

## ALE METHODS FOR DETERMINING STATIONARY SOLUTIONS OF METAL FORMING PROCESSES

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**Abstract.** *In this paper, two efficient convection algorithms are briefly presented in order to update the values stored at the Gauss point during the Eulerian step of an Arbitrary Lagrangian Eulerian computation in solid mechanics. They are based on the finite volume method and on the Streamline Upwind Petrov Galerkin method. Two applications are presented : a cold rolling simulation and a drawbead simulation.*

## 1 INTRODUCTION

The general frame of this paper is in the field of numerical simulation of forming processes by the finite element method. In order to find the solution of steady state processes by numerical simulation with the classical Lagrangian formulation, very large and useless meshes have to be considered. For example, when dealing with rolling simulation, a large part of the sheet has to be discretised even if the results in the first finite elements, which are introduced between the rolls, are not important. However, these finite elements cannot be removed because they are required in order to reach the steady state solution. Consequently, the CPU time is very large. Another approach is the well-known Eulerian formulation: the media flows through the mesh, which is fixed in space. However, boundary conditions are rather difficult to handle particularly frictional contact and free surfaces.

The Arbitrary Lagrangian Eulerian (ALE) formulation was introduced to overcome these problems (see e.g. [7, 8, 2]). The mesh can be handled by the software, irrespective to the body motion, so that both previous formulations can be obtained as particular cases if the mesh sticks to the body or is fixed in space. In such a formulation, time steps are divided into two phases: the first one is Lagrangian and the second one a convective Eulerian phase, where the values stored at the Gauss points have to be updated. In order to avoid oscillations and instability, efficient convection algorithms have to be used. In the present paper, two convection methods will be presented and compared: the first one, called ‘Godunov-type update technique’, is based on a finite volume method and the second is based on the Streamline Upwind Petrov Galerkin (SUPG) method. This work has been introduced in MEFAFOR [7], the non-linear finite element code developed at the LTAS, University of Liège, Belgium.

## 2 THE GODUNOV-TYPE UPDATE TECHNIQUE

Once the first (Lagrangian) step is completed, the studied body is automatically remeshed by the program following the users instructions (it can be a minimization of the meshes distortion or, in the case of stationary processes, most of the nodes are fixed in space). When non linear problems are considered, some important values, like the stress tensor or the equivalent plastic strain, are stored at the Gauss points and have to be updated from the Lagrangian mesh to the new one. This convective step consists of solving a classical convection equation, which can be written :

$$\frac{\partial \sigma}{\partial t} \Big|_{\chi} + w_j \frac{\partial \sigma}{\partial x_j} = 0 \quad (1)$$

where  $\sigma$  is a value stored at the Gauss point (e.g. a component of the stress tensor),  $w_j$  is the relative velocity between the new mesh and the Lagrangian mesh and  $\chi$  is the coordinate system associated with the new mesh.

Although this scalar equation is well-known in fluid mechanics, it is rather difficult to solve it because  $\sigma$  is not a continuous field but is only defined at the Gauss point. Consequently, the gradient cannot be evaluated.

In order to overcome this problem, the values can be extrapolated and averaged to the mesh nodes and the gradient is computed from the resulting continuous field. But this first simple method shows a large amount of numerical diffusion. More sophisticated methods must be used.

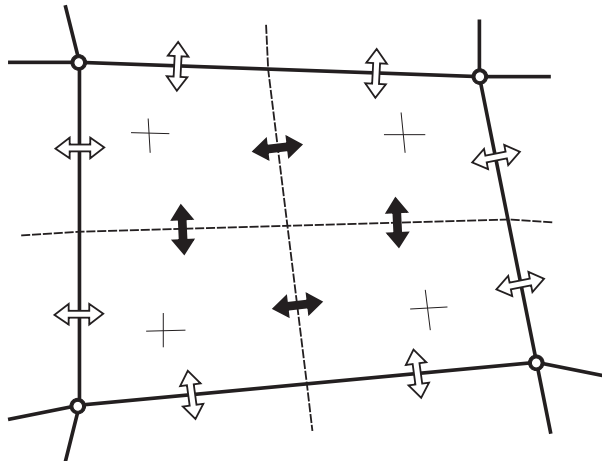


Figure 1: A finite element and its division in 4 finite volumes

The Godunov-update technique was firstly introduced by Casadei, Donéa and Huerta [4, 5]. This method can be useful on structured meshes of Q4P0 hybrid finite elements (4 Gauss points are used to integrate all the values except pressure, for which 1 point is used to prevent locking). It consists of dividing each finite element into four (one for the pressure) cells surrounding each Gauss point (fig. 1). The field to be transferred is assumed to be constant on each cell and thus discontinuous across them. The finite volume problem is solved by the classical Godunov method and an explicit Euler scheme is used for the time integration. The resulting update formula [4] is given below for the cell  $s$  :

$$\sigma_s^{n+1} = \sigma_s^n - \frac{\Delta t}{2 A_s} \sum_{i=1}^{N_s} f_i (\tau_i^c - \tau_s) (1 - \alpha \text{sign}(f_i)), \quad (2)$$

where  $\Delta t$  is the time step,  $A_s$  is the area of the cell,  $N_s$  is the number of boundary lines of the cell (4 in this case),  $\tau_i^c$  is the value of the adjacent cell sharing the boundary  $i$  with the cell  $s$ ,  $\alpha$  is an upwind factor and  $f_i$  is the flux of  $\sigma$  across the boundary  $i$ .

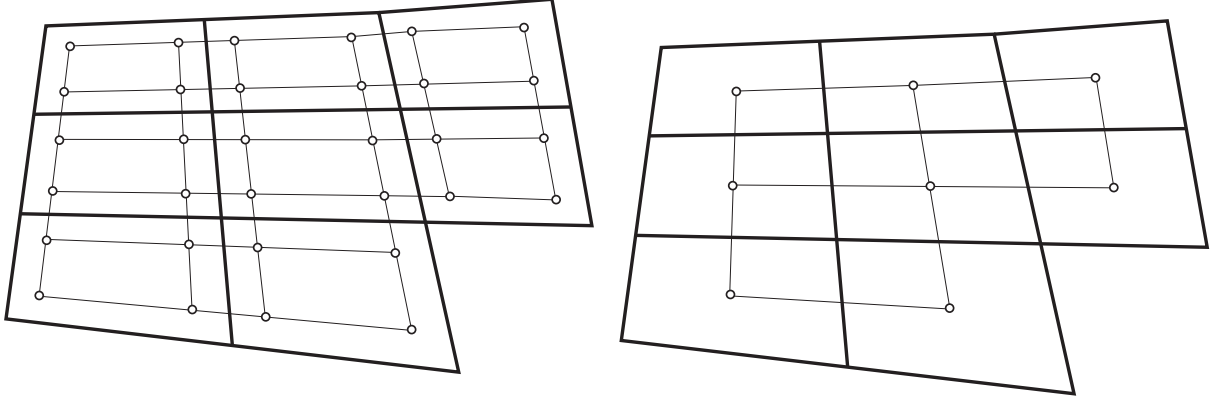


Figure 2: Auxiliary meshes for the SUPG update.

### 3 THE SUPG TECHNIQUE

We compare the latter method with another one based on the Streamline Upwind Petrov Galerkin technique [3], which was introduced in order to avoid the cross-wind diffusion appearing in the solution of the convection problem. The main idea of the method is the definition of a second mesh defined on the Gauss points. The Lagrangian step is solved on the classical finite element mesh and the convection step is performed on the second mesh by the finite element method and the SUPG technique.

The equation 1 can be written in matrix form (see [1] for more details) :

$$\mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = 0, \quad (3)$$

with

$$C_{ij} = \int_V N_i N_j dV + \int_V \frac{k}{\|w\|^2} w_k \frac{\partial N_i}{\partial x_k} N_j dV \quad (4)$$

$$K_{ij} = \int_V w_k N_i \frac{\partial N_j}{\partial x_k} dV + \int_V \frac{k}{\|w\|^2} w_k w_l \frac{\partial N_j}{\partial x_k} \frac{\partial N_j}{\partial x_l} dV \quad (5)$$

where  $N_i$  are the shape functions and  $k$  is a diffusion coefficient computed locally by

$$k = \alpha (|\vec{w} \cdot \vec{h}_1| + |\vec{w} \cdot \vec{h}_2|), \quad (6)$$

where  $\alpha$  is an upwind parameter and  $\vec{h}_1$  and  $\vec{h}_2$  are the median vectors of the element concerned. As in the previous algorithm, an explicit Euler scheme is used for the time integration and the matrix  $\mathbf{C}$  is diagonalized. It can be shown that it improves the stability of the algorithm.

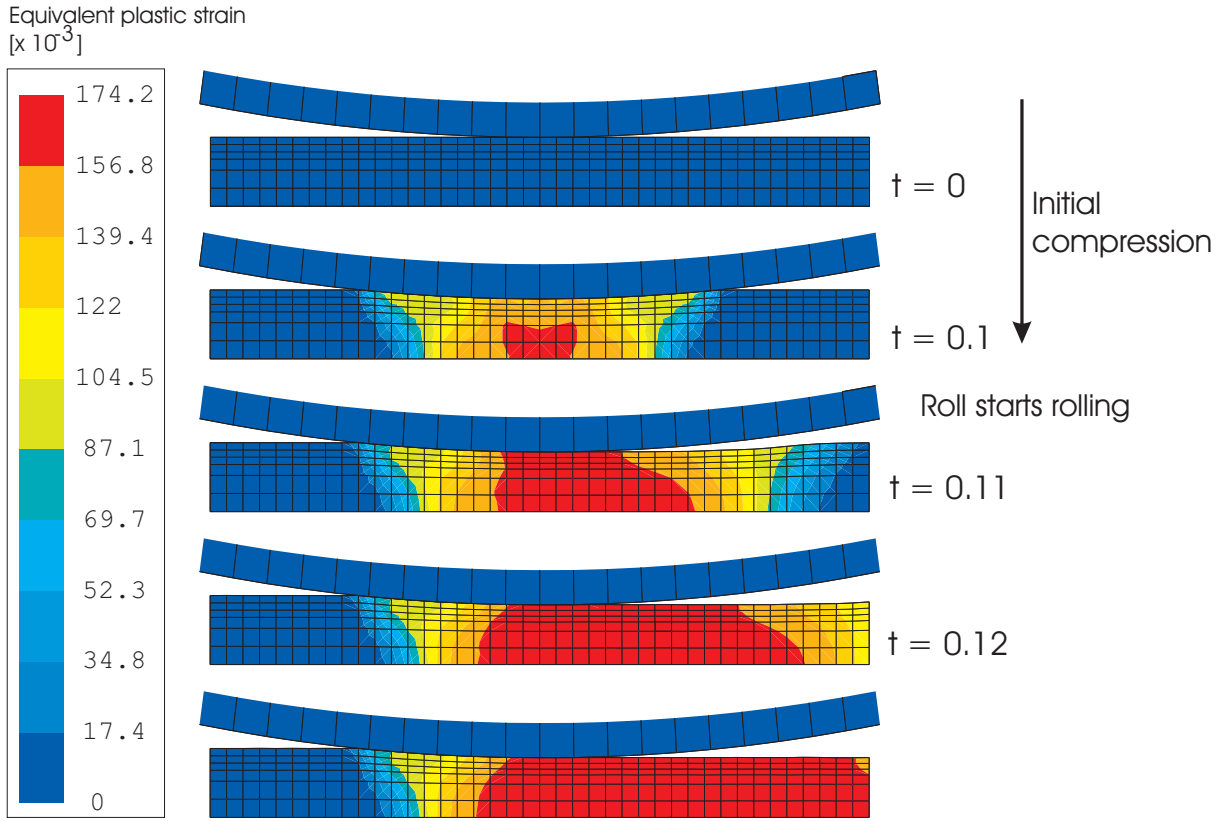


Figure 3: Evolution of the equivalent stress for the Godunov-like update.

#### 4 APPLICATION TO THE ROLLING PROCESS

Both presented algorithms have been tested on a simulation of a cold rolling process. In this case, the ALE formulation is very well suited because we can only study the interesting zone of the process, that is the part of the sheet between the rolls. The problem is symmetric and only one half of the process is studied. The table 1 shows the material properties used for this simulation.

The optimal value for the upwind factor is rather difficult to find. If a low value is chosen, oscillations may appear in the solution. However, a high value introduces too much artificial diffusion.

The roll is a flexible body with the same Young's modulus as the sheet. Only one quarter of the roll is meshed. As far as the mesh of the sheet is concerned, we use an

Young modulus	$E$	$= 6.895 \cdot 10^4$	MPa
Poisson ratio	$\nu$	$= 0.33$	
Yield stress	$\sigma_Y^0$	$= 50.3(1 + \varepsilon_p/0.55)^{0.26}$	MPa
Rolls radius	$R$	$= 158.75$	mm
Half initial thickness	$H_i$	$= 6.274$	mm
Half final thickness	$H_0$	$= 5.385$	mm
Friction coefficient	$\mu$	$= 0.1$	
Upwind factor	$\alpha$	$= 1.0$	

Table 1: Material properties, geometry and parameters of the rolling process.

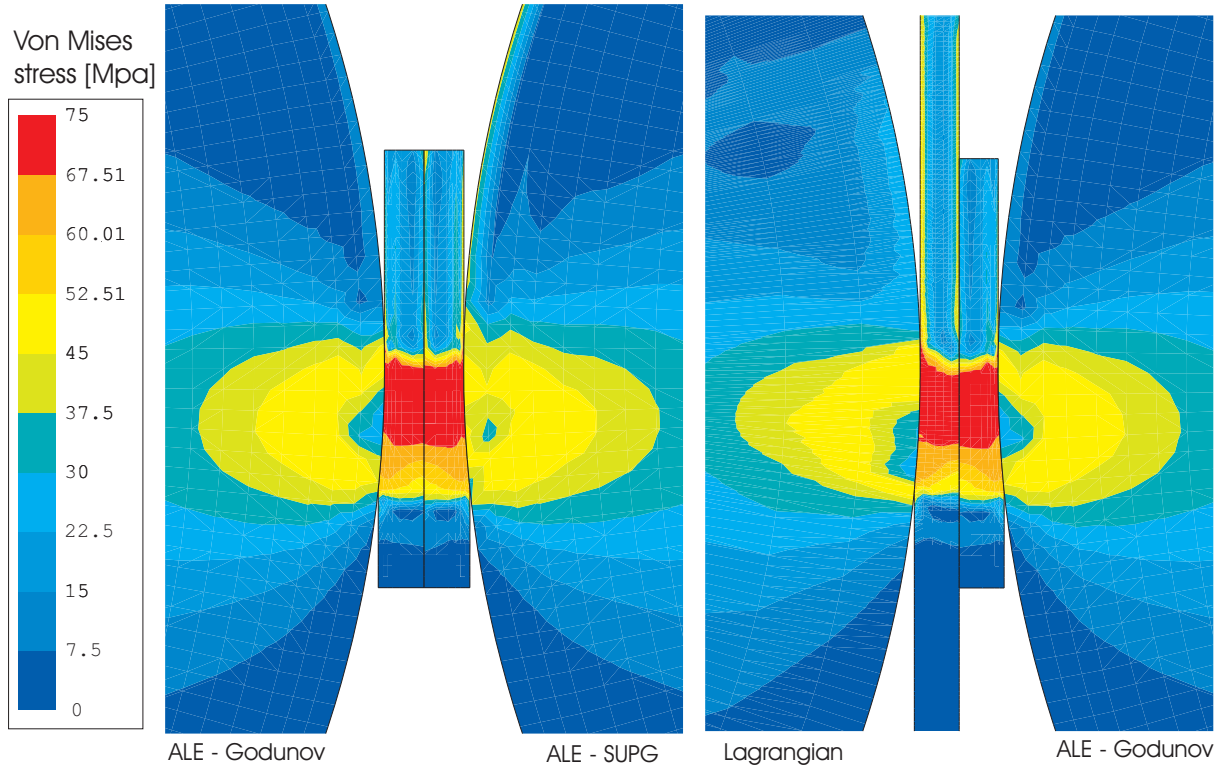


Figure 4: Von Mises stress for the Godunov-like and the SUPG updates compared to the lagrangian case.

Eulerian mesh in the rolling direction and a Lagrangian-Eulerian mesh in the perpendicular direction. Thus, the free surface of the sheet is automatically computed. The ALE method is used for the sheet and the roll.

The Eulerian domain for the sheet is 60 mm long and discretised with 40 x 6 elements and the roll with 120 x 15 elements. The meshes are refined near the contact area in order to get more accurate results.

The calculation is divided into two steps (see figure 3). The first one consists of the clamping of the sheet by the rolls. Once the desired reduction is obtained, the rolls start to rotate with a constant velocity. The computation is stopped after a rotation of 180 degrees in order to check the stability of both algorithms.

Actually, the steady state is reached rather quickly after a 15 degrees rotation. Its obtained after about 440 time steps. The Godunov-like algorithm is approximately 30% faster than the SUPG method (4 min 40 s instead of 6 min 02 s). The equivalent stress is shown on figure 4. On the left, the Godunov and SUPG updates are compared. We see that the results are very similar. However, the SUPG solution shows more numerical oscillations. On the right, the Godunov update is compared to the equivalent Lagrangian simulation and both solutions are very similar.

## 5 APPLICATION TO A DRAWBEAD SIMULATION

Another interesting simulation has been considered to compare our update algorithms. In this case, we try to simulate a drawbead test presented by Nine [6], which consist in clamping a thin sheet of metal between three cylindrical rolls and pulling the sheet to make it bend and unbend through the system. This kind of experiment is very important in the deep drawing industry because the forces of such a system are rather difficult to predict.

The geometry and material properties used for the simulation are presented in the table 2.

Young modulus	$E$	=	200	GPa
Poisson ratio	$\nu$	=	0.3	
Yield stress	$\sigma_Y^0$	=	$516(8.2139 \cdot 10^{-3} + \varepsilon_p)^{0.23}$	MPa
Roll bead radius	$R_1$	=	5.5	mm
Roll shoulder radius	$R_2$	=	5.5	mm
Friction coefficient	$\mu$	=	0.0	
Upwind factor	$\alpha$	=	1.0	

Table 2: Material properties, geometry and parameters of the drawbead simulation.

The frictionless case is chosen here for the comparison. This means that the rolls are not fixed and can roll to follow the sheet motion.

The figure 5 explains the two successive steps during the test. At the beginning, the sheet lies undeformed between the rolls. Then, the clamping phase begins. Once the desired clamping distance is obtained, the drawing phase begins until a stationary solution is computed.

In the case of a Lagrangian simulation, a long part of the sheet must be discretised because the steady state is not obtained immediately. With the ALE formalism, only a small part of the sheet, that is the interesting part, is considered and discretised (this part is called ‘ALE region’ in figure 5). The sheet flows into the ALE mesh.

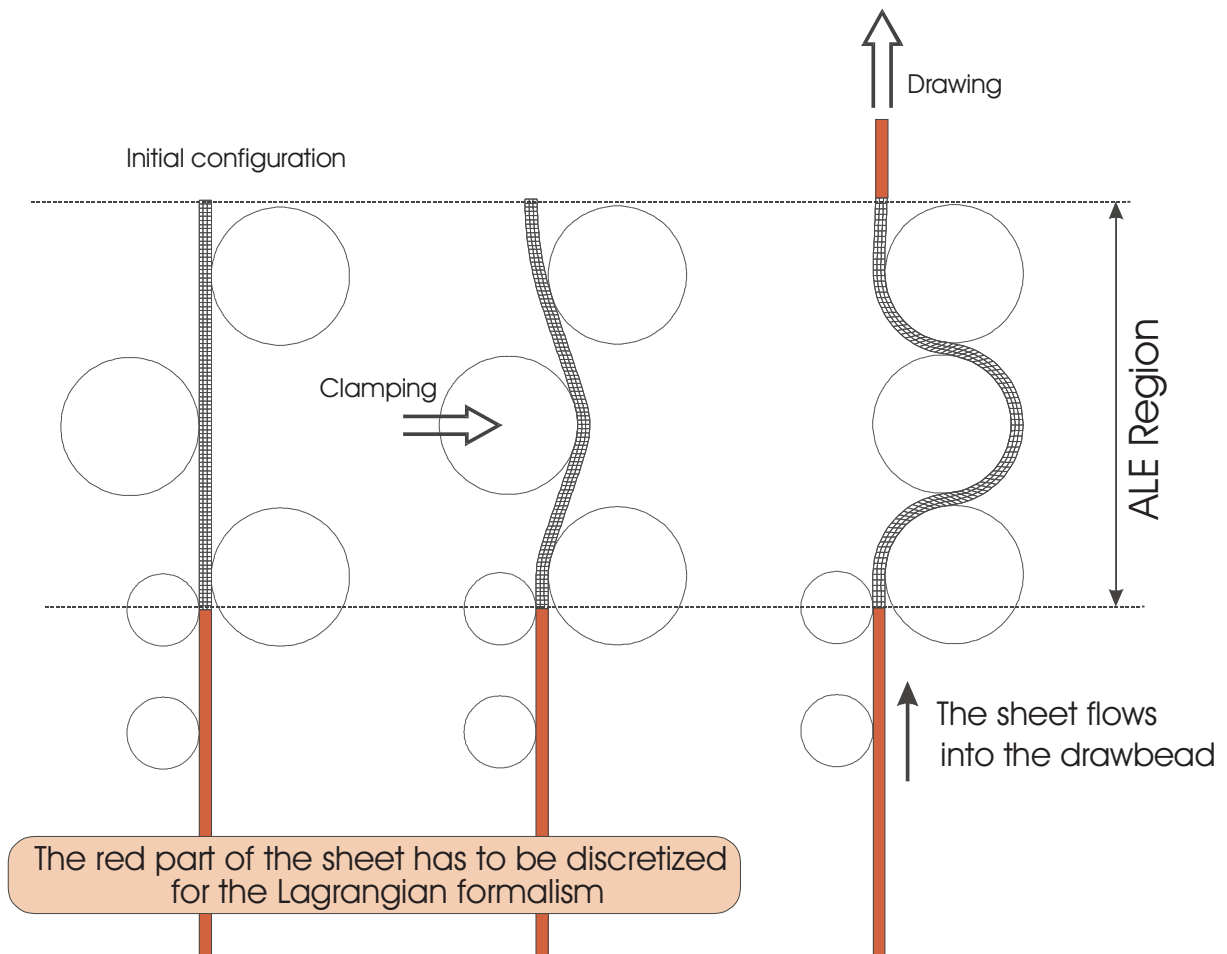


Figure 5: ALE Region for the drawbead simulation.

The figure 6 compares the results obtained by both update methods and the Lagrangian case. Once again, we see that the equivalent plastic strain is very similar in the three cases



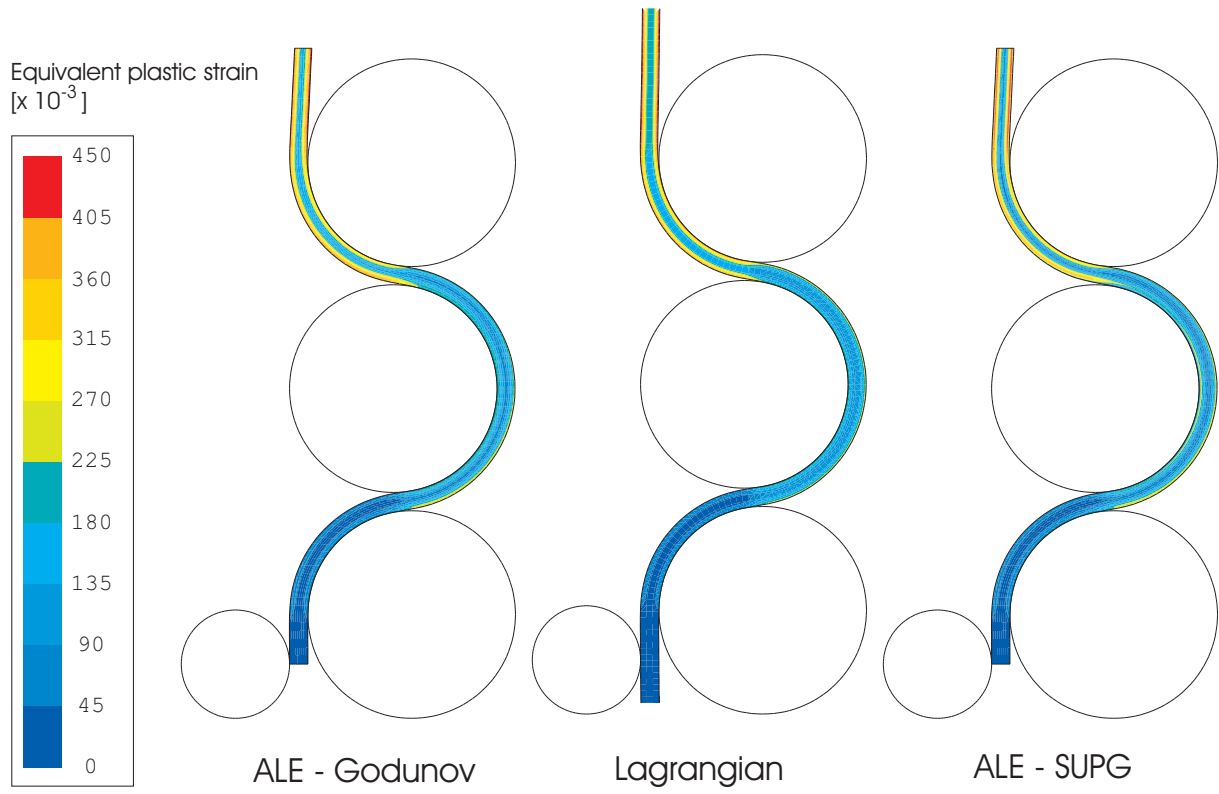


Figure 6: Comparison of the equivalent plastic strain for the Lagrangian and ALE simulations.

## 6 CONCLUSIONS

In this paper, two efficient convection algorithms have been introduced in the frame of Arbitrary Lagrangian- Eulerian methods in solid mechanics. They can be very useful when dealing with stationary processes like rolling. However, these results cannot be extended easily to unstructured meshes.

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