partial x}=g\left(i-J\right) \end{cases} \tag{4}$$

because the diffusion equation becomes trivial, the drift-velocity cancels and mixture variables coincide with water phase variables.

3.2 Sediment Transport

Model (3) may also be used in sedimentation engineering to simulate the transport of a bed load and a wash load along the water flow (Figure 3). For this purpose, let's set the number of layers to two and define the upper layer as the mixture layer (water+wash load) and the lower layer as the bed load layer. The dispersed phase is in this case a granular flow of concentration C.

By denoting relative density s= ρ_s/ρ_w (which varies typically between 2 and 3), the net erosion rate e_b (which accounts for the sediment mass exchange between layers) and the bed porosity p, equations for the mixture (upper) layer are given by:

$$\begin{cases} \dfrac{\partial\left(1+(s-1)C\right)\Omega}{\partial t}+\dfrac{\partial\left(1+(s-1)C\right)\tilde{u}_m\Omega}{\partial x} \\ \qquad =\left[1+\Delta s\left(1-p\right)\right]\dfrac{e_b}{1-p} \\ \dfrac{\partial C\Omega}{\partial t}+\dfrac{\partial C\tilde{u}_m\Omega}{\partial x}=e_b \\ \dfrac{\partial\left(1+(s-1)C\right)\tilde{u}_m\Omega}{\partial t}+\dfrac{\partial\left(1+(s-1)C\right)\tilde{u}_m\tilde{u}_m\Omega+gP_\Omega}{\partial x} \\ \qquad =\left(1+(s-1)C\right)g\left(\dfrac{\partial(-z_b)}{\partial x}-J\right) \end{cases} \tag{5}$$

where z_b is the mobile bed elevation (or the interface between the two layers).

For the bed load layer, we make the further assumptions that the sediment phase in the lower layer is described by a constant sediment concentration C=1-p. Consequently, either the continuity equation and the diffusion equation becomes trivial. What is more, bed load dynamics is specified by a constitutive equation, called transport capacity law, rather than by a momentum equation. Such empirical law states the bed load flux Q_{dx} [25]. On account of these assumptions, equations (3) for the upper layer reduce to a single diffusion equation for the bed load section Ω_d.

$$(1-p)\frac{\partial \Omega_d}{\partial t}+\frac{\partial Q_{dx}}{\partial x}=S \tag{6}$$

which is the one-dimensional form of the Exner equation [4]. The mobile bed elevation z_b is thus computed thanks to bed load section Ω_d via the section geometry.

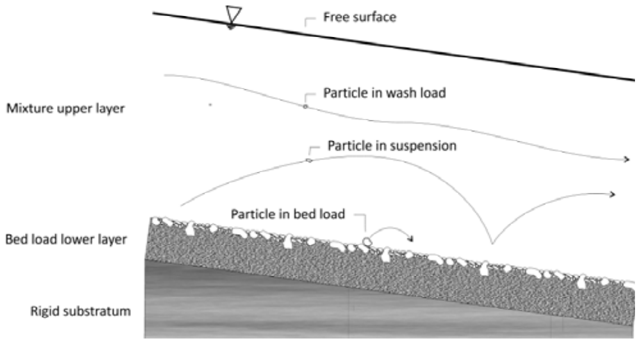

Figure 3: Main sediment transport and sketch of the conceptual model for sediment transport

3.3 Aerated Flow

In many engineering projects a strong interaction is likely to develop between the water flowing on/trough the structure and the air which is adjacent [6]. Accurate prediction of air-water interaction (also called white water) is thus an industrial necessity. the drift-flux equations (3) can be particularized to aerated flows by assuming that a single layer appears and that the air density is negligible in comparison with the water density:

$$\begin{cases} \dfrac{\partial(1-C)\Omega}{\partial t}+\dfrac{\partial(1-C)\tilde{u}_m\Omega}{\partial x}=\dfrac{1}{\rho_w}q_{m,L} \\ \dfrac{\partial C\Omega}{\partial t}+\dfrac{\partial C\tilde{u}_m\Omega}{\partial x}+\dfrac{\partial}{\partial x}\left(\dfrac{C}{1-C}\tilde{U}_{d,j}\Omega\right)=\dfrac{\Gamma_g}{\rho_g}(1-C)+q_{g,L} \\ \dfrac{\partial(1-C)\tilde{u}_m\Omega}{\partial t}+\dfrac{\partial}{\partial x}\left(\beta_{x,x}(1-C)\tilde{u}_m\tilde{u}_m\Omega+gP_\Omega\right) \\ +\dfrac{\partial}{\partial x}\left(\beta_{x,x}\dfrac{C}{(1-C)}\dfrac{\rho_g}{(1-C)}\tilde{U}_{dj}\tilde{U}_{dj}\Omega\right)=(1-C)g(i-J) \end{cases} \quad (7)$$

where C is the air concentration. In (3), the friction slope J aggregates internal and external friction as well as mixture momentum source. It must be computed by means of a two-phase friction correlation [26, 27].

5. Conclusion

In this paper, the drift-flux theory is briefly introduced and proves itself to be a convenient model to naturally unify the mathematical description of most of the transport phenomena encountered in civil and environmental engineering. In particular, the original 1D multi-layer drift-flux model is particularized to single-phase hydrodynamics, sediment transport and air entrainment. To authors' opinion, this offers a convenient way to create a computational code based on a unique set of equations to describe a wide range of environmental flows and its interaction with both the external environment and one or more dispersed phase. Such an algorithm is currently under development within the HACH unit by means of a finite volume scheme.

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