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Coordinate rotation

Bernard Heinesch

Outline

- Need for rotations
- Mathematical tools for coordinate rotation
- Angle determination
  - 'Rotated every-period'
    - Classical 2D rotation
    - 3rd rotation
  - 'Long-term'
    - Planar fit method
    - Lee method
    - Method by sectors
- Comparison between methods

Need for rotations

We start from the sonic coordinate system :

independent of the flow field

(modern sonics output wind components in an orthonormal frame)

We end with the analysis coordinate system :

defined using the flow field

(Aligns the *z*-axis perpendicular to the mean streamlines)

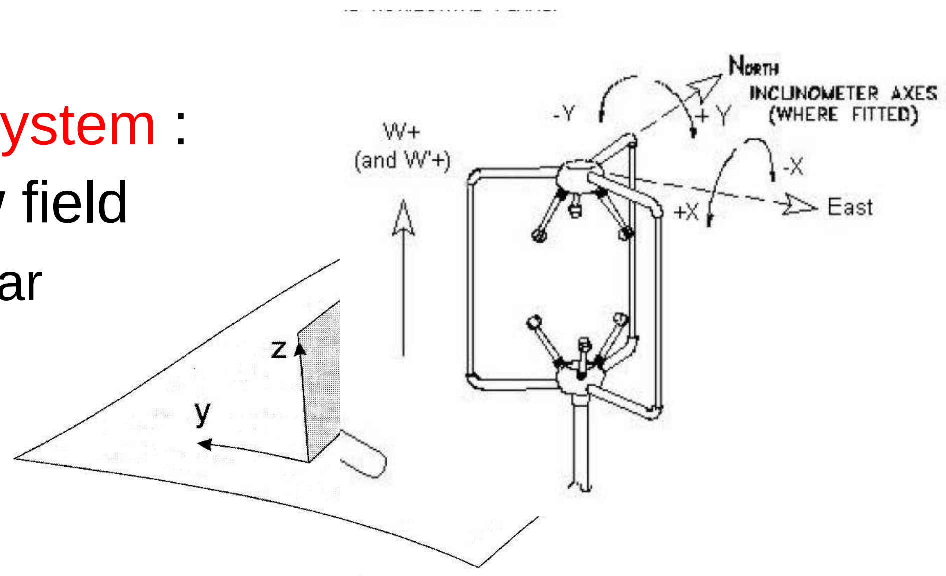

Figure 3.1. The coordinate system should be such that the local normal to the surface and the mean scalar gradient $\nabla \bar{c}$ lie in the x-z plane.

(From Finnigan, 2004)

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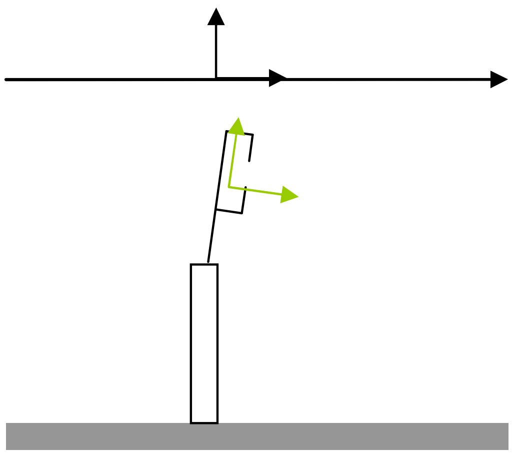
Need for rotations

Illustration with a tilted sonic.

If the sonic is not levelled, a part of the *w*' will be found in *u*'. The rotation scheme is intended to level the sonic anemometer to the terrain surface and thus avoid cross-contamination between the eddy flux components



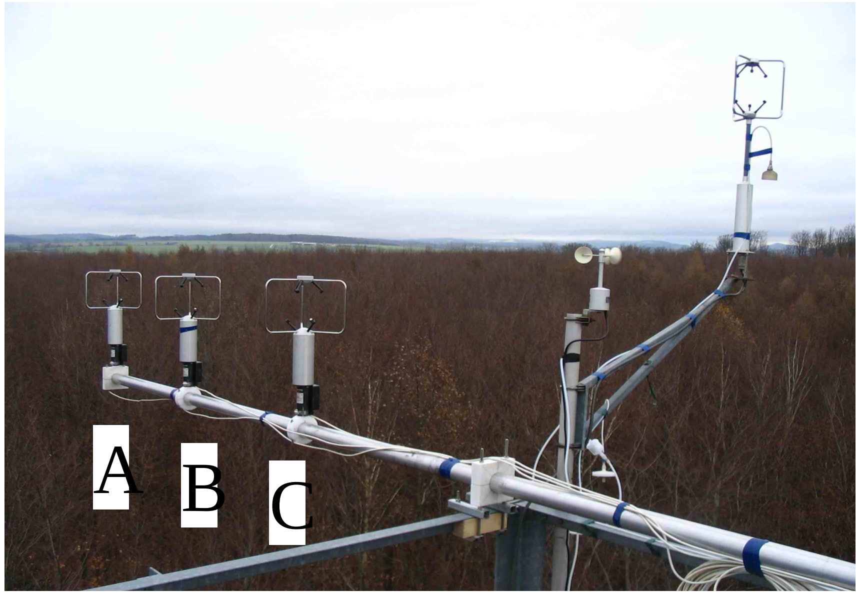
$$\frac{\partial \overline{c}}{\partial t} + \left(\frac{\partial \overline{u'c'}}{\partial x} + \frac{\partial \overline{v'c'}}{\partial y} + \frac{\partial \overline{w'c'}}{\partial z} \right) = \overline{S}_M$$

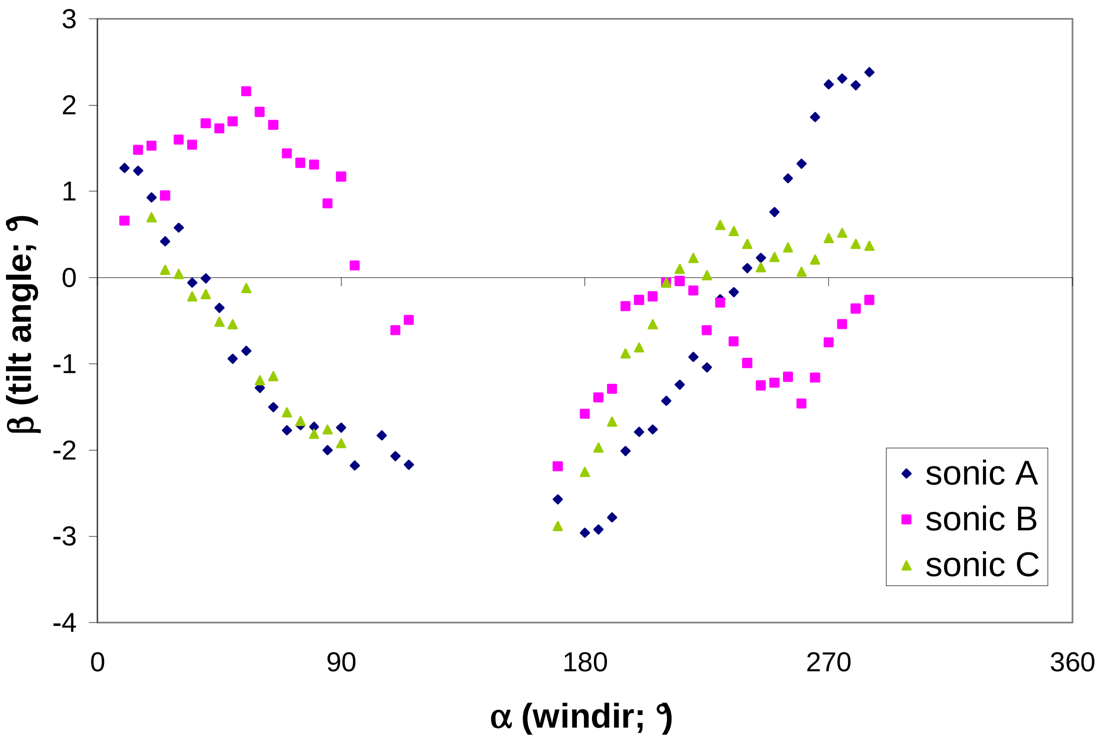
⇒ Coordinate rotation is a necessary step before the observed fluxes can be meaningfully interpreted

Need for rotations

Comparison of sonics (HESSE 2006)

Difficult (impossible) to align the sonic coordinate frame with an objective reference frame reffered to the local terrain





Need for rotations

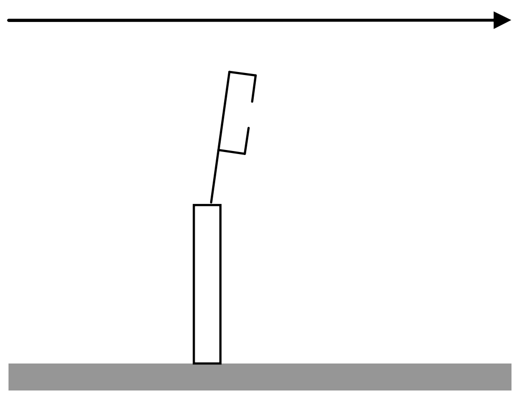
Illustration with a tilted sonic.

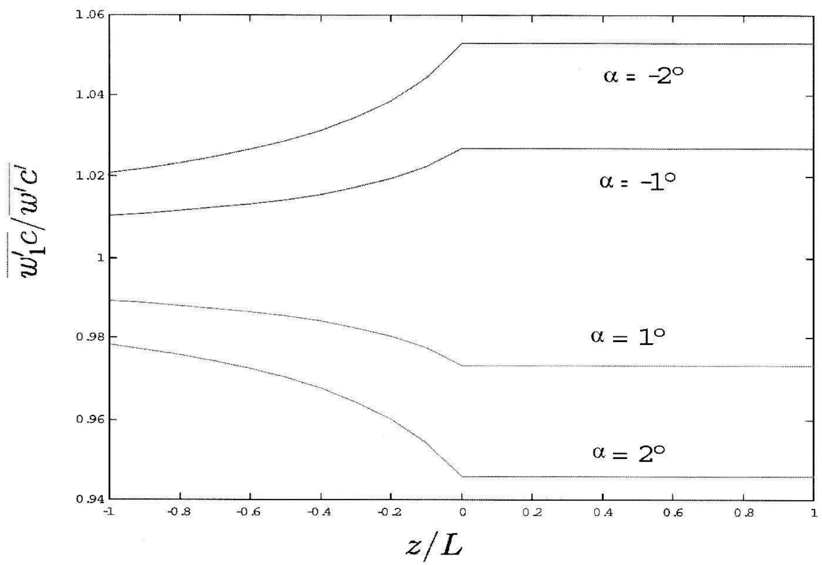
Flux bias due to tilt error :

momentum flux : > 10 % per degree !

scalar eddy flux : < 5 % for tilts below 2°

but possibility of systematic errors (From Lee, Handbook, 2004)





Need for rotations

Why do we rotate coordinates ?

The NEE should not depend on the coordinate frame if we were able to measure accurately all the terms.

In practice, we cannot measure all the terms II+V, thus we have to work in a coordinate frame that will optimizes our ability to estimate II+V , using the terms we can measure.

$$\underbrace{\frac{\partial \bar{c}}{\partial t}}_{I}+\underbrace{\bar{u}\frac{\partial \bar{c}}{\partial x}+\bar{v}\frac{\partial \bar{c}}{\partial y}+\bar{w}\frac{\partial \bar{c}}{\partial z}}_{II}+\underbrace{\left(\frac{\partial \overline{u'c'}}{\partial x}+\frac{\partial \overline{v'c'}}{\partial y}+\frac{\partial \overline{w'c'}}{\partial z}\right)}_{V}=\underbrace{\bar{S}_M}_{IV}$$

Mathematical tools for coordinate rotation

3 rotations are needed to convert the components of a vector ($\vec{U}$) from one coordinate system (sonic frame: subscript '0') to another (analysis frame to be defined later : subscript '3') :

$$\vec{U} \equiv (u_0, v_0, w_0) \Rightarrow \vec{U} \equiv (u_3, v_3, w_3)$$

$$\text{Rotation 1:} \vec{U} \equiv (u_0, v_0, w_0) \Rightarrow \vec{U} \equiv (u_1, v_1, w_1)$$

$$\text{Rotation 2:} \vec{U} \equiv (u_1, v_1, w_1) \Rightarrow \vec{U} \equiv (u_2, v_2, w_2)$$

$$\text{Rotation 3:} \vec{U} \equiv (u_2, v_2, w_2) \Rightarrow \vec{U} \equiv (u_3, v_3, w_3)$$

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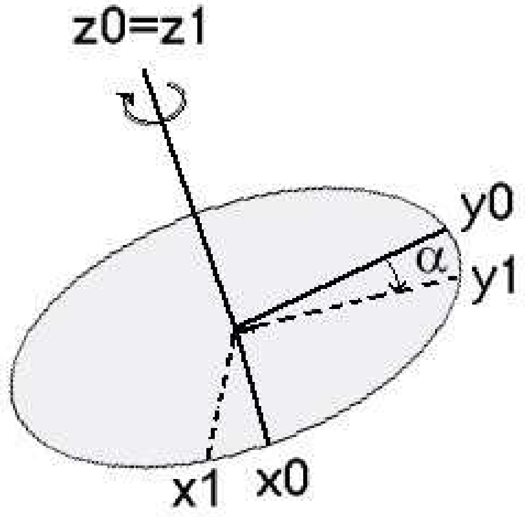
Rotation 1

Around *z*-axis with an angle α (yaw angle) :

$$\bar{u}_1 = \bar{u}_0 \cos\alpha + \bar{v}_0 \sin\alpha$$
$$\bar{v}_1 = -\bar{u}_0 \sin\alpha + \bar{v}_0 \cos\alpha$$
$$\bar{w}_1 = \bar{w}_0$$

$$\begin{pmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{w}_1 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{R_{01}} \begin{pmatrix} \bar{u}_0 \\ \bar{v}_0 \\ \bar{w}_0 \end{pmatrix}$$

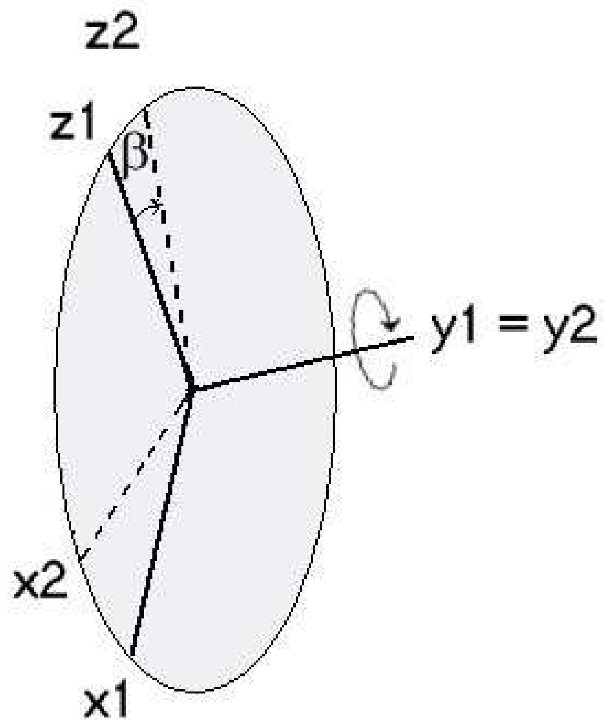


Rotation 2

Around new *y*-axis with an angle β (pitch angle) :

$$\overline{u}_2 = \overline{u}_1 \cos\beta + \overline{w}_1 \sin\beta$$
$$\overline{v}_2 = \overline{v}_1$$
$$\overline{w}_2 = -\overline{u}_1 \sin\beta + \overline{w}_1 \cos\beta$$

$$\begin{pmatrix} \overline{u}_2 \\ \overline{v}_2 \\ \overline{w}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}}_{R_{12}} \begin{pmatrix} \overline{u}_1 \\ \overline{v}_1 \\ \overline{w}_1 \end{pmatrix}$$

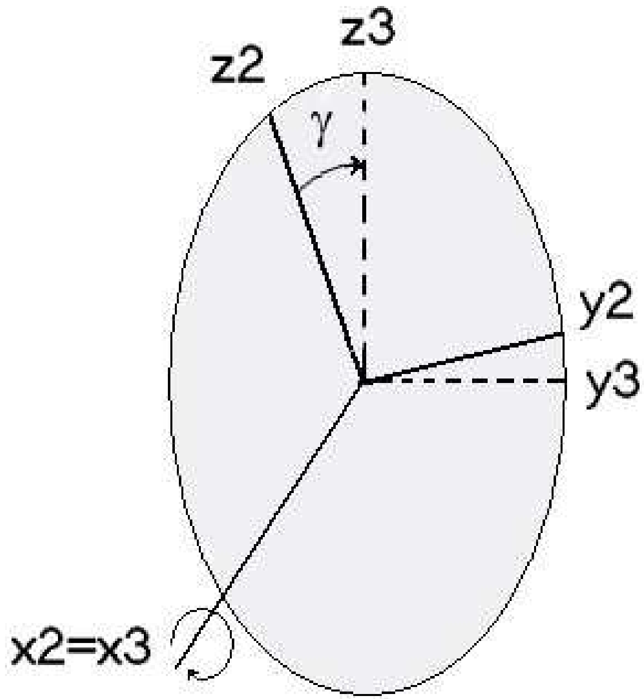


Rotation 3

Around new *x*-axis with an angle γ (roll angle) :

$$\overline{u}_3 = \overline{u}_2$$
$$\overline{v}_3 = \overline{v}_2 \cos\gamma + \overline{w}_2 \sin\gamma$$
$$\overline{w}_3 = -\overline{v}_2 \sin\gamma + \overline{w}_2 \cos\gamma$$

$$\begin{pmatrix} \overline{u}_3 \\ \overline{v}_3 \\ \overline{w}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{pmatrix}}_{R_{23}} \begin{pmatrix} \overline{u}_2 \\ \overline{v}_2 \\ \overline{w}_2 \end{pmatrix}$$



Rotation 123

Transform coordinates between the sonic frame anf the analysis frame :

$$\begin{pmatrix} \overline{u}_3 \\ \overline{v}_3 \\ \overline{w}_3 \end{pmatrix} = R_{23}(\gamma) \cdot R_{12}(\beta) \cdot R_{01}(\alpha) \begin{pmatrix} \overline{u}_0 \\ \overline{v}_0 \\ \overline{w}_0 \end{pmatrix}$$

Possible to go back to the sonic frame by making the reverse rotations :

$$\begin{pmatrix} \overline{u}_0 \\ \overline{v}_0 \\ \overline{w}_0 \end{pmatrix} = R_{01}(\alpha)^{-1} \cdot R_{12}(\beta)^{-1} \cdot R_{23}(\gamma)^{-1} \begin{pmatrix} \overline{u}_3 \\ \overline{v}_3 \\ \overline{w}_3 \end{pmatrix}$$

N.B. Rotations can be applied at the end of the averaging period !

For the second moments

Covariances with scalar :

$$\begin{pmatrix} \overline{u_1'c'} \\ \overline{v_1'c'} \\ \overline{w_1'c'} \end{pmatrix} = R_{01} \begin{pmatrix} \overline{u_0'c'} \\ \overline{v_0'c'} \\ \overline{w_0'c'} \end{pmatrix}$$

(Co)variances of the wind components (Reynolds stress tensor) :

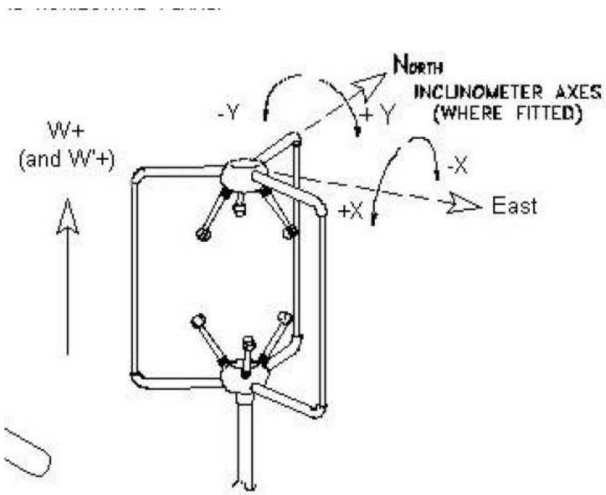
$$\begin{pmatrix} \overline{u_1'u_1'} & \overline{u_1'v_1'} & \overline{u_1'w_1'} \\ \overline{v_1'u_1'} & \overline{v_1'v_1'} & \overline{v_1'w_1'} \\ \overline{w_1'u_1'} & \overline{w_1'v_1'} & \overline{w_1'w_1'} \end{pmatrix} = R_{01} \begin{pmatrix} \overline{u_0'u_0'} & \overline{u_0'v_0'} & \overline{u_0'w_0'} \\ \overline{v_0'u_0'} & \overline{v_0'v_0'} & \overline{v_0'w_0'} \\ \overline{w_0'u_0'} & \overline{w_0'v_0'} & \overline{w_0'w_0'} \end{pmatrix} R_{01}^{\tau}$$

And so on for the rotations 2 and 3...

Angle determination

We start from the sonic coordinate system :
independent of the flow field
(modern sonics output wind components in an orthonormal frame)

We end with the analyses coordinate system :
defined using the flow field



All the story is now to define the angles of rotations α, β and γ!

Angle determination

General aim :

Aligns the *z*-axis perpendicular to the mean streamlines surface

To define the mean streamlines orientation, two approaches are available :

- ‘Rotated every period’ coordinate system

  This coordinate frame is often called the ‘Natural wind system’ and was firstly introduced by Thanner and Thurtell (1969)

- ‘Long-term’ coordinate system

  Different implementations
  - Planar fit
  - Lee method
  - Angle method

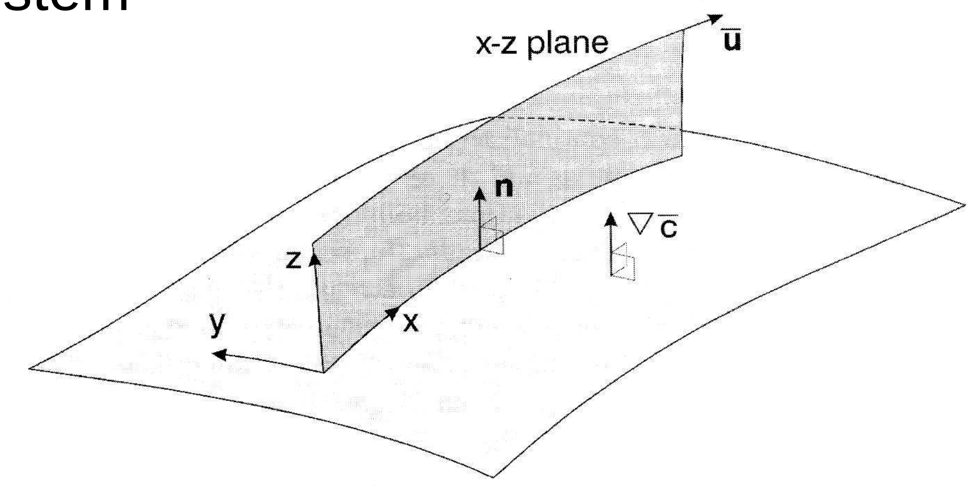


Figure 3.1. The coordinate system should be such that the local normal to the surface and the mean scalar gradient $\nabla\bar{c}$ lie in the x-z plane.

(From Finnigan, 2004)

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Angle determination
'Rotated every-period' (2D rotation)

Aligns the *x*-axis to the short-term (30 min) mean streamline at the measurement point

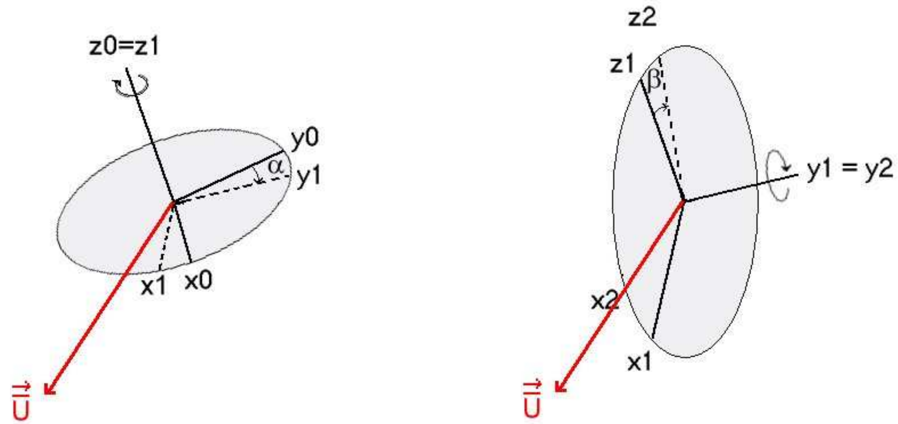
R1 : around *z*-axis, nullifies $\overline{v}$

$$\alpha = \tan^{-1}\left(\frac{\overline{v}_0}{\overline{u}_0}\right)$$

R2 : around new *y*-axis, nullifies $\overline{w}$

$$\beta = \tan^{-1}\left(\frac{\overline{w}_1}{\overline{u}_1}\right)$$

=> *z* is normal to the given streamline but not yet normal to the streamlines surface

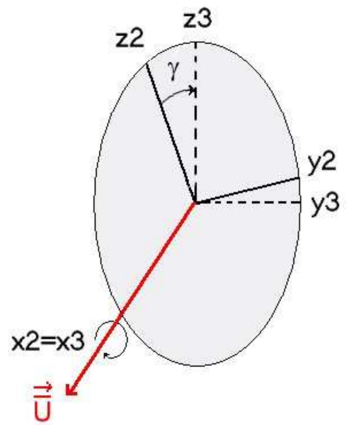


Angle determination
'Rotated every-period' (3rd rotation)

Aligns the *z*-axis normal to and pointed away from the underlying surface

R3 : around *x*-axis, nullifies $\overline{v'w'}$

$$\gamma = \frac{1}{2}\tan^{-1}\left(2\frac{\overline{v_2^{'}w_2^{'}}}{(\overline{v_2^{'2}} - \overline{w_2^{'2}})}\right)$$



Not recommended anymore

Citing Finnigan (2004): '*We find that, in real flows, the standard method has a previously unrecognized closure problem that ensures that the third rotation angle defined using the stress tensor ... will always be in error and often give unphysical results*.'

=> Orientate the sonic *z* axis as nearly normal to the underlying surface as can be achieved and perform only the rotation 1 (yaw) and 2 (pitch).

Angle determination
'Rotated every-period' (2D + 3rd? rotation)

Advantages :

- In a idealized homogeneous flow, it levels the anemometer to the surface
- Allows online computation

Disadvantages :

Limited to a surface layer with a one-dimensionnal flow.
OK only on ideal sites, over selected 'golden days' and fair weather conditions.

- Over-rotation
- Loss of information
  - Useful informations on 3D nature of the flow should be obtained from $\overline{w}$ and $\overline{v'w'}$
- Degradation of data quality
  - Unrealistically large rotation angles in low wind conditions
  - Closure problem on $\overline{v'w'}$

Tilt origine

- Inclination of the sonic relative to the surface
- Flow distorsion
- Electronic offset in the instrument vertical velocity
- Real mean (30 min) vertical motions

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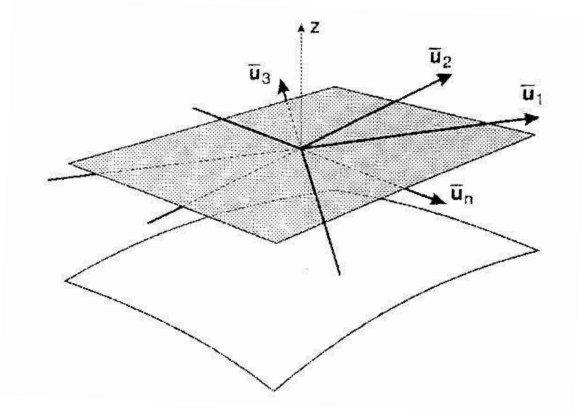
Angle determination
Long-term (Planar Fit)

Aligns the *z*-axis perpendicular to the long-term (compared to 30 min) mean streamline plane

R2 : around *y*-axis with β_{PF}

R3 : around new *x*-axis with γ_{PF}

=> nullifies $\overline{w}_{LT}$



(From Finnigan, 2004)

and y-axis perpendicular to the plane in which the short-term (30 min) **U** and the z axis lie.

R1 : around *z*-axis, nullifies $\overline{v}$

z axis is fixed while *x* and *y* axis are redefined each (30 min) period.

Angle determination
Long-term (Planar Fit)

Determination of β_{LT} and γ_{LT} (angle of rotation 2 and 3)

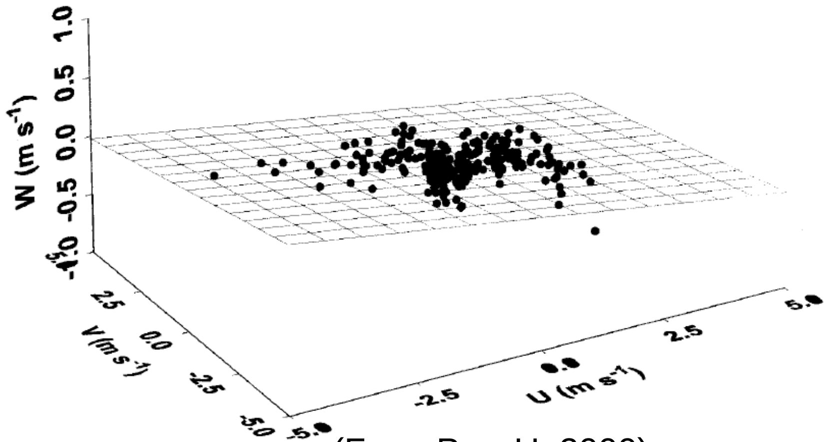
Make a planar regression on wind components in the sonic system

$$\overline{w}_0 = b_0 + b_1\overline{u}_0 + b_2\overline{v}_0$$

w = -0.099998 -0.059016*u -0.043260*v

b_0 accounts for a possible technical offset

b_1 and b_2 define the orientation
of the long-term streamline plane



(From Paw U, 2000)

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Angle determination
Long-term (Planar Fit)

Determination of β_{LT} and γ_{LT} (angle of rotation 2 and 3)

Use these regression coefficients to define $R_{12}(\beta)$ and $R_{23}(\gamma)$

$$\beta_{PF} = \tan^{-1}(-b_1)$$

$$\gamma_{PF} = \tan^{-1}(b_2)$$

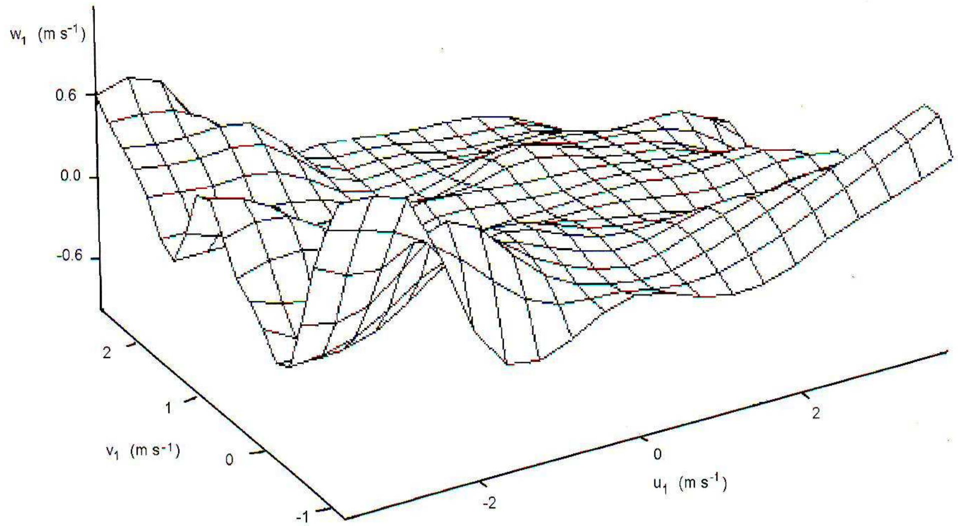
=> *z* is fixed normal to the long-term streamline plane

Make the rotation $R_{01}(\alpha_{PF})$ around *z* after each Reynolds averaging period

$$\begin{pmatrix} u_3 \\ v_3 \\ w_3 \end{pmatrix} = R_{01}(\alpha_{PF}) \cdot R_{23}(\gamma_{PF}) \cdot R_{12}(\beta_{PF}) \begin{pmatrix} u_0 \\ v_0 \\ w_0 \end{pmatrix}$$

Angle determination
Long-term

- In case of a surface different from a plane, sector-wise fit can be used
- Needs a long dataset (several weeks) and post-processing
- Can only be applied to a set of data when the position of the anemometer does not change
- $\overline{w}_{LT} = 0$ but $\overline{w}_{(ST)}$ can be $\neq 0$
- Other methods exist to obtain this 'long
  - 'Lee' method
  - 'angle' method



(From Paw U, 2000)

Angle determination

Summary

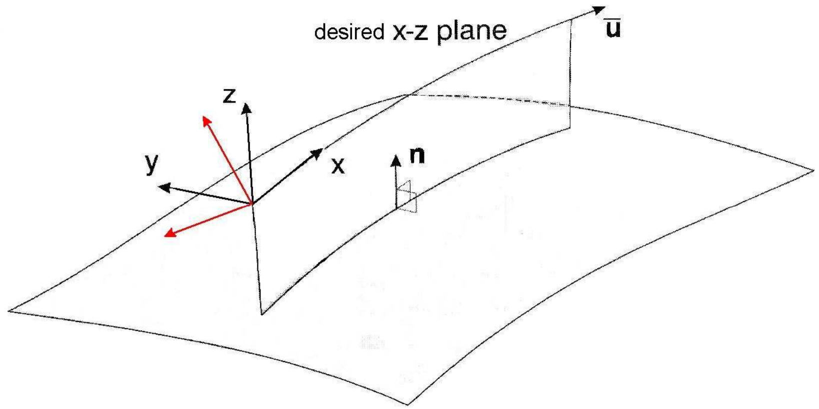
Aligns the *z*-axis perpendicular to the mean streamlines

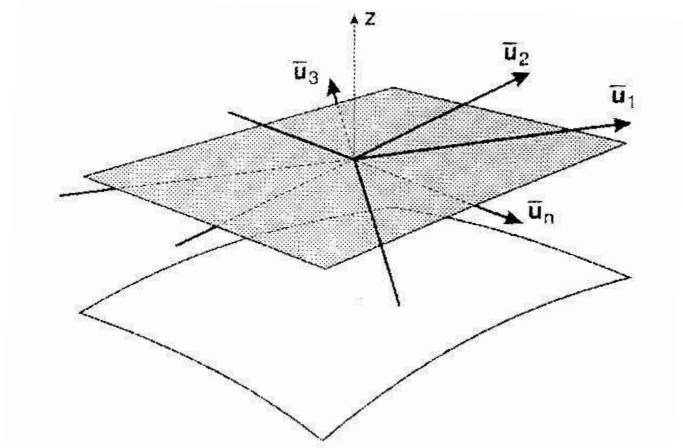
- 'Rotated every period' coordinate system

Use of a unique **U** realization allows to align *z* normal to the short-term streamline (yaw and pitch). It's difficult to extract additional informations from the flow field to align *z* normal to the plane where the short-term streamlines lie (3rd rotation,roll).

- 'Long-term' coordinate system

Use of an ensemble average of **U** allows to define the plane in which the long-term streamlines lies.





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Comparison of coordinate systems 'Rotated every-period' vs 'Long-term'

Momentum flux
3D 'natural wind system' vs PF

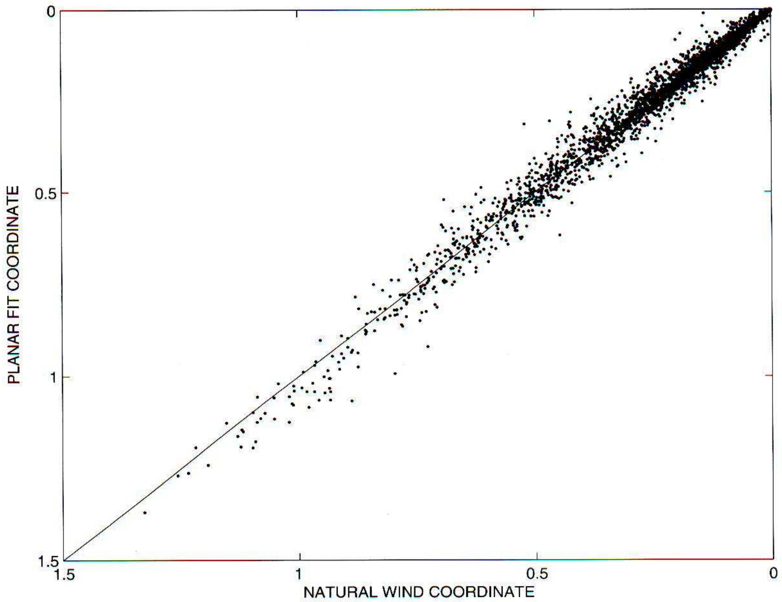

Figure 3.9. Comparison of the streamwise momentum flux ($\overline{u'w'}$, $m^2 s^{-2}$) in the natural wind and planar fit coordinates. Solid line represents 1:1.

CO2 flux
3D 'natural wind system' vs PF

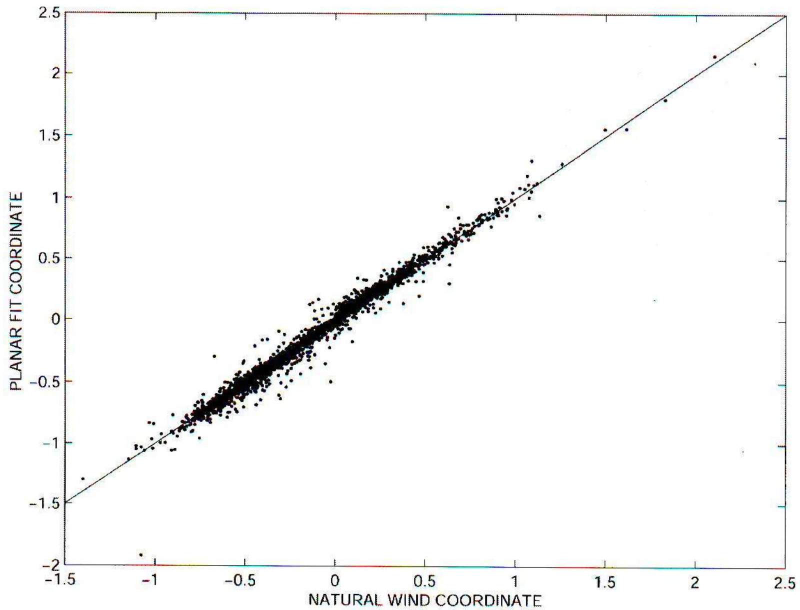

Figure 3.8. Comparison of CO_2 flux ($mg\,m^{-2}\,s^{-1}$) in the natural wind and planar fit coordinates. Solid line represents 1:1.

(From Lee, Handbook, 2004)

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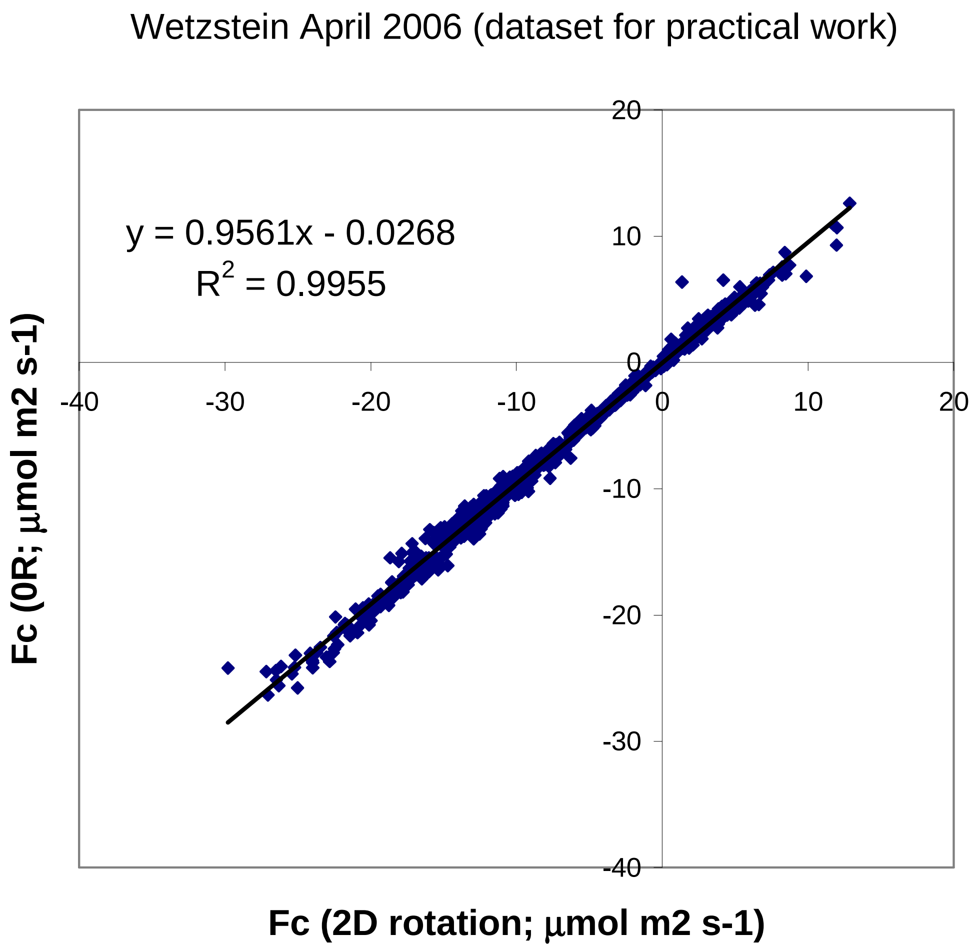
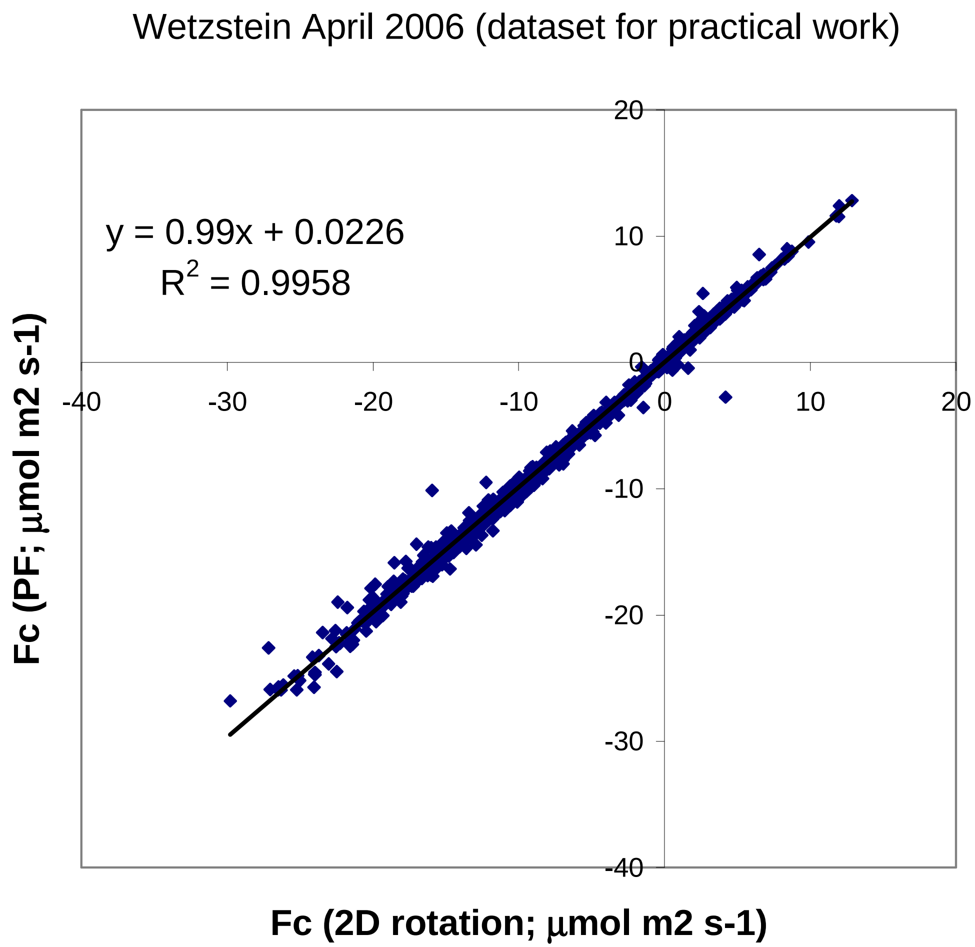
Comparison of coordinate systems
'Rotated every-period' vs 'Long-term'





Conclusions

- Scalar turbulent flux tilt error is usually small for small tilt but this does not negate the need for coordinate rotation to interpret meaningfully this flux
- 2D ‘Rotated every-period’ coordinate system is fine for 1 dimensionnal flows (recommended for agricultural or greengrass with simple topography)
- ‘Long-term’ coordinate system is recommended for more complex flows (more complex topography, mainly forested sites) but the quantitative impact on the scalar turbulent flux is weak compare to 2D rotations
- You should test ‘long-term’ coordinate system on your site to investigate the 3D aspects of the flow
- Impact of rotations on $\overline{w}$ is huge. Crucial for advection estimations (next week lectures)

References

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Extra-material

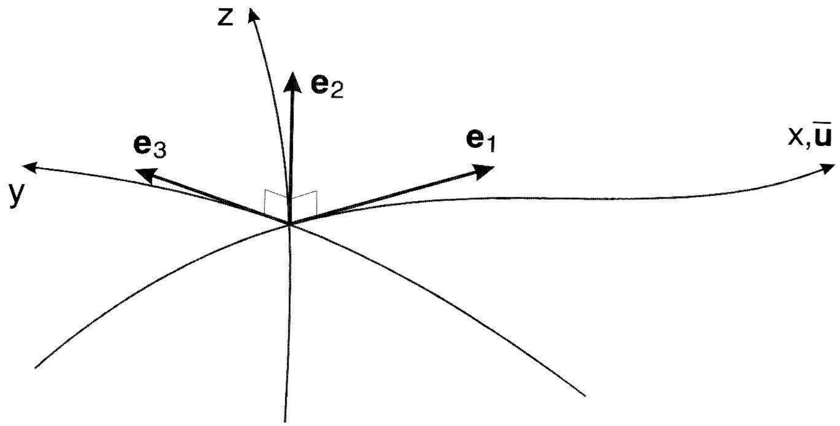
Distinction between vector basis and coordinate frame

- the vector basis : local property of a coordinate system
- the overall coordinate frame consisting of the vector basis and coordinate lines : global property of the flow

Once the vector basis is defined at a point :

Choose the coordinate frame :

- rectangular Cartesian coordinate system
- streamline coordinate system



(From Lee, handbook 2004)

Working in a streamline coordinate system have some theoretical advantages but it's very difficult to define this coordinate system with measurements.

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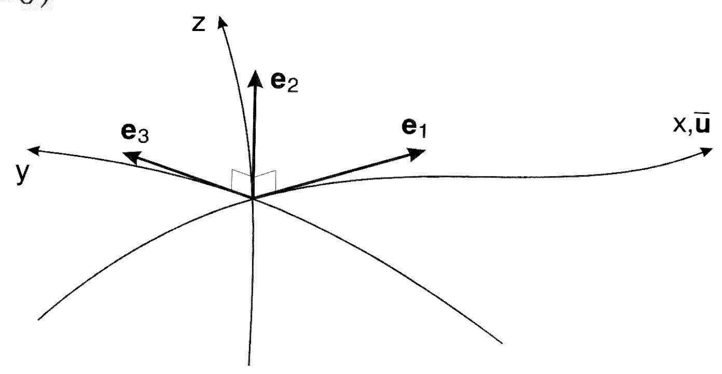
Extra-material

Streamline coordinate system

Working in a streamline coordinate system have some theoretical advantages but it’s very difficult to define this coordinate system with measurements.

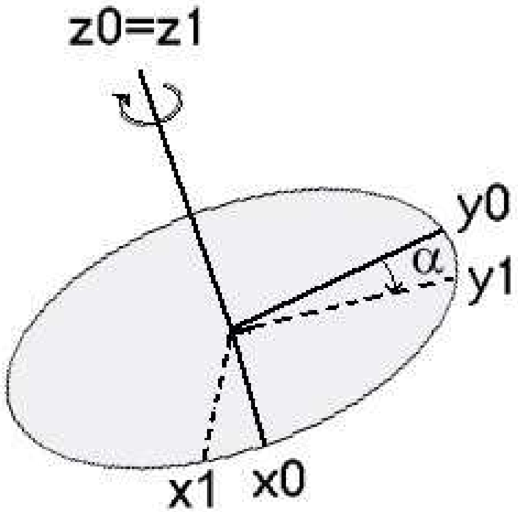
$$\overline{\frac{\partial c}{\partial t}} + \overline{u}\partial_x\overline{c} = -\partial_x\overline{u'c'} - \partial_y\overline{v'c'} - \partial_z\overline{w'c'} - [\frac{1}{L_a}]\overline{u'c'} + [\frac{1}{r}\frac{\partial r}{\partial y}]\overline{v'c'}$$
$$-[\frac{1}{R} + \frac{1}{r}]\overline{w'c'} + S\delta(\vec{x} - \vec{x}_0)$$



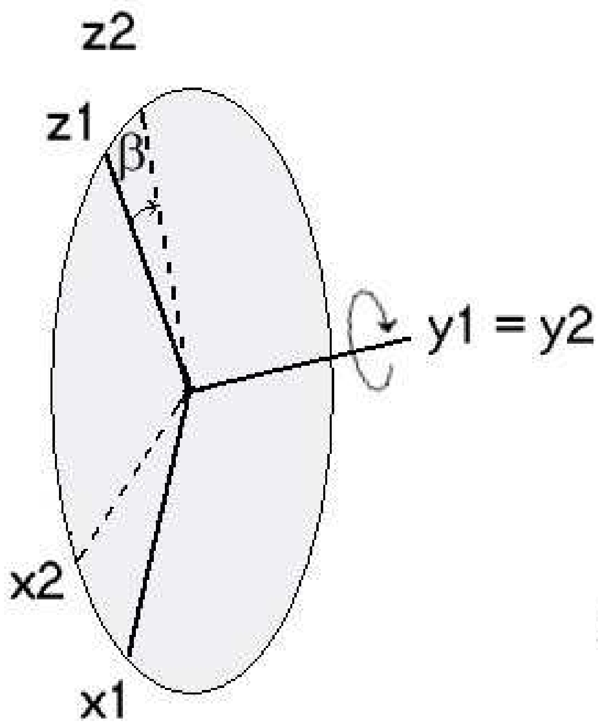
(From Lee, handbook 2004)

Extra-material

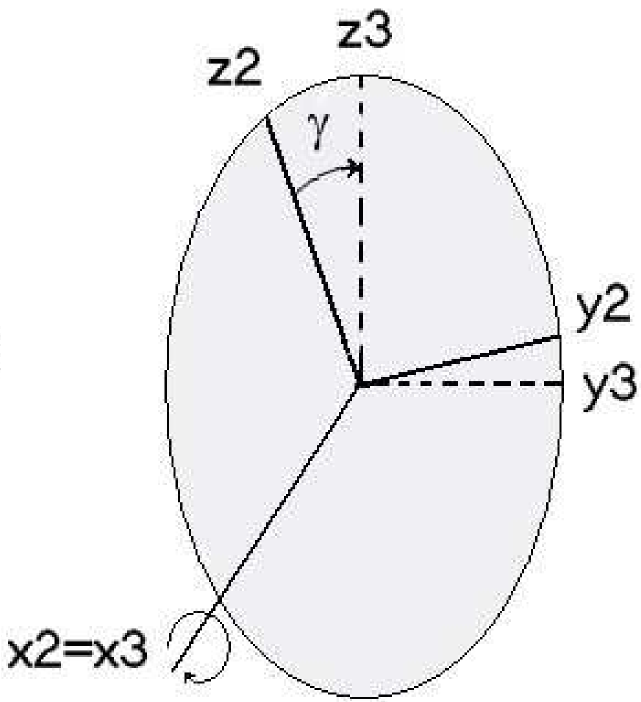
Yaw



Pitch



Roll



Extra-material

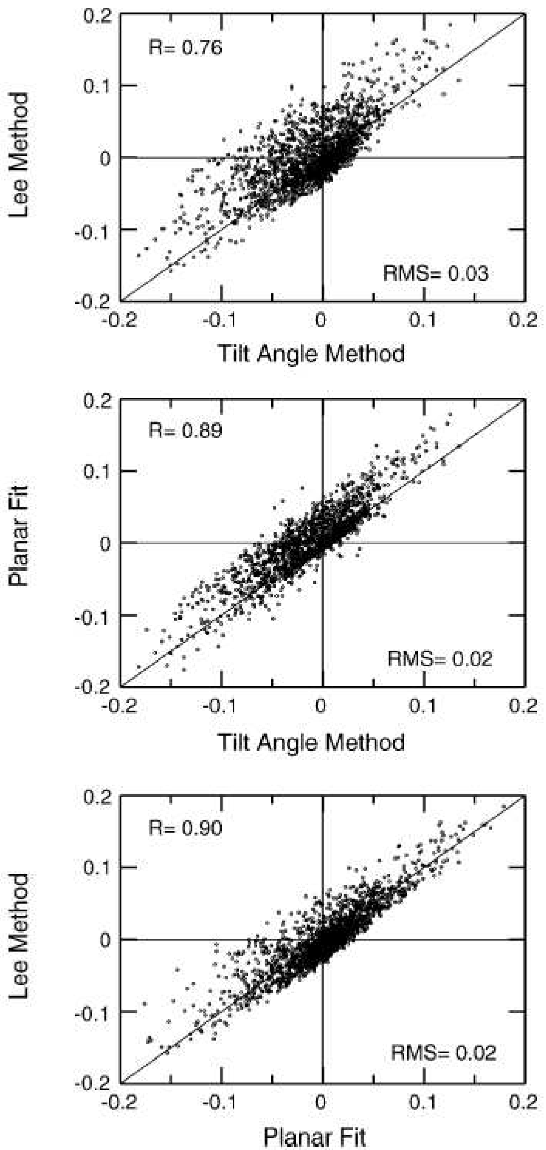


Fig. 2. Comparison of 30-min mean vertical motion (m s^{-1}) at 12 m for three different tilt correction methods.

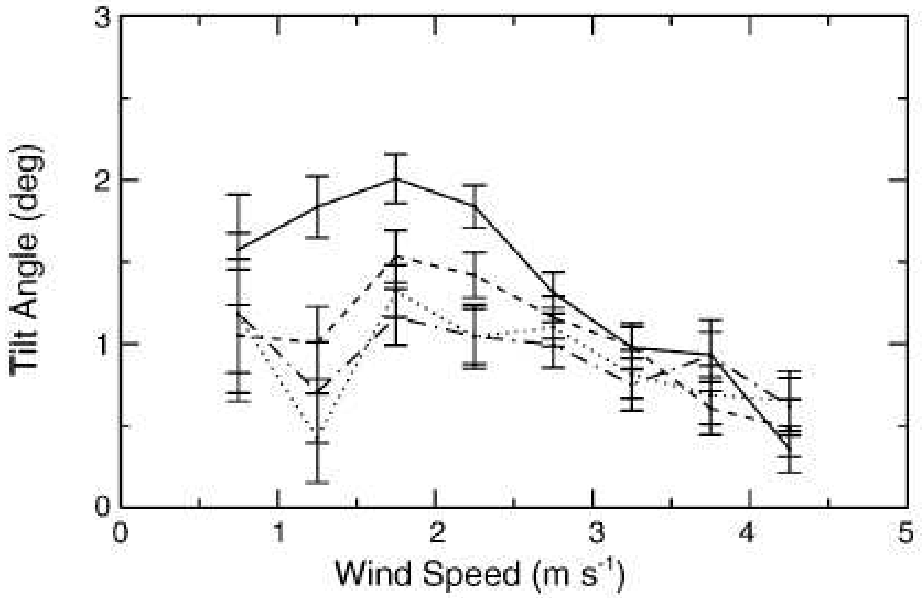


Fig. 4. Mean and standard error of tilt angle as a function of wind speed for wind directions between 290° and 310° (12 m data) for 100-s (solid), 5-min (dash), 10-min (dot) and 30-min (dash-dot) averaging of the wind components.

(From Vickers, AFM 2006)