A sensitivity study of the effects of solar luminosity changes on the Earth's global temperature

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ABSTRACT. A radiative-convective model of the atmosphere extending from the ground to ~ 70 km is used to investigate the response of the surface temperature to increases and decreases in the value of the solar constant. The sensitivity to the choice of the adiabatic lapse rate in the convective region is evaluated. The use of a moist adiabatic lapse rate appears to stabilize the temperature more effectively than in most earlier studies on the subject. At a relative (with respect to the present value) solar luminosity $L/L_\odot = 1.2$, our moist adiabatic model gives only a mean surface temperature of 29 °C, 14 °C above the present average value but over 30 °C less than the corresponding calculation with an adiabatic lapse rate fixed to the usual 6.5 K/km value.
The surface albedo feedback is included in order to study the stability of the climate system with respect to a decrease in the solar constant. This feedback is introduced in the model through a relation linking the global ground albedo to the world mean surface temperature derived from an energy-balance model. As the solar luminosity is lowered, the system becomes unstable and is constrained to jump to a stable solution where the Earth is completely ice-covered. It is found that the location of this discontinuity and the behaviour of the frozen solution strongly depends on the law adopted for the surface albedo-mean temperature relationship. Depending on the choice of this albedo function and, to a lesser extent, on the choice of the adiabatic lapse rate, the critical relative luminosity where the jump to the frozen solution occurs may vary between $L/L_\odot = 0.80$ and $L/L_\odot = 0.96$.

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1. INTRODUCTION

The theory of stellar interiors implies that the solar luminosity must have been lower in the past, during the early history of Earth. The classical models of solar evolution seem to be in particularly good agreement : their results indicate a 25-30 % lower luminosity 4.5 By ago, soon after the Earth's formation, followed by an approximately linear increase until the present time (Endal and Schatten, 1982 ; Newman and Rood, 1977 ; Boury *et al.*, 1975). However, the usual energy-balance climate models (Budyko, 1969 ; Sellers, 1969 ; North, 1975 ; Endal and Schatten, 1982) predict that the Earth would fall in a stable state where the planet is completely ice-covered, if the solar constant was to decrease by only a few percent. This complete glaciation would be irreversible, since these models show that higher luminosities than the present one are required to unfreeze the system. This faint young sun paradox is usually solved by adding a greenhouse gas to the atmosphere. The first attempt in this direction was proposed by Sagan and Mullen (1972), which suggested NH_3 as the greenhouse gas. At present, however, CO_2 is generally accepted as the most likely infrared absorber, since the high NH_3 photolysis rate should have been balanced by an unlikely high ammonia source at ground level (Kasting, 1982). Hart (1978) proposed an evolutionary sequence for atmospheric CO_2 which is compatible with the absence of major glaciations during the whole Earth's history. His work was based on a global energy-balance model coupled to a geochemical one. This CO_2 evolution has later been tested with radiative-convective models (Owen *et al.*, 1979 ; Kuhn and Kasting, 1983). The results indicate that, if Hart's CO_2 evolutionary sequence is adopted, the Earth would never have been frozen but instead, according to the first study, the ancient climate was possibly warmer than at present. A feedback process has even been presented by Walker *et al.* (1981) to explain the tendency of the CO_2 level to increase when the surface temperature is lowered.

However, the climate system includes many other feedbacks which can play a key role in the stabilization of the present climate. Indeed, the small solar luminosity decrease required to completely freeze the present Earth is rather puzzling. Furthermore, the atmospheric CO_2 level which is necessary to prevent an irreversible glaciation of the early Earth is relatively high (Kasting, 1985). In the present paper, we do not intend neither to propose a solution to the faint young sun paradox, nor to analyse the climate response to CO_2 increases. Rather, we present a radiative-convective study of two major climate feedbacks.

The first one is linked to the choice of the moist adiabatic lapse rate (Γ-feedback) and stabilizes the climate system (Chylek and Kiehl (1981), Kasting *et al.* 1984), while the second one, which results from the variation of the ground albedo with temperature (ice-albedo feedback), favours unstability.

The ice-albedo feedback is modelled in some detail in energy-balance models. However, these calculations depend on the adopted parameterization linking the surface temperature to the outgoing infrared flux. On the contrary, one-dimensional radiative-convective models are independent of such a parameterization, since they include an elaborate 1-D treatment of radiative transfer. Nevertheless, these latter models present deficiencies : they seldom account for the ice-albedo feedback and the latitudinal heat transfer is not taken into account. This is why it is of interest to analyse, from a general point of view, the response to solar constant variations of a 1-D radiative-convective model including the ice-albedo feedback. Unfortunately, parameterization of the cloud feedback is highly speculative as it is for all types of climate models.

Another reason which leads us to reexamine the problem of climate stability is the existence of non-steady stellar evolutive models. To explain the apparently low solar neutrino flux, Fowler (1972) proposed that the sun was at present in a transient phase caused by a fast mixing of the core. Such a mixing would have been recurrent, with a period of several hundred million years.

This idea has been investigated by many authors during the last decade (Dilke and Gough, 1972 ; Ulrick, 1975, Gabriel *et al.*, 1976). Acccording to their results, the solar constant would vary strongly during a mixing phase : an initial pulse towards the high luminosities would be followed by a second pulse in the opposite direction. During this event, the solar constant may change by over 50 % of its present value. Although such a process remains speculative, it shows, nevertheless that large fluctuations of the solar luminosity cannot be excluded. Besides, since the present state of the sun would fall on the downward branch of the pulse, it points out the importance of studying the consequences of large solar luminosity increases. Many calculations predict a great sensitivity of the Earth's climate to such increases which, thus, seem unrealistic, so that the interest for recurrent mixing core solar models has declined. However, the numerous feedbacks which control the climate response are still quantitatively unknown.

Previously, several studies have partly discussed the problem of climate stability with a 1-D radiative convective model (Schneider and Dickinson, 1974). First, much work has been devoted to the description of the sensitivity of such a model to CO_2 increases or to small perturbations of the solar constant, in the vicinity of the present luminosity (e.g. Wang *et al.*, 1981). In recent years, however, a few-radiative-convective studies have considered large solar constant variations in describing the role of the various feedbacks. The Γ-feedback has been modelled, for solar luminosities higher than at present, by Lindzen *et al.* (1982), Kasting *et al.* (1984), Lal and Ramanathan (1984) and Vardavas and Carver (1985) using 1-D models and by Wetherald and Manabe (1975) for limited solar constant variations with a simplified three-dimensional climate model.

The results of these studies are very different and they will be compared to our own results. The ice-albedo feedback has been included in the radiative-convective model of Wang and Stone (1980), but this work does not include the Γ-feedback. Rossow *et al.* (1982) described the effect of considering cloud feedback, i.e. the feedback induced by changes in cloud cover and cloud optical depth with surface temperature. This process may be important, since Rossow *et al.* (1982) find that it is potentially suitable to partly solve the faint young sun paradox. Nevertheless, cloud feedback is not included in our model, in order to isolate the effects of the other feedbacks, especially because the response of cloud properties to temperature changes remains largely unknown.

In this paper, we study the climate system both for high and low luminosities. We show that for solar constants larger than at present, this system is possibly more stable than suggested by the results of earlier workers. Furthermore, since it accounts for the ice-albedo feedback, our model is used to determine the critical luminosities where the transitions between the ice-covered and the ice-free solutions occur. It is shown that the values of these critical luminosities are strongly dependent on the assumed relation between surface temperature and ground albedo. Note that, in this paper, we do not intend to study the problem of the appearance of a runaway greenhouse at high solar luminosities. However, we nevertheless report some results corresponding to high surface temperature.

2. MODEL DESCRIPTION

Principles

The radiative-convective model used in this study is conceptually similar to the model described by Manabe and Wetherald (1967). The radiative codes are partly based on the work of Ramanathan (1976) and will thus not be described in details. The pressure grid extends from the ground to 4×10^{-2} mbar which roughly corresponds to 72 km in the present atmosphere. The atmosphere is divided into 26 layers equally spaced in log-pressure coordinate. The one-dimensional thermodynamic equation

$$\frac{dT}{dt} = -\frac{1}{\rho c_p}\frac{dF}{dz} \qquad (1)$$

is used to compute the temperature at the intermediate pressure in each layer. The symbol ρ represents the atmospheric density and c_p is the specific heat at constant pressure. The F term denotes the radiative flux at altitude z and is given by :

$$F(z) = F_{\odot}^{\uparrow} - F_{\odot}^{\downarrow} + F_{IR}^{\uparrow} - F_{IR}^{\downarrow} \qquad (2)$$

where $F_{\odot}$ is the upward or downward integrated solar flux and F_{IR} the infrared terrestrial flux.

A forward time-marching method is used to solve (1) until a steady-state is reached. To evaluate the terms of (2), infrared absorption of terrestrial radiation by CO_2, O_3 and H_2O and absorption of solar radiation by H_2O and O_3 are considered.

The CO_2 absorption is calculated following the formalism given by Ramanathan (1976). The method suggested by Alimandi and Visconti (1979) to account for the temperature dependence of the absorptivity is adopted.

In order to account for the overlap between the CO_2 15 μm band and the H_2O rotational lines, it is assumed that the total transmission due to both gases T_t may be computed in each interval as the product of the individual transmissivities :

$$T_t = (1 - \bar{A}_{CO_2})\, T_{H_2O} \qquad (3)$$

where $\bar{A}_{CO_2}$ is the total CO_2 absorptivity. The transmissivity of water vapour T_{H_2O} near 15 μm is taken from Sasamori (1968).

The O_3 absorption at 9.6 μm is calculated as in Ramanathan (1976) and Donner and Ramanathan (1980). The H_2O absorptance parameterization for 6.3 μm and the rotational band given by Sasamori (1968) is adopted. However, to account for the inconsistency noticed by Dickinson *et al.* (1978), the emissivity ε is related to the H_2O pathlength u (g cm^{-2}) and the absorptivity $\bar{A}_{H_2O}$ by the relationship :

$$\varepsilon = \frac{\bar{A}_{H_2O}}{0.847\, u^{0.022}} \qquad (4)$$

which was derived by Ramanathan *et al.* (1983).

The H_2O « *e*-continuum » in the 8-12 μm window is calculated using the expression given by Roberts *et al.* (1976) for the absorption coefficient and its temperature dependence.

Since Sasamori's parameterization provides a good fit only for values of u less than 10 g cm^{-2}, the emissivity and the absorptivity of H_2O have been limited to an upper value of 1.

Absorption of solar radiation in the visible and ultraviolet regions by H_2O and O_3 is parameterized using the formulation of Lacis and Hansen (1974). This parameterization includes the effects of Rayleigh scattering and surface albedo.

As usual, the tropospheric temperature gradient is not allowed to exceed a critical lapse rate Γ_c corresponding to a convective profile as will be discussed below. This convective adjustment is made using the method described by Manabe and Wetherald (1967).

A mean ozone profile for the present atmosphere is adopted (Vardavas and Carver, 1984). This profile shows a volume mixing ratio of 4 ppm at the ground, peaks at 3 ppm near 36 km and decreases to 1 ppm at the top of the model.

The choice of the H_2O profile is critical for models with increased solar constant. In this study, a Manabe and Wetherald (1976) vertical distribution for the relative humidity r_{H_2O} is adopted :

$$r_{H_2O} = 0.80 \left[\frac{\frac{P}{P_0} - 0.02}{1 - 0.02} \right]^{\Omega} \qquad (5)$$

where $P_0 = 1$ atm is the surface pressure, P the total pressure and Ω a temperature-dependent parameter introduced by Cess (1976) to account for the temperature control of the relative humidity. However, in view of the large uncertainties concerning this dependence for temperature structures very different from the present conditions, a fixed value $\Omega = 1$ is adopted here. In order to prevent negative values in equation (5), when the mixing ratio reaches 3 ppm, the relative humidity is fixed at the value calculated at this level up to the tropopause i.e. the altitude of minimum temperature. Above the tropopause, the mixing ratio is fixed to the tropopause value up to the top of the model. Consequently, the stratospheric mixing ratio may become less than 3 ppm in the case of a cold tropopause. This distribution simulates the effect of a tropopause cold trap. For warm models, the 3 ppm level is not reached below the tropopause. In this case, relation (5) is used up to the tropopause above which the H_2O mixing ratio is kept to the tropopause value.

Lapse rate

Two formulations were adopted to test the sensitivity of the calculations to the value of the temperature lapse rate in the convective region. A fixed adiabatic lapse rate of 6.5 K/km is usually adopted in radiative-convective models of the present atmosphere and has been used for some of these calculations. A moist adiabatic lapse rate has also been used in the troposphere. The moist lapse rate Γ_s is calculated from the expression (Iribane and Godson, 1981) :

$$\Gamma_s = \Gamma_d \frac{\left(\frac{P}{P - P_s}\right)\left(1 + \frac{L_w}{RTP}\right)}{1 + \frac{0.387\, L_w^2}{c_p R T^2 P}} \qquad (6)$$

where

Γ_d is the dry adiabatic lapse rate (9.8 K/km),
L_w, the water vapour latent energy,
P_s, the saturated water vapour pressure at temperature T,
R, the perfect gas constant.

The temperature dependence of the latent heat of sublimation and condensation is given by :

$$L_w = 2.834 \times 10^{10}\ \text{erg g}^{-1} \quad \text{for} \quad T < 273.15\ \text{K}$$

$$= 2.497 \times 10^{10} - 2.31 \times 10^{7}(T - 273.15)$$

for $T > 273.15$ K.

The Magnus formula (see Iribane and Godson, 1981) :

$$\log_{10} P_s = \frac{-2937.4}{T} - 4.9283 \log_{10} T + 23.5518 \qquad (7)$$

gives a good fit to the experimental saturation pressures (in mbar) in the range of temperatures considered. Due to the rapid decrease of the water vapour content in the upper troposphere and stratosphere, the calculated Γ_s rapidly tends to the dry air lapse rate value of 9.8 K/km.

Albedo

The total planetary albedo results from two main components : the atmospheric albedo with a cloud and a Rayleigh scattering contributions and the ground albedo which is a global average on ice-covered and ice-free areas. The atmospheric contribution is obtained from the formulation by Lacis and Hansen (1974). The magnitude of the solar radiation absorption in the visible and near infrared regions depends on the quantity of H_2O in the troposphere. Consequently, for large amounts of water vapour, the atmospheric albedo decreases to values below the average value of the present atmosphere. For example, in the model, α decreases from 0.24 to 0.20 when the solar relative luminosity increases from 1 to 1.2 in the moist adiabatic case. The choice of a ground albedo and various parameters describing the cloud optical properties is also required for the Lacis and Hansen formulation. These cloud parameters are fixed in this model and will be specified in section 3. The surface albedo feedback is included by using a relationship :

$$A_g = A_g(\bar{T}_s) \qquad (8)$$

between the mean ground albedo A_g and the mean surface temperature $\bar{T}_s$. This function must correspond to the ice albedo $A_{g,\,\text{ice}}$ for temperatures low enough to cause glaciation of the complete planet. It tends to the ice-free mean albedo for high values of $\bar{T}_s$, when the polar caps are totally melted. Between these two limits, the relation (8) is obtained from an energy balance model, following a procedure similar to that developed by Wang and Stone (1980).

In such a model, an energy balance equation is written for each latitudinal belt, the vertical dimension being neglected. This equation states that the absorbed solar flux must be balanced by the outgoing infrared flux to space and the net latitudinal heat transfer. The model calculates basically the outgoing infrared flux, from which the zonal surface temperature may be derived by a known parametric relationship. The latitudinal dependence of the calculated infrared flux is given as an expansion of Legendre polynomials of even order (which states a perfect symmetry between the two hemispheres). The zero order coefficient of this development gives the mean global infrared flux from which the mean global temperature $\bar{T}_s$ may be derived.

The model requires the knowledge of several parameters and parametric functions. First, the latitudinal dependence of the total albedo α (ground + atmosphere) must be known. Here, the following relationship is used to describe the dependence of the coalbedo $a = 1 - \alpha$ on the sine of latitude x :

$$\begin{aligned} a(x, x_s) &= a_0 + a_2 P_2(x) \quad &\text{for } x < x_s \\ &= b_0 &\text{for } x \geqq x_s \end{aligned}$$

In this equation, x_s is the sine of the mean latitude of the equatorward boundary of the polar caps corresponding to a zonal surface temperature of − 10° C (North, 1975), $P_2(x)$ is the second Legendre polynomial and a_0, a_2 and b_0 are constants taken from North (1975). Secondly, the latitudinal heat transport is modelled by a dimensionless heat diffusion coefficient D. Its value is fixed from the requirement that a model with a mean global temperature equal to the present value $\bar{T}_s = 15°$ C has polar caps extending up to $x_s = 0.95$. In the present model, the value $D = 0.322$ has been derived, intermediate between $D = 0.267$ of Endal and Schatten (1982) and $D = 0.382$ of North (1975) and in close agreement with $D = 0.31$, given by North *et al.* (1981). Thirdly, the mean annual meridional distribution of solar radiation $S(x)$ at the top of the atmosphere must be specified. Following North *et al.* (1981), the second order Legendre development $S(x) = 1 - 0.477\, P_2(x)$ has been adopted.

Since $S(x)$ is normalized to unity, the mean global albedo may be expressed by

$$\bar{\alpha}(x_s) = \int_0^1 S(x)\, \alpha(x, x_s)\, \mathrm{d}x \qquad (9)$$

In the present study, it is assumed that the same average is valid for the ground albedo A_g, provided that the function $\alpha(x, x_s)$ is replaced by the zonal ground coalbedo $a_g(x, x_s)$. In other words, this means that the same latitudinal distribution of solar radiation is assumed for the ground and for the top of the atmosphere. Strictly speaking, this assumption is not entirely correct since the albedo of the atmosphere and the cloud cover are weakly latitude-dependent. Furthermore, the function $a_g(x, x_s)$ is considered to have the same form as a (x, x_s). The constant $a_2 = -0.0779$ is unchanged, while b_0 and a_0 are fixed such as the global ground albedo of the ice-covered planet is equal to $A_{g,\,\text{ice}}$ and that of the present Earth ($x_s = 0.95$) is equal to 0.1, a value usually adopted in radiative-convective models. Consequently, this method enables the calculation of the ground albedo as a function of x_s and, since the relation between x_s and $\bar{T}_s$ is known from the solution of the energy-balance model, it also provides the form of the function $A_g(\bar{T}_s)$ in relation (8).

The calculated temperature dependence of the ground albedo is presented in figure 1, for $A_{g,\,\text{ice}} = 0.7$ and $A_{g,\,\text{ice}} = 0.5$. (The first value corresponds to newly deposited snow and the second to more ancient one). The solution of our energy-balance model gives $\bar{T}_s = -15°$ C, as the maximum global temperature of a completely frozen Earth. That is the reason why, for both $A_{g,\,\text{ice}}$ values, the albedo begins to vary only above this temperature. Wang and Stone (1980) derived the function $A_g(\bar{T}_s)$ by a method similar to ours, but with slightly different values for several parameters. Their results are also displayed in figure 1. The major difference with our curves is that, in their model, the maximum global temperature of an ice-covered Earth is much lower ($\bar{T}_s \simeq -29°$ C), this

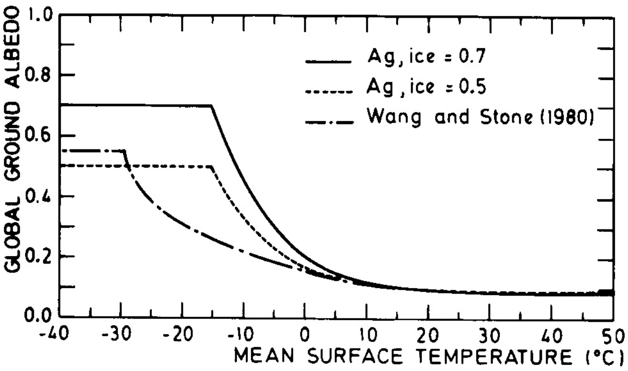

Figure 1

Dependence of the global ground albedo on the mean surface temperature in the present model and in the study of Wang and Stone (1980). *The parameter $A_{g,\,ice}$ represents the global ground albedo of an ice-covered Earth.*

being due in part to their lower adopted temperature ($T = -13°$ C) of the ice cap edges.

3. RESULTS

The model has been run with a 6-h time step until convergence was achieved. The clear-sky approximation gives a surface temperature higher than the present mean surface temperature of 15° C, so that it is necessary to introduce a cloud layer to reproduce the present conditions. Since the purpose of this paper is not the study of cloud effects, a model with a single cloud layer is justified, provided it produces an equilibrium surface temperature close to 15° C. This temperature depends on the altitude and the opacity τ_c (to solar radiation) of the cloud layer. The fractional cloud cover A_c has been assumed equal to 0.5, a value close to the observed one. With this value of A_c and with the adiabatic lapse rate fixed to 6.5 K/km, the best agreement with present conditions is obtained at $\tau_c = 4.0$, for a low altitude cloud layer between 816 and 666 mb. In this case, the surface temperature is equal to 13.3° C when the function with $A_{g,\,ice} = 0.7$ is adopted for the ground albedo variation (see fig. 1). The net infrared flux at the top of the atmosphere is equal to 259.9 W m^{-2}, corresponding to a mean global albedo of 0.24. This value is lower than the actual value of the Earth's albedo. Consequently, the model does not depict exactly the present conditions, but, it is nevertheless useful to study the sensitivity of a radiative-convective model, with the ice albedo feedback, to solar constant variations. The temperature profile is displayed in figure 5 (curve C). It is close to the 1976 U.S. standard atmosphere profile, except that the tropopause is slightly colder. However, it is in better agreement with the hemispherically-averaged profile given by Ramanathan and Dickinson (1979). Furthermore, this discrepancy is generally present in radiative-convective models ; in particular in the model of Kasting *et al.* (1984) and, to a lesser extent, in that of Vardavas and Carver (1984).

In table 1, the values of the sensitivity parameter (in the vicinity of the present luminosity)

$$\beta = L_\odot \left(\frac{\mathrm{d}\dot{T}}{\mathrm{d}L} \right)_{L = L_\odot} \qquad (10)$$

Table 1

Sensitivity parameter β, surface temperature T_s and planetary albedo $\bar{\alpha}$ for several models with present solar luminosity. The low altitude cloud layer means between 816 *and* 666 mb *and the mean altitude one between* 666 *and* 544 mb.

Model	β (°C)	T_s (°C)	$\bar{\alpha}$
1° Low altitude cloud layer, $\tau_c = 4$; $\Gamma = 6.5$ K/km Ice-albedo feedback	233	13.3	0.238
2° Low altitude cloud layer, $\tau_c = 4$; Γ = moist adiabatic Ice-albedo feedback	93	10.7	0.245
3° Low altitude cloud layer, $\tau_c = 8$; $\Gamma = 6.5$ K/km Fixed ground albedo ($A_g = 0.1$)	172	14.5	0.233
4° Mean altitude cloud layer, $\tau_c = 8$; Γ = moist adiabatic Ice-albedo feedback	102	14.1	0.305
5° Mean altitude cloud layer, $\tau_c = 8$; Γ = moist adiabatic Fixed ground albedo ($A_g = 0.1$)	90	14.5	0.302

are displayed for four models with different assumptions on cloud cover and convective adjustment. The surface temperature T_s and the mean global albedo $\bar{\alpha}$ at $L = L_\odot$ are also listed in the same cases. For the $\Gamma = 6.5$ K/km model mentioned above, the sensitivity β is equal to 223° C, while it is reduced by a factor of 2.5 if a moist adiabatic lapse rate is adopted. These results are obtained with a cloud layer of relatively low altitude. As suggested in table 1, a higher cloud layer located between 666 and 544 mb with $\tau_c = 8$ (mean altitude cloud layer) can reproduce almost exactly the actual values of both the surface temperature T_s and the planetary albedo $\bar{\alpha}$. With such a cloud layer type, a moist convective model gives $\beta = 102°$ C, close to the value obtained with the corresponding low cloud model. To show the importance of the ice-albedo feedback (at $L/L_\odot = 1$), β is also displayed for a moist adiabatic calculation with fixed ground albedo ($A_g = 0.1$).

It is observed that β is less affected by inclusion of this feedback than in classical energy-balance models (Budyko, 1969 ; Sellers, 1969). A similar result was obtained by Wang and Stone (1980) with their fixed cloud altitude model. However these authors used $\Gamma = 6.5$ K/km for the convective adjustment. Thus, it is of interest to test the importance of the ice-albedo feedback, in the $\Gamma = 6.5$ K/km case. In this context, model 3 in table 1 should be compared with model 1. It is observed that the neglect of the ice-albedo feedback reduces the sensitivity from $\beta = 233°$ C to 172° C, a value noticeably larger than in the moist adiabatic calculation.

Several runs with various solar luminosities were performed. The results are presented in figure 2. Both a moist and a fixed 6.5 K/km adiabatic lapse rates have been considered. The ground albedo variation is

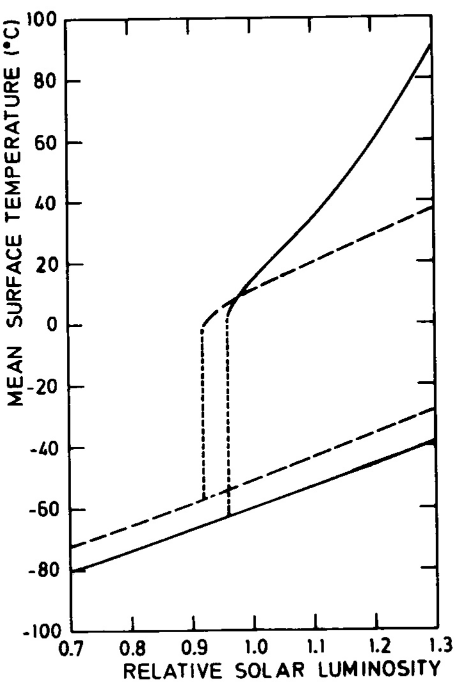

Figure 2
Equilibrium mean surface temperature as a function of relative solar luminosity $L/L_\odot$ for two different convective adjustments. In the first case, the adiabatic lapse rate is fixed to 6.5 K/km (full line), while, in the second one, it is fixed to its moist value (dashed line). Also indicated is the approximate location of the downward jump to the frozen solution.

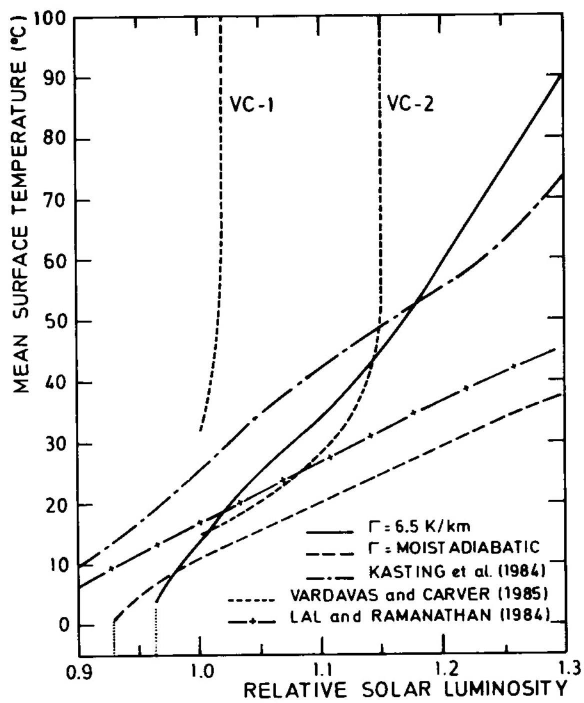

Figure 3
Equilibrium mean surface temperature as a function of relative solar luminosity $L/L_\odot$ for the partially frozen and the unfrozen solution. The results of the present study (full and long-dashed line) are compared to the models of Kasting et al. *(1984), Vardavas and Carver* (1985), *and Lal and Ramanathan* (1984). *The characteristics of these models are given in the text.*

the one obtained with $A_{g,\,ice} = 0.7$ (see fig. 1). These results are similar to those of an energy-balance model. Indeed, two different solutions appear : one where the Earth is completely frozen and another where it is completely unfrozen or where small ice caps are present. These solutions correspond to the stable branches in an energy-balance model. A jump from one solution to the other is possible. For example, as the solar constant is decreased from its present value, the ice caps grow progressively. During their growth, they may reach a critical size where the solution becomes unstable, so that the system is constrained to jump to the stable frozen solution. Similarly, a jump from the frozen to the unfrozen solution may be observed as the solar constant is increased. In the present model, this upward jump would occur as the surface temperature of the frozen solution reaches – 15° C, the temperature where the ground albedo begins to vary. From figure 2, it is seen that the $L/L_\odot$ ratio must be much larger than 1.3, the higher relative luminosity displayed.

Unfrozen solutions for $L/L_\odot > 1$

First, the unfrozen solution for $L/L_\odot > 1$ will be examined. Probably the most interesting question for this domain is whether or not a runaway greenhouse may be achieved for reasonably low solar luminosities. In figure 3, our results have been reported to be compared with those of Kasting *et al.* (1984) and Vardavas and Carver (1985). For $L/L_\odot = 1$, the equilibrium temperature is lower for a moist adiabatic lapse rate than for the fixed 6.5 K/km adjustment model, but the difference is very small. The reason of the fairly good agreement between the two models is that the moist adiabatic lapse rate has an average value of ~ 6.5 K/km in the convective region. For example, in our moist adiabatic model, $\Gamma = 6$ K/km at the ground, whereas it becomes larger than 6.5 K/km higher in the troposphere. When the solar luminosity is increased from the present value, the moist adiabatic lapse rate is decreased. This is due to a larger release of latent heat during convective uprising, resulting from a higher water vapour mixing ratio. Consequently, at a given surface temperature. the infrared radiation emited to space from the upper troposphere is higher than for $\Gamma = 6.5$ K/km. If we reverse this reasoning, we see that, for a fixed solar luminosity (and, thus an approximately fixed infrared flux to space), the surface temperature must be lower in the moist adiabatic case. This feedback process is obvious from our results in figure 3. As the solar constant is increased from the present value, the curves for moist and fixed 6.5 K/km adiabatic lapse rate become more and more separated. At $L/L_\odot = 1.2$, the difference is larger than 30° C. The temperature and water vapour mixing ratio profiles are presented for both models in figure 5 and 6 respectively. In the 6.5 K/km adjustment model, the surface temperature is equal to ~ 59.5° C, corresponding to an effective water column of 21.9 g cm^{-2}. It is also to be stressed that in such a case, the emissivity of H_2O given by Sasamori's formula is limited to unity. As mentioned before, these warm cases (models A and B) exhibit stratospheric H_2O mixing ratios in excess of 3 ppm (17 ppm in model A) whereas in cold model D, the value above the tropopause is 0.09 ppm.

Lindzen *et al.* (1982) have studied the feedback process linked to the adiabatic lapse rate. In their model, the solar luminosity was varied by changing the net solar flux at the top of the atmosphere. At $L/L_\odot = 1.29$, they found a surface temperature $T_s = 48.7°$ C for the fixed 6.5 K/km adjustment model, whereas their moist adiabatic model gives $T_s = 35.9°$ C, in good agreement with the 37.4° C value calculated here for $L/L_\odot = 1.3$. This agreement is probably fortuitous, since Lindzen *et al.* (1982) did not take the 8-12 μm « *e*-continuum » absorption of H_2O into account.

Lindzen *et al.*'s results for large values of the water vapour column were criticized by Lal and Ramanathan (1984) since their emissivity parameterization was based on Rodgers' (1967) approximation for long wavelength absorption. Rodgers' emissivity exceeds unity for H_2O contents in excess of 10 g cm^{-2}. However, this limitation only affects results for large values of T_s (fig. 2). In figure 3, we have reported the results of Lal and Ramanathan (1984), Kasting *et al.* (1984) and Vardavas and Carver (1985).

Lal and Ramanathan (1984) calculated the surface temperature dependence on solar luminosity assuming either a moist adiabatic adjustment or a fixed lapse rate. In the first case, as shown in figure 3, their sensitivity is similar to that of our model, whereas with the $\Gamma = 6.5$ K/km assumption, their results show a much smaller increase of T_s with L than ours. Their calculations exhibit a remarkably small difference (5 K at $L/L_\odot = 1.2$) between the 6.5 K/km and the moist adiabatic cases. Kasting *et al.* (1984) assumed a saturated water vapour profile and a moist adiabatic lapse rate. Their results show a progressive increase of the surface temperature with solar luminosity. For example, at $L/L_\odot = 1.31$, they found $T_s \simeq 76°$ C. On the contrary, in the corresponding model of Vardavas and Carver (1985) (curve VC-1 in fig. 3), a runaway greenhouse is already achieved for $L/L_\odot \geqq 1.02$. This comparison points out the extreme sensitivity of the results on the adopted calculation. Another curve (VC-2) of Vardavas and Carver (1985) is reported in figure 3. It has been obtained with a moist adiabatic lapse rate in a non-saturated atmosphere (eq. (5) above). The effect is to decrease the equilibrium surface temperature : at $L/L_\odot = 1$, the model yields exactly the present value for T_s. As a result, the runaway greenhouse appears for slightly higher luminosities ($L/L_\odot \simeq 1.15$) than in the saturated case. This behaviour is nevertheless very different from the progressive increase observed in the model of Kasting *et al.* (1984), also apparent in our own results.

Our 6.5 K/km adjustment model seems to agree well with the moist adiabatic curve of Kasting *et al.* (1984), except that the slope dT_s/dL is appreciably larger in our model, due to the lack of the negative Γ feedback.

The same non-linearity, obvious from the curvature changes, may be observed for both curves. In the present model, however, a negative curvature is observed around $L/L_\odot = 1$, owing to the ice-albedo feedback which constrains the temperature to fall more rapidly when the luminosity decreases. In the model of Kasting *et al.* (1984), such a negative curvature appears only for $L/L_\odot > 1$, but is due to the negative Γ feedback, which is absent in our fixed 6.5 K/km adjustment model. For high $L/L_\odot$, the curvature becomes positive for both models, as a result of the H_2O greenhouse. This result is at variance with Lal and Ramanathan's (1984) figure 3 which exhibits a negative curvature above $L/L_\odot$. In our moist adiabatic model, the Γ feedback seems to be more effective than in the one of Kasting *et al.* (1984). The reason is that the temperature remains lower, since the water profile is not saturated in the present model, thus lessening the importance of the H_2O greenhouse. However, the choice of the water vapour content is not the only reason for the difference. Indeed, adopting a saturated water vapour profile in our moist adiabatic model increases the temperature up to 55° C at $L/L_\odot = 1.3$, a value which is still ~ 20° C lower than in the work of Kasting *et al.* (1984).

In conclusion, the comparison of our work with previous models shows that the response of the Earth climate to an increase in solar luminosity remains largely unknown, since the results of the more complete models are quite different. Presumably, this disagreement does not originate only in the treatment of the convective adjustment and the water vapour profile, but it seems that the results are very sensitive to small differences in the adopted model. The explanation of these different behaviours is not possible without a careful intercomparison study of the details of the radiative codes. In the case of the present model, some radiative approximations have been used :

— the H_2O continuum outside the 8-12 μm region has been neglected ;

— Doppler broadening for stratospheric and mesospheric H_2O has not been included ;

— the isothermal emissivity formulation has been adopted in Sasamori's formulation (Ramanathan and Downey, 1986).

These simplifications are possibly sources of disagreement between various authors. Besides, the choice of parameters characterizing the cloud layer is also different and frequently not fully explicited. Rossow *et al.* (1982) demonstrated the importance of clouds in stabilizing the Earth's climate.

Partially frozen solutions for $L/L_\odot < 1$

In this subsection, the partially frozen solutions for $L/L_\odot < 1$ will be discussed. To describe the variation of the ground albedo with surface temperature, the parameter $A_{g,\,ice}$ was first fixed to 0.7 (see fig. 1). The equilibrium surface temperature for both the moist and the fixed 6.5 K/km adiabatic lapse rate is presented in figure 3, as a function of the relative solar luminosity. The two curves cross each other at $L/L_\odot \simeq 0.98$. The surface temperature varies more slowly for the moist adiabatic model, so that the jump to the frozen solutions occurs for lower luminosities. Indeed, in this case, this jump is located between $L/L_\odot = 0.9$ and $L/L_\odot = 0.93$, whereas a 0.96 relative solar luminosity is low enough to make the system unstable in the 6.5 K/km adjustment model.

Furthermore, slightly lower temperatures are possible in the moist adiabatic case. These observations may be explained by taking into closer consideration the stability of the climate system.

To warrant stability, the condition

$$\frac{\mathrm{d}F_{\mathrm{IR}}^{\mathrm{NET}}}{\mathrm{d}T_s} > \frac{\mathrm{d}F_{\odot}^{\mathrm{NET}}}{\mathrm{d}T_s} \tag{11}$$

must be fulfilled. In this equation, $F_{\mathrm{IR}}^{\mathrm{NET}}$ and $F_{\odot}^{\mathrm{NET}}$ represent the net (upward) infrared and net (downward) solar fluxes at the top of the atmosphere and T_s is the surface temperature. If the climate system is perturbed from its equilibrium state, it will return to the equilibrium when condition (11) is satisfied. For example, if the temperature is increased, condition (11) implies that the upward flux of infrared radiation to space will become larger than the absorbed solar flux, so that the system will cool and return to equilibrium. For a given solar luminosity, the stability condition (11) may be rewritten as

$$\frac{\mathrm{d}F_{\mathrm{IR}}^{\mathrm{NET}}}{\mathrm{d}T_s} > -\frac{L}{4}\frac{\mathrm{d}\bar{\alpha}}{\mathrm{d}T_s} \tag{12}$$

The temperature dependence of $\bar{\alpha}$ includes the variation of the ground albedo T_s as well as the atmospheric contribution. In figure 7, the net infrared flux $F_{\mathrm{IR}}^{\mathrm{NET}}$ at the top of the atmosphere has been plotted as a function of T_s, both for the moist adiabatic and the fixed 6.5 K/km adjustment model. These curves are based on all the models which have reached equilibrium. For $T_s > 0°$ C, the derivative $\mathrm{d}F_{\mathrm{IR}}^{\mathrm{NET}}/\mathrm{d}T_s$ is larger in the moist adiabatic case. Consequently, the use of a moist adiabatic lapse rate increases the stability of the climate system. This is why the jump to the frozen solution arises at a lower temperature in the moist adiabatic model. As a result, the critical luminosity where the jump occurs is lower. This process is important since our results show that the adoption of a moist adiabatic lapse rate accounts for more than a 3 % decrease in the critical luminosity.

The condition (12) shows that the stability and consequently the location of the downward jump to the frozen solutions is closely tied to the assumed dependence of the ground albedo on the surface temperature. Thus, it is interesting to analyse the sensitivity of the system to the choice of the ground albedo function. Various runs were performed with the albedo curves displayed in figure 1. The behaviours of all these functions are similar : as the surface temperature is decreased, the ground albedo increases more and more rapidly until it reaches the value $A_{g,\,\mathrm{ice}}$ for an ice-covered Earth. However, at a given temperature, the derivatives $\mathrm{d}A_g/\mathrm{d}T_s$ are very different for the various models. According to the above discussion, this point is critical for the stability of the system. In fact, as the solar constant is decreased, the temperature decreases, so that $\mathrm{d}A_g/\mathrm{d}T_s$ and, hence, $\mathrm{d}\bar{\alpha}/\mathrm{d}T_s$ become more negative. This tendency cannot be compensated by the lower L value. Consequently, the right-hand side of equation (12) increases progressively. A point may be reached where this stability condition is no longer satisfied. This point corresponds to the critical luminosity at which the jump to the frozen solution occurs. As it is obvious from figure 1, the surface temperature of this critical point will be higher for the model with $A_{g,\,\mathrm{ice}} = 0.7$ than for $A_{g,\,\mathrm{ice}} = 0.5$ or for the albedo function of Wang and Stone (1980). In the latter case, it is clear that the surface temperature may become very negative before the critical point is reached. As a test, our model has been run with the ground albedo of Wang and Stone (1980). It is found that the jump to the frozen solution is located between $L/L_{\odot} = 0.78$ and $L/L_{\odot} = 0.82$, in good agreement with the fixed cloud altitude model of Wang and Stone (1980). For $L/L_{\odot} = 0.82$, the system is always stable with a surface temperature equal to $-12°$ C.

As a conclusion to this problem, it is important to note that the exact location of the downward jump to the frozen branch is highly sensitive to the convective adjustment method as well as to the adopted function describing the dependence of the ground albedo with respect to temperature. The range of values for the critical luminosity extends from $L/L_{\odot} \simeq 0.80$ to $L/L_{\odot} = 0.96$, even without considering cloud feedback.

Completely frozen solutions

Several runs were made in order to describe the stable solution for a completely frozen Earth. These models depict a situation which has probably never occurred during the existence of the Earth, since high solar luminosities are required to force the system to leave the frozen solution. In addition, the evidences of major glaciations are rare in the geologic record for the early history of Earth (Crowley, 1983). Consequently, the study of the frozen branch of the solution is only interesting from a theoretical point of view, to make the description of the climate system more complete.

The equilibrium surface temperature is presented in figure 4 as a function of the relative solar luminosity $L/L_{\odot}$. As above, the ground albedo of the ice covered Earth has been set to the values $A_{g,\,\mathrm{ice}} = 0.$ and $A_{g,\,\mathrm{ice}} = 0.5$. In the first case, both the moist and the fixed 6.5 K/km adiabatic lapse rate have been considered for the convective adjustment. Owing to the low surface temperature and hence the low water vapour content, the moist adiabatic lapse rate tends to be equal to its value $\Gamma_D = 9.8$ K/km in a dry atmosphere. Consequently, for the same solar luminosity (i.e. approximately the same net infrared flux at the top of the atmosphere), the moist adiabatic model will be warmer than the 6.5 K/km adjustment model. From figure 4, it may be observed that the temperature difference is almost independent of the solar luminosity and is equal to ~ 10 °C. However, it is clear that, over a completely frozen Earth, there is no obvious reason to fix the adiabatic lapse rate to the present 6.5 K/km value. That is why the moist adiabatic model must be thought as being more representative of the real system. The result of the model with $A_{g,\,\mathrm{ice}} = 0.5$ are also presented in figure 4. For this calculation, the moist adiabatic lapse rate was used in the convective adjustment. Much higher surface temperatures are obtained, with respect to the results for $A_{g,\,\mathrm{ice}} = 0.7$. This observation strengthens the idea that the choice of the albedo function is very important in climate modelling.

In figure 4, we have reported the surface temperatures corresponding to the energy balance equation

$$\frac{L}{4}(1 - \bar{\alpha}) = F_{\mathrm{IR}}^{\mathrm{NET}}(T_s) \qquad (13)$$

The net infrared flux at the top of the atmosphere is parameterized as

$$F_{\mathrm{IR}}^{\mathrm{NET}}(T_s) = A + BT_s \qquad (14)$$

Such a linear relationship is common in the classical energy-balance models. For A and B, the values of Budyko (1969) were adopted: $A = 202\ \mathrm{W\,m^{-2}}$ and $B = 1.45\ \mathrm{W\,m^{-2}}\ (^{\circ}\mathrm{C})^{-1}$. The mean global albedo $\bar{\alpha}$ was set equal to 0.62 and to 0.5. The first value was used by North *et al.* (1981) and may represent an average between clouds and ice for $A_{g,\,\mathrm{ice}} = 0.7$. The second is useful to compare with the $A_{g,\,\mathrm{ice}} = 0.5$ model. The most striking difference between equation 13) and the results of the radiative-convective model is the slope of the temperature curves. Indeed, the slope $\mathrm{d}T_s/\mathrm{d}L$ is slightly lower in the radiative-convective case. This difference is partially due to somewhat lower values for $\mathrm{d}F_{\mathrm{IR}}^{\mathrm{NET}}/\mathrm{d}T_s = B$ in the case of equation (14), with respect to the radiative-convective model. This may be observed in figure 7, where the Budyko (1969) outgoing infrared flux has been plotted as a function of surface temperature and may be compared to the results of the radiative-convective model.

The approximate locations of the upward jump to the unfrozen solution has been indicated in figure 4. These jumps occur when the mean ground albedo A_g becomes temperature-dependent, i.e. when the surface temperature reaches − 15 °C. At this temperature, the derivative $\mathrm{d}A_g/\mathrm{d}T_s$ is discontinuous, so that the function $\bar{\alpha}(T_s)$ becomes suddenly very sharp (see fig. 1) and makes the climate system unstable. The luminosity of the upward jump strongly depends on the ice-covered Earth albedo value $A_{g,\,\mathrm{ice}}$.

The temperature profile over an ice-covered Earth is an interesting result, since it cannot be obtained with an energy-balance model. In figure 5, we have reported the temperature profile in the fixed 6.5 K/km adjustment model, for $L/L_{\odot} = 0.9$ and $A_{g,\,\mathrm{ice}} = 0.7$. The corresponding water vapour profile is displayed in figure 6. The most striking feature is the small size of the convective region and the low tropopause altitude (8 km). As a result, the temperature at the tropopause is not much lower than at the surface. For the corresponding moist adiabatic model, the tropopause is even lower, since its altitude is $z \simeq 7$ km. Another interesting feature of this ice-covered model is a small increase of the stratopause temperature.

Relation linking the outgoing IR flux to the surface temperature

In the above discussion, we have emphasized the importance of the net infrared flux at the top of the

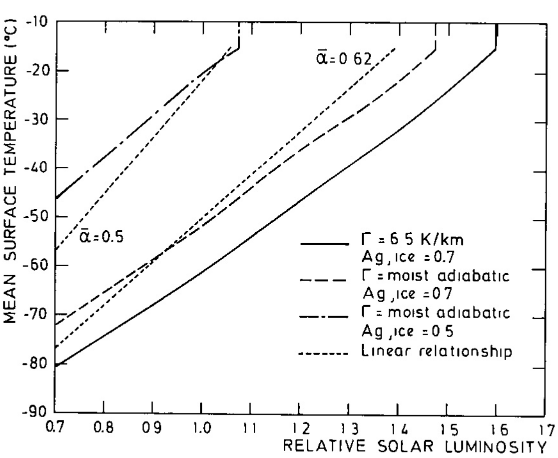

Figure 4
Equilibrium mean surface temperature as a function of relative solar luminosity $L/L_{\odot}$ for the completely frozen solution. The results of the radiative-convective model are compared with the energy-balance equation (14) *(curve labelled « linear relationship ») in which the linear parameterization of Budyko* (1969) *is adopted for the infrared flux. The parameters $A_{g,\,\mathrm{ice}}$ and $\bar{\alpha}$ are respectively the mean ground and mean total albedos of the system.*

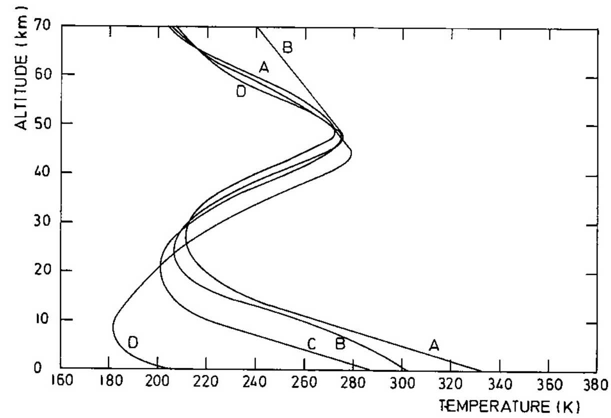

Figure 5
Vertical temperature profile in four different models.

A : $L/L_{\odot} = 1.2$, $\Gamma = 6.5$ K/km
B : $L/L_{\odot} = 1.2$, Γ = moist adiabatic
C : $L/L_{\odot} = 1.0$, $\Gamma = 6.5$ K/km
D : $L/L_{\odot} = 0.9$, $\Gamma = 6.5$ K/km (frozen branch).

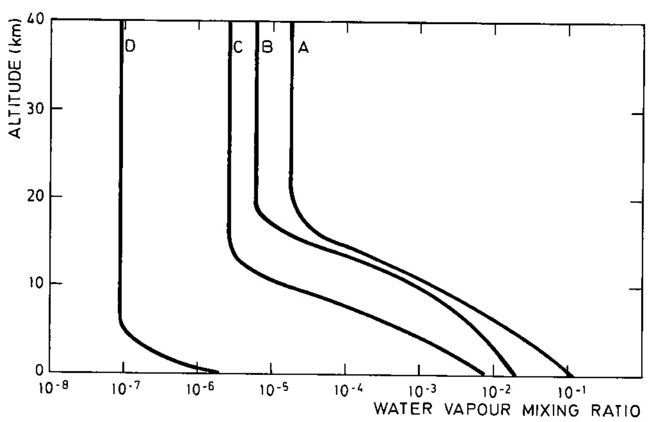

Figure 6
Vertical profiles of the water vapour mixing ratio for the models labelled A *to* D *in figure* 5.

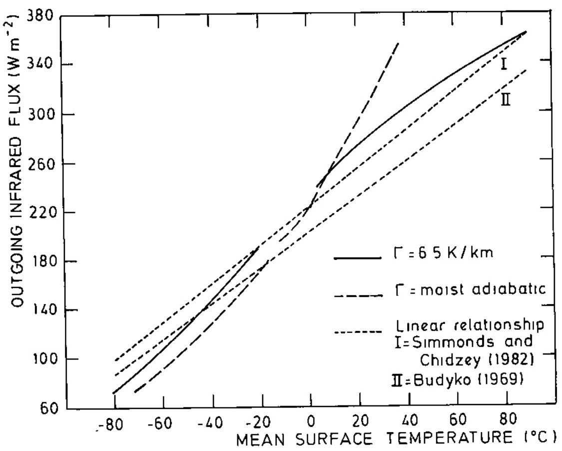

Figure 7
Outgoing infrared flux F_{IR}^{NET} as a function of surface temperature T_s. The results of the radiative-convective model are compared to two linear relationships of the form $F_{IR}^{NET} = A + BT_s$.

atmosphere, i.e. the outgoing infrared radiation flux. This flux (as well as its derivative with respect to T_s) is a critical parameter of the climate model. Generally, simple linear relationships like equation (14) are used in the energy-balance models, to describe the IR flux dependence on the surface temperature. For example, the widely-used parameterization of Budyko (1969) is displayed in figure 7 together with the more recent one of Simmonds and Chidzey (1982).

A radiative-convective model is a useful tool to calculate the outgoing infrared flux as a function of T_s, since it accounts for the vertical dimension of the atmosphere. The results of the present radiative-convective model are presented in figure 7. The curves are composed of two branches : the low temperature branch on which the Earth is completely ice-covered and the high temperature one corresponding to the ice-free or partially frozen solution. In the intermediate temperature domain, the curves are interrupted, since the climate system is unstable. These results show that the relation between F_{IR}^{NET} and T_s is non-linear. However, as a first approximation, the fixed 6.5 K/km curve agrees well with the parameterization of Simmonds and Chidzey (1982). However, when the purpose is the analysis of the consequences of a solar constant variation, the moist adiabatic curve is presumably more realistic. In this latter case, figure 7 shows that the discrepancies with usual parameterizations of the form of equation (14) are important. A similar result was obtained by Lal and Ramanathan (1984) who displayed the relation between the mean surface temperature and the incoming solar flux.

4. SUMMARY AND CONCLUSIONS

In this study, we have attempted to analyse the climate response of the atmosphere of the Earth to substantial variations in the solar luminosity. Two important feedback processes have particularly retained our attention, the Γ- and the ice-albedo feedbacks.

The first one appears when a moist adiabatic lapse rate is used for the convective adjustment. Since the low troposphere is sufficiently wet for water to condense into clouds, it is probable that a moist value is more realistic than the arbitrary 6.5 K/km lapse rate, commonly used in radiative-convective models. The Γ-feedback results in a greater stability, which corresponds to an increase of the slope of the curve giving the outgoing infrared flux as a function of the surface temperature. The comparison of our results with previous model shows that the climate response to a solar constant increase is not easy to describe, since the existing models provides very different solutions. In this case of a higher luminosity, our calculations indicate that the Γ-feedback is more effective than in the models of Vardavas and Carver (1985) and Kasting *et al.* (1984), even if complete saturation is assumed for the water vapour profile. Since it favours climate stability, this situation makes more possible the high luminosity variations predicted by recurrent mixing theories of the sun and delays the establishment of a runaway greenhouse in the early atmosphere of Venus. However, any conclusions on these subjects would remain speculative and more work is required to correctly describe the surface temperature in this range of luminosities. In the case of a solar constant decrease, the stabilization due to the Γ-feedback causes the jump toward the frozen solution to occur for somewhat lower (3-6 %) L values.

The second feedback especially studied in this paper is due to the variation of the ground albedo with surface temperature corresponding to changes in the area of the ice polar caps. This problem has largely been described with energy-balance models. In the present work, another approach has been used. Indeed, in order to link internally the outgoing infrared flux to the surface temperature, our one-dimensional radiative-convective model has been used. It is shown that the critical luminosity where the jump to the frozen solution appears is strongly dependent on the shape of the albedo-surface temperature function $A_g(T_s)$. If the slope dA_g/dT_s is not too negative, the system may remain stable for temperatures much lower than 0 °C and, thus, for relatively low luminosities. On the contrary, a sharply-varying $A_g(T_s)$ function may make unstable some systems with temperatures slightly larger than 0 °C. In the present study, it was shown that different realistic choices of the albedo function can lead to a variation of ~ 10 % in the location of the downward jump to the frozen solution. This observation would be useful in explaining the faint young sun paradox, since larger fluctuations of the climate system around the present state are permitted.

The stable solution for an ice-covered Earth has also been examined. It was observed that this solution is very sensitive to $A_{g,\text{ice}}$, the global ground albedo of the ice-covered Earth. Especially, the upward jump toward the unfrozen solution occurs at much lower solar constants for $A_{g,\text{ice}} = 0.5$ than for $A_{g,\text{ice}} = 0.7$: a difference in the critical luminosity of about 40 % has been found. If the A_g function of Wang and Stone (1980) was used, this critical luminosity would even be lower, since the climate system would become unstable as soon as the surface temperature reaches − 29 °C. From figure 4, it may be guessed that, in this case, the jump would occur for $L \simeq 0.9$. This conclusion is important to mention since it shows that, depending on the adopted albedo function, a glaciation of the complete planet may be reversible or irreversible (i.e. it would or not demand a large increase of L for deglaciation). In particular, this result is interesting in connexion with recurrent mixing theories of solar evolution. Indeed on the early Earth, when the solar constant was lower, a luminosity decrease during a transient solar phase might have frozen the whole planet, since this event would be followed by deglaciation.

In conclusion, the radiative-convective calculations of this paper indicate that the response of the Earth to solar constant changes is complex, due to the interference of several feedback processes. In view of the system complexity, we have attempted to separate the effects of these various feedbacks. The present work deals specifically with the Γ- and the ice-albedo feedbacks. However, the cloud albedo and opacity feedbacks are also important and could play a further role in stabilizing or destabilizing the Earth's climate. This implies that the actual response of the ocean-atmosphere system to large variations of the solar constant may have been different from the results of this and other previous model studies.

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