

# An Extended LOTOS for the design of Real-Time Systems

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## 1. Introduction

We give in the following a brief presentation of ET-LOTOS [Lél 95a, Lél 95b]. ET-LOTOS extends with quantitative time the formal description technique LOTOS [ISO 8807]. Other proposals for a "time extended" LOTOS exist. Let us mention [QMF 94] and [BLT 94]. ET-LOTOS serves as basis for the time extension part of E-LOTOS, the new standard for LOTOS currently developed within ISO (ISO/IEC JTC1/SC21).

We assume in the sequel that the reader has a basic knowledge of the syntax and the semantics of LOTOS.

## 2. Formal semantics and properties of ET-LOTOS

### 2.1. Datatypes and time domain

In ET-LOTOS, like in LOTOS, datatypes are described in the Abstract Datatype language ACT ONE, that has an initial semantics.

The time domain, denoted  $D$ , is defined as the set of values of a given data sort  $\text{time}$  ( $D = Q(\text{time})$ ). Its definition is left free to the will of the specifier provided that the following elements be defined.

- A total order relation represented by " $>$ ".
- An element  $0 \in D$  such that:  $\forall r \in D: r \neq 0 \Rightarrow r > 0$
- An element  $\infty \in D$  such that:  $\forall r \in D: r \neq \infty \Rightarrow \infty > r$
- A commutative and associative operation " $+$  :  $D, D \rightarrow D$ " such that:
  - $\forall r, r1 \in D: r > r1 \Leftrightarrow \exists r' > 0 \bullet (r' + r1) = r$
  - $\forall r, r1 \in D: r > 0 \text{ and } r1 \neq \infty \Rightarrow r + r1 > r1$
  - $\forall r \in D: r + 0 = r$
  - $\forall r \in D: r + \infty = \infty$

The relations " $\leq$ ", and " $-$ " can be derived easily as follows :

- $\forall r, r1 \in D \bullet r \leq r1 \Leftrightarrow (r < r1 \vee r1 = r)$
- $\forall r, r1, r2 \in D \bullet r1 \leq r \Rightarrow (r - r1 = r2 \Leftrightarrow r1 + r2 = r)$
- $\forall r, r1 \in D \bullet r \leq r1 \Rightarrow r - r1 = 0$

In particular, the time domain can be dense as well as discrete, but to be able to give the operational semantics of ET-LOTOS in terms of Labelled Transition Systems (LTS), it must be countable, such as the rational numbers.

## 2.2 Notations

The following notations hold for the remainder of the paper.  $G$  denotes the countable set of common observable gates.  $L = G \cup \{\delta\}$  denotes the alphabet of observable gates where  $\delta$  is the special action denoting successful termination ( $\delta \notin G$ ).  $\delta$  does not appear explicitly in the syntax of LOTOS.  $S$  denotes the set of sorts,  $V$  denotes the set of ground terms in the initial algebra associated with the ACT ONE specification:  $V = \bigcup_s Q(s)$ .  $CL = L \times V^*$  denotes the set of observable actions.  $A = CL \cup \{i\}$  denotes the alphabet of actions, where the symbol  $i$  is reserved for the unobservable internal action ( $i \notin L$ ).  $g$  (resp.  $a$ ) denotes an element of  $G$  (resp.  $A$ ):  $g \in G$ ,  $a \in A$ .  $gv_1 \dots v_n$  and  $\delta v_1 \dots v_n$  denote elements of  $CL$ , with the  $v_i$ 's  $\in V$ . Capital Greek letters such as  $\Gamma$  will be used to denote subsets of  $G$ .  $D$  denotes the countable time domain which is the alphabet of time actions.  $D_{0\infty} = D - \{0, \infty\}$ .

## 2.3 Syntax of the behaviour part of ET-LOTOS

The collection of ET-LOTOS behaviour expressions is defined by the following BNF expressions. In these expressions,  $\tilde{x}$  represents a vector of process names,  $SP$  is a selection predicate, the  $e_i$ 's represent a term<sup>1</sup>  $tx$ , the  $o_i$ 's represent either  $?x:s$  (with  $x$  a variable of sort  $s$ ) or  $!tx$  (with  $tx$  a ground term), the  $x_i$ 's (resp.  $tx_i$ 's) are variables (resp. ground terms) of sorts  $s_i$ 's,  $d \in D$  and in  $@t$ ,  $t$  is a variable of sort time. The new features are printed in italics:

$$P ::= Q \text{ where } \tilde{X} := \tilde{Q}^2$$

$$Q ::= \text{stop} \mid \text{exit}(e_1, \dots, e_n) \{d\} \mid go_{o_1 \dots o_n} @t [SP]; Q \mid i @t \{d\}; Q \mid \Delta^d Q \mid Q [] Q \mid Q | [\Gamma] | Q \mid \\ \text{hide } \Gamma \text{ in } Q \mid Q \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \mid Q [>Q \mid X \mid [SP] \rightarrow Q \mid \\ \text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } Q \mid \text{choice } x_1:s_1, \dots, x_n:s_n [] Q \mid \text{inf} \quad ||| P$$

Remark: in  $go_{o_1 \dots o_n} @t [SP]; Q$  we let both  $@t$  and  $[SP]$  be optional, and use the convention that, if omitted,  $[SP] = [true]$ . In  $i @t \{d\}; Q$ , both  $@t$  and  $\{d\}$  are optional. If omitted,  $d = 0$ . Similarly  $\{d\}$  is optional in  $\text{exit}\{d\}$ , and  $\text{exit}$  means implicitly  $\text{exit}\{\infty\}$ .

The binding powers of the operators are like in LOTOS. For the new operators,  $\Delta^d$  has the same power as action-prefix and  $\text{inf} \quad |||$  the same as  $\text{choice } x_1:s_1, \dots, x_n:s_n []$ .

**An additional shorthand notation:** We define the notation  $go_{o_1 \dots o_n} \{d\}; Q$ , for  $go_{o_1 \dots o_n} @t [t \leq d]; Q$ , provided that  $t$  be fresh in  $Q$ . Under the same restriction, we also introduce the notation

<sup>1</sup> This term can be: 'any  $s$ ' (with  $s \in S$ )

<sup>2</sup> For convenience, we suppose, without lack of generality, that there is a single where-clause that gathers all the process declarations of the specification.

$g \circ 1 \dots \circ n \{d_1, d_2\}; P$  to mean  $g \in t[d_1 \leq t \leq d_2]; P$ . The meaning of these rewritings will become clear in the next section.

## 2.4 Semantics of ET-LOTOS

The operational semantics of ET-LOTOS, presented in the following, is of the so-called "time/actions" type. This means that the occurrence of actions and the passing of time are considered as separate concerns, each one being described by a dedicated set of rules.

### 2.4.1 Notations

$P, P', Q, Q'$  denote ET-LOTOS behaviour expressions.

$P \xrightarrow{a} P'$ , with  $a \in A$ , means that process  $P$  may engage in action  $a$  and, after doing so, behave like process  $P'$ .  $P \xrightarrow{g} P'$  means  $\exists P', a \bullet P \xrightarrow{a} P' \wedge \text{name}(a) = g$ .  $P \not\xrightarrow{g}$  means  $\neg (P \xrightarrow{g})$  i.e.  $P$  cannot perform an action on gate  $g$ .  $P \xrightarrow{d} P'$ , with  $d \in D_{0\infty}$ , means that process  $P$  may idle (i.e. not execute any action in  $A$ ) during a period of  $d$  units of time and, after doing so, behave like process  $P'$ .  $P \not\xrightarrow{d}$ , with  $d \in D_{0\infty}$ , means that  $\nexists P' \bullet P \xrightarrow{d} P'$ , i.e.  $P$  cannot idle during a period of  $d$  units of time. In these expressions, it is required that  $P$  and  $P'$  be closed, i.e. they do not contain free variables.

### 2.4.2 Inference rules

In the following inference rules,  $d \in D_{0\infty}$ ,  $d_1 \in D$ ,  $d' \in D_\infty$ ,  $g \in G$  and  $a \in A$ .

We introduce a process, denoted `block`, which has no axiom and no inference rules. This process cannot perform any action and blocks the progression of time.

#### Inaction

$$(S) \quad \text{stop} \xrightarrow{d} \text{stop}$$

Remark that `stop` cannot perform any action but can idle.

#### Exit

$$(Ex1) \quad \text{exit}(e_1, \dots, e_n) \{d_1\} \xrightarrow{\delta v_1 \dots v_n} \text{stop}$$

where  $v_i = [t_i]$  if  $e_i = t_i$  (a ground term)

$v_i \in Q(s_i) = \{[t] \mid t \text{ is a ground term of sort } s_i\}$  if  $e_i = \text{any } s_i$

$$(Ex2) \quad \text{exit}(e_1, \dots, e_n) \{d_1+d\} \xrightarrow{d} \text{exit}(e_1, \dots, e_n) \{d_1\}$$

$$(Ex3) \quad \text{exit}(e_1, \dots, e_n) \{d_1\} \xrightarrow{d} \text{stop} \quad (d > d_1)$$

The  $\{d_1\}$  attribute is called the life reducer. Its role is to restrict the time period during which the process can terminate successfully: `exit` $\{d_1\}$  can only perform  $\delta$  during the next  $d_1$  time units. If `exit` $\{d_1\}$  has not performed  $\delta$  yet after  $d_1$  time units, it is too late and the process turns into `stop` (rule Ex3).

**Observable action-prefix**

$$\begin{aligned}
 \text{(AP1)} \quad & g o_1 \dots o_n @t[SP];P \xrightarrow{g v_1 \dots v_n} [v_1/o_1, \dots, v_m/o_m, 0/t]P \\
 & \text{if } \vdash [v_1/o_1, \dots, v_m/o_m, 0/t]SP \\
 & \quad v_i = [w] \quad \text{if } o_i = !w \\
 & \quad v_i \in Q(s) = \{[w] \mid w \text{ is a ground term of sort } s\} \quad \text{if } o_i = ?x:s \\
 & \text{and where } v_i/o_i = v_i/x \quad \text{if } o_i = ?x:s \\
 & \quad v_i/o_i \text{ is void} \quad \text{if } o_i = !w \\
 \text{(AP2)} \quad & g o_1 \dots o_n @t[SP];P \xrightarrow{d} g o_1 \dots o_n @t[[t+d/t]SP];[t+d/t]P
 \end{aligned}$$

In  $@t$ ,  $t$  is a variable of sort  $\text{time}$ . This variable is used to measure the delay actions were being offered on  $g$  when one occurred. When an action occurs (rule AP1),  $t$  is instantiated. Instantiating  $t$  by 0 is logical:  $g o_1 \dots o_n @t[SP];P$  describes a process at a given instant and the counting of  $t$  starts at that instant. So,  $t$  is still at 0 if the process immediately does an action on gate  $g$ . The way the value of  $t$  is kept up to date if  $g o_1 \dots o_n @t[SP];P$  idles is defined by AP2.

The  $t$  variable can appear in the selection predicate  $SP$ , if there is one. The conditions joined with AP1 express that the only possible instantiations for the attributes of  $g$  are the ones that make  $SP$  true at that instant.

**Internal action-prefix**

$$\text{(I1)} \quad i @t\{d1\};P \xrightarrow{i} [0/t]P \qquad \text{(I2)} \quad i @t\{d1+d\};P \xrightarrow{d} i @t\{d1\};[t+d/t]P$$

There is no rule like Ex3 for the internal action-prefix.  $i @t\{d1\};P$  cannot idle more than  $d1$  time units. If it reaches this limit, time is blocked. The only solution left is to accomplish  $i$ . This means that, in Timed Extended LOTOS, the occurrence of  $i$  is compulsory. The semantics of  $i @t\{d1\};P$  is that  $i$  *shall* occur during the next  $d1$  time units<sup>3</sup>. On the other hand, the semantics of  $exit(d1)$  is that  $\delta$  *may* occur within the next  $d1$  time units.

**Delay prefixing**

$$\begin{aligned}
 \text{(D1)} \quad & \frac{P \xrightarrow{a} P'}{\Delta^0 P \xrightarrow{a} P'} & \text{(D2)} \quad \Delta^{d1+d} P \xrightarrow{d} \Delta^{d1} P \\
 & & \text{(D3)} \quad \frac{P \xrightarrow{d} P'}{\Delta^{d1} P \xrightarrow{d+d1} P'}
 \end{aligned}$$

$\Delta^d;P$  expresses that  $P$  will be delayed by  $d$  time units.

**Choice**

$$\begin{aligned}
 \text{(Ch1)} \quad & \frac{P \xrightarrow{a} P'}{P [ ] Q \xrightarrow{a} P'} & \text{(Ch1')} \quad \frac{Q \xrightarrow{a} Q'}{P [ ] Q \xrightarrow{a} Q'} & \text{(Ch2)} \quad \frac{P \xrightarrow{d} P', Q \xrightarrow{d} Q'}{P [ ] Q \xrightarrow{d} P' [ ] Q'}
 \end{aligned}$$

Remark rule Ch2: the passing of time does not resolve a choice. Rule Ch2 also states that both operands evolve in time at the same pace.

<sup>3</sup> Of course, in a choice context, the occurrence of  $i$  could be prevented by another offered action.

### Generalized choice

The semantics of choice  $x_1:s_1, \dots, x_n:s_n[]P$  is defined via an auxiliary operator, denoted  $Achoice(d)$   $x_1:s_1, \dots, x_n:s_n[]P$ , where  $d \in D_\infty$ .  $Achoice$  stands for  $AgedChoice$ . By definition, choice  $x_1:s_1, \dots, x_n:s_n[]P = Achoice(0) x_1:s_1, \dots, x_n:s_n[]P$ .

$$(GC1) \quad \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{a} P'}{Achoice(0) x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{a} P'}$$

$$(GC2) \quad \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{d} P'', P'' \xrightarrow{a} P'}{Achoice(d) x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{a} P'} \quad \text{if } d > 0$$

where the  $tx_i$  are ground terms with  $[tx_i] \in Q(s_i)$

$$(GC3) \quad \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{d+d'} \quad \forall \langle tx_1, \dots, tx_n \rangle \bullet [tx_i] \in Q(s_i), i = 1, \dots, n}{Achoice(d') x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{d} Achoice(d+d') x_1:s_1, \dots, x_n:s_n[]P}$$

### Parallel composition

$$(PC1) \quad \frac{P \xrightarrow{a} P'}{P | [\Gamma] | Q \xrightarrow{a} P' | [\Gamma] | Q} \quad (\text{name}(a) \notin \Gamma \cup \{\delta\}) \quad (PC3) \quad \frac{P \xrightarrow{d} P', Q \xrightarrow{d} Q'}{P | [\Gamma] | Q \xrightarrow{d} P' | [\Gamma] | Q'}$$

$$(PC1') \quad \frac{Q \xrightarrow{a} Q'}{P | [\Gamma] | Q \xrightarrow{a} P | [\Gamma] | Q'} \quad (\text{name}(a) \notin \Gamma \cup \{\delta\})$$

$$(PC2) \quad \frac{P \xrightarrow{a} P', Q \xrightarrow{a} Q'}{P | [\Gamma] | Q \xrightarrow{a} P' | [\Gamma] | Q'} \quad (\text{name}(a) \in \Gamma \cup \{\delta\})$$

### Infinite parallel composition

$$(IP1) \quad \frac{P \xrightarrow{a} P'}{\text{inf} ||| P \xrightarrow{a} P' |||} \quad (\text{inf} ||| P) \quad (IP2) \quad \frac{P \xrightarrow{d} P'}{\text{inf} ||| P \xrightarrow{d} \text{inf} ||| P'}$$

$\text{inf} ||| P$  corresponds to an infinity of occurrences of  $P$  evolving in parallel. In ET-LOTOS, such a behaviour cannot be described by a recursive process like  $P_s := P ||| P_s$ , because unguarded recursions block time (see [LéL 95b]).

### Hide

$$(H1) \quad \frac{P \xrightarrow{a} P'}{\text{hide } \Gamma \text{ in } P \xrightarrow{a} \text{hide } \Gamma \text{ in } P'} \quad (a \notin \Gamma)$$

$$(H2) \quad \frac{P \xrightarrow{a} P'}{\text{hide } \Gamma \text{ in } P \xrightarrow{i} \text{hide } \Gamma \text{ in } P'} \quad (a \in \Gamma)$$

$$(H3) \quad \frac{P \xrightarrow{d} P', \forall g \in \Gamma \bullet (P \xrightarrow{g} \wedge \forall P'' \forall d' < d \bullet (P \xrightarrow{d'} P'' \Rightarrow P'' \xrightarrow{g} P))}{\text{hide } \Gamma \text{ in } P \xrightarrow{d} \text{hide } \Gamma \text{ in } P'}$$

Rule (H3) expresses the *maximal progress* principle adopted for ET-LOTOS. This principle states that the hidden events must occur as soon as possible. So, the process can only idle if no hidden action is possible.

### Enabling

$$(En1) \quad \frac{P \xrightarrow{a} P'}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{a} P' \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q} \quad (\text{name}\{a\} \neq \delta)$$

$$(En2) \frac{P \xrightarrow{\delta v_1 \dots v_n} P'}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{i} [v_1/x_1, \dots, v_n/x_n]Q} \quad \forall j \leq n \bullet v_j \in Q(s_j)$$

$$(En3) \frac{P \xrightarrow{d} P', P \not\xrightarrow{\delta}, \forall P'' \forall d' < d \bullet (P \xrightarrow{d'} P'' \Rightarrow P'' \not\xrightarrow{\delta})}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{d} P' \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q}$$

The occurrence of  $\delta$  is hidden by the enabling operator. According to the maximal progress principle, it must occur as soon as possible.

### Disabling

$$(Di1) \frac{P \xrightarrow{a} P'}{P[>Q] \xrightarrow{a} P'[>Q]} \quad (\text{name}(a) \neq \delta) \quad (Di2) \frac{Q \xrightarrow{a} Q'}{P[>Q] \xrightarrow{a} Q'}$$

$$(Di3) \frac{P \xrightarrow{a} P'}{P[>Q] \xrightarrow{a} P'} \quad (\text{name}(a) = \delta) \quad (Di4) \frac{P \xrightarrow{d} P', Q \xrightarrow{d} Q'}{P[>Q] \xrightarrow{d} P'[>Q]}$$

### Guard

$$(G1) \frac{P \xrightarrow{a} P'}{[SP] \rightarrow P \xrightarrow{a} P'} \quad \text{if } DS \vdash SP \quad (G2) \frac{P \xrightarrow{d} P'}{[SP] \rightarrow P \xrightarrow{d} P'} \quad \text{if } DS \vdash SP$$

$$(G3) [SP] \rightarrow P \xrightarrow{d} \text{stop} \quad \text{if } \neg DS \vdash SP$$

### Let

$$(L1) \frac{[tx_1/x_1, \dots, tx_n/x_n] P \xrightarrow{a} P'}{\text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } P \xrightarrow{a} P'} \quad (L2) \frac{[tx_1/x_1, \dots, tx_n/x_n] P \xrightarrow{d} P'}{\text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } P \xrightarrow{d} P'}$$

### Process instantiation

$$(In1) \frac{[g_1/h_1, \dots, g_n/h_n] P \xrightarrow{a} P', Q[h_1, \dots, h_n] := P}{Q[g_1, \dots, g_n] \xrightarrow{a} P'} \quad (In2) \frac{[g_1/h_1, \dots, g_n/h_n] P \xrightarrow{d} P', Q[h_1, \dots, h_n] := P}{Q[g_1, \dots, g_n] \xrightarrow{d} P'}$$

Let us outline some interesting features of the semantic rules defined above:

- The LOTOS rules are kept unchanged.
- The alphabet  $A$  of actions is kept as is (e.g. no additional time stamps in action labels). It is just extended with time actions from a separate set  $D$ .

## 2.5. Properties

ET-LOTOS exhibits many interesting properties (the proofs can be found in [LéL 95b]):

- The operational semantics of ET-LOTOS is consistent.
- Time transitions are deterministic:  $\forall P \bullet (P \xrightarrow{d} P' \wedge P \xrightarrow{d} P'') \Rightarrow P' = P''$ .
- Time transitions are closed under the relation  $\leq$ :  $P \xrightarrow{d} \Rightarrow \forall d' \in ]0, d[ \bullet P \xrightarrow{d'}$ .  
Furthermore,  $P \xrightarrow{d} P' \Rightarrow \forall d' \in ]0, d[ \bullet \exists d'' \bullet P \xrightarrow{d'} P'' \xrightarrow{d''} P' \wedge d = d' + d''$ .
- Time transitions are additive:  $P \xrightarrow{d} P'$  and  $P' \xrightarrow{d'} P''$  implies  $P \xrightarrow{d+d'} P''$ .
- Strong bisimulation  $\sim$  is a congruence.
- ET-LOTOS is upward compatible with LOTOS, according to the definition given in [NiS92], but for guarded specifications only.

## References

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