



Laboratoire de Techniques Aéronautiques et Spatiales (ASMA)
Milieux Continus & Thermomécanique

Contributions aux algorithmes d'intégration temporelle conservant l'énergie en dynamique non-linéaire des structures.

Travail présenté par

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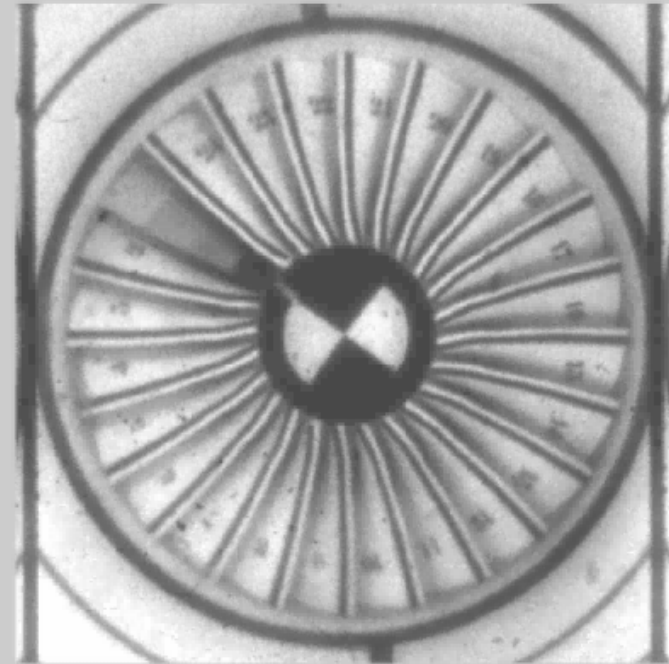
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Introduction

Industrial problems

- Industrial context:
 - Structures must be able to resist to crash situations
 - Numerical simulations is a key to design structures
 - Efficient time integration in the non-linear range is needed
- Goal:
 - Numerical simulation of blade off and wind-milling in a turboengine
 - Example from SNECMA





Introduction

Original developments

- Original developments in implicit time integration:
 - Energy-Momentum Conserving scheme for elasto-plastic model (based on a hypo-elastic formalism)
 - Introduction of controlled numerical dissipation combined with elasto-plasticity
 - 3D-generalization of the conserving contact formulation
- Original developments in implicit/explicit combination:
 - Numerical stability during the shift
 - Automatic shift criteria
- Numerical validation:
 - On academic problems
 - On semi-industrial problems



Scope of the presentation

1. Scientific motivations
2. Consistent scheme in the non-linear range
3. Combined implicit/explicit algorithm
4. Complex numerical examples
5. Conclusions & perspectives



1. Scientific motivations

- Dynamics simulation
- Implicit algorithm: our opinion
- Conservation laws
- Explicit algorithms
- Implicit algorithms
- Numerical example: mass/spring-system

2. Consistent scheme in the non-linear range

3. Combined implicit/explicit algorithm

4. Complex numerical examples

5. Conclusions & perspectives



1. Scientific motivations

Dynamics simulations

- Scientific context:
 - Solids mechanics
 - Large displacements
 - Large deformations
 - Non-linear mechanics

- Spatial discretization into finite elements:

- Balance equation
$$M \ddot{\vec{x}} + \vec{F}^{\text{int}} = \vec{F}^{\text{ext}}$$

- Internal forces formulation
$$\vec{F}^{\text{int}} = \int_{V_0} \Sigma \mathbf{f}^T \vec{D} J dV_0$$

Σ : Cauchy stress; \mathbf{F} : deformation gradient; $\mathbf{f} = \mathbf{F}^{-1}$;

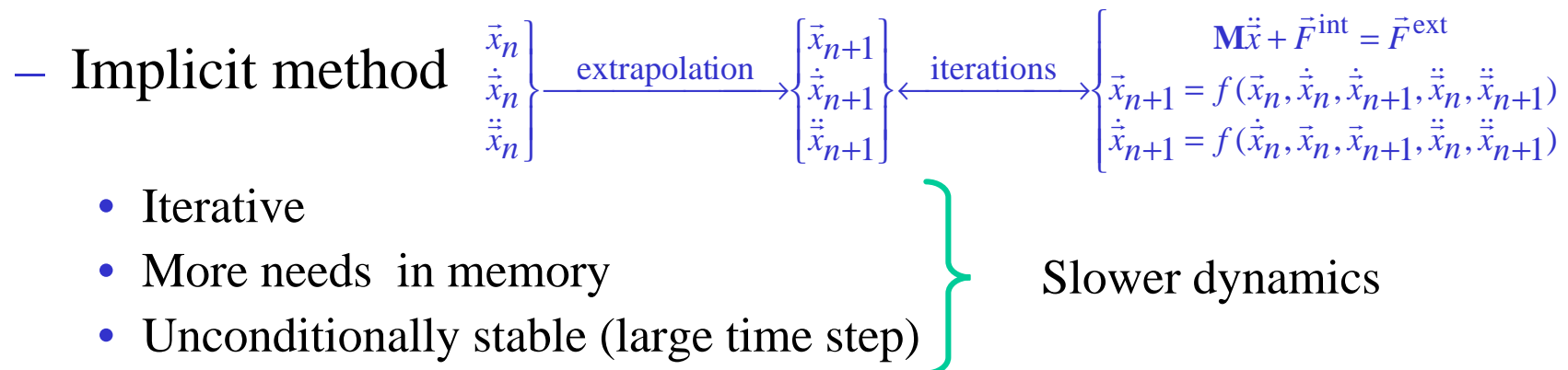
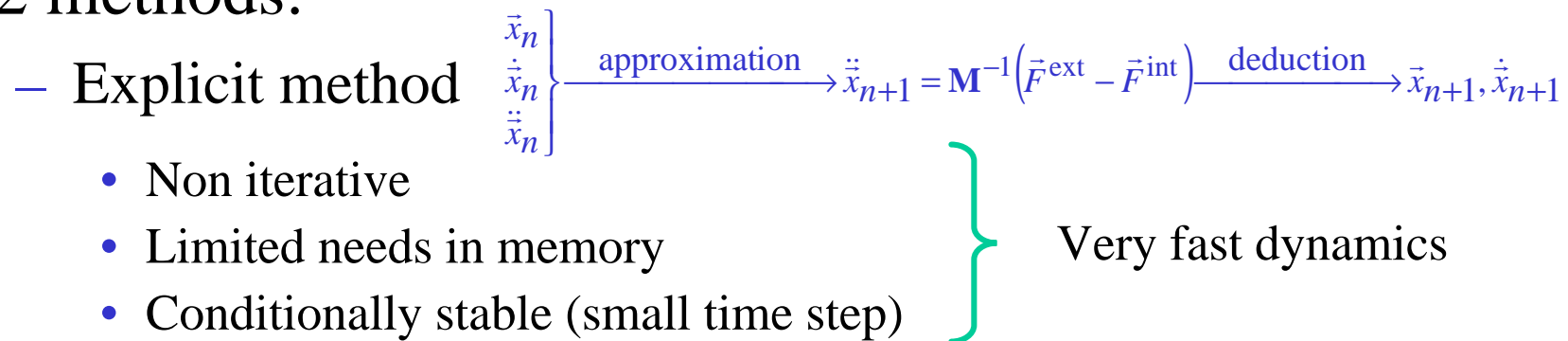
$\vec{D} = \partial \mathbf{j} / \partial \vec{x}_0$: derivative of the shape function; J : Jacobian



1. Scientific motivations

Dynamics simulations

- Temporal integration of the balance equation
- 2 methods:





1. Scientific motivations

Implicit algorithm: our opinion

- If wave propagation effects are negligible
 - Implicit schemes are more suitable
 - Sheet metal forming (springback, superplastic forming, ...)
 - Crashworthiness simulations (car crash, blade loss, shock absorber, ...)
- Nowadays, people choose explicit scheme mainly because of difficulties linked to implicit scheme:
 - Lack of smoothness (contact, elasto-plasticity, ...)
 - convergence can be difficult
 - Lack of available methods (commercial codes)
- Little room for improvement in explicit methods
- Complex problems can take advantage of combining explicit and implicit algorithms



1. Scientific motivations

Conservation laws

- Conservation of linear momentum (Newton's law)

- Continuous dynamics $\frac{\partial \mathbf{M}\dot{\vec{x}}}{\partial t} = \vec{F}^{\text{ext}}$

- Time discretization $\sum_{\text{nodes}} \mathbf{M}\dot{\vec{x}}_{n+1} - \mathbf{M}\dot{\vec{x}}_n = \Delta t \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{ext}} \quad \& \quad \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{int}} = 0$

- Conservation of angular momentum

- Continuous dynamics $\frac{\partial \vec{x} \wedge \mathbf{M}\dot{\vec{x}}}{\partial t} = \vec{x} \wedge \vec{F}^{\text{ext}}$

- Time discretization $\sum_{\text{nodes}} \sum_{\text{nodes}} \vec{x}_{n+1} \wedge \mathbf{M}\dot{\vec{x}}_{n+1} - \vec{x}_n \wedge \mathbf{M}\dot{\vec{x}}_n = \Delta t \sum_{\text{nodes}} \vec{x}_{n+1/2} \wedge \vec{F}_{n+1/2}^{\text{ext}}$

- $\& \quad \sum_{\text{nodes}} \vec{x}_{n+1/2} \wedge \vec{F}_{n+1/2}^{\text{int}} = 0$

- Conservation of energy

- Continuous dynamics $\frac{\partial}{\partial t} K + \frac{\partial}{\partial t} W^{\text{int}} = \frac{\partial}{\partial t} W^{\text{ext}} + \dot{D}^{\text{int}}$

W^{int} : internal energy;

W^{ext} : external energy;

D^{int} : dissipation (plasticity ...)

- Time discretization $W_{n+1}^{\text{int}} - W_n^{\text{int}} - \Delta D^{\text{int}} = \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{int}} \cdot [\vec{x}_{n+1} - \vec{x}_n] \quad \&$

$$\sum_{\text{nodes}} \frac{1}{2} \mathbf{M}\dot{\vec{x}}_{n+1} \cdot \dot{\vec{x}}_{n+1} - \frac{1}{2} \mathbf{M}\dot{\vec{x}}_n \cdot \dot{\vec{x}}_n + W_{n+1}^{\text{int}} - W_n^{\text{int}} - \Delta D^{\text{int}} = \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{ext}} \cdot [\vec{x}_{n+1} - \vec{x}_n]$$





1. Scientific motivations

Explicit algorithms

- Central difference (no numerical dissipation)

$$\dot{\vec{x}}_{n+1/2} = \dot{\vec{x}}_{n-1/2} + \Delta t \ddot{\vec{x}}_n$$

$$\vec{x}_{n+1} = \vec{x}_n + \Delta t \dot{\vec{x}}_{n+1/2}$$

$$\ddot{\vec{x}}_{n+1} = M^{-1} [\vec{F}_{n+1}^{\text{ext}} - \vec{F}_{n+1}^{\text{int}}]$$

- Hulbert & Chung (numerical dissipation) [CMAME, 1996]

$$[1 - \mathbf{a}_M] \ddot{\vec{x}}_{n+1} = M^{-1} [\vec{F}_n^{\text{ext}} - \vec{F}_n^{\text{int}}] - \mathbf{a}_M \ddot{\vec{x}}_n$$

$$\vec{x}_{n+1} = \vec{x}_n + \Delta t \dot{\vec{x}}_n + \Delta t^2 \left[\frac{1}{2} - \mathbf{b} \right] \ddot{\vec{x}}_n + \Delta t^2 \mathbf{b} \ddot{\vec{x}}_{n+1}$$

$$\dot{\vec{x}}_{n+1} = \dot{\vec{x}}_n + \Delta t [1 - \mathbf{g}] \ddot{\vec{x}}_n + \Delta t \mathbf{g} \ddot{\vec{x}}_{n+1}$$

- Small time steps \longrightarrow conservation conditions are approximated
- Numerical oscillations may cause spurious plasticity



1. Scientific motivations

Implicit algorithms

- α -generalized family (Chung & Hulbert [JAM, 1993])

- Newmark relations:
$$\begin{cases} \ddot{\bar{x}}_{n+1} = \frac{1}{\mathbf{b} \Delta t^2} \left[\bar{x}_{n+1} - \bar{x}_n - \Delta t \dot{\bar{x}}_n - \left[\frac{1}{2} - \mathbf{b} \right] \Delta t^2 \ddot{\bar{x}}_n \right] \\ \dot{\bar{x}}_{n+1} = \frac{\mathbf{g}}{\mathbf{b} \Delta t} \left[\bar{x}_{n+1} - \bar{x}_n + \left[\frac{\mathbf{b}}{\mathbf{g}} - 1 \right] \Delta t \dot{\bar{x}}_n + \left[\frac{\mathbf{b}}{\mathbf{g}} - \frac{1}{2} \right] \Delta t^2 \ddot{\bar{x}}_n \right] \end{cases}$$

- Balance equation:
$$\frac{1 - \mathbf{a}_M}{1 - \mathbf{a}_F} M \ddot{\bar{x}}_{n+1} + \frac{\mathbf{a}_M}{1 - \mathbf{a}_F} M \ddot{\bar{x}}_n + [\vec{F}_{n+1}^{\text{int}} - \vec{F}_{n+1}^{\text{ext}}] + \frac{\mathbf{a}_F}{1 - \mathbf{a}_F} [\vec{F}_n^{\text{int}} - \vec{F}_n^{\text{ext}}] = 0$$

- $\alpha_M = 0$ and $\alpha_F = 0$ (no numerical dissipation)

- Linear range: consistency (i.e. physical results) demonstrated
- Non-linear range with small time steps: consistency verified
- Non-linear range with large time steps: total energy conserved but without consistency (e.g. plastic dissipation greater than the total energy, work of the normal contact forces > 0 , ...)

- $\alpha_M \neq 0$ and/or $\alpha_F \neq 0$ (numerical dissipation)

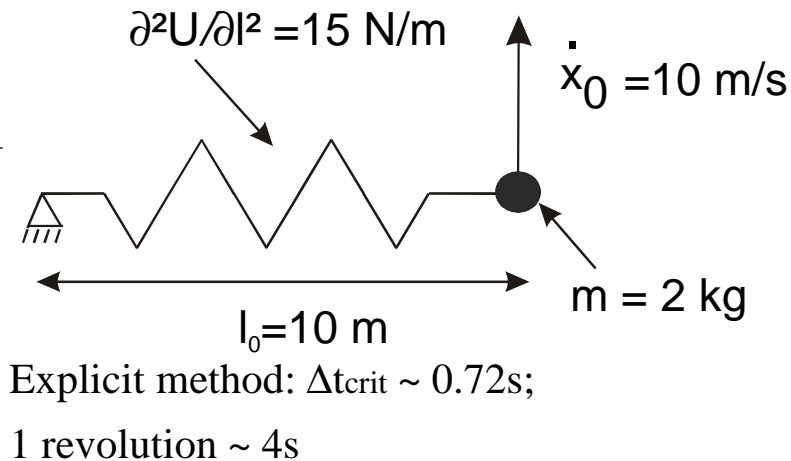
- Numerical dissipation is proved to be positive only in the linear range



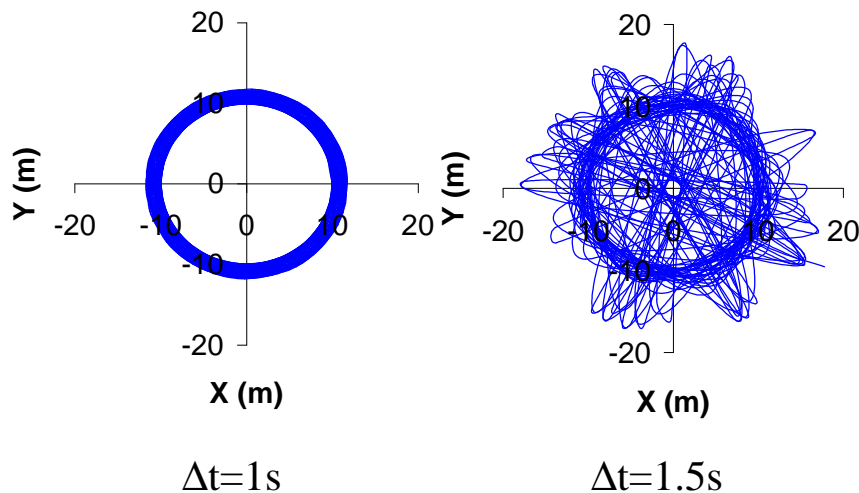
1. Scientific motivations

Numerical example: mass/spring-system

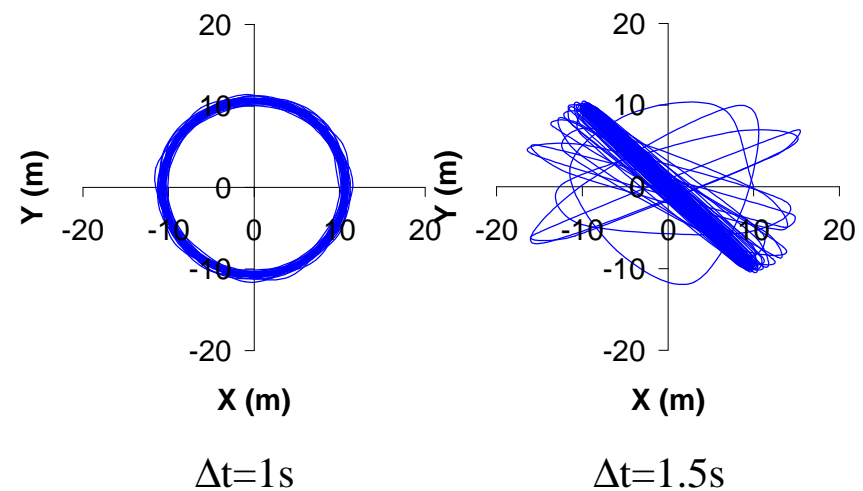
- Example: Mass/spring system (2D) with an initial velocity perpendicular to the spring (Armero & Romero [CMAME, 1999])



- Newmark implicit scheme (no numerical damping)



- Chung-Hulbert implicit scheme (numerical damping)





Scope of the presentation

1. Scientific motivations
2. **Consistent scheme in the non-linear range**
 - Principle
 - Dissipation property
 - The mass/spring system
 - Formulations in the literature: hyperelasticity
 - Formulations in the literature: contact
 - Developments for a hypoelastic model
 - Numerical example: Taylor bar
 - Numerical example: impact of two 2D-cylinders
 - Numerical example: impact of two 3D-cylinders
3. Combined implicit/explicit algorithm
4. Complex numerical examples
5. Conclusions & perspectives



2. Consistent scheme in the non linear range Principle

- Consistent implicit algorithms in the non-linear range:
 - The Energy Momentum Conserving Algorithm or EMCA
(Simo et al. [ZAMP 92], Gonzalez & Simo [CMAME 96]):
 - Conservation of the linear momentum
 - Conservation of the angular momentum
 - Conservation of the energy (no numerical dissipation)
 - The Energy Dissipative Momentum Conserving algorithm or EDMC (Armero & Romero [CMAME, 2001]):
 - Conservation of the linear momentum
 - Conservation of the angular momentum
 - Numerical dissipation of the energy is proved to be positive



2. Consistent scheme in the non linear range Principle

- Based on the mid-point scheme (Simo et al. [ZAMP, 1992])

– Relations displacements
/velocities/accelerations

$$\left\{ \begin{array}{l} \frac{\ddot{\vec{x}}_{n+1} + \ddot{\vec{x}}_n}{2} = \frac{\dot{\vec{x}}_{n+1} - \dot{\vec{x}}_n}{\Delta t} \\ \frac{\dot{\vec{x}}_{n+1} + \dot{\vec{x}}_n}{2} (+ \dot{\vec{x}}_{n+1}^{\text{diss}}) = \frac{\vec{x}_{n+1} - \vec{x}_n}{\Delta t} \end{array} \right.$$

– Balance equation

$$M \frac{\ddot{\vec{x}}_{n+1} + \ddot{\vec{x}}_n}{2} = \vec{F}_{n+1/2}^{\text{ext}} - \vec{F}_{n+1/2}^{\text{int}} \left(- \vec{F}_{n+1/2}^{\text{diss}} \right)$$

– EMCA:

- With $\vec{F}_{n+1/2}^{\text{int}} \neq \int_{V_0} \Sigma^{n+1/2} \mathbf{f}_0^{n+1/2 T} \bar{D}J_0^{n+1/2} dV_0$ and $\vec{F}_{n+1/2}^{\text{ext}}$ designed to verify conserving equations

- No dissipation forces and no dissipation velocities

– EDMC:

- Same internal and external forces as in the EMCA
- With $\vec{F}_{n+1/2}^{\text{diss}}$ and $\dot{\vec{x}}_{n+1}^{\text{diss}}$ designed to achieve positive numerical dissipation without spectral bifurcation



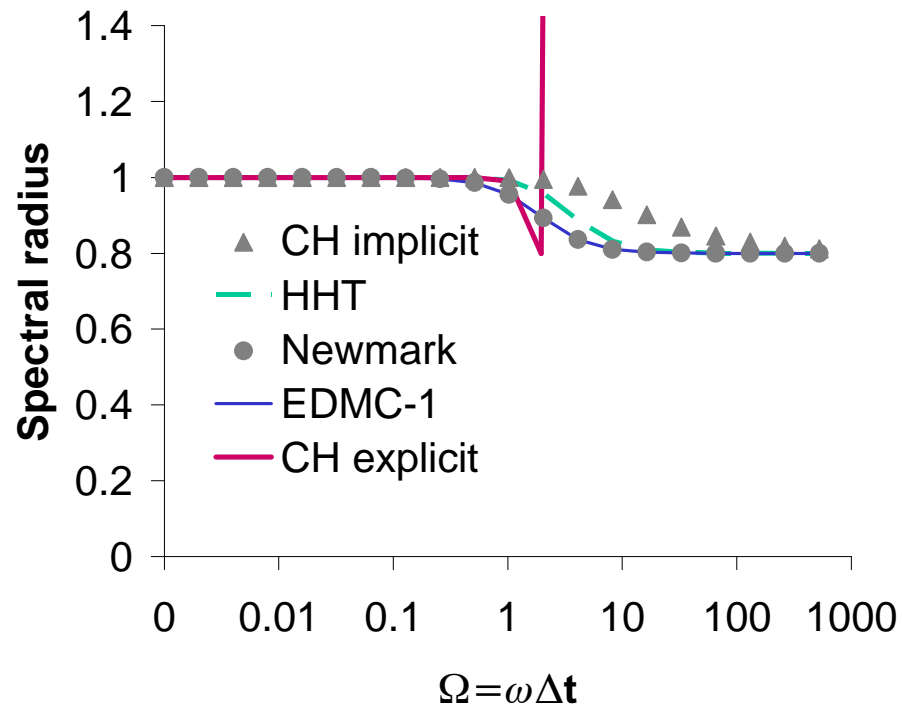
2. Consistent scheme in the non linear range Dissipation property

■ Comparison of the spectral radius

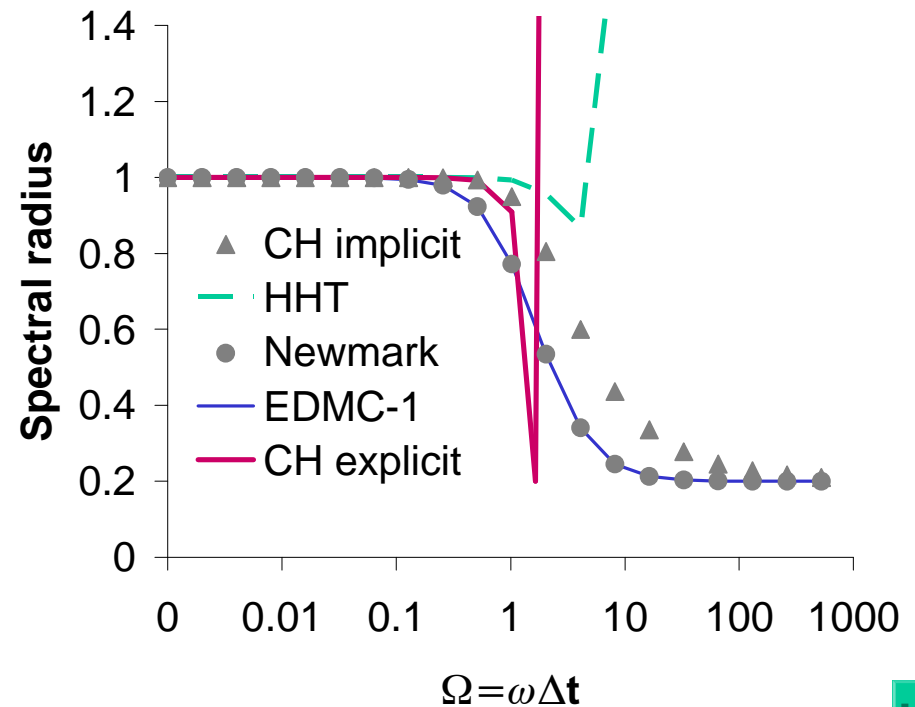
- Integration of a linear oscillator:
$$\vec{x}_{n+1} = \exp\left(\frac{\ln r^2}{\Delta t} t_{n+1}\right) \cos\left(\omega t_{n+1}\right)$$

ρ : spectral radius; ω : pulsation

Low numerical dissipation



High numerical dissipation



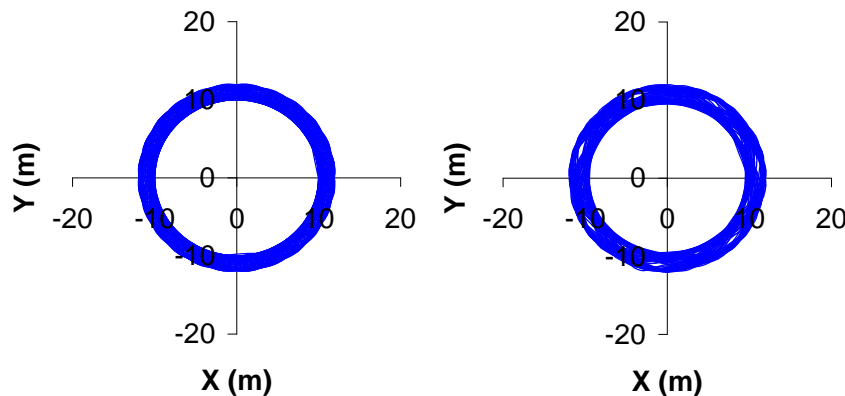


2. Consistent scheme in the non linear range The mass/spring system

- Forces of the spring for any potential V

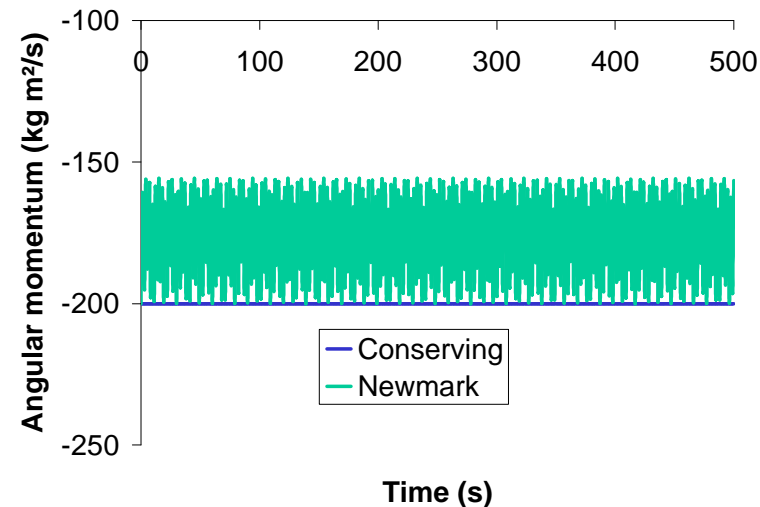
- Without numerical dissipation (EMCA) (Gonzalez & Simo [CMAME, 1996])

$$\vec{F}_{n+1/2}^{\text{int}} = \frac{V(l_{n+1}) - V(l_n)}{l_{n+1}^2 - l_n^2} [\vec{x}_{n+1} + \vec{x}_n]$$



EMCA, $\Delta t=1s$

EMCA, $\Delta t=1.5s$



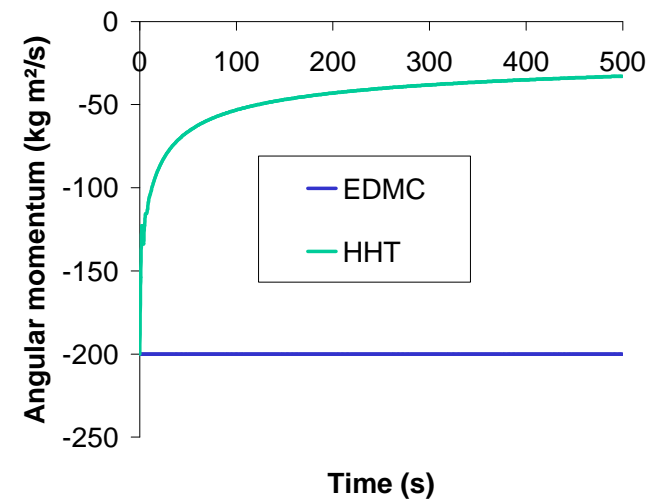
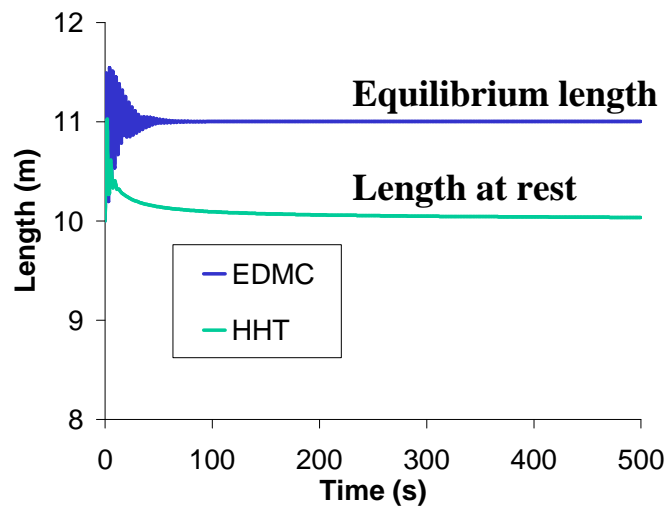
- The consistency of the EMCA solution does not depend on Δt
- The Newmark solution does-not conserve the angular momentum



2. Consistent scheme in the non linear range The mass/spring system

- With numerical dissipation (EDMC 1st order) with dissipation parameter $0 < \chi < 1$ (Armero & Romero [CMAME, 2001]), here $\chi = 0.111$

$$\left\{ \begin{aligned} \dot{\vec{x}}_{n+1}^{\text{diss}} &= c \frac{\left\| \dot{\vec{x}}_{n+1} \right\| - \left\| \dot{\vec{x}}_n \right\|}{\left\| \dot{\vec{x}}_{n+1} \right\| + \left\| \dot{\vec{x}}_n \right\|} \frac{[\vec{x}_{n+1} + \vec{x}_n]}{2} \\ \vec{F}_{n+1/2}^{\text{diss}} &= c \frac{V''\left(\frac{l_{n+1} + l_n}{2}\right) [l_{n+1} - l_n]}{l_{n+1} + l_n} \frac{[\vec{x}_{n+1} + \vec{x}_n]}{2} \end{aligned} \right.$$



- Only EDMC solution preserves the driving motion:
 - The length tends towards the equilibrium length
 - Conservation of the angular momentum is achieved



2. Consistent scheme in the non linear range Formulations in the literature: hyperelasticity

- Hyperelastic material (stress derived from a potential V):
 - Saint Venant-Kirchhoff hyperelastic model (Simo et al. [ZAMP, 1992])
 - General formulation for hyperelasticity (Gonzalez [CMAME, 2000]):

$$\vec{F}_{n+1/2}^{\text{int}} = \int_{V_0} \frac{\mathbf{F}_0^n + \mathbf{F}_0^{n+1}}{2} \left[\frac{\partial V}{\partial \mathbf{GL}} \left(\frac{\mathbf{GL}_0^n + \mathbf{GL}_0^{n+1}}{2} \right) + \frac{o\left(\|\mathbf{GL}_0^{n+1} - \mathbf{GL}_0^n\|^2\right)}{f(V, \mathbf{s}_{vm}, \mathbf{e}^p)} \right] \vec{D} dV_0$$

\mathbf{F} : deformation gradient

\mathbf{GL} : Green-Lagrange strain

V : potential

$\vec{D} = \partial \mathbf{j} / \partial \vec{x}_0$

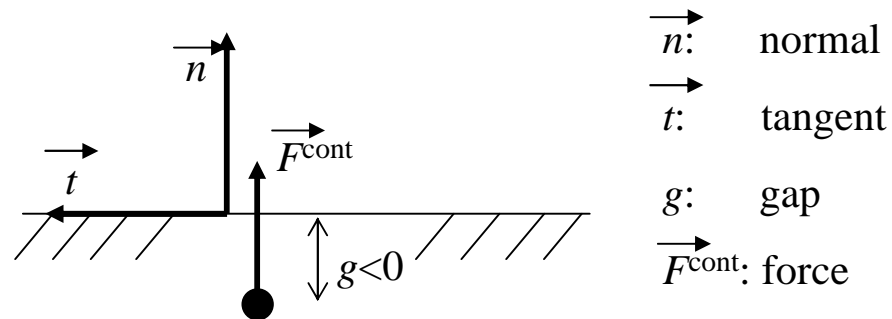
\mathbf{j} : shape functions

- Classical formulation: $\vec{F}^{\text{int}} = \int_{V_0} \mathbf{F} \frac{\partial V}{\partial \mathbf{GL}} \vec{D} dV_0$
- Hyperelasticity with elasto-plastic behavior: energy dissipation of the algorithm corresponds to the internal dissipation of the material (Meng & Laursen [CMAME, 2001])



2. Consistent scheme in the non linear range Formulations in the literature: contact

- Description of the contact interaction:



- Computation of the classical contact force:

- Penalty method

$$\vec{F}^{\text{cont}} = -k_N g \vec{n}$$

k_N : penalty

- Augmented Lagrangian method

$$\vec{F}^{\text{cont}} = \Lambda^{(k)} \vec{n} - k_N g \vec{n}$$

Λ : Lagrangian

- Lagrangian method

$$\vec{F}^{\text{cont}} = \Lambda \vec{n}$$



2. Consistent scheme in the non linear range Formulations in the literature: contact

- Penalty contact formulation (normal force proportional to the penetration “gap”) (Armero & Petöcz [CMAME, 1998-1999]):

- Computation of a dynamic gap for slave node \vec{x} projected on master surface $\vec{y}(u)$

$$g_{n+1}^d = g_n^d + \vec{n}_{n+1/2} \bullet [\vec{x}_{n+1} - \vec{x}_n - \vec{y}_{n+1}(u_{n+1/2}) + \vec{y}_n(u_{n+1/2})]$$

- Normal forces derived from a potential V

$$\vec{F}_{n+1/2}^{\text{cont}} = \frac{V(g_{n+1}^d) - V(g_n^d)}{g_{n+1}^d - g_n^d} \vec{n}_{n+1/2}$$

- Augmented Lagrangian and Lagrangian consistent contact formulation (Chawla & Laursen [IJNME, 1997-1998]):

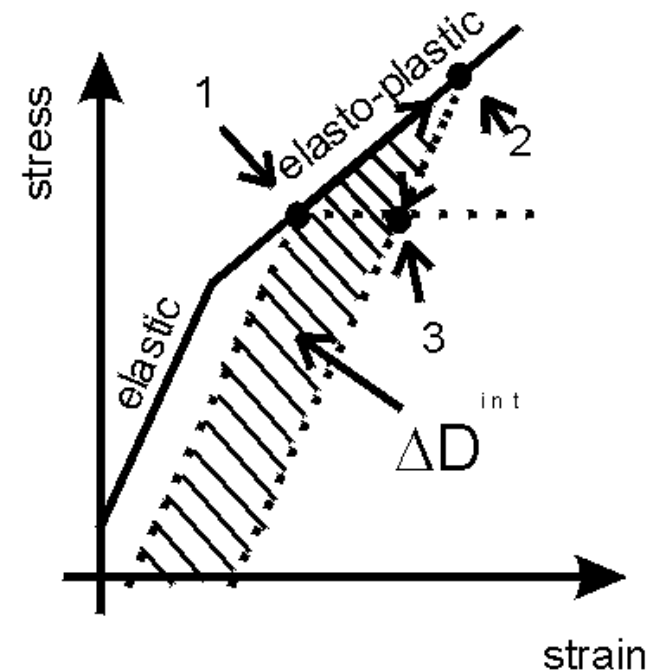
- Computation of a gap rate

$$\Delta t \ g_{n+1/2}^r = \vec{n}_{n+1/2} \bullet [\vec{x}_{n+1} - \vec{x}_n - \vec{y}_{n+1}(u_{n+1/2}) + \vec{y}_n(u_{n+1/2})]$$



2. Consistent scheme in the non linear range Developments for a hypoelastic model

- The EMCA or EDMC for hypoelastic constitutive model:
 - Valid for hypoelastic formulation of (visco) plasticity
 - Energy dissipation from the internal forces corresponds to the plastic dissipation
- Hypoelastic model:
 - stress obtained incrementally from a hardening law
 - no possible definition of an internal potential!
 - Idea: the internal forces are established to be consistent on a loading/unloading cycle





2. Consistent scheme in the non linear range Developments for a hypoelastic model

- Incremental strain tensor:

\mathbf{E} : natural strain tensor; \mathbf{F} : deformation gradient

$$\mathbf{E}_n^{n+1} = \frac{1}{2} \ln \left(\mathbf{F}_n^{n+1 \text{ T}} \mathbf{F}_n^{n+1} \right)$$

- Elastic incremental stress:

\mathbf{S} : Cauchy stress; \mathbf{H} : Hooke stress-strain tensor

$$\Delta \Sigma_n^{n+1} = \mathbf{H} : \mathbf{E}_n^{n+1}$$

- Plastic stress corrections:

(radial return mapping: Wilkins [MCP, 1964],
Maenchen & Sack [MCP, 1964], Ponthot [IJP, 2002])

\mathbf{s}^c : plastic corrections; \mathbf{s}^{vm} : yield stress;

\mathbf{e}^p : equivalent plastic strain

$$\mathbf{S}_n^{c n+1} = f(\mathbf{s}^{vm}, \mathbf{e}^p)$$

- Final Cauchy stresses:

(final rotation scheme: Nagtegaal &
Veldpaus [NAFP, 1984], Ponthot [IJP, 2002])

\mathbf{R} : rotation tensor;

$$\Sigma^{n+1} = \mathbf{R}_n^{n+1} \left[\Sigma^n + \Delta \Sigma_n^{n+1} - \mathbf{s}^c \right] \mathbf{R}_n^{n+1 \text{ T}}$$

- Classical forces formulation:

$\mathbf{f} = \mathbf{F}^{-1}$; D derivative of the shape function;

J : Jacobian

$$\vec{F}_{n+1}^{\text{int}} = \int_{V_0} \Sigma^{n+1} \mathbf{f}_0^{n+1 \text{ T}} \vec{D} J dV_0$$



2. Consistent scheme in the non linear range Developments for a hypoelastic model

- EMCA (without numerical dissipation):

- Balance equation $M \frac{\ddot{\bar{x}}_{n+1} + \ddot{\bar{x}}_n}{2} = \vec{F}_{n+1/2}^{\text{ext}} - \vec{F}_{n+1/2}^{\text{int}}$

- New internal forces formulation:

$$\vec{F}_{n+1/2}^{\text{int}} = \frac{1}{4} \int_{V_0} [\mathbf{I} + \mathbf{F}_n^{n+1}] [\boldsymbol{\Sigma}^n + \mathbf{C}^*] \mathbf{f}_0^{nT} \bar{D}J^n + [\mathbf{I} + \mathbf{f}_n^{n+1}] [\boldsymbol{\Sigma}^{n+1} + \mathbf{C}^{**}] \mathbf{f}_0^{n+1T} \bar{D}J^{n+1} dV_0$$

\mathbf{F} : deformation gradient; \mathbf{f} : inverse of \mathbf{F} ; D derivative of the shape function; J : Jacobian = det \mathbf{F} ;
 \mathbf{S} : Cauchy stress

- Correction terms \mathbf{C}^* and \mathbf{C}^{**} :
 (second order correction in the plastic strain increment)

$$\mathbf{C}^* = \frac{\Delta D^{\text{int}} / J_0^n - \mathbf{GL}_n^{n+1Pl} : \boldsymbol{\Sigma}^n}{\mathbf{GL}_n^{n+1} : \mathbf{GL}_n^{n+1}} \mathbf{GL}_n^{n+1}$$

$$\mathbf{C}^{**} = \frac{\Delta D^{\text{int}} / J_0^{n+1} - \mathbf{A}_n^{n+1Pl} : \boldsymbol{\Sigma}^{n+1}}{\mathbf{A}_n^{n+1} : \mathbf{A}_n^{n+1}} \mathbf{A}_n^{n+1}$$

ΔD^{int} : internal dissipation due to the plasticity; \mathbf{A} : Almansi incremental strain tensor ($\mathbf{A} = \mathbf{A}^{\text{pl}} + \mathbf{A}^{\text{el}}$);
 \mathbf{GL} : Green-Lagrange incremental strain ($\mathbf{GL} = \mathbf{GL}^{\text{pl}} + \mathbf{GL}^{\text{el}}$)

- Verification of the conservation laws

$$\left\{ \begin{array}{l} \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{int}} = 0 \quad \& \quad \sum_{\text{nodes}} \vec{x}_{n+1/2} \wedge \vec{F}_{n+1/2}^{\text{int}} = 0 \\ \Delta D^{\text{int}} = \int_{\text{cycle nodes}} \sum \vec{F}_{n+1/2}^{\text{int}} \cdot [\vec{x}_{n+1} - \vec{x}_n] \end{array} \right.$$



2. Consistent scheme in the non linear range Developments for a hypoelastic model

- EDMC (1st order accurate with numerical dissipation):

- Balance equation
$$M \frac{\ddot{\vec{x}}_{n+1} + \ddot{\vec{x}}_n}{2} = \vec{F}_{n+1/2}^{\text{ext}} - \vec{F}_{n+1/2}^{\text{int}} - \vec{F}_{n+1/2}^{\text{diss}}$$

- New dissipation forces formulation:

$$\vec{F}_{n+1/2}^{\text{diss}} = \frac{1}{4} \int_{V_0} \left[\mathbf{I} + \mathbf{F}_n^{n+1} \right] \mathbf{D}^* \mathbf{f}_0^{nT} \bar{D} + \left[\mathbf{I} + \mathbf{f}_n^{n+1} \right] \mathbf{D}^{**} \mathbf{f}_0^{n+1T} \bar{D} dV_0$$

- Dissipating terms \mathbf{D}^* and \mathbf{D}^{**} :

$$\left\{ \begin{array}{l} \mathbf{D}^* = \frac{c/2 \mathbf{E}_n^{\text{el}^{n+1}} : H : \mathbf{E}_n^{\text{el}^{n+1}} J_0^n}{\mathbf{GL}_n^{n+1} : \mathbf{GL}_n^{n+1}} \mathbf{GL}_n^{n+1} \\ \mathbf{D}^{**} = \frac{c/2 \mathbf{E}_n^{\text{el}^{n+1}} : H : \mathbf{E}_n^{\text{el}^{n+1}} J_0^{n+1}}{\mathbf{A}_n^{n+1} : \mathbf{A}_n^{n+1}} \mathbf{A}_n^{n+1} \end{array} \right.$$

- Verification of the conservation laws

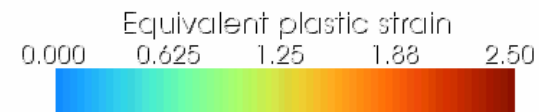
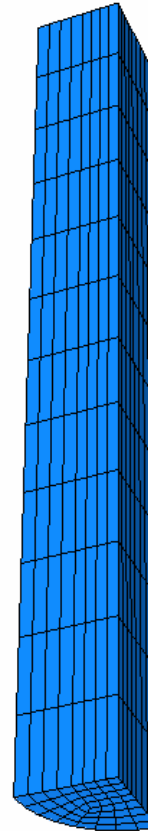
$$\left\{ \begin{array}{l} \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{diss}} = 0 \quad \& \quad \sum_{\text{nodes}} \vec{x}_{n+1/2} \wedge \vec{F}_{n+1/2}^{\text{diss}} = 0 \\ \Delta D^{\text{num}} = \sum_{\text{nodes}} \vec{F}_{n+1/2}^{\text{diss}} \cdot [\vec{x}_{n+1} - \vec{x}_n] + \mathbf{M} \dot{\vec{x}}_{n+1/2}^{\text{diss}} \cdot [\dot{\vec{x}}_{n+1} - \dot{\vec{x}}_n] \end{array} \right.$$

ΔD^{num} : numerical dissipation



2. Consistent scheme in the non linear range Numerical example: Taylor bar

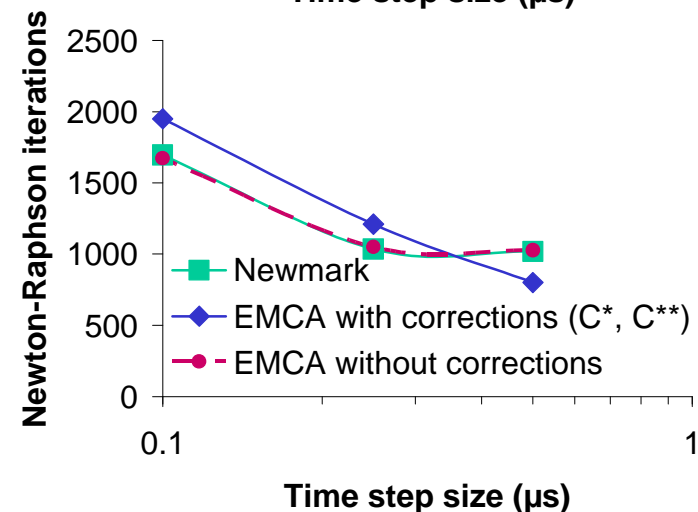
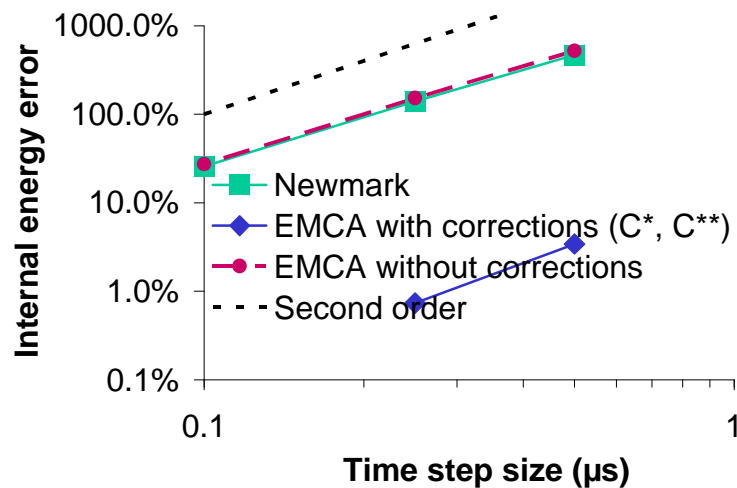
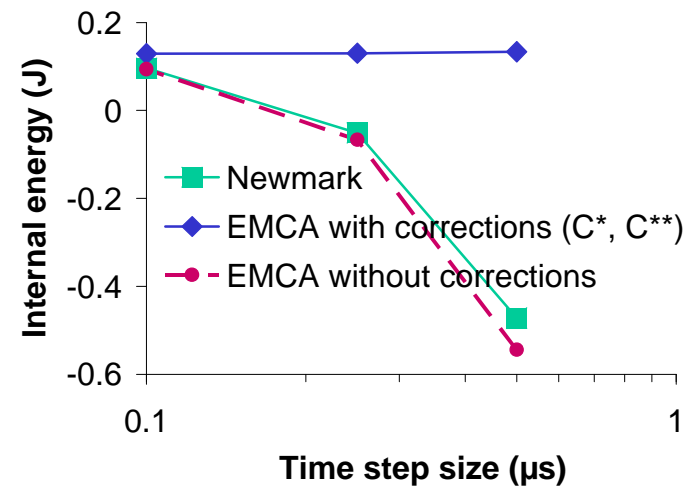
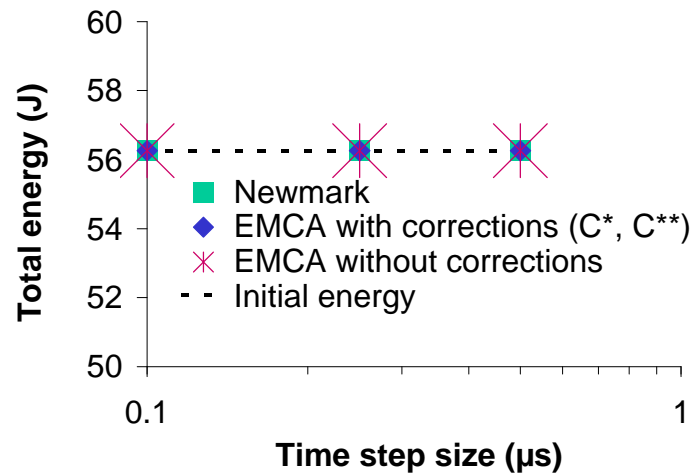
- Impact of a cylinder :
 - Hypoelastic model
 - Elasto-plastic hardening law
 - Simulation during $80 \mu\text{s}$





2. Consistent scheme in the non linear range Numerical example: Taylor bar

- Simulation without numerical dissipation: final results

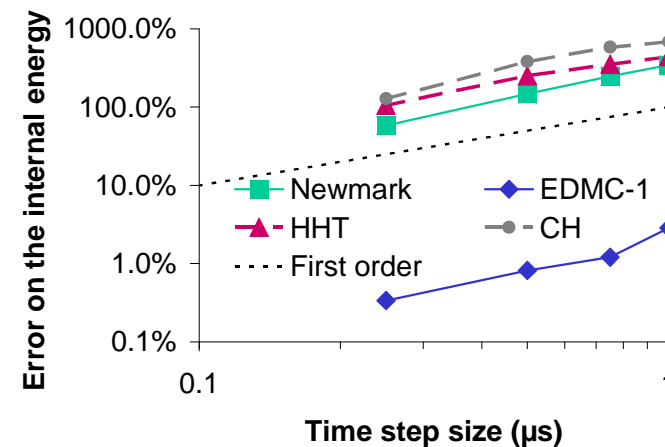
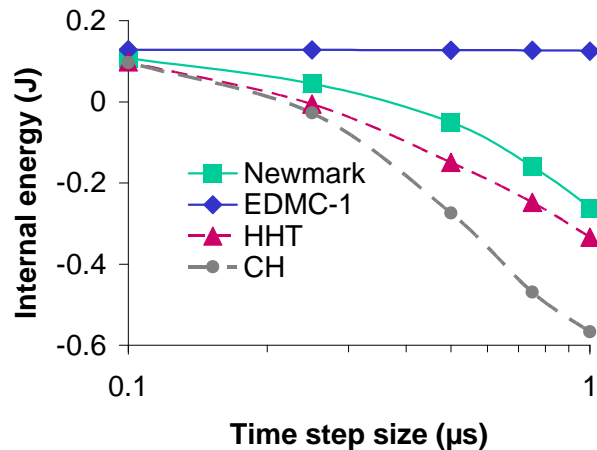




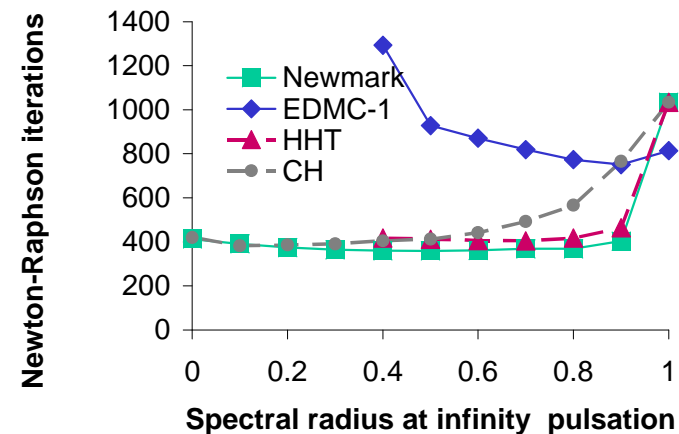
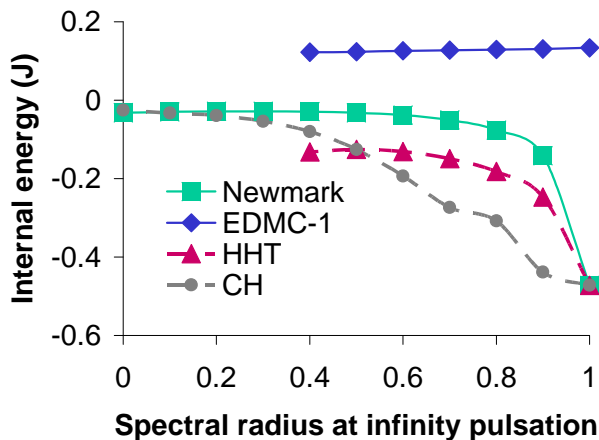
2. Consistent scheme in the non linear range Numerical example: Taylor bar

■ Simulations with numerical dissipation: final results

– Constant spectral radius at infinity pulsation = 0.7



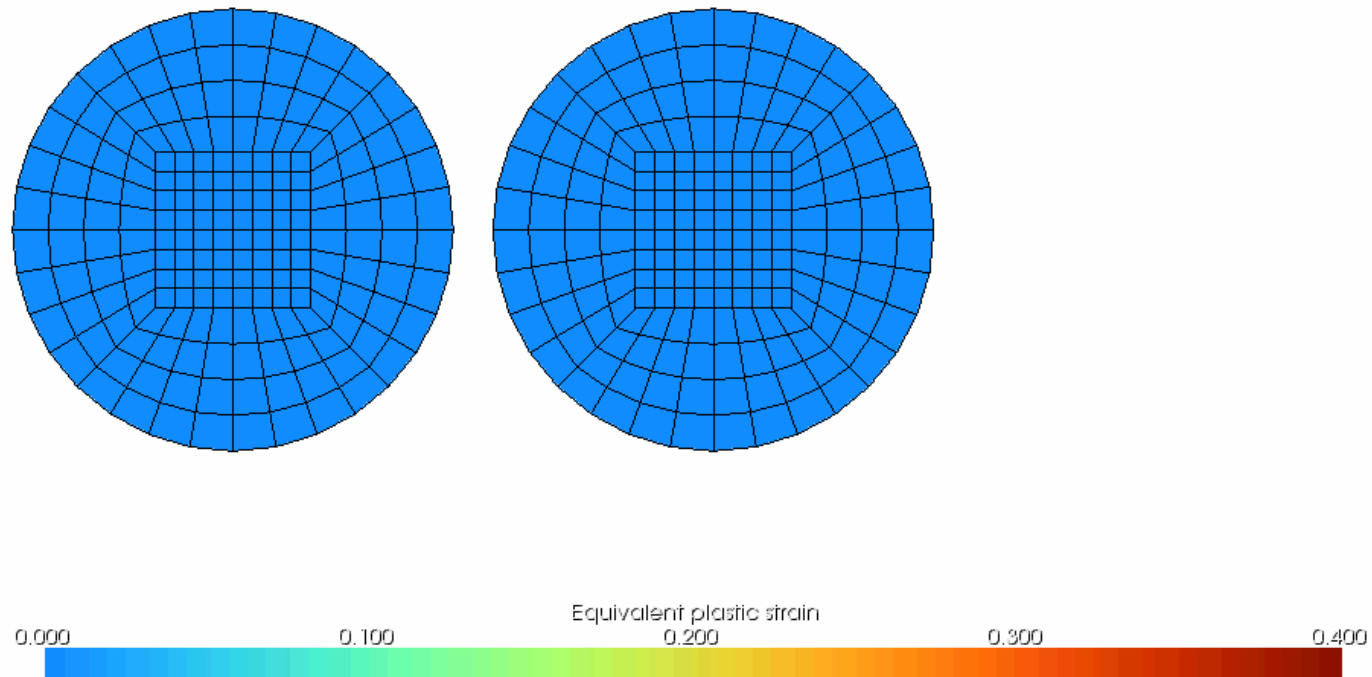
– Constant time step size = 0.5 μs





2. Consistent scheme in the non linear range Numerical example: impact of two 2D-cylinders

- Impact of 2 cylinders (Meng&Laursen) :
 - Left one has a initial velocity (initial kinetic energy 14J)
 - Elasto perfectly plastic hypoelastic material
 - Simulation during 4s

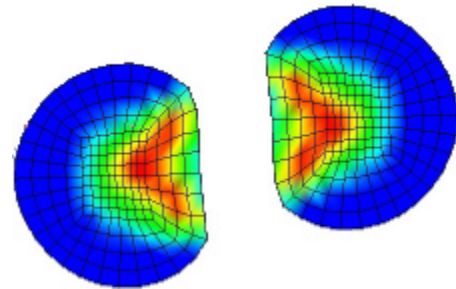




2. Consistent scheme in the non linear range Numerical example: impact of two 2D-cylinders

- Results comparison at the end of the simulation

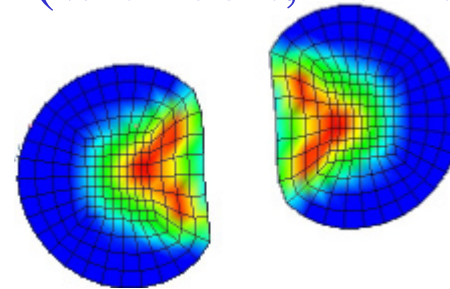
Newmark($\Delta t=1.875$ ms)



Equivalent plastic strain
0 0.089 0.178 0.266 0.355



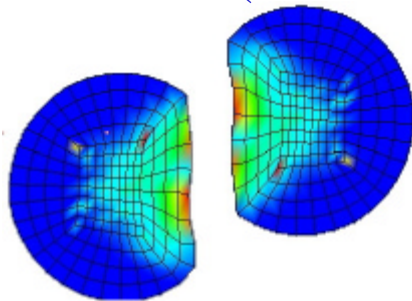
EMCA (with cor., $\Delta t=1.875$ ms)



Equivalent plastic strain
0 0.090 0.180 0.269 0.359



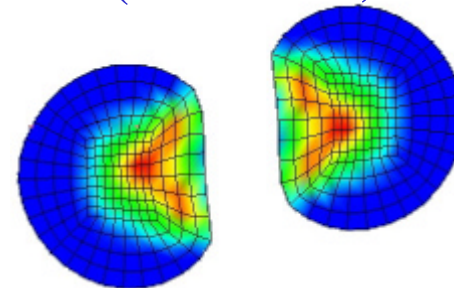
Newmark($\Delta t=15$ ms)



Equivalent plastic strain
0 0.305 0.609 0.914 1.22



EMCA (with cor., $\Delta t=15$ ms)



Equivalent plastic strain
0 0.094 0.187 0.281 0.374

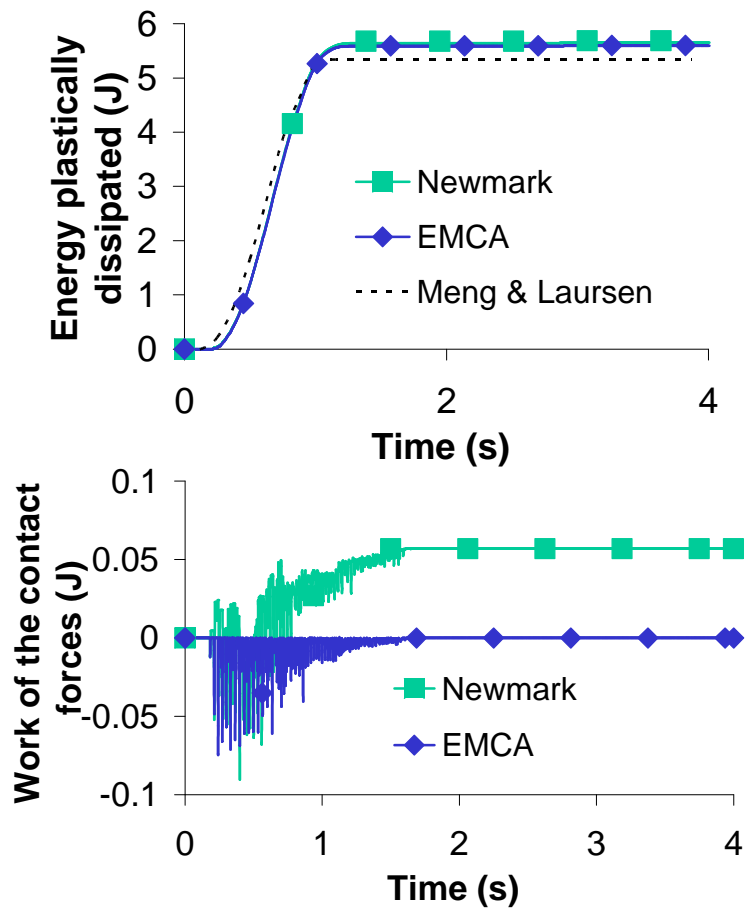




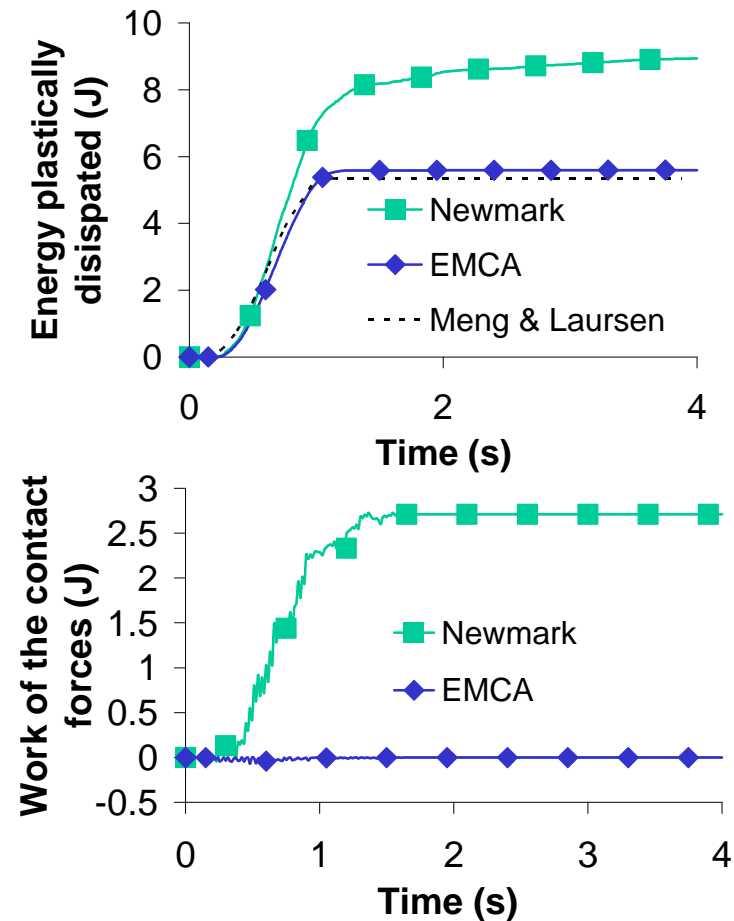
2. Consistent scheme in the non linear range Numerical example: impact of two 2D-cylinders

■ Results evolution comparison

$\Delta t = 1.875$ ms



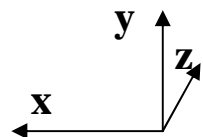
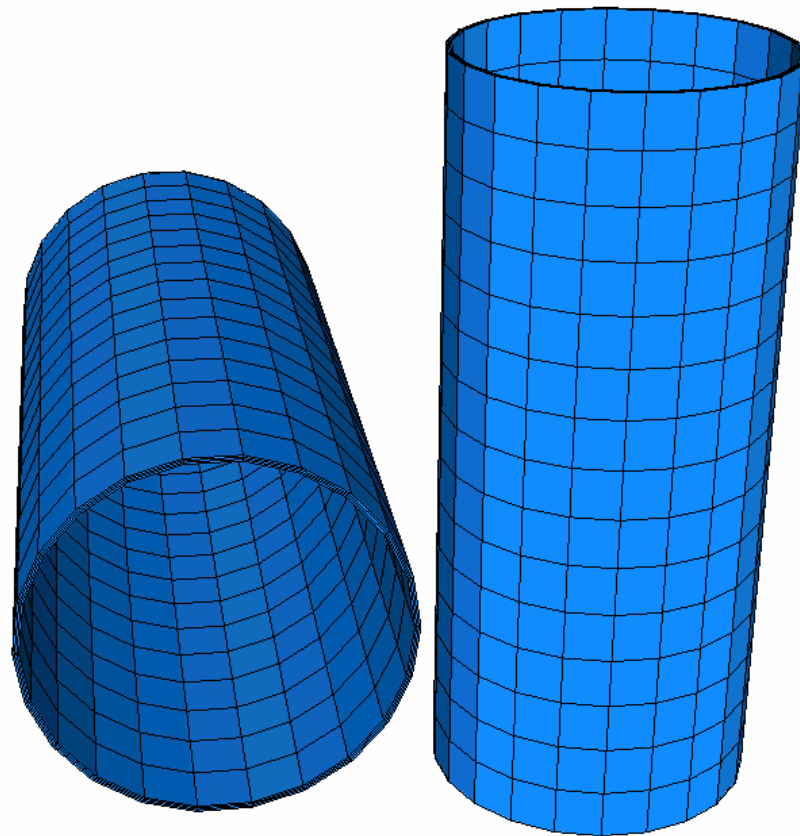
$\Delta t = 15$ ms





2. Consistent scheme in the non linear range Numerical example: impact of two 3D-cylinders

- Impact of 2 hollow 3D-cylinders:
 - Right one has a initial velocity
($\dot{x}_{0X} = 10\dot{x}_{0Y}$)
 - Elasto-plastic hypoelastic material (aluminum)
 - Simulation during 5ms
 - Use of numerical dissipation
 - Frictional contact

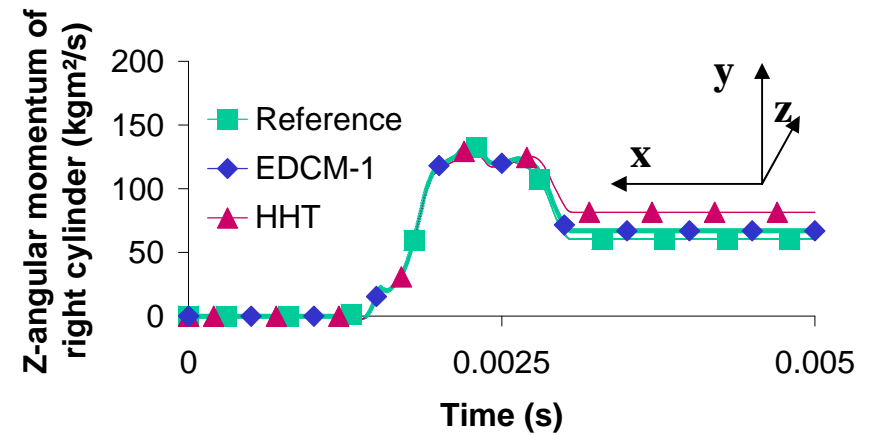
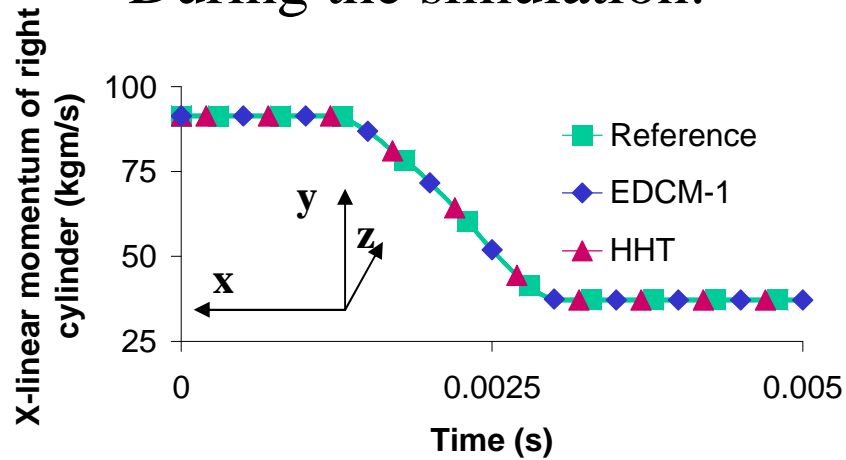




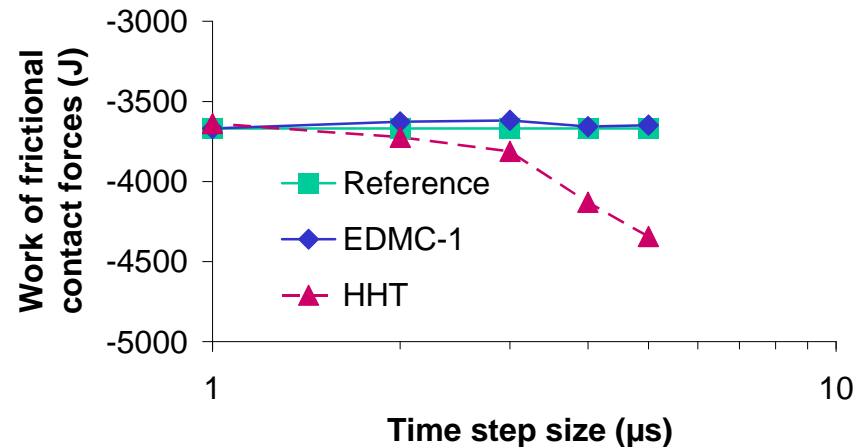
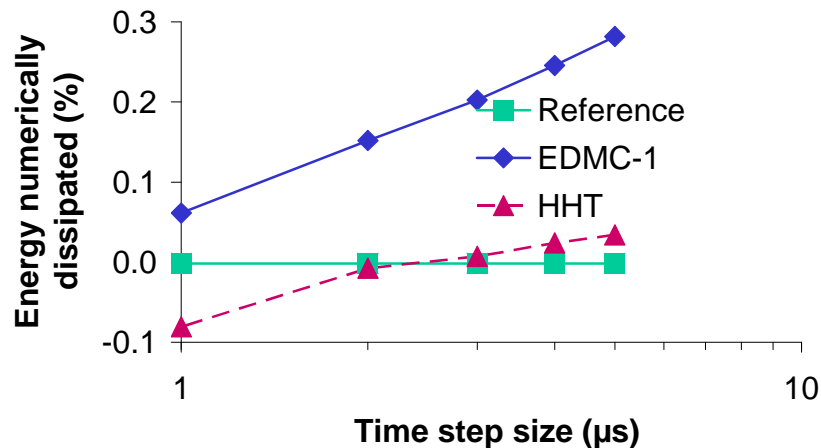
2. Consistent scheme in the non linear range Numerical example: impact of two 3D-cylinders

- Results comparison with a reference (EMCA; $\Delta t=0.5\mu s$):

— During the simulation:



— At the end of the simulation:





Scope of the presentation

1. Scientific motivations
2. Consistent scheme in the non-linear range
3. **Combined implicit/explicit algorithm**
 - Automatic shift
 - Initial implicit conditions
 - Numerical example: blade casing interaction
4. Complex numerical examples
5. Conclusions & perspectives



3. Combined implicit/explicit algorithm Automatic shift

- Shift from an implicit algorithm to an explicit one:

- Evaluation of the ratio r^*

$$r^* = \frac{CPU \ 1 \ \text{implicit} \ \text{step}}{CPU \ 1 \ \text{explicit} \ \text{step}}$$

- Explicit time step size depends on the mesh

$$\Delta t_{\text{expl}} = \frac{\Omega_b}{w_{\text{max}}}$$

Ω_b : stability limit;

w_{max} : maximal eigen pulsation

- Implicit time step size depends on the integration error (Géradin)

e_{int} : integration error;

Tol : user tolerance

$$\left\{ \begin{array}{l} e_{\text{int}} = \Delta t^3 \ddot{x} \approx \frac{\Delta t_{\text{impl}}^2 \sum_{i=\text{nodes}} \Delta |\ddot{x}|_i}{e_{\text{ref}}} \\ \frac{\Delta t_{\text{impl}}^{\text{new}}}{\Delta t_{\text{impl}}^{\text{old}}} = \left[\frac{2e_{\text{int}}}{Tol} \right]^{1/2.5} \end{array} \right.$$

- Shift criterion

$$\Delta t_{\text{impl}} < \frac{r^* \Delta t_{\text{expl}}}{m}$$

m : user security



3. Combined implicit/explicit algorithm Automatic shift

- Shift from an explicit algorithm to an implicit one:

- Evaluation of the ratio r^*

$$r^* = \frac{\text{CPU } 1 \text{ implicit step}}{\text{CPU } 1 \text{ explicit step}}$$

- Explicit time step size depends on the mesh

$$\Delta t_{\text{expl}} = \frac{\Omega_b}{w_{\text{max}}}$$

- Ω_b : stability limit;

- w_{max} : maximal eigen pulsation

- Implicit time step size interpolated from an acceleration difference

$$\Delta t_{\text{impl}} = \left(\frac{\text{Tol } e_{\text{ref}} \sqrt{\Delta t_{\text{expl}}}}{2 \sum_{i=\text{nodes}} \Delta \left| \ddot{x} \right|_i} \right)^{2/5}$$

- Tol : user tolerance

- Shift criterion

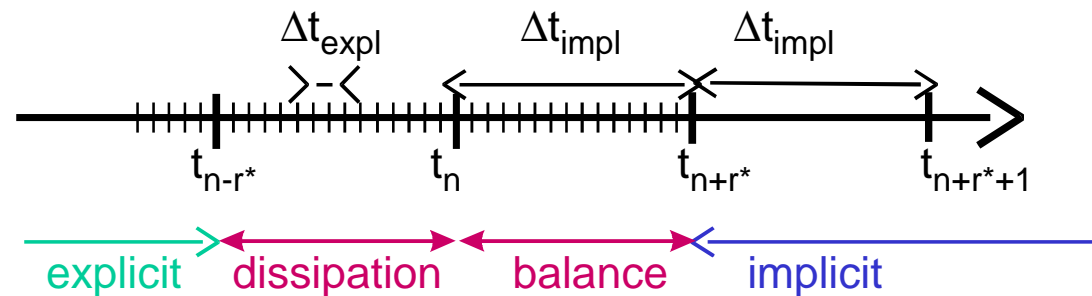
$$\Delta t_{\text{impl}} > m^* \Delta t_{\text{expl}}$$

- m : user security



3. Combined implicit/explicit algorithm Initial implicit conditions

- Stabilization of the explicit solution:



- Dissipation of the numerical modes: spectral radius at bifurcation equal to zero.
- Consistent balance of the r^* last explicit steps:

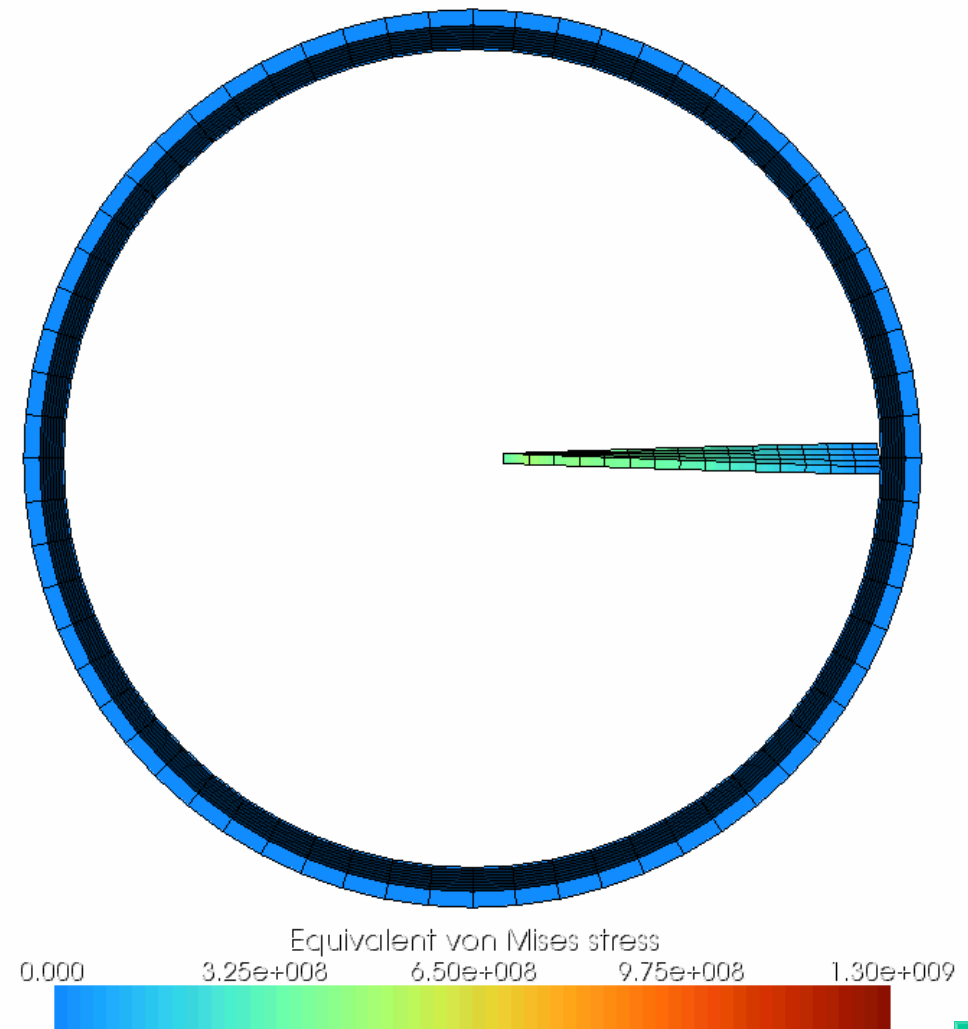
$$M \frac{\ddot{\vec{x}}_{n+r^*} + \ddot{\vec{x}}_n}{2} = \vec{F}_{n+r^*/2}^{\text{ext}} - \vec{F}_{n+r^*/2}^{\text{int}} \left(-\vec{F}_{n+r^*/2}^{\text{diss}} \right)$$



3. Combined implicit/explicit algorithm

Numerical example: blade casing interaction

- Blade/casing interaction :
 - Rotation velocity
3333rpm
 - Rotation center is moved
during the first half
revolution
 - EDMC-1 algorithm
 - Four revolutions
simulation

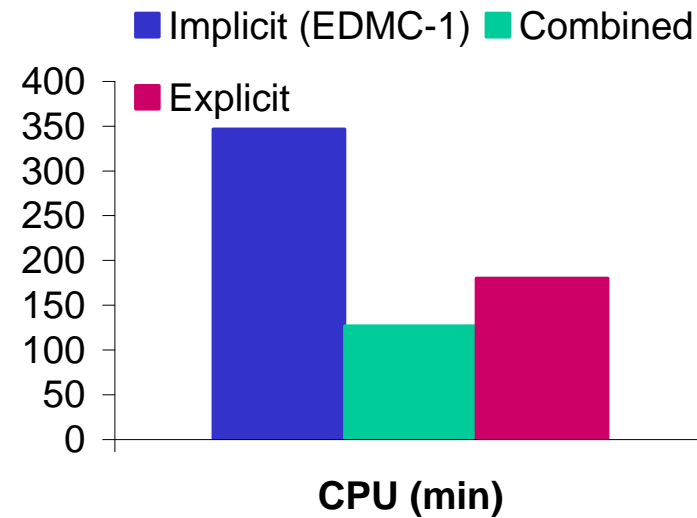
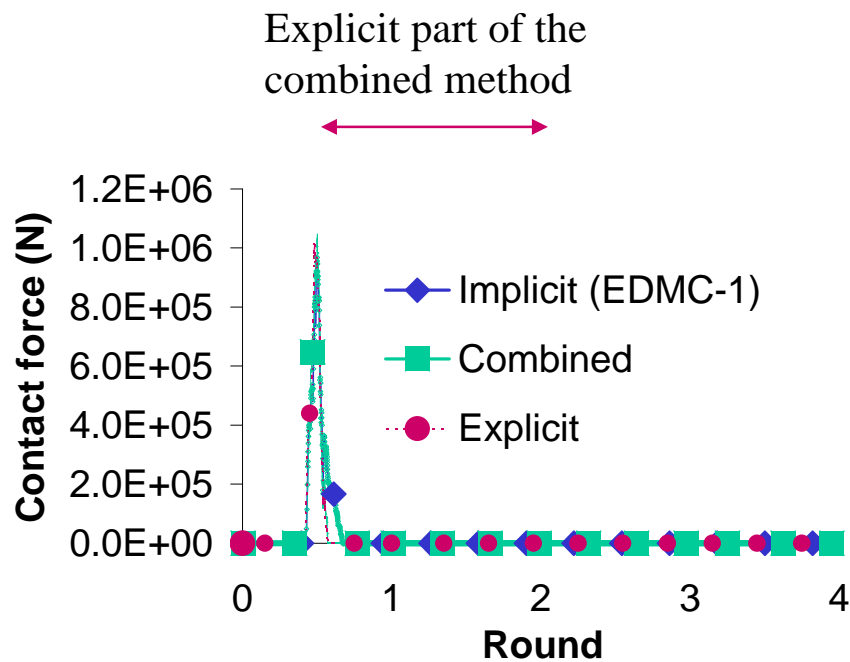




3. Combined implicit/explicit algorithm

Numerical example: blade casing interaction

- Final results comparison:





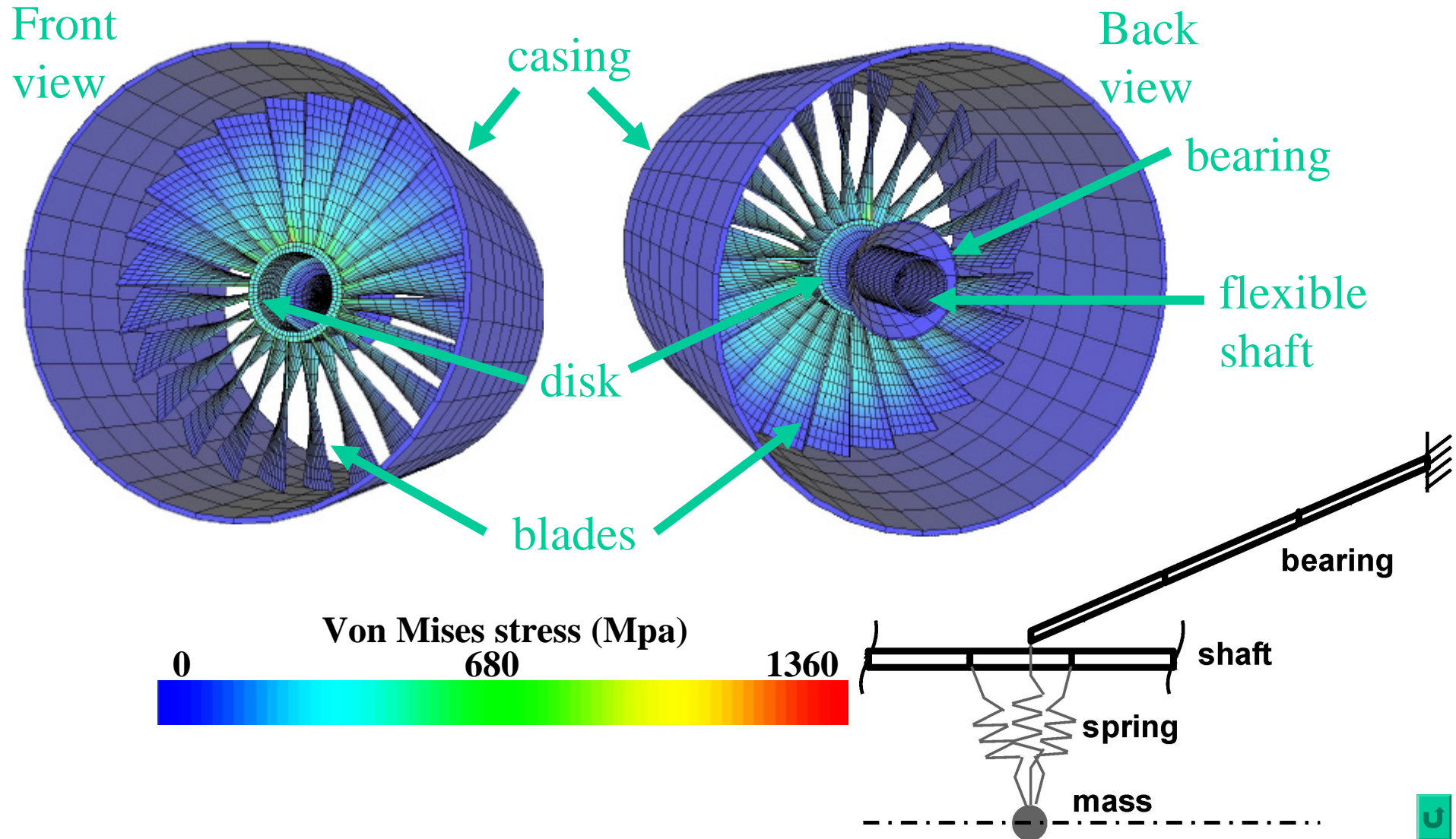
Scope of the presentation

1. Scientific motivations
2. Consistent scheme in the non-linear range
3. Combined implicit/explicit algorithm
4. **Complex numerical examples**
 - Blade off simulation
 - Dynamic buckling of square aluminum tubes
5. Conclusions & perspectives



4. Complex numerical examples Blade off simulation

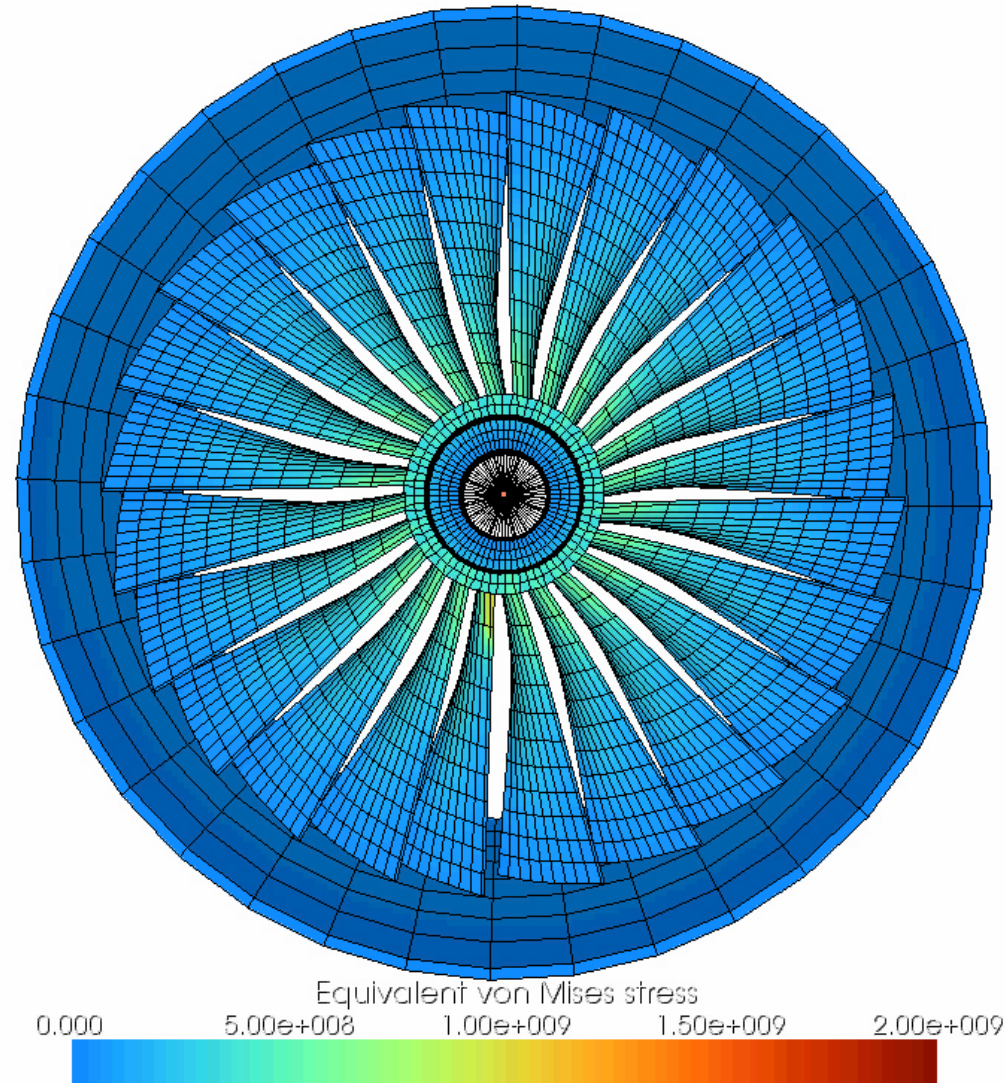
- Numerical simulation of a blade loss in an aero engine





4. Complex numerical examples Blade off simulation

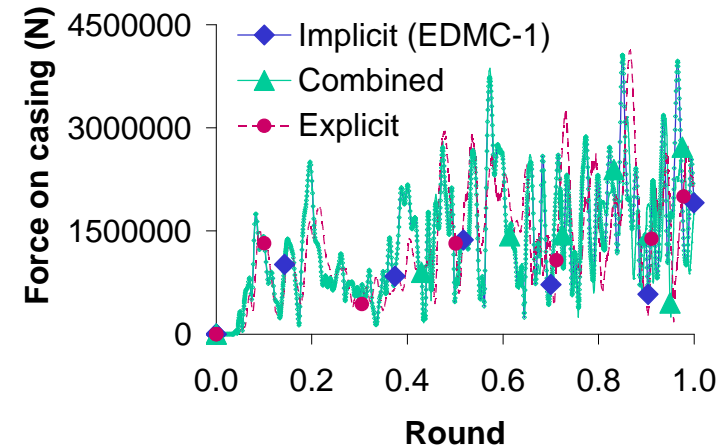
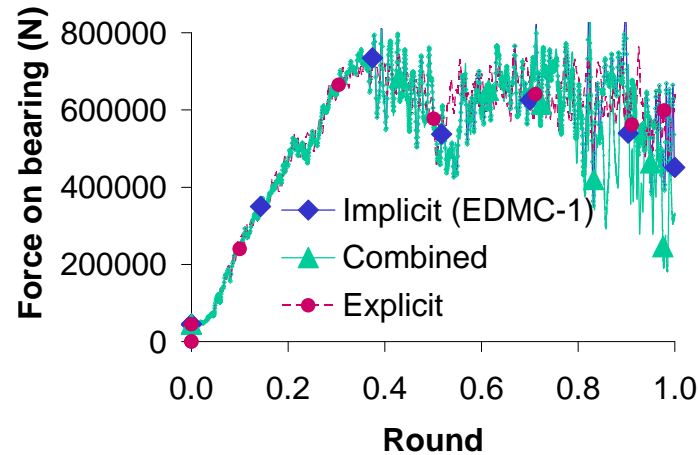
- Blade off :
 - Rotation velocity
5000rpm
 - EDMC algorithm
 - 29000 dof's
 - One revolution
simulation
 - 9000 time steps
 - 50000 iterations (only
9000 with stiffness
matrix updating)





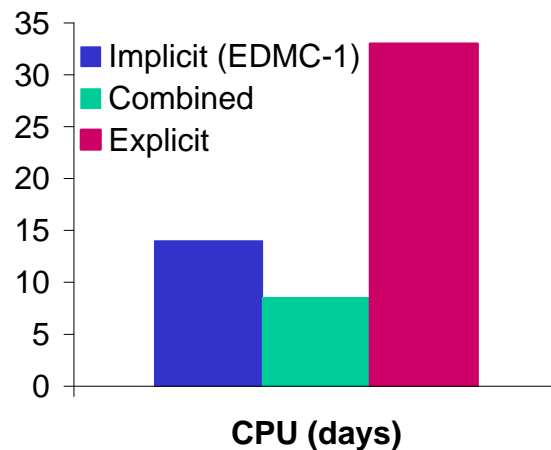
4. Complex numerical examples Blade off simulation

Final results comparison:

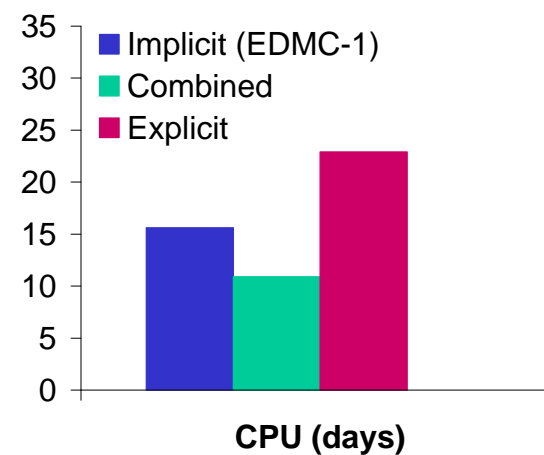


CPU time comparison before and after code optimization:

Before optimization



After optimization

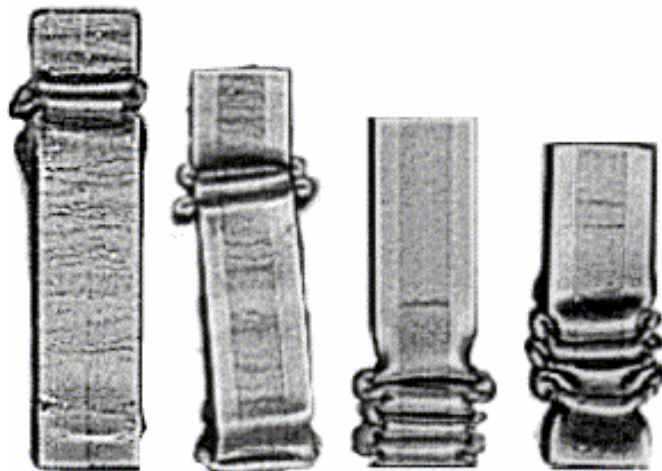




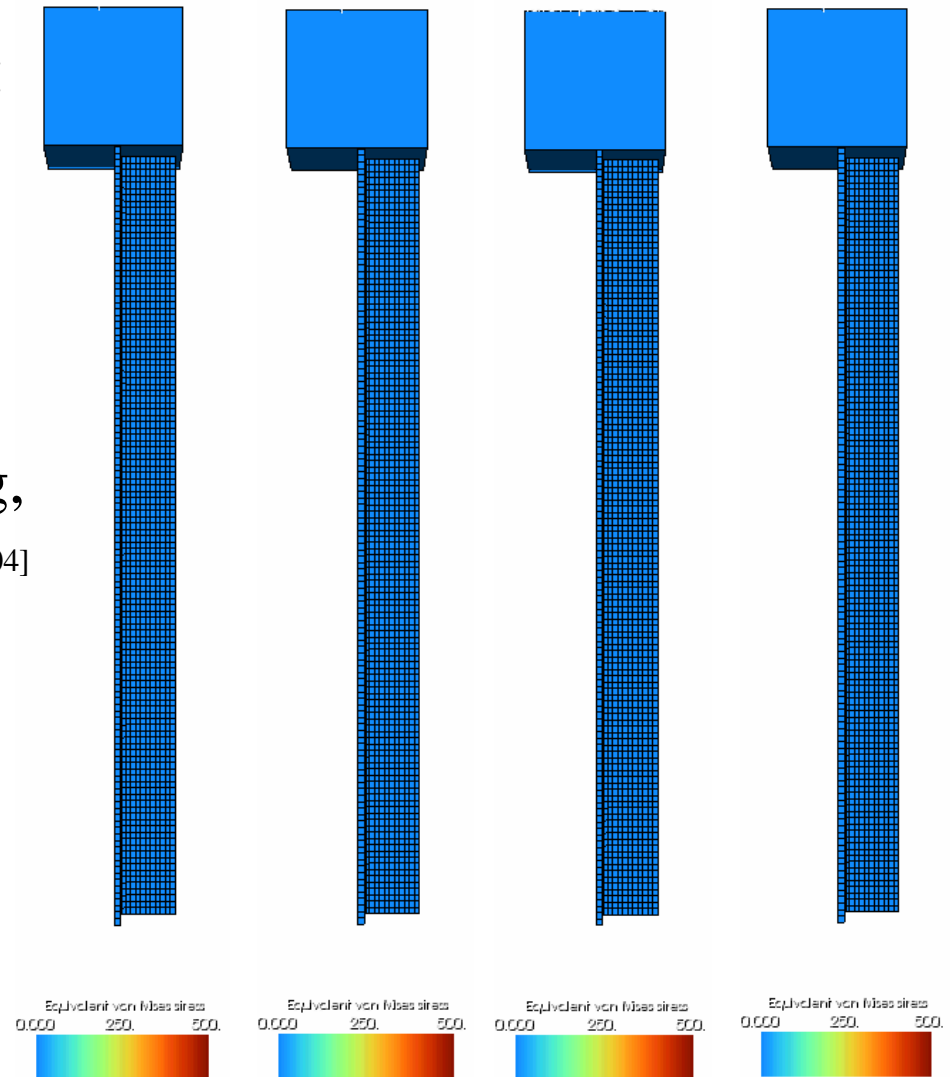
4. Complex numerical examples

Dynamic buckling of square aluminum tubes

- Absorption of 600J with different impact velocities :
 - EDMC algorithm
 - 16000 dof's / 2640 elements
 - Initial asymmetry
 - Comparison with the experimental results of Yang, Jones and Karagiozova [IJIE, 2004]



Impact velocity :



98.27 m/s

64.62 m/s

25.34 m/s

14.84 m/s

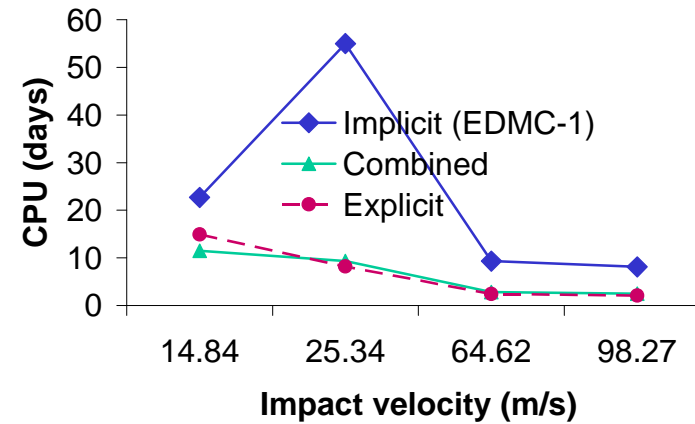
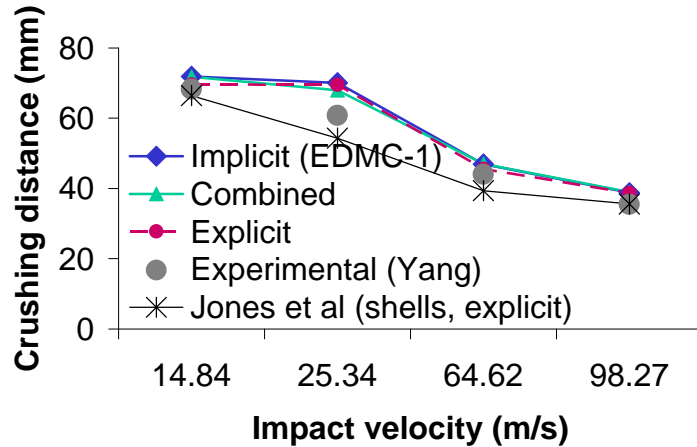




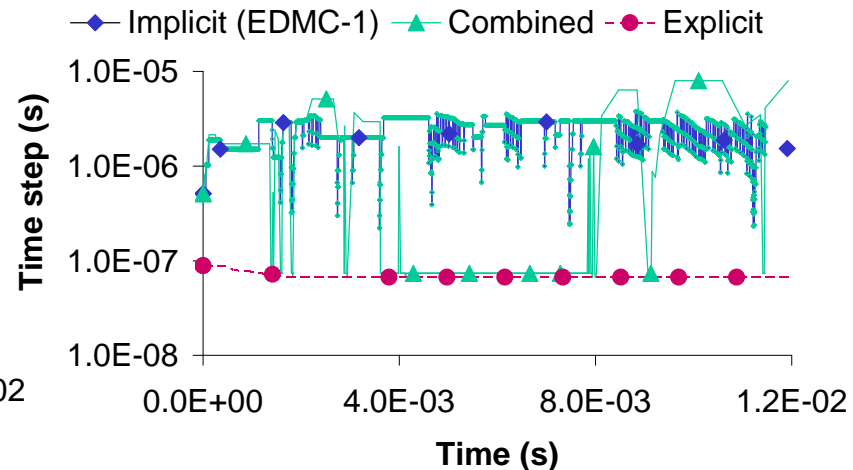
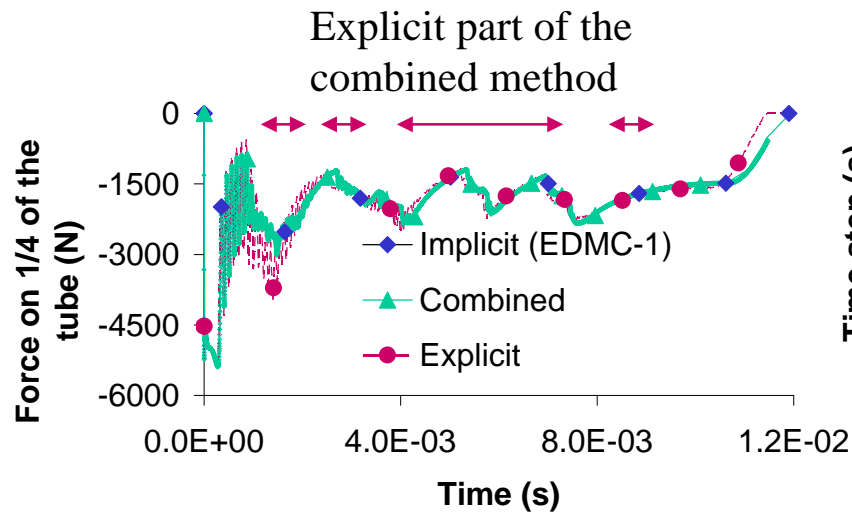
4. Complex numerical examples

Dynamic buckling of square aluminum tubes

Final results comparison:



Time evolution for the 14.84 m/s impact velocity:





Scope of the presentation

1. Scientific motivations
2. Consistent scheme in the non-linear range
3. Combined implicit/explicit algorithm
4. Complex numerical examples
5. **Conclusions & perspectives**
 - Improvements
 - Advantages of new developments
 - Drawbacks of new developments
 - Futures works



5. Conclusions & perspectives Improvements

- Original developments in consistent implicit schemes:
 - New formulation of elasto-plastic internal forces
 - Controlled numerical dissipation
 - Ability to simulate complex problems (blade-off, buckling)
- Original developments in implicit/explicit combination:
 - Stable and accurate shifts
 - Automatic shift criteria
 - Reduction of CPU cost for complex problems (blade-off, buckling)



5. Conclusions & perspectives

Advantages of new developments

- Advantages of the consistent scheme:
 - Conservation laws and physical consistency are verified for each time step size in the non-linear range
 - Conservation of angular momentum even if numerical dissipation is introduced
- Advantages of the implicit/explicit combined scheme:
 - Reduction of the CPU cost
 - Automatic algorithms
 - No lack of accuracy
 - Remains available after code optimizations



5. Conclusions & perspectives

Drawbacks of new developments

- Drawbacks of the consistent scheme:
 - Mathematical developments needed for each element, material...
 - More complex to implement
- Drawback of the implicit/explicit combined scheme:
 - Implicit and explicit elements must have the same formulation



5. Conclusions & perspectives

Future works

- Development of a second order accurate EDMC scheme
- Extension to a hyper-elastic model based on an incremental potential
- Development of a thermo-mechanical consistent scheme
- Modelization of wind-milling in a turbo-engine
- ...



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Merci de votre attention



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