Energy and transmission allocation in overlapping electricity markets : incorporating N-1 security and accounting for losses

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Abstract: The possibility for market participants to place their bids in markets where they are not geographically located is investigated in this paper. An iterative procedure enabling the simultaneous clearing of those overlapping markets by transaction schedulers while managing the resulting pre and post-contingency congestion is proposed. Transmission losses are calculated during the iterations and are accounted for by allocating them to the various transaction schedulers. The procedure is illustrated and commented on a test system.

Keywords: electricity spot markets, market coupling, congestion management, overlapping markets, transmission loss allocation.

I. Introduction

The creation of a functioning Internal Electricity Market in Europe has been subject to lots of discussions in the last years [1]. Presently, inter-area trade is basically held through *explicit auction* mechanisms for allocation of scarce interconnection resources. This is performed via the posting by TSOs of Available Transfer Capacity (ATC) values for importing and/or exporting at each interconnection and the selling of consistent transmission rights to the market actors. Although attractive in theory, this approach has been found in practice to yield some inefficient use of the network. The main reasons are: (a) it is difficult for the participants to anticipate what the value of each transmission line will be for them, (b) some participants tend to hoard capacity that they don't finally use, and (c) pancaking of allocations appears when several borders are involved in a transaction [2].

Hence, the *implicit auction* approach for congestion management, where the scarce transmission resources are allocated implicitly at the time the energy market is cleared, is gaining more and more ground [1]. This is the main way intra-area congestion management is treated in some parts of North America, with the several pool based Locational Marginal Pricing (LMP) approaches [3]. Another implicit auction approach, called market splitting, has been used for years in Scandinavian countries where in case of congestion the market is split in two or more price areas [4]. Both the LMP and the market splitting approaches require a centralized market operator that combines the bids in a market clearing procedure. But setting up a centrally operated single electricity spot market covering the various areas in Europe does not seem practically possible at present.

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Overcoming this issue, a method that has been put recently into practice is the so-called *market coupling* method with the famous Trilateral Market Coupling (TLC) approach that couples the day-ahead markets of France, Belgium and the Netherlands since 2006. Market coupling is an implicit auction similar to market splitting but performed in reverse order: first each sub-market is cleared and then these markets are coupled. It is thus a method performing coordination among different markets, each using its own rules inside its area [5].

In the TLC approach, the day-ahead markets of the three countries are coupled. Those markets are operated by three local Power Exchanges (PXs), namely APX, Belpex and Powernext, who provide the IT systems and run the common coupling algorithm. The involved Transmission System Operators (TSOs), namely RTE, Elia and TenneT, calculate cross border capacities, set up physical exchanges, share congestion revenues and pay the market coupling service fee that is determined locally. The basic idea is that a market participant interacts only with the PX of its area, while some central calculations take care of energy being exported from low to high price areas, within the limits of transfer capacity. The extension of TLC to the five countries of the Central Western Europe region (i.e. involving also Germany and Luxembourg) has been announced for 2010.

The big advantage of TLC is that it offers to each market participant of an area an available liquidity extended throughout all three involved areas. On the other hand, one could argue that the three PXs eventually make up a monopoly on the inter-area day-ahead electricity trade; two or more market participants cannot settle an inter-area transaction without passing via their respective PX.

In [6], another way for coupling markets and handling inter-area congestion has been proposed. The approach consists in allowing participants (generators, large consumers) to bid directly in any market of the interconnection they wish, without being obliged to do so via their local operator. Transaction Schedulers (TS), each corresponding to an electricity market, settle multilateral transactions among participants, while a central entity coordinates the overall operation through interactions with the TSs. The resulting *overlapping markets* are cleared simultaneously via an iterative algorithm, with the available transmission capacity fairly shared among the TSs and the feasibility of the final schedules ensured at the same time.

In [7], the algorithm proposed in [6] has been extended, allowing market participants to place their bids simultaneously in more than one TS's markets. Transmission capacity is iteratively shared among the TSs as in [6], but an additional level of iterations is added, allocating market participants to the TSs finally scheduling them in their markets.

This paper extends the algorithm presented in [7], so that it takes into account $N-1$ security constraints, expressed in terms of linear Line Outage Distribution Factors (LODF), as well as losses, approximated as proportional to the square of a branch active power flow.

The remaining of the paper is organized as follows. First, the problem is formulated in Section II. In Sections III and IV the iterative algorithm that was developed in [7] is recalled. Section III also includes the extension to deal with $N-1$ security constraints as well. Section V presents the proposed treatment of losses. The overall algorithm is illustrated through a small example in Section VI, while some conclusions are provided in Section VII.

II. Problem formulation

Let M be the number of TSs. Each TS clears the market it represents, using its own rules. The outcomes are scheduled generation-load quantities together with the corresponding prices offered to each generator or asked by each load.

The clearing is typically formalized as an optimization problem. For the mth TS it takes on the form:

$$\min_{\mathbf{g}_m,\mathbf{d}_m} \{\mathbf{c}_m^T\mathbf{g}_m - \mathbf{b}_m^T\mathbf{d}_m\} \tag{1}$$

$$\text{s. t.} \qquad \mathbf{1}^T\mathbf{g}_m = \mathbf{1}^T\mathbf{d}_m \tag{2}$$

$$\mathbf{0} \leq \mathbf{g}_m \leq \overline{\mathbf{g}}_m \tag{3}$$

$$\mathbf{0} \leq \mathbf{d}_m \leq \overline{\mathbf{d}}_m \tag{4}$$

where $\mathbf{c}_m$ (respectively $\mathbf{b}_m$) is a vector containing the bids of all generators (consumers) bidding in market m, $\mathbf{g}_m$ ($\mathbf{d}_m$) contains the quantities of generators (consumers) dispatched by the mth TS, $\mathbf{1}$ is a unit column vector, $(\overline{\mathbf{g}}_m)_i$, i.e. the ith element of vector $\overline{\mathbf{g}}_m$, is the maximum power that generator i is willing to produce for market m and equivalently for $(\overline{\mathbf{d}}_m)_j$. Equation (2) expresses that each TS has a balanced schedule.

The vector of net bus power injections is obtained as the summation of all the TS schedules:

$$\mathbf{n} = \sum_m \mathbf{n}_m = \sum_m \mathbf{g}_m - \sum_m \mathbf{d}_m \tag{5}$$

Once this vector is known, branch power flows can be computed using a model of the entire network. In this paper a DC model of the interconnection is used. This is a commonly used model in market clearing problems and it is well suited to the linear computations presented in the remaining of the paper. It is assumed that the various TSOs in the interconnection assemble and share such a network model, which is used to coordinate the overlapping markets simultaneous clearings.

Let B be the number of branches and N the number of buses in the system. In order to assess the impact of the power injection schedule on branch flows, we resort to the well-known Power Transfer Distribution Factors (PTDF)[4]. This yields the following relationship in matrix form:

$$\mathbf{p} = \mathbf{S}\ \mathbf{n} \tag{6}$$

where $\mathbf{p}$ is the vector of branch power flows and $\mathbf{S}$ is the $B\times N$ matrix relating branch power flows to bus power injections, and defined by:

$$(\mathbf{S})_{bk} = \frac{X_{ik} - X_{jk} - X_{iN} + X_{jN}}{x_b} \tag{7}$$

where i and j are the terminal buses of branch b, x_b its reactance, X_{ik} the entry in the ith row and kth column of the $N \times N$ bus reactance matrix $\mathbf{X}$, and similarly for the other entries. Assuming that bus N is the slack bus, the Nth row and the Nth column of $\mathbf{X}$ have all zero elements.

The congestion management problem dealt with in [6], [7] consists in coordinating the M independent and simultaneous market clearings so that the resulting injection schedules $\mathbf{n}_m$ yield feasible branch flows, i.e. satisfy the constraints:

$$-\overline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \tag{8}$$

where $\overline{\mathbf{p}}$ denotes the branch maximum acceptable flows.

Additionally, a basic security requirement in power system operation is that the system should be able to withstand any loss of one single element without entering into an emergency situation. For the here-presented problem of clearing overlapping markets, the above $N-1$ security requirement has been translated into the constraint that the power flows resulting from the loss of any branch do not overload any of the remaining branches.

Following the choice of a linear network model, we resort to well-known Line Outage Distribution Factors (LODF) [4]. For each branch, these factors result from the PTDFs of the system configuration with and without the branch under question. The LODFs are linear sensitivities, each of them giving the fraction of the power flowing in a branch v before its outage, that is flowing in branch b after the outage. Let $\mathbf{L}$ the $B \times B$ matrix of LODFs and p_b^v the flow in branch b that results after the outage of branch v. We have:

$$\Delta p_b^v = p_b^v - p_b = (\mathbf{L})_{bv} p_v \tag{9}$$

where p_b and p_v are the bth and vth branch flows before the outage of branch v. By definition of $\mathbf{L}$, we have $(\mathbf{L})_{bb} = -1$.

In (9) the pre-outage flows can be substituted by (6), which yields the post-outage flow as a linear function of the injection schedule:

$$p_b^v = (\mathbf{L})_{bv}\mathbf{s}_v\mathbf{n} + \mathbf{s}_b\mathbf{n} = \left((\mathbf{L})_{bv}\mathbf{s}_v + \mathbf{s}_b\right)\mathbf{n} \tag{10}$$

where $\mathbf{s}_b$ is the bth row of the $\mathbf{S}$ matrix.

The $N-1$ security criterion requires to check for each of the B branches the $B-1$ power flows that take place after the outage of another branch. Thus, for each pair (b, v) we check a security constraint of the type:

$$-\alpha\ \overline{p}_b \leq p_b^v \leq \alpha\ \overline{p}_b \tag{11}$$

where $\alpha \geq 1$ accounts for possible overload allowed in post-contingency situation (typically $1.05 \leq \alpha \leq 1.1$).

Using (10) for every post-outage flow p_b^v yields a linear relationship between the post-contingency flows and the pre-contingency bus power injections.

The satisfaction of the B constraints in (8) as well as the $B \times (B-1)$ constraints of type (11) makes up the congestion management problem dealt with in this paper.

III. Transmission allocation loop

Initially the M TSs clear their markets independently of each other. The bus injection vector defined in (5) is thus available. The corresponding branch flows, as well as all post-outage flows, can be computed. If no limit is violated, the TS schedules can be approved. Otherwise, congestions are managed through an iterative procedure, referred to as "Transmission allocation loop", and detailed hereafter.

Assume that the power flow p_b in the bth branch exceeds its upper limit:

$$p_b = \widehat{p}_b > \overline{p}_b \tag{12}$$

Using Eqs. (5) and (6), this inequality can be rewritten as:

$$\sum_m \mathbf{s}_b\, \widehat{\mathbf{n}}_m > \overline{p}_b \tag{13}$$

where $\widehat{\mathbf{n}}_m$ is the schedule of the mth TS, obtained as described in the previous section. It turns out that $\mathbf{s}_b\widehat{\mathbf{n}}_m$ is the participation of the mth TS in the bth branch flow. Obviously, all TS participations add up to the infeasible branch flow $\widehat{p}_b$.

From there on, the mth TS is required to change its schedule from $\widehat{\mathbf{n}}_m$ to a new value $\mathbf{n}_m$ so that its contribution to the branch flow p_b is decreased by at least a specified amount Δp_m:

$$\mathbf{s}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq -\Delta p_m \tag{14}$$

with the sum of all Δp_m values being equal to the branch overload to be corrected:

$$\sum_m \Delta p_m = \sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m \; - \; \overline{p}_b \tag{15}$$

If both (14) and (15) hold true, the power flow will become:

$$\sum_m \mathbf{s}_b\mathbf{n}_m \leq \sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m - \sum_m \Delta p_m = \overline{p}_b \tag{16}$$

i.e. it will be decreased below or at its limit.

The same constraint decomposition is performed for all overloaded branches. Thus, each TS is required to incorporate constraints of type (14) into its market clearing problem (1-4). In fact, (14) can be rewritten as:

$$\mathbf{s}_b\mathbf{n}_m \leq \mathbf{s}_b\widehat{\mathbf{n}}_m - \Delta p_m \tag{17}$$

where the left-hand side represents the flow produced in branch b by the schedule of the mth TS, and the right-hand side can be interpreted as a reduced capacity allocated to that TS. Branch overloads with opposite signs, i.e. with $\widehat{p}_b < -\overline{p}_b$, are treated in a similar way.

It is suggested that a coordinator is given the role to collect the various TS intended schedules, check for possible resulting constraint violations and, if needed, communicate to the TSs constraints of type (14). Clearly, the choice of the Δp_m values dictates the way transmission capacity will be eventually allocated to the TSs.

As discussed in the Introduction, an implicit auction approach, in which limited transmission capacity is allocated in the course of clearing the (multiple) markets, is chosen to deal with congestion management. In addition, only little commercially-sensitive information, such as the cleared schedules from TSs, are expected to be communicated between involved parties for reasons of confidentiality and of practical implementability. In this context, it is proposed to allocate transmission capacity to TSs in proportion to their respective utilizations of the congested branches.

Coming back to the overloaded branch b, this choice suggests that the constraint (17), reflecting the share of the transmission capacity among the TSs, should be:

$$\mathbf{s}_b\mathbf{n}_m \leq \frac{\mathbf{s}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m}\overline{p}_b \tag{18}$$

The above equation is equivalent, as can be shown by using (15) and (17), to choosing Δp_m such that:

$$\frac{\Delta p_m}{\sum_m \Delta p_m} = \frac{\mathbf{s}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m} \tag{19}$$

Equation (18) suggests that the more a TS is using a congested branch the more it has the right to keep on using it. This goes towards increasing efficiency: the more a TS uses a branch, the more this is likely to be valuable for its schedule.

On the other hand, (18) can be rewritten as

$$\mathbf{s}_b(\mathbf{n}_m - \widehat{\mathbf{n}}_m) \leq \frac{\mathbf{s}_b\widehat{\mathbf{n}}_m}{\sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m}(\overline{p}_b - \widehat{p}_b)$$

which shows that the more a TS participates in a congestion, the more it has to participate in its alleviation. This meets the objective of fairness and practical acceptability of the policy: the larger the responsibility of a TS in a flow, the larger the correction requested from this TS.

These two interpretations of (18) may look contradictory at a first glance but are mathematically equivalent owing to the choice of proportionality. Further discussion regarding this choice can be found in [7].

Let us assume now that an $N-1$ constraint is violated, i.e. for the given injections $\widehat{\mathbf{n}}_m$ we have:

$$\sum_m \left((\mathbf{L})_{bv}\mathbf{s}_v + \mathbf{s}_b \right)\, \widehat{\mathbf{n}}_m > \alpha\, \overline{p}_b \tag{20}$$

for a pair (b, v).

One can see that this constraint violation depends on the values of two branch flows, namely $\widehat{p}_b = \sum_m \mathbf{s}_b\widehat{\mathbf{n}}_m$ and $\widehat{p}_v = \sum_m \mathbf{s}_v\widehat{\mathbf{n}}_m$. The post-outage overload can be managed by decreasing the pre-outage flow in either of the two involved branches.

In the same way as we previously defined the participation of the mth TS in the bth branch's flow as $\mathbf{s}_b\widehat{\mathbf{n}}_m$, we can now define the participation of the mth TS in the overload of the bth branch after the outage of the vth one as $\left((\mathbf{L})_{bv}\mathbf{s}_v + \mathbf{s}_b \right)\, \widehat{\mathbf{n}}_m$. Again, all TS participations add up to the post-outage overload (20).

The effort to alleviate the congestion is again shared among the TSs, with the coordinator being assigned the task to communicate to every TS a constraint involving only its own injection schedule in a way that if all TS satisfy their constraints then the initial overload is cleared, as was the case with (17). Let us call $\Delta\widetilde{p}_m$ the amount by which the mth TS is requested to contribute to the congestion alleviation. Note that this change refers to a post-outage flow, while the TS is requested to modify its pre-outage schedule. This means that $\Delta\widetilde{p}_m$ can be obtained from a $\Delta\widetilde{p}_m$ change of the TS's participation in the bth branch flow, or by a $\Delta\widetilde{p}_m/(\mathbf{L})_{bv}$ change of its participation in the vth branch flow, or by an equivalent change involving both flows.

The policy used to allocate the transmission capacity remains that of contributing proportionally to the participation in the (now post-outage) overload, i.e. $\Delta\widetilde{p}_m$ is such that:

$$\frac{\Delta\widetilde{p}_m}{\sum_m((\mathbf{L})_{bv}\mathbf{s}_v+\mathbf{s}_b)\widehat{\mathbf{n}}_m-\alpha\overline{p}_b}=\frac{((\mathbf{L})_{bv}\mathbf{s}_v+\mathbf{s}_b)\widehat{\mathbf{n}}_m}{\sum_m((\mathbf{L})_{bv}\mathbf{s}_v+\mathbf{s}_b)\widehat{\mathbf{n}}_m} \tag{21}$$

and the mth TS will have to clear again its market with the additional constraint:

$$((\mathbf{L})_{bv}\mathbf{s}_v+\mathbf{s}_b)\left(\mathbf{n}_m-\widehat{\mathbf{n}}_m\right)\le-\Delta\widetilde{p}_m \tag{22}$$

A similar approach is followed for branches with $\sum_m\left((\mathbf{L})_{bv}\mathbf{s}_v+\mathbf{s}_b\right)\ \widehat{\mathbf{n}}_m<-\alpha\ \overline{p}_b$.

Iterations are performed between market clearings by the TSs, on one hand, and Transmission allocation by the coordinator, on the other hand, until an equilibrium is reached. At every iteration constraints of type (14) and/or (22) may be added if needed. After a branch overload has been solved as described above, it should be prevented from taking place again in subsequent iterations, for instance when other branch overloads are handled. To this purpose, the inequality constraints (14) and (22) stemming from previous congestion managements remain in effect when dealing with new congestions.

IV. Energy allocation loop

As discussed in [7], an issue that could be raised with the above described overlapping market structure has to do with the risk for the final schedule to be far from what could be reached by optimizing the whole system as a single market. The reason is that some attractive market participants (e.g. cheap generators), having placed their bids in a market, may be excluded when the latter is cleared, and thus remain inactive while they could still be used by another TS to reach a better schedule.

The proposed solution consists in allowing market participants to place their bids in more than one market simultaneously. After the market clearings, the participant should be allocated to the TS from which it received the best offer (the highest price to be paid if it is a generator, or the lowest price to pay if it is a consumer). Price is the criterion used to eventually select which TS a particular participant will be assigned to.

An iterative procedure, referred to as “Energy allocation loop”, is implemented by the coordinator to allow the previously mentioned simultaneous dispatching of the market participants by all the TSs.

The procedure starts with the market participants placing their bids, each consisting of a maximum quantity (corresponding to available generation or to load asking to be served) and one price per TS. After having cleared its market, the mth TS communicates to the coordinator its demanded bus generation vector $\mathbf{g}^o_m$ and consumption vector $\mathbf{d}^o_m$, together with the corresponding price vectors $\boldsymbol{\pi}^g_m$ and $\boldsymbol{\pi}^d_m$.

For a given generator i, if the total power demanded by the various TSs is below (or equal to) its capacity, i.e. $\sum_m(\mathbf{g}^o_m)_i\ \le\ (\overline{\mathbf{g}})_i$, that power is simply allocated to the various TSs as they requested. Otherwise, there is a conflict, and the role of the coordinator is to take care that the generator is finally dispatched at the most profitable possible prices. To this purpose, the coordinator allocates the power to one or several of the involved TSs by decreasing order of offered price. In case several TSs compete for the same generator with equal offered prices, the available power is shared in proportion with the requested quantities.

Hence, generally, some TSs will be left with power imbalances, and the markets have to be cleared again. In order the power just allocated to a TS not to be available to the others, the coordinator communicates reduced bounds $(\overline{\mathbf{g}}_m)_i$ to the latter TSs.

Thus, the TSs come up with new demanded quantities and offered prices. At this stage, the coordinator repeats the above procedure, with the following two additional rules:

1) what was previously allocated to a TS and is still requested remains with that TS;
2) what was previously allocated to a TS and is not requested any longer is made available to the other TSs.

These iterative adjustments lead to a gradual allocation of all demanded generations. Loads are handled in a similar way, but with the allocation performed by increasing order of prices requested by the TSs in order to serve them.

The procedure terminates when each market is balanced, no TS has incentive to further improve its schedule by dispatching available generation or load, and no conflict is left for any resource. The coordinator can now use the bus injections stemming from TS allocated power quantities to proceed with the execution of the Transmission allocation loop.

The overall procedure, thus, consists of two loops, an inner (the Energy allocation) and an outer one (the Transmission allocation)[7].

V. Accounting for losses

In the Energy and Transmission allocation procedure presented down to here the transmission system has been assumed lossless. However, losses correspond to a non negligible percentage of the energy production. Thus, it is appropriate, when scheduling generation and allocating transmission capacity, to also account for losses. The viewpoint adopted in this paper is that each TS should be assigned the responsibility for the losses it “creates” due to its schedule.

Typically, in DC model-based operations, losses are estimated and redistributed as negative bus power injections throughout the system. In the absence of an accurate estimate of losses, we resort to an approach where the estimation of losses is performed iteratively while clearing the market [8], [9].

Initially the branch flows are computed according to a lossless model, as in (6). Then, the losses l_b in each branch b are calculated using the approximation $l_b = r_b p_b^2$, where r_b is the branch's series resistance. Those branch losses are translated into bus power withdraws, to be treated as loads at the next iteration. To this purpose, an additional power withdraw $l_b/2$ is assigned to each end bus of the branch. New generation schedules are then computed in order to compensate for the additional “loads” and branch flows are again computed using (6). The branch losses can then be updated based on the new flows, and so on. The procedure is fast, it usually converges after three iterations.

The above technique can be easily applied to the overlapping market problem, taking advantage of the already iterative market clearing procedure to incorporate the updated bus withdraws accounting for losses. This is easily added to the Transmission allocation loop; the coordinator, after computing the branch flows, calculates the corresponding losses as well. But, since each of the M markets is power balanced, a mechanism is needed to share among the various TS the additional generation needed to cover the bus withdraws stemming from losses.

It is well known that responsibility for losses cannot be assigned to market participants in a undisputable way (they depend on the combined result of all bus injections). This is easily seen by using (5), (6) in the branch losses formula:

$$\begin{aligned} l_b &= r_b \left(\sum_m \mathbf{s}_b \mathbf{n}_m \right)^2 \\ &= r_b \left(\sum_m (\mathbf{s}_b \mathbf{n}_m)^2 + \sum_m \sum_{k \neq m} (\mathbf{s}_b \mathbf{n}_m)(\mathbf{s}_b \mathbf{n}_k) \right) \end{aligned} \tag{23}$$

When allocating the losses to the various TSs, it seems straightforward to allocate each term $r_b(\mathbf{s}_b\mathbf{n}_m)^2$ to the mth TS. On the other hand, terms involving two TSs, i.e. $r_b(\mathbf{s}_b\mathbf{n}_m)(\mathbf{s}_b\mathbf{n}_k)$, need to be shared among them. In [10], the authors argue that it is not always fair to just equally divide each such term between the two TSs, which would, for instance, suggest that the mth TS is allocated $\frac{r_b}{2}(\mathbf{s}_b\mathbf{n}_m)(\mathbf{s}_b\mathbf{n}_k)$ of responsibility for the term it shares with the kth one. Different ways for allocating the bilinear terms have been proposed.

We have followed the idea of allocating the bilinear term in proportion to the square of each TS participation in the branch flow. The motivation for this choice is the quadratic relationship between power flows and losses. Hence, every bilinear term is assigned as follows:

$$\text{to the } m\text{th TS:} \quad \frac{(\mathbf{s}_b\mathbf{n}_m)^2}{(\mathbf{s}_b\mathbf{n}_m)^2 + (\mathbf{s}_b\mathbf{n}_k)^2}\, r_b(\mathbf{s}_b\mathbf{n}_m)(\mathbf{s}_b\mathbf{n}_k)$$

$$\text{to the } k\text{th TS:} \quad \frac{(\mathbf{s}_b\mathbf{n}_k)^2}{(\mathbf{s}_b\mathbf{n}_m)^2 + (\mathbf{s}_b\mathbf{n}_k)^2}\, r_b(\mathbf{s}_b\mathbf{n}_m)(\mathbf{s}_b\mathbf{n}_k)$$

Thus, coming back to the loss allocation mechanism performed in the Transmission allocation loop, the coordinator, after computing the branch flows, allocates the branch losses to the various TS and, together with the congestion management constraints, it communicates to the TSs the corresponding bus withdraws to cover in their new market clearings.

VI. Illustrative example

For clarity, we illustrate the features of the proposed approach on a problem where: (i) the loads are considered inelastic, i.e. only the generators are bidding, and (ii) each of the involved TSs serves the load of an area. Note that the method is generally able to handle situations where each TS serves loads dispersed throughout the system, or some loads place their bids simultaneously to more than one TSs. Thus, each TS dispatches generation, located anywhere in the interconnection, so as to satisfy the load located in its area.

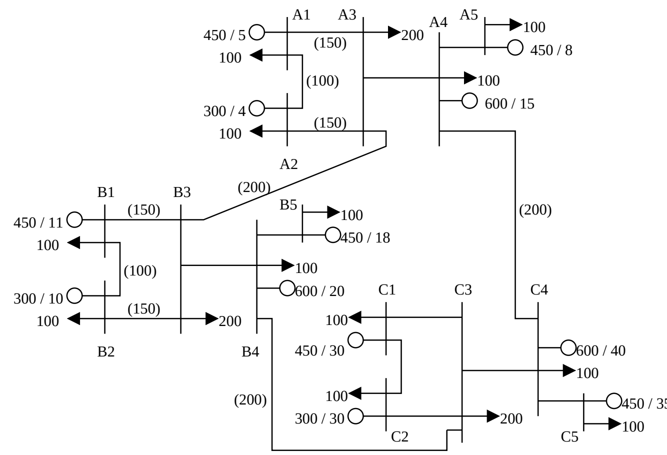

Fig. 1. Three-area test system (some branch capacities (in MW) are shown in parenthesis next to the corresponding branches, while next to each generator its available capacity (in MW) as well as its bid (in €/h) are shown).

The three-area 15-bus system shown in Fig. 1 is used. It consists of three five-bus areas, identified from the bus names, each of them serving 600 MW of load. The three areas are identical as regards the distribution of loads and the location and capacity of generators. However, they differ by the generator bids, which are the cheapest in area A and the most expensive in area C. Each generator is assumed to make the same bid to all three TSs (i.e. $(\mathbf{c}_A)_i = (\mathbf{c}_B)_i = (\mathbf{c}_C)_i$). A market clearing pricing mechanism has been used by all TSs to come up with their offered to generators prices.

The generation schedules resulting after an execution of the procedure are shown in Tables I and II, with and without losses accounted for, respectively. Columns 3-5 present the generation quantities that each TS finally managed to dispatch, while column 6 is the sum of the three, making up the total quantity dispatched for each generator. In column 7, the generation schedule obtained by performing a single market clearing over the whole system is shown. In this single clearing all branch and post-outage branch flow limits are incorporated as inequality constraints in the optimization problem. To this purpose, the same PTDFs and LODFs are used as for the proposed iterative procedure. Losses are accounted for by resorting to the iterative method explained in Section V. The last column of the table presents, for comparison, the total generation schedules that resulted after an execution of the procedure without considering $N-1$ constraints. The resulting generation costs, for all the above mentioned cases, are grouped in Table III.

The fact that each TS finally allocated most of its generation from inside the area where its load is located is indicative of the followed congestion management rule; during the iterations

TABLE I
FINAL GENERATION ALLOCATION (IN MW); NO ACCOUNT FOR LOSSES

Gen	Bid	incorporating $N-1$ constraints TS A	TS B	TS C	Total	Single	without $N-1$
gA1	5	105	25	25	155	155	249
gA2	4	110	50	50	210	210	250
gA5	8	232	115	74	421	420	382
gB1	11	43	100	0	143	155	157
gB2	10	110	100	0	210	210	300
gB5	18	0	210	71	281	270	262
gC1	30	0	0	210	210	210	0
gC2	30	0	0	155	155	155	0
gC5	35	0	0	15	15	15	200

TABLE II
FINAL GENERATION ALLOCATION (IN MW); ACCOUNTING LOSSES

Gen	Bid	incorporating $N-1$ constraints TS A	TS B	TS C	Total	Single	without $N-1$
gA1	5	105	25	25	155	155	250
gA2	4	110	50	50	210	210	251
gA5	8	229	116	83	428	428	391
gB1	11	48	100	7	155	155	156
gB2	10	65	100	45	210	210	300
gB5	18	46	214	12	272	273	267
gC1	30	0	0	210	210	210	0
gC2	30	0	0	155	155	155	0
gC5	35	0	0	17	17	17	207

TABLE III
FINAL GENERATION COSTS (IN €/H)

$N-1$	losses	TS A	TS B	TS C	Total	Single
yes	no	4395	7127	13675	25197	25115
	yes	4817	7210	13271	25298	25300
no	no	4093	6467	11184	21743	21300
	yes	4155	6572	11416	22142	21568

of the procedure each TS had been assigned the effort to alleviate congestion caused by itself trying to import in its area. Expectedly, the additional consideration of post-outage branch flow limits (see Eq. (11)) decreases, compared to the case when only pre-outage limits are considered (Eq. (8)), the inter-area transmission capacities and, thus, obliges the TSs to resort to generation located "closer" to the loads they serve. This results in higher generation costs. Finally, one can see that accounting for transmission losses results in all TSs dispatching some additional generation, leading to a small augmentation of the resulting costs.

The intention of the proposed method is not to perform a single system-wide market clearing in a decentralized way, but rather to allow participants to freely settle transactions over a common transmission grid. However, the fact that it seems to result in schedules with overall cost close to the theoretical minimum (i.e. single overall clearing) appears to be an attractive feature, since the opposite would suggest that interesting generation capacities were not exploited.

VII. CONCLUSION

A transaction-based, decentralized market paradigm has been outlined in this paper. In its essence, it is an approach where market participants are left free to settle multilateral transactions (done via the intervention of generic entities called TSs) without them being dispatched by a central entity. Whether it is preferable to operate a market in a centralized manner or coordinate multilateral trades, has been extensively discussed in the literature. A major advantage of the centralized approach is that transmission network constraints are taken care of implicitly when clearing the energy market. The choice/need for centralization stems from the difficulty to efficiently coordinate multilateral trades being simultaneously scheduled; it is not an objective by itself. On the contrary, it goes with the principle of free trading to let market participants the option to buy and sell electric energy in the terms they agree between themselves. However, given the transmission network constraints that couple the different transactions, it is more challenging to coordinate them in a decentralized way.

This is the intention of the procedure proposed in this paper. Both pre- and post-outage contingencies are implicitly handled in the approach, while losses are also taken care of, at the same time, by updating them within the iterations and allocating them to the various TSs. According to the presented results, the transmission network is efficiently used.

It should be noted that the Energy allocation loop can be viewed as an optional feature of the procedure, offering the extra possibility of market participants being a priori available for the most profitable dispatch. The algorithm works as well with the Energy allocation omitted; in this case, the remaining Transmission allocation loop coordinates the use of the network among the TSs, each of them dispatching purely its own participants.

Issues of further interest are the possibly constraining time for the iterations to converge, the exposure of the procedure to gaming, as well as the accommodation of complex bid structures.

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