PREDICTION OF THE ACOUSTICS OF COUPLED SPACES WITH THE ACOUSTIC-DIFFUSION MODEL

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ABSTRACT

A general model is proposed to simulate the acoustics of coupled rooms. It is based on a diffusion equation, solved numerically to perform acoustic predictions. The presence of scattering objects –or the "fittings"– is also taken into account. Distinct sub-volumes can be defined, representing either coupled volumes or zones with different fitting characteristics. Some sample results are presented, and compared with ray-tracing results and experimental data. Two situations are assessed: two coupled classrooms, and a room divided into two zones, one empty, one fitted [1]. The diffusion-model predictions match the other data satisfactorily, both in terms of sound attenuation and sound decay. Diffusion-based results are obtained with the advantage of low computational time compared to ray-tracing results.

INTRODUCTION

Coupled volumes systems, composed of two or more spaces that are connected through acoustically transparent openings (i.e. coupling apertures), have attracted considerable attention in architectural acoustics. This configuration can be found in various buildings such as concert halls, industrial halls or office spaces.

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Several models have been proposed, such as statistical theory [2], modal theory [3], or ray-tracing technique [4]. In a recent paper [5], a generalization and a numerical implementation of the so-called room-acoustic diffusion model has been proposed to predict the reverberant field in single rooms. The main interest of this method is its ability to give satisfactory estimations of the spatial distribution of sound pressure level and reverberation time, with low computation times.

In this study, this model is extended to the coupled-room configuration. In addition, a solution for describing a density of obstacles scattering sound (e.g. furniture, machines etc. –the “fittings”–) within this model is also proposed. Numerical results are provided for two cases: two coupled classrooms and a room divided into two zones, one fitted, one empty. Results are then compared with experimental data, together with ray-tracing based data.

GENERAL THEORY OF COUPLED SPACES WITH AN ACOUSTIC-DIFFUSION MODEL

Single room theory. The acoustic-diffusion model has been derived by using the sound-particle concept, and details about this analogy can be found in references [5, 6]. Let us consider first a single volume in which perfectly diffuse sound reflection –i.e., described by the Lambert’s law [2]– on the surfaces is assumed. This room may be filled with objects scattering sound and randomly located in the room –the “fittings”. The input parameters for the room-acoustic diffusion model are the following:

- V and S, the room volume and the area of its surfaces, respectively, and consequently its mean free path when empty, $\lambda_r = 4V/S$;
- the complete geometry of the room surfaces;
- the local absorption coefficient of the surfaces α_r. Arbitrary spatial variations of this coefficient are possible;
- the sound absorption coefficient α_f of the fitting objects, and their scattering area per unit volume f_d –or the fitting density, in 1/m [1, 7];
- an arbitrary number M of omnidirectional sound sources assumed to be punctual, with output acoustic power $P_i(t)$ and location $\mathbf{r}_i$ ($i = 1...M$).

The time- and space-dependent reverberant acoustic energy density $w(\mathbf{r}, t)$ can then be shown to be described by a diffusion equation [6, 5]:

$$\frac{\partial w(\mathbf{r},t)}{\partial t} - D\nabla^2 w(\mathbf{r},t) + \sigma w(\mathbf{r},t) = \sum_{i=1}^{M} P_i(t)\delta(\mathbf{r}-\mathbf{r}_i) \qquad \text{in} \quad \mathcal{V}, \tag{1}$$

$$D\frac{\partial w(\mathbf{r},t)}{\partial n} + \frac{c\alpha}{4} w(\mathbf{r},t) = 0 \qquad \text{on} \quad \mathcal{S}. \tag{2}$$

In these equations, ∇^2 is the Laplace operator, $\mathcal{V}$ denotes the domain delimited by the room surfaces, $\mathcal{S}$ denotes the room boundaries, and α is the local surface absorption coefficient. The term D is the so-called diffusion coefficient, with expression

$$D = \frac{\lambda c}{3}, \tag{3}$$

λ being the mean free path of the sound particles between two collisions, either on a surface or on a fitting object [8]:

$$\lambda = \frac{(\lambda_r/f_d)}{\lambda_r + 1/f_d}. \tag{4}$$

The term σ in Eq. (1), with expression:

$$\sigma = c\alpha_f f_d, \tag{5}$$

represents the absorption of sound energy by the fittings. The right-hand term of the diffusion equation, denoted below as $F(t)$, is a source term which models the acoustic sources in terms of power output and location [5]. The boundary condition defined by Eq. (2) models the sound energy absorption by the room surfaces using the Sabine's absorption coefficient. The numerical solving of Eqs. (1) and (2) –by using a finite-element-model (FEM) solver– permits the reverberant sound field in a single volume to be predicted [5, 8]. The direct sound field radiated by the source can also be easily integrated into the solution.

Coupled-rooms theory. This theory can be extended to an arbitrary number N of acoustically coupled sub-volumes $\mathcal{V}_i$. Each sub-volume is described by its volume V_i, the area of its surfaces S_i, the characteristics of its fittings, f_{di} and α_{di}, and a source term $F_i(t)$ depending on the point-sources located within its volume, as written in Eq. (1). Each sub-volume is then characterized by its own mean free path λ_i, depending on its shape, dimensions, and fitting characteristics. Diffuse reflections on surfaces are still assumed. The time and space-dependent acoustic energy density $w(\mathbf{r}, t)$ is described in each sub-domain $\mathcal{V}_i$ by a well-defined diffusion equation:

$$\frac{\partial w(\mathbf{r}, t)}{\partial t} - D_i \nabla^2 w(\mathbf{r}, t) + \sigma_i w(\mathbf{r}, t) = F_i(t) \qquad \text{in} \quad \mathcal{V}_i, \tag{6}$$

with $\cup_i \mathcal{V}_i = \mathcal{V}$, $\mathcal{V}$ being the total calculation domain over which the sound energy density is to be calculated. $D_i = \lambda_i c/3$ and $\sigma_i = c\alpha_{fi} f_{di}$ are the diffusion coefficient and fitting-absorption term for a given sub-volume, as presented above for a single volume. The absorption of sound by surfaces is still described by the boundary condition of Eq. (2). Two kind of different practical situations can lead, in the context of the room-acoustic diffusion theory, to the occurrence of different sub-volumes $\mathcal{V}_i$:

- acoustically coupled rooms: two sub-volumes are connected through a small aperture (see Fig. 1a in the case of two empty coupled rooms). In this case the mean free paths λ_i of the empty sub-volumes –leading to the determination of the diffusion coefficient D_i– is evaluated as if the two coupled sub-volumes were uncoupled –the aperture is closed. This implies that this aperture does not affect much the mean free path of the sub-volumes [9, 10];

- different sub-volumes can model different parts of a room with different fitting characteristics, in terms of absorption and density (see Fig. 1b for the case of a room split into two parts with different fitting densities).

Both cases, in fact, can be seen as particular cases of non-homogeneous diffusion, leading to sub-volumes with different diffusion parameters D and σ.

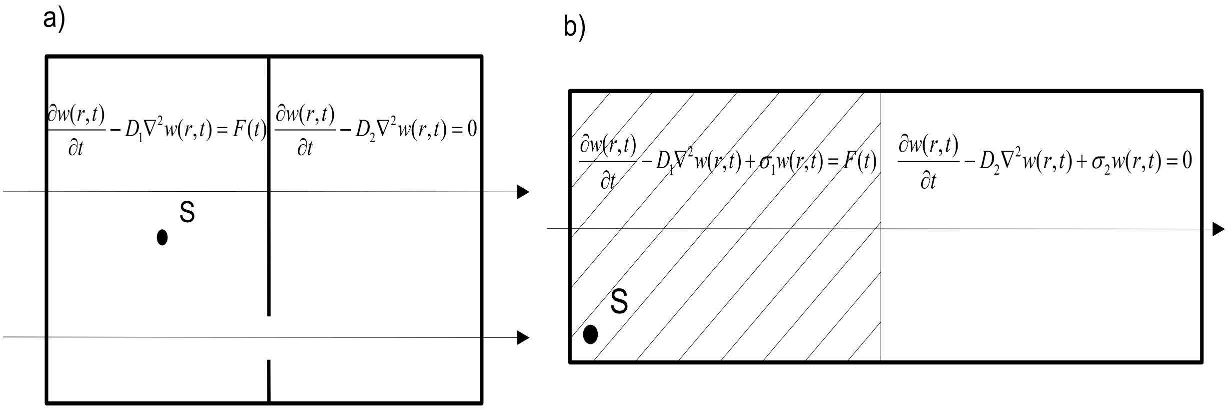

Figure 1: *a) sketch of two coupled rooms; b) sketch of a room with two zones with different fittings characteristics.*

SOME CALCULATION RESULTS AND THEIR COMPARISON WITH RAY-TRACING AND EXPERIMENTAL DATA

Case studies. In this part, some predictions by using the room-acoustic diffusion model are presented and compared to experimental data and ray-tracing-based results. Two cases are considered. The first is the case of two acoustically coupled classrooms [9, 10]. A simplified sketch of this configuration is presented in Fig. 1a; their dimensions are nearly identical $(9.4 \times 6.7 \times 3)$ m^3. They are both empty ($f_d = 0$), so that the diffusion coefficients D_1 and D_2 are in this case identical. They are coupled through a (0.8×2.1) m^2 door aperture. An omnidirectional sound source is located at the center of the left room, at height 1.5 m. The absorption coefficient of these rooms has been preliminarily estimated by reverberation time (RT) measurements when uncoupled – i.e. the door was shut. The rooms volume has been meshed by using 5400 elements with the FEM solver for obtaining diffusion results. For comparison, the spatial variations of sound field has been carefully measured in terms of sound pressure level (SPL) and RT, with about 160 measurement locations in each coupled room (with a finer discretization around the coupling aperture). For further comparison, numerical simulations were also carried out with the ray-tracing based software CATT-Acoustic. 20×10^6 sound rays were emitted to calculate the spatial variations of SPL, and 200×10^3 sound rays per receptor were used for the RT predictions.

The second case investigated is a room with dimensions $(30 \times 8 \times 3.85)$ m^3, split into two equal parts with different fitting densities, initially studied by Ondet et al. [1] (simplified sketch in Fig. 1b). An omnidirectional sound source, with output power 100 dB, is located close to the left bottom corner, at height

0.85 m. The absorption coefficients are well-defined (see reference [1] for further details), and the fittings were polystyrene blocks ($\alpha_f = 0.3$) of dimensions $(0.5 \times 0.5 \times 3)$ m^3. In configuration A, the left part is fitted with an estimated fitting density of about 0.26 m^{-1}, and the right one is empty. Configuration B is the reverse (left part empty and right part fitted). Some SPL measurements has been performed along the line indicated by the arrow on figure 1b for both configurations. For further comparison, numerical results are given by using the ray-tracing based Rayscat software [1]. The number of rays emitted is 20×10^5. It is emphasized here that in the experimental case studied, the reflection law of the room surfaces has been found to be purely specular [1]. As the diffusion model assumes purely diffuse reflections, the use of the diffusion model necessitates a preliminary empirical estimation of the mean free path of the room, when empty [8]; the extension of the acoustic-diffusion theory to rooms with arbitrary reflection laws is currently being investigated.

Stationary sound attenuation. For the two coupled classrooms, Figs. 2a and 2b plot respectively the SPL along two lines passing through the coupling aperture and through the wall, as indicated on Fig. 1a, for frequency 1 kHz (the separation wall is at position 6.5 m). The diffusion method and the ray-tracing method both give satisfactory results concerning the transition of SPL through the door (maximum discrepancy 2 dB with the measurement data); results are better with ray-tracing concerning the line passing through the wall. On the other hand, ray-tracing calculation requires a much higher computational time –at least ten times greater– since obtaining consistent results in the neighbouring room requires a high number of emitted rays.

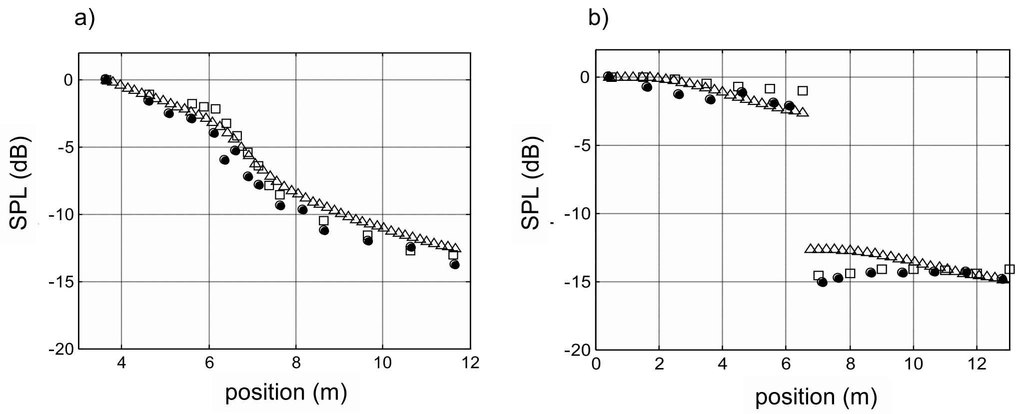

Figure 2: *SPL at frequency* 1 *kHz along the two arrows passing a) through the coupling aperture and (b) through the separation wall, as shown in Fig. 1a. (•) Experimental data, (△) room-acoustic diffusion model, (□) ray-tracing.*

For the half-fitted room, results are given in Figs. 3a and 3b for configurations A and B, respectively, at frequency 2 kHz. The change of sound attenuation at the separation line between the two zones, together with the sound attenuation slopes, are equally well predicted by the room-acoustic diffusion and ray-tracing models –the ray-tracing method, again, giving higher computational time.

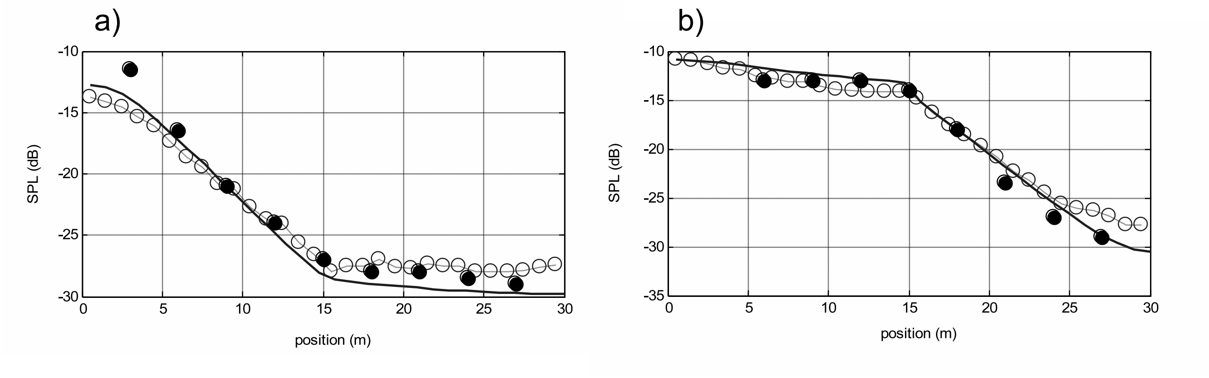

Figure 3: *SPL along the arrow shown in Fig. 1b, at frequency* 2 *kHz.* ($\bullet$), *Measurements data [1]; Solid line, room-acoustic diffusion model;* ($\circ$) *ray-tracing model. (a) Configuration A; (b) Configuration B.*

Sound decay. Simulated and experimental RT (RT20 are calculated in this example) as a function of frequency, averaged over all receiver locations for each room, are presented for the case of the two coupled classrooms, in Figs. 4a (source room) and 4b (neighbouring room). Data also show the experimental values of the RT20 of the rooms when uncoupled: the RT20 is not much affected by the coupling in the source room. Conversely it increases significantly in the neighbouring room, due to the coupling with the source room, more reverberant –this is due to the presence of a concrete wall in this room. The diffusion-model and ray-tracing results are both in good agreement with the experimental data, with mean discrepancies of about 8 % for both rooms. The room-diffusion model predicts well the influence of coupling on the RT20 in this experimental case; the calculation time is 8 min and gives the RT20 variations over the whole calculation domain; on the same computer the ray-tracing requires about 45 min per receptor.

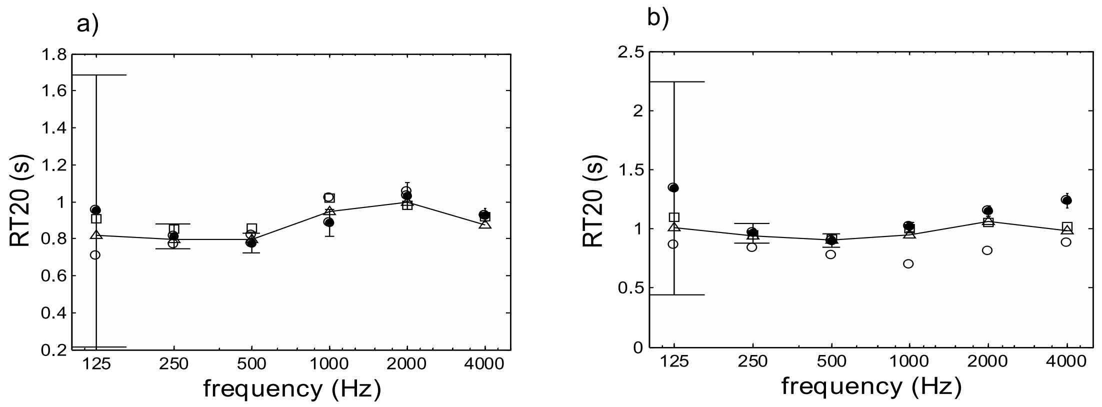

Figure 4: *Reverberation time (RT20). (a) Source room, (b) neighbouring room. Coupled rooms:* ($\bullet$) *experimental data,* ($\triangle$) *diffusion model,* ($\square$) *ray-tracing; Uncoupled rooms:* ($\circ$) *experimental data. The vertical bars indicate the dispersion of the RT measurements at all locations.*

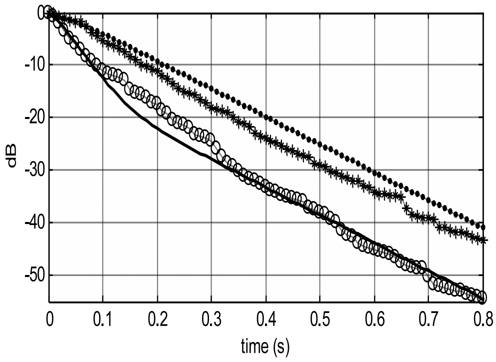

Figure 5: *Temporal sound decay for Configuration A. Solid line, room-acoustic diffusion model in the fitted (left) part;* (•) *room-acoustic diffusion model in the empty (right) part;* (○) *ray-tracing model in the fitted part;* ($*$) *ray-tracing model in the empty part.*

A sample example is also given for the calculation of RT30 in the case of the half-fitted room, configuration A. No experimental data are available for this case. Temporal sound decay curves, as given by the diffusion and ray-tracing models, are presented in Fig. 5. Whereas the decay is quasi-linear in the empty zone, a phenomenon of double decay is predicted in the fitted zone, containing the source: the shorter reverberation, due to the presence of absorbing fittings, is first heard; the later part of the sound decay is then dominated by the longer reverberation of the empty zone. Both methods predict this double decay with similar slopes, although the transition between them is differently predicted.

Double decay is usually associated with coupled rooms acoustics, but this example shows that two parts of a same room fitted very differently is also favorable to double decay occurrence. As mentioned earlier, coupled rooms and rooms with variable fittings are both particular cases of non-homogeneous diffusion in the context of room-acoustic diffusion theory.

SUMMARY

A model – the so-called room-acoustic diffusion model– is proposed to simulate the acoustics of coupled rooms. It is based on a diffusion equation, solved numerically to perform acoustic predictions. The presence of scattering objects within the rooms –or the "fittings"– is also taken into account. Distinct sub-volumes can be defined, representing either coupled volumes or zones with different fitting characteristics. Some sample results are presented, and compared with ray-tracing results and experimental data. Two situations are assessed: two coupled classrooms, and a room divided into two zones, one empty, one fitted. The diffusion-model predictions match the other data satisfactorily, both in terms of sound attenuation and sound decay. Diffusion-based results are obtained with the advantage of low computational time, compared to ray-tracing results. Further work will aim at simulating the acoustics of volumes containing both acoustically coupled sub-volumes and zones with different fitting characteristics. Practical applications could be industrial workrooms, and educational or professional buildings for instance, where offices or rooms with a high amount of fittings are connected to empty and more reverberant halls.

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