The Hybrid Finite Element Mixing Cell Method

A new flexible method for large-scale groundwater modelling

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Introduction
Large-scale groundwater models are more and more used as managing and understanding tools for policy makers.

At this scale, groundwater models range from black-box to distributed physically-based models.

Each technique has advantages and disadvantages.
## Introduction

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<th>Advantages</th>
<th>Disadvantages</th>
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<td>▪ Easy to use</td>
<td>▪ Not spatially distributed results</td>
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Introduction

- The HFEMC method is a new and pragmatic approach which allows:
  - combining, in a single model and in a fully integrated way, different mathematical and numerical approaches
  - keeping the advantages of the spatial representation using finite element mesh

- The HFEMC method was implemented in the SUFT3D (Saturated and Unsaturated Flow and Transport in 3D) finite element code
Concepts and equations of the HFEMC method
Concepts and equations of the HFEMC method

- The flexibility of the HFMEC method allows solving the groundwater flow and transport problem using simultaneously different types of equation (one by subdomain)

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<th>FLOW</th>
<th>TRANSPORT</th>
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<td>Simple Linear Reservoir</td>
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<td>Distributed Linear Reservoir</td>
<td>Distributed Mixing Model</td>
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<td>Flow in porous media</td>
<td>Advection - Dispersion</td>
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Concepts and equations of the HFEMC method

- Choice of flow and transport equations depends on:
  - level of knowledge of hydrogeological conditions in the subdomain
  - hydrogeological interest in the subdomain

**Poor level of knowledge/interest**
- simple linear reservoir

**High level of knowledge/interest**
- flow in porous media
- advection - dispersion
Concepts and equations of the HFEMC method

- Fundamental principle of the HFEMC method consists in dividing the modelled zone into several subdomains
Concepts and equations of the HFEMC method

- Internal boundary conditions defined at the interfaces between subdomains allow:
  
  - considering groundwater fluxes between subdomains
  
  - calculating separate groundwater budgets for each subdomain
  
  - considering implicitly thin and low permeable formations (e.g. clay or shale) in order to avoid irregular elements and limit the number of unknowns
Concepts and equations of the HFEMC method

- **Flow equations:**
  
  - **Simple linear reservoir:**
    
    \[ Q_{LR} = S_{LR} \times A_{LR} \times \frac{\partial H_{LR}}{\partial t} = -\alpha_{LR} \times (H_{LR} - H_{ref}) + Q \]
    
    - \( Q_{LR} \) = flow rate entering or leaving the linear reservoir
    - \( \alpha_{LR} \) = exchange coefficient of the linear reservoir
    - \( H_{LR} \) = mean water level in the linear reservoir
    - \( H_{ref} \) = reference level
    - \( Q \) = source/sink term

N nodes but 1 unknown
(e.g. mean water level in the reservoir)
Concepts and equations of the HFEMC method

- Flow equations:
  - Distributed linear reservoir:
    - Generalisation of the simple linear reservoir to multiple interconnected reservoirs (along the three dimensions)
    \[ Q_{LR,I} = S_{LR,I} \times A_{LR,I} \times \frac{\partial H_I}{\partial t} = \sum_{j \in \eta_I} \alpha_{IJ} \times (H_J - H_I) + Q_I \]
    - Reservoirs are finite element control volumes
  - Flow in porous media equation (CVFE formulation of the classical groundwater flow equation based on Darcy’s law)
Concepts and equations of the HFEMC method

- Transport equations:
  - Simple linear reservoir:
    \[
    \frac{\partial (V_{\text{eff, res}} \bar{C})}{\partial t} = Q_{\text{in}} C_{\text{in}} - Q_{\text{out}} \bar{C}
    \]
  - Distributed mixing model:
    \[
    V_{\text{res},I} \frac{\partial (\theta_{I} C_{I})}{\partial t} - \sum_{J \in \eta_{I}} \alpha_{IJ} C_{\text{ups}}^{(I,J)} (H_{J} - H_{I}) - Q_{CI} = 0
    \]
    - Unconditionally stable
    - Dispersion only governed by the size of the mixing cells. However, for diffuse pollution, the spatial dispersion of the source is dominant.
  - Advection-diffusion transport equation
Application of the HFEMC method
Applications of the HFEMC method

- Abandoned coalfield of Cheratte
- Chalk aquifer of the Geer basin
Applications of the HFEMC method

Abandoned coalfield of Cheratte

- Abandoned underground coalfield exploitation
- Five exploited zones
- Stop of dewatering operations from mine closure
Applications of the HFEMC method

Objective:

- Modelling of the current piezometry (groundwater rebound almost completed)

→ Development of a 3D spatially distributed groundwater flow model

→ For groundwater flow:
  
  → Use of simple linear reservoirs for exploited zones
  
  → Use of flow in porous media equation for adjacent and overlying unexploited zones
Applications of the HFEMC method

- Calibrated steady-state model

- Linear reservoirs

- Porous media
Applications of the HFEMC method

Chalk aquifer of the Geer basin

- Chalk aquifer constitutes important groundwater resources for the city of Liège and its suburbs (around 30 millions m³/y)
- 65% of the basin occupied by agriculture
- Nitrate concentrations in groundwater increasing for 50 years
Applications of the HFEMC method

- Objectives:
  - modelling of the spatial distribution of the observed nitrate trends in the aquifer
  - prediction of nitrate trends evolution in groundwater
    - Development of a 3D spatially distributed groundwater flow and solute transport model
    - For groundwater flow:
      - use of flow in porous media equation
    - For transport:
      - use of the distributed mixing model (nitrate = diffuse pollution)
Applications of the HFEMC method

- Calibration of the groundwater flow model on two contrasted steady state situations

Steady-state calibration for the high groundwater level period 1983-1984
Applications of the HFEMC method

- Calibration of the solute transport model using tritium data
Applications of the HFEMC method

- Calibration of the solute transport model using estimated trends and prediction of trends
Conclusions and perspectives
Conclusions and perspectives

Conclusions:

- The HFEMC method is a new flexible modelling tool for large-scale groundwater modelling

- Applications on the abandoned Cheratte coalfield and the Geer basin show promising results

Perspectives

- Other applications on large-scale basins
- Coupling of the groundwater model with a soil model to represent more accurately the nitrate input
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Thanks for your attention!