VIBRATION ASSESSMENT OF BEAM STRUCTURES AND STIFFENED PANELS

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Abstract: - Nowadays, noise and vibration problems tend to become an important part of the design process in the automotive and naval industries. Vibrations often affect the passengers comfort, but more dangerously may damage the structure, embarked merchandise and equipments. A simple way to avoid vibrations is to prevent the resonance conditions. The paper presents a study about the vibration of beam structures and stiffened panels. The main application is to determine the eigenfrequencies of structures like platforms, trailer chassis and as well as stiffened shell - beam assemblies. The research work covers analytical vibration modeling of 3D beam structures and 3D stiffened shells, as well as the finite element analyses necessary for the validation. The analysis combines the assumption of undamped free vibrations with the simple harmonic motion for the displacement. The 3D numerical model (6 degrees of freedom per node) uses Euler-Bernoulli beam equations in the axial, torsional and flexural cases. This modeling allows to easily take into account the concentrated masses distributed on the panel surface. This approach has been implemented in FORTRAN into a numerical module and will be integrated in the near future with the LBR-5 generic stiffened structure optimization tool.

Key-Words: - vibration, natural frequency, resonance, stiffened panel, concentrated mass, Lapack

1 General overview

Vibrations acting into the mechanical systems can cause many problems at different levels such as mechanical and performance degradation. If we include the human factor, the study of the vibration becomes extremely important.

The work presented in this paper is devoted only to the case of beams structures, stiffened panels, their assemblies, and other connected problems. For this type of structures the contact with the human beings are very limited. The most affected are the structural fatigue level and the functioning of the embarked installations.

In addition of the marine engineering field (decks, ship tanks, offshore structures), the stiffened panels and beam structures are the base element of many other engineering domains. For example, taking the case of a platform vehicle equipped with a military shooting system, the resonant displacement of the platform can affect the fire precision. Imagine also an armored tank equipped with a balance-bridge damaged by the vibration during military actions.

Another important application connecting the marine and vehicle fields refers to the dynamics vehicle/ship-deck investigations. The experiences demonstrate that the dynamic interactions between the vehicles and the vessel deck (for example, a roll-on/roll-off RO-RO vessel with vehicle cargo) may be very different from that of static case. In this case, it was found that the vehicle cargoes can work as mass dumpers to reduce at least one mode shape response of the deck [1].

Nowadays, the vibrational comportment is often verified in the design process of this type of structures. Therefore, in the preliminary design stage or during the structural design phase, the stakeholders will carry out adequate vibration analysis for each type of structures. These analyses have a dual aim:

- The first goal is purely theoretical and supposes to determine their natural frequencies;
- The second aim assumes to measure and compare, during the normal operating, the vibrational characteristics with the agreed limit values because despite careful analysis vibrations cannot be avoided completely.

The parameters calculated or measured for the second goal are vibrational displacement **s**, vibrational velocity **v** and vibrational acceleration **a**. The acceptable values are indicated in ISO standards, directives and intern specifications. Magnitudes exceeding these values or falling short of them do not necessarily indicate an admissible state of vibration [2]. These criteria may be used as bounds of admissibility, for instance if made a part of the contractor or some other form of obligation. Concerning this paper, criteria for vibrations are stipulated with respect to the overstressing of structural members (deformation, fatigue, strength), engines and equipments (failure, malfunctions) and to physiological effects on people (if it is the case). Concerning the first category (structural vibrations) for each type of structures assessment diagrams can be obtained experimentally (Fig. 1).

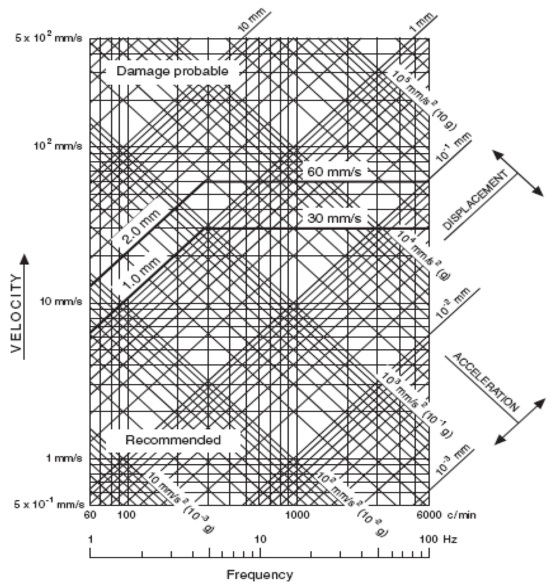

Fig. 1 – Assessment diagram for vibration of structures [2]

The series ISO 7919 and ISO 10816 specify the limitations concerning engines and connected aggregates. These parts should generally not be subjected to vibrations exceeding 0.71 mm in amplitude, 14 mm/s in velocity amplitude and 0.7 g in acceleration amplitude [3]. This paper will not cover these aspects, but it will give you some general indications to calculate the vibrational magnitudes. It is important to know that, in simulations, these magnitudes can be correctly estimated only when all structural details, including also mass distribution, are known.

Thereby, this paper will present only the numerical modeling necessary to obtain the natural frequencies of beams structures, stiffened panels and their assemblies. The numerical model constitutes the base of a vibration module written in FORTRAN. FE simulations were carried out in order to validate and asses the limitations of this module. We have chosen an analytical way to obtain these resonant frequencies. In this way, the vibration module can be effortlessly implemented under an EF code through external subroutines (i.e. UVARM user variables routines in ABAQUS).

2 Numerical model

To calculate analytically the eigenfrequencies of a stiffened panel we employ a virtual artifice that consists in the decomposition of the panel into a beam grid (Fig. 2). The mass of the plate (without stiffeners) will be distributed along the longitudinal stiffeners in order to keep the beam aspect approximation and to preserve the total mass of the structure. The main condition is to preserve the global inertia of the stiffened panel.

This choice allows us to use the beam theory to solve the problem. At the same time it will be easily to assess vibration for complex structures like stiffened panels - beams assemblies and also to take into account concentrated masses distributed on the panel surface.

The beam vibration analysis combines the assumption of undamped free vibrations combined with the simple harmonic motion for the displacement. The mathematical model uses Euler-Bernoulli beam equations in the axial, torsional and flexural cases and allows considering 6 degrees of freedom per node. The torsion and the flexion are uncoupled in this study.

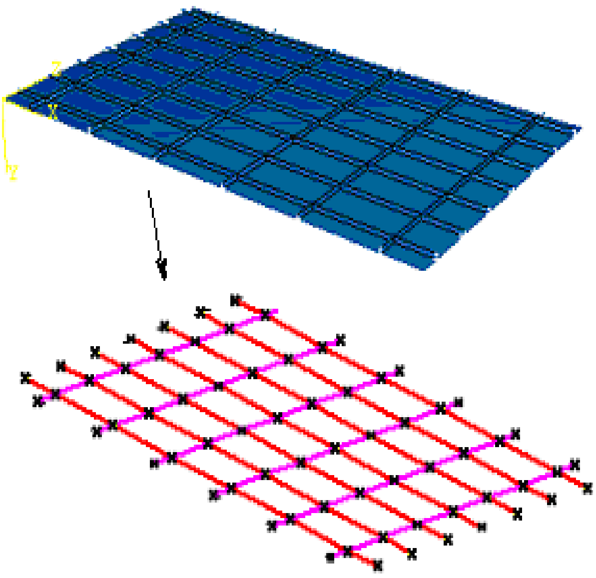

Fig. 2 – Decomposition of a stiffened panel into a beam grid

2.1 Analytical method

The analytical method is based on the elastic, homogeneous and isotropic material hypothesis. The Euler-Bernoulli formulation assumes that cross-section, which are initially plane and perpendicular to the axis of the beam, remain plane and perpendicular to this axis. The transverse shear deformation is thereby neglected.

Considering the dynamic equilibrium of an elementary section of the beam, we obtain the next equations of motion for the three fundamental cases [4] :

- axial vibration

$$-\rho A\frac{\partial^2 u}{\partial t^2}+\frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)=0 \qquad (1)$$

- flexural vibration

$$\rho A\frac{\partial^2 v}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 v}{\partial x^2}\right)=0 \qquad (2)$$

- torsional vibration

$$-\rho I_p\frac{\partial^2 \theta}{\partial t^2}+\frac{\partial}{\partial x}\left(GJ\frac{\partial \theta}{\partial x}\right)=0 \qquad (3)$$

where u is the axial displacement, v is the transversal displacement and θ is the angular rotation (twist). A represents the cross-section area, I - the second moment of area, Ip – the polar second moment of area, G – the shear modulus and J is the torsional constant.

Since the system is supposed undamped, we assume that a mode of vibration is harmonic, as for discrete systems. Thus the general solution of eq. (1) to eq. (3) is one of the form:

$$h(x,t)=\phi(x)\sin(\omega t+\Psi) \qquad (4)$$

where h is the displacement or rotation, ω a pulsation, Ψ a constant and $\Phi(x)$ an eigenfunction, which describe the mode shape at the frequency ω.

Substituting eq. (4) in the previous equations and factoring out the sine terms, we get for each equation the next solutions (5, 6, 7):

- axial vibration

$$U(x)=A_1\sin(a\,x)+A_2\cos(a\,x) \qquad (5)$$

- flexural vibration

$$V(x)=B_1\sin(b\,x)+B_2\cos(b\,x)+B_3\sinh(b\,x)+B_4\cosh(b\,x) \qquad (6)$$

- torsional vibration

$$\Theta(x)=C_1\sin(c\,x)+C_2\cos(c\,x) \qquad (7)$$

where $a=\omega\sqrt{\frac{\rho}{E}}$ (pulsation multiplied by the velocity of propagation of extensional waves in the beam), $b=\sqrt{\omega^2\frac{\rho A}{E I}}$ and $c=\omega\sqrt{\frac{\rho Ip}{G J}}$. In theory, the constants A_i, B_i, C_i are evaluated by the boundary conditions. Knowing the displacements and rotation expressions, we can calculate the internal nodal forces in the nodes of the beam by the next formulae:

- normal effort

$$N=EA\frac{\partial U(x)}{\partial x} \qquad (8)$$

- transversal effort

$$T=-EI\frac{\partial^3 V(x)}{\partial x^3} \qquad (9)$$

- torque and bending moments

$$M_\theta=GJ\frac{\partial\Theta(x)}{\partial x} \text{ and } M=EI\frac{\partial^2 V(x)}{\partial x^2} \qquad (10)$$

For a single beam (Fig. 3) it is possible to eliminate the constants in order to obtain an expression between nodal local forces (N^L, six per node) and nodal local displacements (U^L, six per node).

$$\left[N^L\right]_{12x1}=\left[K^L(\omega,C_{mp})\right]_{12x12}\left[U^L\right]_{12x1} \qquad (11)$$

C_{mp} represents the mechanical and physical characteristics of the beam.

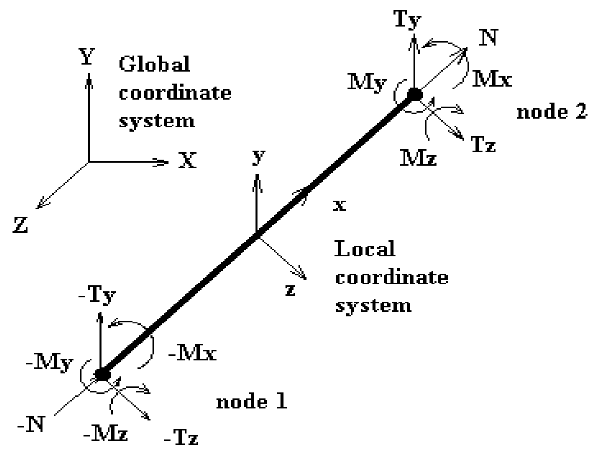

Fig. 3 – Efforts and sign convention for a single beam

The matrix K^L represents the stiffness and mass matrix. It is considered a continuous matrix because a beam represents in this case a continuum system. A continuous system has its mass, elasticity and damping distributed. Therefore its mass is inseparable from the elasticity of the beam. In this way, this matrix $[K^L(\omega, C_{pm})]$ cannot be decomposed into a separate mass matrix and a separate stiffness matrix without losing in accuracy. This matrix is non-symmetrical and the pulsation ω is located inside the *sin*, *cos*, *sinh* and *cosh* functions.

In the case of a 3D multi-beam structure, the nodal local efforts and displacement must be projected into a global coordinate system. A global continuous stiffness and mass matrix will be obtained. This matrix connects the global nodal effort with the global nodal displacements and allows us to calculate the eigenfrequencies of the system:

$$\left[N^G\right]_{dof\,x\,1} = \left[K^G\left(\omega, C_{mp}\right)\right]_{dof\,x\,dof}\left[U\right]_{dof\,x\,1} \quad (12)$$

where "*dof*" is the total number of degrees of freedom. In reality the resonant phenomena express by very important structural displacements, but numerically the displacements is supposed to tend towards the infinite one. This condition is accomplished by the cancellation of the determinant of the matrix K^G:

$$\det\left(\left[K^G\left(\omega, C_{mp}\right)\right]_{dof\,x\,dof}\right) = 0 \quad (13)$$

Calculating this determinant, an equation function of the pulsation ω (or function of the frequency f) is obtained. The solutions of this equation represent the eigenfrequencies of the beam structure. The total number of the natural frequencies is equal to the degree of the characteristic equation (13).

Knowing the external forces that acts into the nodes structure, form the equation (12) we can obtain the vibrational displacements of each node. For harmonic excitations, the vibrational parameters are connected through the following relations:

$$v = 2\pi \cdot f \cdot s \quad \text{and} \quad a = 2\pi \cdot f \cdot v \quad (14)$$

2.2 Concentrated masses

The beam modeling can easily take into account the concentrated masses on the stiffened panel surface. The masses must be distributed into the beam structure nodes. The presence of these auxiliary masses will determine a decrease of the eigenfrequencies values because the pulsation ω is proportional to the square root of *k/m* (stiffness/mass): $\omega \approx \sqrt{\frac{k}{m}}$

To implement the concentrated masses into the vibration calculus, we write the dynamic equilibrium for each node that has an associated mass. We obtain:

$$\begin{aligned}\left[N^G\right] &= \left[K^G\left(\omega, C_{mp}\right)\right]\left[U(x)\right] \\ \left[N^G\right] &= F_{dynamic}\left(= m\ddot{U}(x,t)\right)\end{aligned} \quad (15)$$

where $F_{dynamic}$ are the dynamic inertial forces dues to the concentrated masses. Numerically $F_{dynamic}$ is represented by a diagonal matrix. Considering a harmonic function of time for the displacements, the above relation become:

$$\left(\left[K^G\left(\omega, C_{mp}\right)\right] - \omega^2\left[M_a\right]\right)\left[U(x)\right] = 0 \quad (16)$$

where $[M_a]$ is the additional mass matrix. To calculate the eigenfrequencies of a beam structure with additional masses, we compute the determinant of the next expression:

$$\det\left(\left[K^G\left(\omega, C_{mp}\right)\right] - \omega^2\left[M_a\right]\right) = 0 \quad (17)$$

2.3 CPU time dissemination

The resolution of equations (13) and (16) (the unknown is the pulsation ω) makes possible to find the eigenfrequencies of the beam structure. Numerically, two methods were tested.

2.3.1 Classic dichotomy

The first method supposes to divide the relevant frequency interval into small fixed intervals and calculate the determinant at each frequency step. A change of the determinant sign indicates a solution of the characteristic equation.

The accuracy of this method is influenced by the frequency step dimension, but smaller is the step larger is the CPU calculation time.

We have tried 5 common numerical methods to calculate the determinant of the global mass and stiffness continuous matrix (eq.13). The next table summarizes the CPU time machine necessary to find the first natural frequency of a beam structure with 300 degrees of freedom. The program carries out 37 calculations of the determinant (frequency range from 0.1 Hz to 3.7 Hz with an increment of 0.1 Hz, the first eigenfrequency being between 3.6 Hz and 3.7 Hz).

Method (single precision)	**CPU time**
Leverrier algorithm	hours
Product of eigenvalues (Lapack)	57.00 s
Gauss partial-pivoting scheme	14.9 s
Classic LU decomposition	11.5 s
Optimized LU decomposition (Lapack)	7.9 s

The optimized LU decomposition using Lapack libraries of linear algebra routines proves to be the fastest method with 0.21 seconds per increment for this case. Using the same method, for a beam structure with 990 dof, the time was around 1 s per increment (on a laptop with Intel Centrino Core 2 Duo, 4 Gb RAM). All methods give identically results.

2.3.1 Discrete approach

The second method supposes to dissociate the matrix $K^G(\omega, C_{mp})$ into a mass matrix $M^G(C_{mp})$ and a static stiffness matrix $K^G(C_{mp})$, similar to the discrete systems. To obtain the static stiffness matrix, we carry out series expansions at least 15 terms of the matrix $K^G(\omega, C_{mp})$ and of his double derivation with regard to ω in the vicinity of $\omega = 0$.

$$K^G(C_{mp}) = \lim_{\omega \to 0} K^G(\omega, C_{mp}) \qquad (18)$$

$$M^G(C_{mp}) = \lim_{\omega \to 0} \frac{\partial K^G(\omega, C_{mp})}{\partial \omega} \qquad (19)$$

These matrices are independents of the frequency *f*. With this approach, the resonant frequencies are obtained solving the eigenvalue problem (eq. 12).

$$K^G - \omega^2 M^G = 0 \qquad (20)$$

The CPU time is considerably reduced: 1.4 s instead 7.9 s for the case characterized by 300 dof.

The last method represents an approximation of the analytical method and it is valid for lower frequency domain (< 100 Hz). For a single analysis, the first method is desirable. Is not the case for the optimization software that requires very small CPU time. In this case, the second method shall be used.

3 Particularities and tests

Finite element simulations with the industrial software ABAQUS were carried out to validate our numerical tool. Then, the results are compared.

3.1 3D beam structures

The first validation of the vibration tool was realized on 3D beam structures. The finite element simulations used a beam modeling. For both simulations (FE and vibration tool), all connected nodes are rigidly joined, instead all non-connected nodes are clamped. All the beam sections are identically. We have treated two typical of structures, i.e. a 3D type (masts, Fig. 4) and a planar type (planar grid, Fig. 5). The material is steel (E = 2.1e11 N/m^2, ρ = 7800 kg/m^3, ν = 0.3).

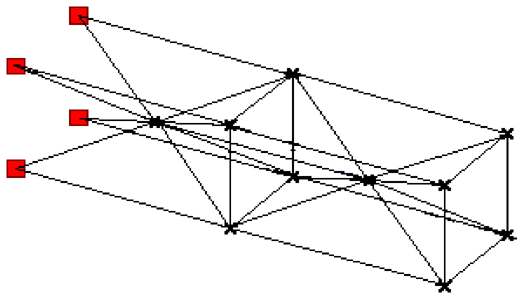

Fig. 4 – 3D multi-beam structure
length 24 m, width = 6 m
annular section - Φ = 100 mm, thickness = 10 mm

	Frequency			
Method	f_1	f_2	f_3	f_4
Classic dichotomy	0.76	1.40	-	-
Discrete method	1.24	1.35	1.50	1.61
ABAQUS	0.52	1.47	3.09	3.20

Table 1 – 3D multi-beam structure results

Generally, only FE calculus using very fine mesh can supply results closed to those of analytical continuum methods. Using classic dichotomy method, the natural frequencies agree well with those determined with ABAQUS. The differences can be justified by unheeded of the constrained torsion. Moreover, we don't use the Timoshenko's beam model, so we neglect shear and rotational inertia effects.

The results obtained with discrete method are different. This second method is adapted for complex three-dimensional structure only using many beams and many beam connections.

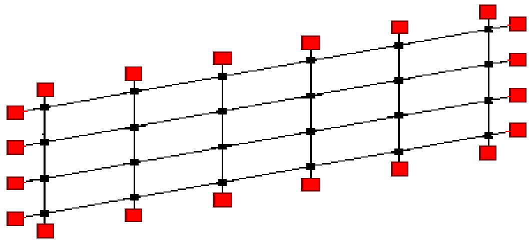

Fig. 5 – Complex planar beam structure
Length 17 m, width = 8 m
Rectangular section - h = 30 mm, b = 40 mm

The next tables(Which ones) present the results obtained with classic dichotomy method, discrete approach (see §2.3) and ABAQUS for the second problem test.

	Frequency		
Method	f_1	f_2	f_3
Classic dichotomy	1.55	2.15	2.68
Discrete method	1.56	2.16	2.80
ABAQUS	1.72	2.22	3.15

Table 2 – Vibration module methods comparison

As we can see in the Table 2, for planar structures the both methods give practically the same results, but with different CPU times (greater for the classic dichotomy). These results are also in good correlation with ABAQUS results.

3.2 3D stiffened panels

The second validation of our vibration module uses planar stiffened panels. For stiffened panels having only one direction stiffeners, the mass of the plate will be distributed on the second direction (perpendicularly on stiffeners) by creating virtual beam on this direction.

After the split of the stiffened panel into a beam grid (Fig. 2), it is necessary to calculate the second moment of area of each beam section of the new structure. To second moment of area about y-axis I_y is calculated with respect to the vertical principal axis of the considered cross section (Fig. 6).

To calculate the second moment of area about z-axis I_z, we tested 3 feasible cases for which this moment is calculated to respect of:

- the horizontal principal axis of the considered section (C1);
- the horizontal principal axis of the undivided section of the stiffened panel (C2);
- the horizontal plane passing through the virtual centroid C_z of the whole stiffened panel (eq. 21), (C3).

$$C_z = \frac{z_c{}^T \cdot A^T + z_c{}^L \cdot A^L}{A^T + A^L} \tag{21}$$

where $z_c{}^T$ and $z_c{}^L$ are the centroid (C) of the transversal and horizontal cross-section area of the entire stiffened panel, and A^T and A^L the correspondents cross-section areas.

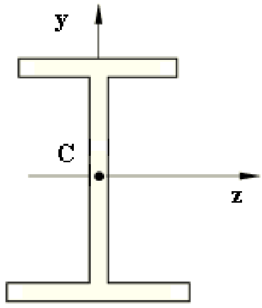

Fig. 6 – Notations of transversal cross-section

The first type of tested problems refers to stiffened panels with clamped edges. We consider a rectangular panel (20 x 30 x 0.006 m, Fig. 7) with 11 identical transversal frames (T section, web 0.25 x 0.01 m, flange 0.2 x 0.012 m) and 15 different longitudinal frames (the same T section and L section - web 0.16 x 0.008 mm, flange 0.05 x 0.02 m). The material is steel.

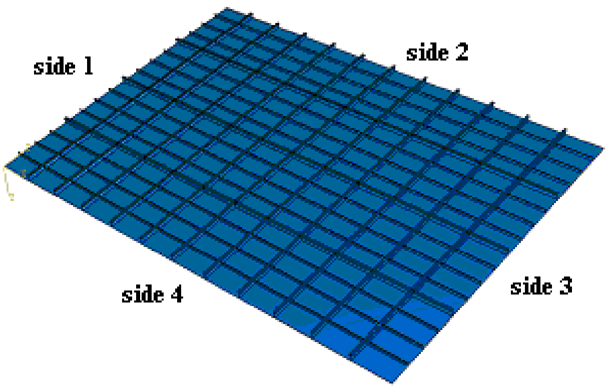

Fig. 7 – Complex stiffened panel

For this problem, we apply 3 different boundary conditions on free edges:

- BC1 – all sides clamped;
- BC2 – sides 1 and 2 clamped, 3 and 4 free;
- BC3 – sides 1 and 3 clamped, 2 and 4 free.

The vibration tool values (Table 3) are obtained using the discrete method.

Boundary condition	Freq. [Hz]	Vibration tool			Abaqus
		C 1	C 2	C 3	
BC1	f_1	**3.99**	4.02	4.02	**3.89**
	f_2	**5.34**	5.46	5.46	**5.31**
	f_3	**8.29**	8.59	8.55	**8.15**
BC2	f_1	**0.60**	0.60	0.60	**0.60**
	f_2	**1.35**	1.40	1.40	**1.33**
	f_3	**2.84**	2.85	2.85	**3.20**
BC3	f_1	**1.16**	1.21	1.22	**1.09**
	f_2	**1.31**	1.37	1.37	**1.27**
	f_3	**2.87**	2.87	2.87	**2.88**

Table 3 – Comparisons with FE results

All results are in good agreement with those of ABAQUS, but those of case 1 (second moment of area about z-axis I_z is calculated with respect to the horizontal principal axis of the considered section) are very closed. However, the vibration tool results remain higher than ABAQUS results.

The second type of studied problems relates to platforms. The boundary conditions impose to block the displacements and rotations for 4 nodes (Fig. 8). The panel (4 x 3 x 0.016 m) has identical transversal and longitudinal frames disposed symmetrically. The section of the frames is I profile with web 50 x 10 mm and no flange. The material is steel.

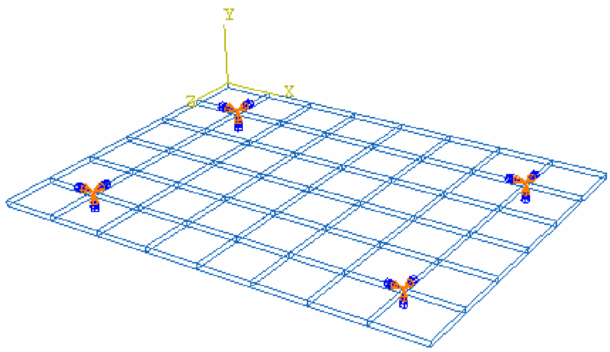

Fig. 8 – Complex stiffened panel - beam assembly

Table 4 gives the results obtained with our numerical tools (classic dichotomy method and the discrete approach) and those of ABAQUS. In this case, the panel and the stiffeners are meshed with shell elements.

Method	f_1	f_2	f_3	f_4
Classic dichotomy	8.52	10.52	13.67	18.52
Discrete approach	8.54	10.52	13.73	18.59
Abaqus	8.67	10.59	17.53	19.21

Table 4 – Vibration module comparisons

Excepting the mode 3, the numerical tool results are in very good agreement with ABAQUS results. In ABAQUS, all 4 modes are global modes of vibration. Unfortunately, we can only to compare the natural frequency values. To view the vibration mode associate to each natural frequency, we must model under an FE code the equivalent beam grid associate to considered panel. Our simulations using the equivalent beam grid ensure that the first modes are global modes.

3.3 Vibration module limitations

As we have described in § 2.1, the vibration tool uses a beam modeling. As a consequence the deformation of the shell between beam grid cannot be take into account. If the beams of the panel have a very high moment of inertia, the first vibration modes are represented by local modes (Fig. 9). If the stiffeners' width is very large the first mode of vibration can also be a local mode. Certainly, the boundary conditions can also affect the vibration modes.

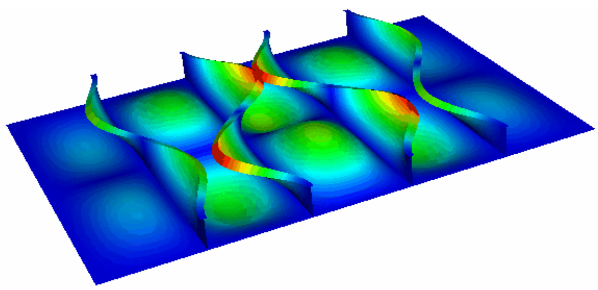
Fig. 9 – Local vibration modes of a stiffened panel

To identify some limitations of this numeric tool, we tested a type of stiffened panel in relevant configurations.

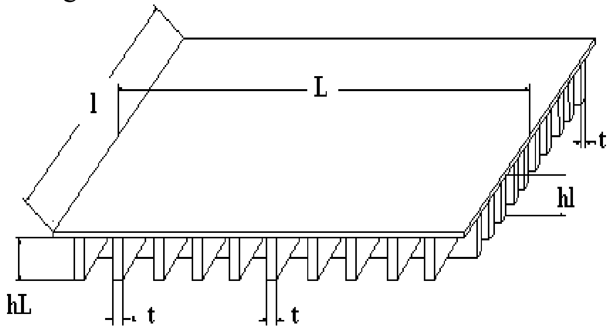

Fig. 10 – Stiffened panel notations

The first three configurations refer to a square panel, and the next five configurations refer to a rectangular panel. The both structures have 10 transversal and 10 longitudinal stiffeners. The stiffeners are disposed symmetrically with respect to the symmetry axis of the panel and the distance between them is equally. Figure 10 and Table 5 present the characteristic dimensions of each case.

Case	L	l	h_L	h_l	t
1	6.6 m	6.6 m	0.22 m	0.22 m	8 mm
2	6.6 m	6.6 m	0.44 m	0.22 m	8 mm
3	6.6 m	6.6 m	0.66 m	0.22 m	8 mm
4	19.8 m	6.6 m	0.22 m	0.22 m	8 mm
5	19.8 m	6.6 m	0.44 m	0.22 m	8 mm
6	19.8 m	6.6 m	0.66 m	0.22 m	8 mm
7	19.8 m	6.6 m	0.22 m	0.44 m	8 mm
8	19.8 m	6.6 m	0.22 m	0.66 m	8 mm

Table 5 – Panel dimensions

Concerning the results, we compared our results with those of COSMOS only for the first global mode of vibration (Table 5). Apart from the cases 3 and 8 for which the first mode of vibration is a local mode (vibration of stiffeners), all other results are in good concordance.

	Vibration tool		COSMOS	
Case	Natural frequency [Hz]			Mode
	C 1	C 2		
1	32.08	32.10	30.10	1
2	56.97	57.02	53.36	1
3	86.43	86.49	77.21	111
4	15.41	15.41	15.40	1
5	14.96	14.97	15.15	1
6	15.83	15.83	15.77	1
7	39.09	39.26	34.89	1
8	62.86	63.38	53.01	87

Table 5 – Panels dimensions

It is very difficult to give the real limitations of this vibration tool. After few tests, we consider that the dimensions of the stiffeners must remain smaller in front of the dimension of the shell and the distance between stiffeners.

4 Conclusion

In this paper, a numerical approach to calculate the resonant frequencies for beam structures, stiffened panels and their assemblies is presented. Finite element simulations were carried out to validate the numerical tool.

Two methods can be used to obtain the natural frequencies. The first, named classic dichotomy is based on Euler-Bernoulli equations and is purely analytical. In this case it is necessary to divide the relevant frequency interval into small intervals and to calculate the determinant of the characteristic mass and stiffness continuum matrix. The main advantage of this method is the accuracy of the results. Nevertheless, this accuracy is limited by the modeling and is influenced by frequency step size. The main inconvenience is the large CPU calculation time in case of complex structures (over 600 dof). In some cases, the numerical tool delivers some parasite frequencies in the vicinity of a natural frequency. This may be due to some numerical errors, a non-dimensionless model and/or to single precision.

To integrate the numerical model into an optimization design process (that requires reduced CPU calculation time) a second method was developped, named discrete approach. The calculation time becomes very small even for structures with many degrees of freedom (2000 dof) and the parasite frequencies disappear. This method was validated with simplified FEA (beam mesh which contains only one element).

Due to beam modeling, the both methods allow to obtain only the resonant frequencies corresponding to global vibration modes. The modeling of the concentrated masses was tested and validated only in the case of simple structures.

In practical dimensioning, the two first natural frequencies are the most relevant. These two values calculated with the vibration tool are very close to those given by ABAQUS for all problems treated in this paper. In conclusion, taking into account the limitations of these methods, it is appreciated that the numerical tool can be successfully used to calculate correctly at least the two first resonant frequencies for beam structures and stiffened panels.

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