*Quality and validation of numerical resolutions of unsteady flow equations for river networks used as a tool for water management*

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# QUALITY AND VALIDATION OF NUMERICAL RESOLUTIONS OF UNSTEADY FLOW EQUATIONS FOR RIVER NETWORKS USED AS A TOOL FOR WATER MANAGEMENT

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## ABSTRACT

Two computational programs handling unsteady flow problems in networks with branches of complex geometries are compared. The first one uses a Petrov-Galerkin finite element method, the second one a fixed grid method of characteristics. The simulation of the Belgian river Meuse during variable conditions induced by hydraulic power plants shows for both codes a very satisfying agreement with recorded measurements. Their use in a complex network submitted to a sudden important flood highlights accurate and useful informations for water management.

## INTRODUCTION

The increasing water supply, arisen from the technical and social evolution of our industrial societies, calls out for a fit management of the available resources. Water becomes more than ever a scarce good to be spared. The antagonist aims to come up (electric energy production, nuclear power cooling, drinking water management, aquatic sports...) prompt the manager to use mathematical tools in order to keep a clear view of the transient flows induced in the system by very disturbed situations with sharp floods or very low water levels [1]. The constant renewal of interest for several years in searching new numerical schemes for the fluid dynamic equations can be explained by the merit of each of them only in particular situations. An efficient resolution of the Navier-Stokes equations, able to modelize most of the physical processes involved in the flow field, remains a fundamental concern in the modern hydraulic engineering. The two presented softwares were developed to cover such various topics as dam-break flood-wave propagation, blood circulation and water resources management. The numerical resolution of the set of differential equations expressed in a section averaged form is carried out in the first case by a Petrov-Galerkin finite element method with special test functions and prediction-correction. The second one resorts to a fixed grid method of characteristics.

## THEORETICAL MODEL

In another study [2], we demonstrated the existence of an analogy between the equations that we used to simulate waves propagation in very distensible tubes applied to a network of arteries and the ones used to describe continuous and discontinuous open channel flows. Besides, the main assumptions ensure a suitable modelization of water resources management. In particular, the squares of the ratios between the speeds in the directions perpendicular to the main flow and the axial speed are insignificant. This implies a hydrostatic diagram of pressure, changes the system in a quasi two-dimensional one and allows the integration of the equations in the section. The second main assumption is the no-slip condition that is considered along the walls. They lead to the following system of fully non-linear quasi two-dimensional matrix system :

$$\frac{\partial X}{\partial t} + A\,(X)\frac{\partial X}{\partial x} + B\,(X) = 0 \qquad (1)$$

In the case of open channel flows, this set of equations is written :

$$\frac{\partial\,\Omega}{\partial\,t} + \frac{\partial\,Q}{\partial\,x} = 0 \quad \text{and} \quad \frac{\partial\,Q}{\partial\,t} + \frac{\partial(\varphi U\,Q)}{\partial\,x} + g\,\Omega\frac{\partial Z}{\partial x} + gF = 0 \qquad (2,3)$$

where we call respectively t and x the time and space variables, h(x,t) the height of water, Ω(x,h) the wet cross-section, defined as a function of the space and the height of water, Q(x,t) the flow [Ω and Q are both included in the general vectorial expression X(x,t)], U(x,t) the average speed in the section (axial speed), g the gravity acceleration, Z(x,t) the elevation of free surface and F the friction term. Moreover, the parameter φ characterizes the unequal distribution of the axial speed in the section [2].

## NUMERICAL RESOLUTION

Resolution by finite element method

The finite element method divides the domain into subdomains wherein every variable is represented as a polynominal function of its values at the nodes of the element. The general formulation consists of an orthogonal projection of the residual error due to discretization approximation to a set of linearly independent complete functions included in a vector P. Due to the wide range of applications (management simulations, dam-break flood wave propagation...), the weighting functions vector P adopts a generalized form in accordance with the works of Katopodes [3] :

$$P = N + \varepsilon\,A\,(X^*)^T\frac{\partial N}{\partial x} \qquad (4)$$

with ε the parameter setting the degree of dissipation. It controls undesirable oscillations arising in the simulations with significant convective terms or with propagation of sharp fronts [4]. Raymond and Garder [5] have studied its very selective action on wavelengths. The temporal discretization of the equations is obtained by a variable-weighted implicit approximation on two time steps while the difference on these steps discretizes temporal derivatives. Moreover, the well-known method of the Lagrange multipliers allows to take into account any junction in networks. For a fast resolution of the system of 2n equations with 2n unknowns, the code proceeds with splitting up the equations and using an iterative method of prediction-correction. In both systems composed first with n discretized continuity equations, then with n discretized momentum equations, the unknown variables and non-linear terms are estimated either by a linear interpolation from the previous intervals at the first iteration, or, afterwards, by the values at the iteration that precedes.

Resolution by the method of characteristics
The method of characteristics is founded on the description of waves propagation [2]. The unsteadiness of the studied phenomena implies a high spatial and temporal variability of the celerities and the speeds, and consequently of the slopes of the characteristic curves. Thus, the resolution of the equations cannot be based on a simple process. The chosen method is the fixed grid pattern method, which is the most qualified for the treatment of junctions and the control of numerical accuracy. The dependence of the characteristics shape on the unknowns as well as the integration with trapezia of the equations terms that do not correspond to total derivates, make an iterative calculation procedure indispensible. The resolution of the equations at a limit or at a junction of several channels is either a simplification or a generalization of the procedure inside a channel. The equations and the numerical techniques are detailed in [2].

## SIMULATIONS AND RESULTS

Variable conditions at the ends of a single channel
In the hydraulic power plants of the Belgian river Meuse, a sudden turbine starting can induce a very important flow variation in a few seconds. The channel considered in this simulation is ended by two movable dams, each one associated with a hydraulic power plant. It is described in figure 1.

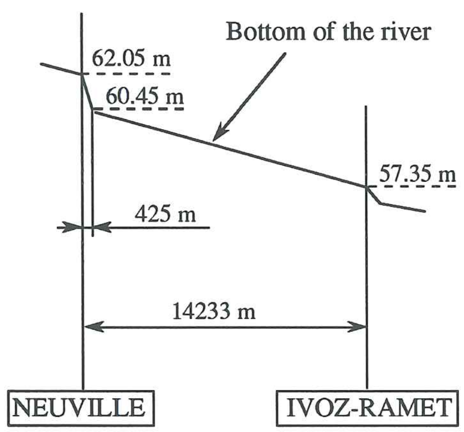


Figure 1. Studied channel limited by the movable dams and the power plants of Neuville and Ivoz-Ramet.

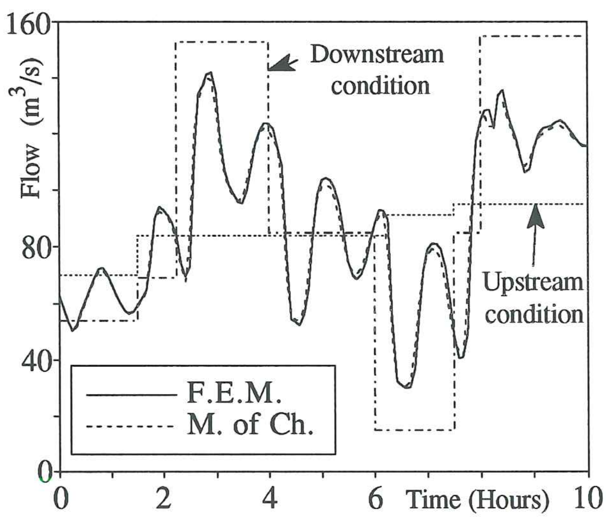


Figure 2. Flow curves imposed at the limit of the channel and flow curves calculated by both methods at the abscissa 6036 m.

The real cross-sections were used as well as the well-known friction Manning formula [4] with a coefficient of 30 (International units) previously determined in the river. Flows at both ends through the dams and the power plants were reconstituted during 10 hours. Those curves, given in figure 2 were used as limit conditions. In default of precise data about the channel management before the studied period, the chosen initial conditions were linear free surface and flow variations between the known initial values at each end. Figure 2 also shows simulated flow curves at the abscissa 6036 m. Figure 3 and 4 compare recorded water levels at Hermalle (abscissa 2956 m) and Ivoz-Ramet (abscissa 14233 m) with the curves simulated by both codes. They all show very similar behaviours of the two codes as well as a high correlation between the models and reality. The amplitudes and propagation celerities are adequately reproduced in spite of some uncertainties. The approximative initial conditions explain the imprecision of the levels at Ivoz-Ramet during the first hours of the simulation. On the other hand, the flow curves imposed at the ends involve imprecisions, particularly on the flow through the dams and the speeds of variations through the plants.

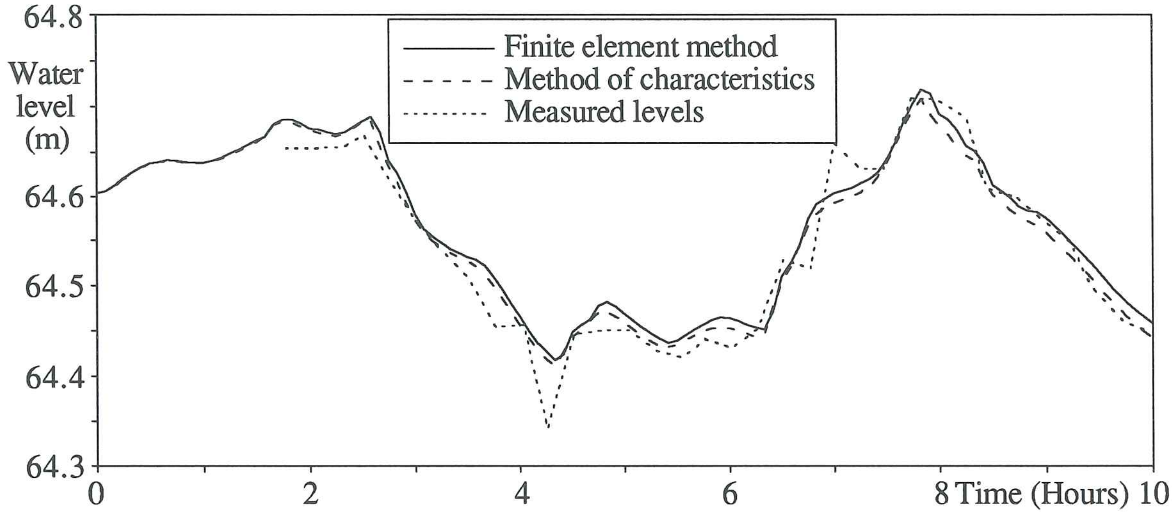


Figure 3. Temporal recorded and simulated evolutions of the free surface at Hermalle.

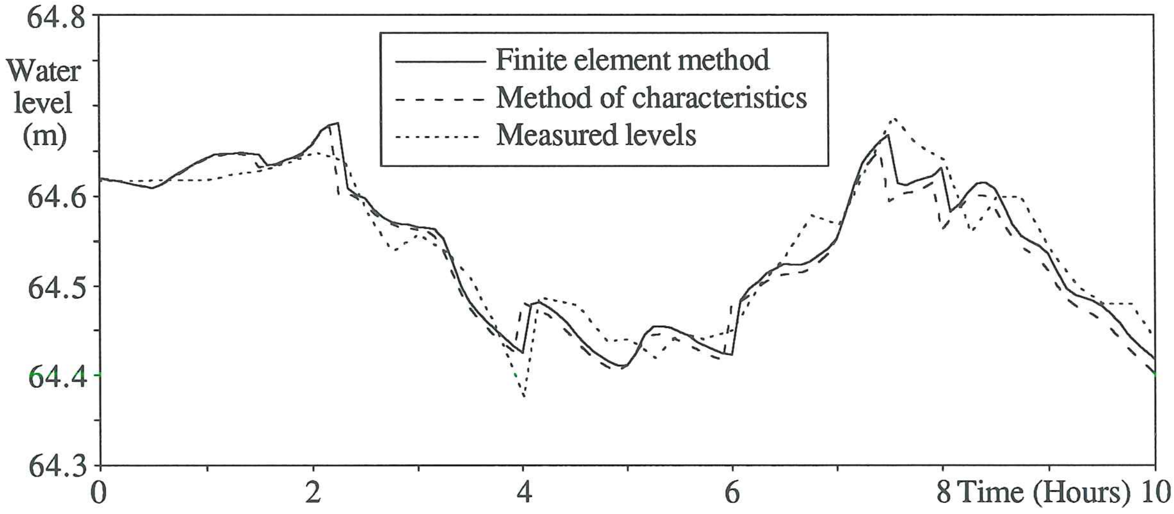


Figure 4. Temporal recorded and simulated evolutions of the free surface at Ivoz-Ramet.

Example of a network of rivers

The following simulation is inspired from the confluence of the Meuse and the Ourthe rivers, the Meuse derivation and the King Albert canal in Liège, Belgium. This network lies between the dams of Ivoz-Ramet, Chênée and Monsin and the locks of Monsin. Figure 5 presents the studied geometry.

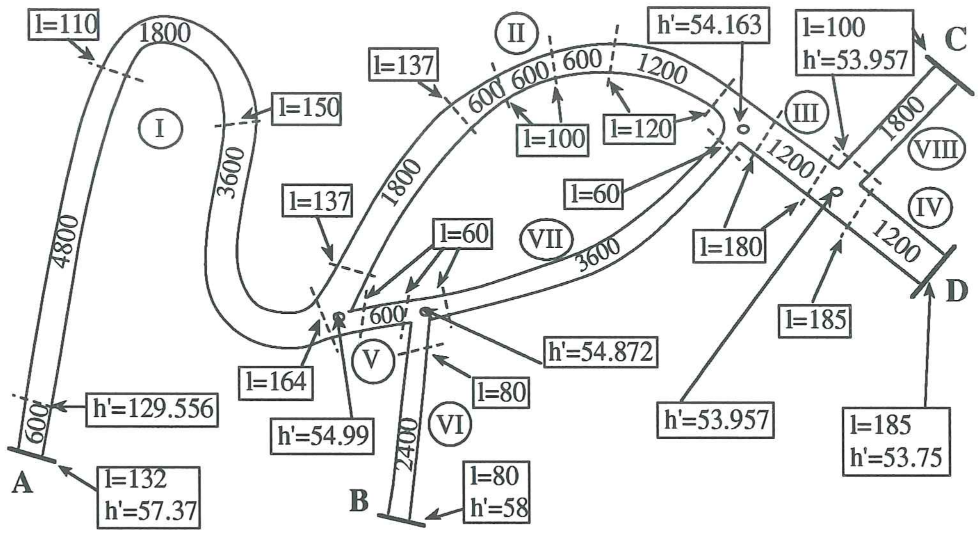


Figure 5. Geometry of the problem. The Roman numerals designate the branches. Their length is written in the branches (in meters), l designates the width of the sections (in meters) and h' the level of the bottom (in meters).

The sections are rectangular, the width and the level of the bottom vary linearily between given values. The Manning coefficient is also 30. At the beginning, there is no flow and the water level is fixed everywhere at 60 meters. During the first 20 minutes, a flow is linearily introduced up to the value of 200 $m^3/s$ at the extremity A and 40 $m^3/s$ at B. Then, they are maintained at these values. At the other limits of the network, the following conditions are imposed. In D, the dam is supposed adequately controlled so that the water level is fixed at 60 m and in C, the presence of locks suggests a no-flow condition. After 200 minutes of simulation, an important wave is introduced in B and we impose a linear variation of the flow from 40 $m^3/s$ to 140 $m^3/s$ in 10 minutes. Then, the flow is maintained to 140 $m^3/s$. The evolution of the waves is studied. In particular, two inversions of flow are observed in the branch V. This is illustrated in figure 6.

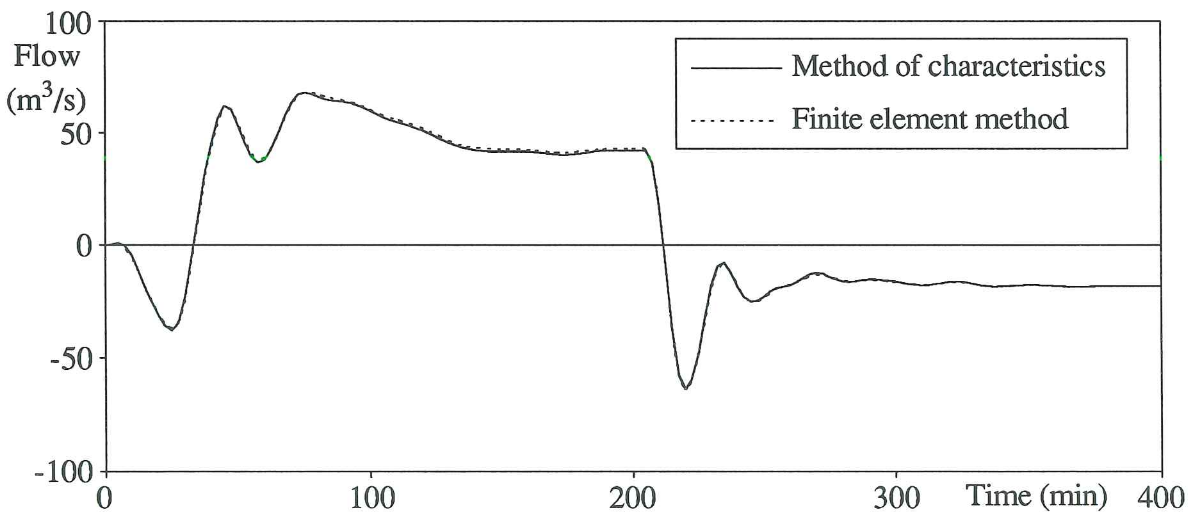


Figure 6. Temporal evolution of the flow at the downstream end of the branch V.

## DISCUSSION AND CONCLUSIONS

The good behaviour of the codes in these applications do not lead to conclude that hydraulic softwares can be used as "black box" tools. On the contrary, the good agreement between the results has to be found in a deep knowledge of factors affecting the numerical accuracy. One of the most important purpose is to keep a behaviour energetically as neutral as possible without introducing spurious oscillations, even if the way to follow completely differ into each method.

The implicit finite element technique shows a classical non-dissipative character that has restricted for a long time its field of application. The temporal decentering prevents the growth of most unreasonable oscillations while introducing controlled dissipation for better stability features. The upstream-weighted functions, controlled by the parameter $\varepsilon$, wipe out over-estimated oscillations that disrupt discontinuous simulations and capture sharp transitions by a selective damping of high frequency waves. Their effect is not enhanced here because of the lack of a large wavelength spectrum.

Sensitivity to current number is the most outstanding feature of the explicit method of characteristics. In [2], its behaviour energetically non neutral is demonstrated, without any doubt the most often a trend to the dissipation. The practical rule is to choose correctly the relation between time and space intervals by approaching the current numbers to the unit, rather than to refine the spatial and temporal meshes to the utmost. Besides, the convergence and the stability impose limits to the usable current numbers. Practically, the unit cannot be reached. In the computations, the physical reality was simulated by keeping a constant time step with a fit adjustment of the space interval, in order to maintain the current numbers into a suitable range.

Thus, only a discerning choice of simulation parameters makes of each code a powerful software package. Advantages and shortcomings of both methods point out their ideal context of application, as also the cautions aimed to ensure their best numerical efficiency. However, the general character of the mathematical formulation allows their application without any restriction to any geometry and network, with noticeable influences of a full non-linear expression, for the working out of an optimal management of hydraulic resources.

## ACKNOWLEDGMENTS

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## REFERENCES

1. La gestion automatisée des cours d'eau canalisés et ses problèmes hydrauliques, Rapport du Groupe de Travail n° 8 du Comité technique permanent I, Bulletin de l'A.I.P.C.N. (suppl 68), 1990.
2. Pochet, Th. Contribution to the mathematical modelization of hemodynamics in the cardiovascular system, PhD Dissertation, University of Liège, 1990.
3. Katopodes, N.D. A dissipative Galerkin Scheme for Open-Channel Flow, Journal of hydraulic Engineering, ASCE (110, n°4), 450-466, 1984.
4. Pirotton, M. Calcul de la propagation d'une onde de rupture de barrage, Tribune de l'eau (44, n° 551), 23-32, 1991.
5. Raymond, W.H. and Garder, A. Selective Damping in Galerkin Method for Solving Wave Problems with Variable Grids, Monthly Weather Review (104), 1583-1590, 1976.