Modelling the sound transmission through partition walls using a diffusion model

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ABSTRACT
The diffusion model has been used successfully to evaluate the acoustic behaviour of a system of coupled rooms connected through a coupling aperture. In this paper, an extension of this model is proposed to deal with the propagation of sound energy through a partition wall. The diffusion model can be considered as an extension of the statistical theory to none diffuse sound fields. Numerical comparisons with the statistical theory are then carried out. The following parameters are varied: its transmission loss of the separation wall, its surface, the coupled room’s absorption coefficient and the coupled room’s volume. The agreement between the statistical theory and the diffusion model is very good.

INTRODUCTION
In two adjacent rooms separated by a partition wall (Fig. 1), exchanges of acoustical energy occur. This phenomenon is of great significance for buildings comfort. The ISO 140-4 norm [1], which permits to evaluate the transmission loss of a partition wall through measurements, and the EN 12354-1 norm [2], used for predicting the sound transmission through partition walls, are based on the statistical theory. However, this theory becomes invalid when the reverberant sound field departs from the diffuse field assumptions [3], like in long or flat rooms. For such geometries, Valeau et al. [4] showed that the transmission through a partition wall can be evaluated by using a model based on a diffusion equation. The so-called diffusion model can be viewed as an extension to the statistical theory with the main interest that the predicted reverberant sound field is no longer tied to the diffused field assumptions. This model introduced by Ollendorff [5] has been validated, experimentally or numerically, in configurations like rooms with proportionate dimensions [4,6,7], long rooms [4,6-8], flat rooms [4,6,7,9] and a system of rooms coupled through an aperture [10]. Nevertheless, in the model of Valeau et al.[4], the transmission through a partition wall is treated as a boundary condition: only the reverberant field within the room without the sound source (noted in the following coupled room) is evaluated. The room containing the source (noted in the following source room) is reduced to a surface sound source located at the partition wall, which energy is deduced from the statistical theory.

In this paper, a more accurate extension of the diffusion model is proposed to deal with the complete phenomenon of the reverberant sound field transmission between two rooms through a partition wall. In Section 2, the statistical theory and the modified diffusion model are presented. The proposed modifications of the diffusion model are numerically validated by comparison to the statistical theory in Section 3.

MODELS PRESENTATION
The characteristics of the problem under consideration are as follows (Fig. 1): a first room (the source room) of volume $V_1$ and surface area $S_1$ (without the partition wall area) contains a sound source of power $P$. This room is separated from a second room (the coupled room, of volume $V_2$ and surface area $S_2$ without the partition wall area) by a partition wall. The part of the wall which allows sound transmission is noted down coupling area. The coupling area is defined by its surface $S_c$ and its transmission loss $R$. The absorption coefficients in the source room and in the coupled room are noted down $\alpha_s$ and $\alpha_c$, respectively.
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In the statistical theory, two constant energy densities, $E_1$ in the source room and $E_2$ in the coupled room are considered. The energy balance in the coupled room can be written as [11]:

$$-A_2 E_2 \frac{c}{4} + \tau S_e E_1 \frac{c}{4} - \tau S_e E_2 \frac{c}{4} = 0,$$

where $\tau = 10^{-R/10}$ is the transmission coefficient and $c$ the sound velocity. The first term depicts the absorption of sound energy by the walls, $A_2$ being the equivalent absorption area of the coupled room. The second one represents the energy transfer from the source room to the coupled room through the coupling area and the third one, the energy transfer from the coupled room to the source room. Considering now the sound levels, $L_1$ in the source room and $L_2$ in the coupled room, we obtain [11]:

$$L_2 - L_1 = R - 10 \log \left( \frac{S_e}{A_2} \right).$$  (2)

The ISO 140-4 and EN 12354-1 standards [1,2] use both Eq. (2) to relate the sound pressure level difference ($L_2-L_1$) to the partition wall transmission loss $R$. In this norm the equivalent absorption area is evaluated by using the Sabine absorption coefficient. In this paper, the Eyring absorption coefficient, more accurate for high absorptions, is used instead.

Diffusion model

Based on the diffusion of particles in a scattering medium [12], the acoustic energy density distribution in an enclosure of volume $V$ and surface $S$ can be described by the following diffusion equation [4]:

$$\frac{\partial w(r,t)}{\partial t} - D \nabla^2 w(r,t) = P(r,t) \text{ in } V,$$  (3)

where $w(r,t)$ is the acoustic energy density, $\nabla^2$ the Laplace operator, $P(r,t)$ the source term and $D = \lambda c / 3$ with $\lambda = 4V / S$ the room mean free path. The energy exchanges at the boundaries (walls absorptions) can be expressed as a mixed boundary condition, in the following form [4]:

$$\mathbf{J} \cdot \mathbf{n} = -D \frac{\partial w(r,t)}{\partial \mathbf{n}} = hw(r,t) \text{ on } S,$$  (4)

where $h$ is the local exchange coefficient, $\mathbf{n}$ the outgoing normal vector and $\frac{\partial}{\partial \mathbf{n}}$ the normal derivative. The exchange coefficient can be expressed using $\alpha$, the Eyring absorption coefficient [6,7], as:

$$h = -\frac{c \ln (1 - \alpha)}{4}$$  (5)
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The sound pressure level (SPL) can be expressed as [13] $\text{SPL}(r,t) = 10 \log \left( \frac{w(r,t) \rho c^2}{P_{\text{ref}}^2} \right)$ where $P_{\text{ref}}$ is equal to $2 \times 10^5$ Pa. The diffusion equation (4) associated to the boundary conditions of Equation (5) are solved numerically by means of the finite element method; details can be found in reference [4].

To extend this model to two enclosures coupled through a partition wall, two diffusion equations must be considered in each enclosure:

$$-D_1 \nabla^2 w_1(r) = P(r) \text{ in } V_1,$$

$$-D_2 \nabla^2 w_2(r) = 0 \text{ in } V_2,$$

where $D_1$ and $D_2$ are the respective diffusion coefficients of the source and coupled rooms. Note that only the stationary problem is considered here, the time related terms being discarded. The boundary conditions can be expressed as:

$$D_1 \frac{\partial w_1(r)}{\partial n_1} + h_1 w_1(r) = 0 \text{ on } S_1,$$

$$D_2 \frac{\partial w_2(r)}{\partial n_2} + h_2 w_2(r) = 0 \text{ on } S_2,$$

where $n_1$ and $n_2$ are the exterior normal vectors. At the coupling area, the energy exchanges between the enclosures must be expressed. To model the energy transmission from the source room to the coupled one, Valeau et al. [4] treat the partition wall as a sound source in the coupled room. The ingoing energy flux $J_m$ can be written, as [4]:

$$\mathbf{J} \cdot \mathbf{n} = -D \frac{\partial w}{\partial n} = -J_m,$$

the negative sign accounting for the ingoing nature of the energy in the coupled room. The value of $J_m$ is related to the mean density $w_0$ in the source room and the transmission coefficient through [4]:

$$J_m = -\frac{\tau c w_0}{4}.$$

In this last model, $w_0$ was evaluated by using the statistical theory. The sound pressure level, obtained in the coupled room by using this diffusion model, matched satisfactorily the one obtained by using the statistical theory. Here, this boundary condition is extended on either sides of the coupling area. If we consider also the energy absorption occurring at the surface of the coupling area, we obtain now:

$$D_1 \frac{\partial w_1(r)}{\partial n_1} + h_1 w_1(r) = \frac{\tau c}{4} w_2 \text{ on } S_c, \text{ side source room},$$

$$D_2 \frac{\partial w_2(r)}{\partial n_2} + h_2 w_2(r) = \frac{\tau c}{4} w_1 \text{ on } S_c, \text{ side coupled room}.$$

Equation (12) accounts for the energy transfer from the coupled room to the source room and Eq. (13) accounts for the energy transfer from the coupled room to the source room. Equations (7) and (8) together with the boundary conditions (8), (9), (12) and (13) form a system which must solved by a no-linear method.
NUMERICAL VALIDATION
In this section, the diffusion model is compared to the statistical theory. The tested geometry is composed of 5x5x5 m room containing the sound source (Fig. 2). This sound source is modeled has a 17 cm sphere locate at (2, 2, 2) m with sound power level of 100dB. The source room is coupled to a room with a 5X5m cross section and a varying length. In each room, the walls absorption coefficient is homogenous. The comparison criterions are the difference $L_1- L_2$ (dB) of the sound pressure level of reverberant sound between the source and the coupled rooms and the sound pressure level SPL (dB) along the line C. For the diffusion model, the sound pressure levels are evaluated by averaging the sound pressure level in the whole volume of each room. Several parameters are investigated: the transmission loss $R$ and the surface $S_c$ of the coupling area, $\alpha_c$ the absorption coefficients of the coupled rooms as well as the length $l_x$ of the coupled room.

To perform the diffusion model simulations, a maximum number of 6000 linear Lagrange elements are used, giving a computational time inferior to 30 s on a personal computer.

![Simulated configuration (dimensions in m).](image)

Transmission loss
The coupled room dimensions are 5x5x5 m and the absorption coefficients of both rooms are set to 0.1. The coupling area surface is equal to 25 m$^2$ and its transmission loss is varied between 10 and 50 dB. Fig. 3(a) shows a good agreement between the models, with a maximal discrepancy inferior to 0.5 dB.

![Sound pressure level difference as a function of the transmission loss(a) and of the coupling area surface: statistical theory (●), diffusion model (○).](image)

Coupling area surface
The coupled room dimensions are set to 5x5x5 m and the absorption coefficients to 0.1 for both rooms. The coupling area transmission loss is to 20 dB and its surface is varied between 1 and 25 m$^2$. Fig. 3(b) shows again a very good agreement between the models, with a maximal discrepancy of about 0.3 dB.

Absorption area of the coupled room
The coupled room dimensions are set to 5x5x5 m. The coupling area transmission loss is 20 dB and its surface is equal to 25 m$^2$. The absorption coefficient of the source room is equal to 0.1 while the coupled room absorption is varied between 0.05 and 0.5. The agreement between the models is very good, with a maximal discrepancy inferior to 0.7 dB (Fig. 4(a)).
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Figure 4.- Sound pressure level difference as a function of the coupled room’s absorption coefficient (a) and the coupled room length: statistical theory (▬) and diffusion model (●).

**Volume of the coupled room**
The coupled room length is varied from 1 to 30 m. The coupling area transmission loss is 20 dB and its surface to 25 m². The absorption coefficient of both rooms is equal to 0.1. Again, the agreement between the models is good, with a maximal discrepancy inferior to 2.3 dB (Fig. 4(b)). However, the difference between the models increases with the coupled room length. As expected, when the coupled room length increases, the reverberant sound field is not strictly diffuse, meaning that the statistical theory is no longer suitable. As an example, for \( l_x = 30 \) m, the coupled room length is six times greater than the other dimensions: it can be then assimilated to a long room where it is known that the diffuse field assumptions are not valid [14]. Thus, Fig. 5 shows one of the main interests of the diffusion model: the predicted reverberant sound field within the coupled room (as well as in the source room) is not diffuse. In the present configuration, the sound pressure level varies of about 14 dB along the length of the coupled room. For a more absorbent room or a more disproportionate room, this variation will be greater. The diffusion model allows one to predict the sound transmission through rooms whose the diffuse sound field assumptions are not valid, like long rooms (e.g. corridors) and flat rooms (e.g. workshops).

![Figure 5.- Distribution of the sound pressure along line C with \( l_x=30 \) m: statistical theory (▬) and diffusion model (●).](image)

**CONCLUSIONS**
A modification of the diffusion model for room-acoustics has been proposed to account for sound transmission between two rooms coupled through a partition wall. In this model, the reverberant sound fields are evaluated with two diffusion equations, one for each room. A coupling condition at the partition wall is introduced and the considered system is none linearly solved. This modified diffusion model has been validated by comparison with the statistical theory. The different parameters governing the sound transmission: the transmission loss and the surface of the coupling area, the absorption coefficients of the coupled room as well as the length of the coupled room, have been varied. A very good agreement was found with the statistical theory, with a maximal discrepancy inferior to 2.3 dB. The diffusion model can be then used an alternative to the statistical theory as a prediction tool in building acoustic. While the statistical theory is limited to configurations where the diffuse field assumptions hold, the diffusion model can predict the transmission in configurations where the reverberant sound field is not diffuse in expense of a very low computation time.

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