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LIMITS ON THE TOP QUARK MASS FROM $N(W \rightarrow e\nu)/N(Z \rightarrow e^+e^-)$ AND $B_d^0\,\bar{B}_d^0$ MIXING.*

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Abstract

With a new UA1 lower limit of 41 GeV on the top quark mass m_t, one might think that the best available bounds come from $B_d^0\,\bar{B}_d^0$ mixing. We show however that most analyses have been too optimistic and that the best lower limit (at the 90% C.L.) that one can extract from the ARGUS measurement is of the order of 30 GeV. We also discuss the status of the upper limit on m_t that one can derive from the $p\bar{p}$ collider measurement of the ratio $R = N(W \rightarrow e\nu)/N(Z \rightarrow e^+e^-)$ and combine the two to get $25\ \text{GeV} \leq m_t \leq m_W$ at the 95% C.L.

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I. Upper Limit on m_t from the $\mathcal{R}$ Ratio:

The $\mathcal{R}$ ratio, experimentally defined by $\mathcal{R} = N(W \to e\nu)/N(Z \to e^+e^-)$, is theoretically given by the following expression:

$$\mathcal{R} = \left[\frac{\Gamma(W \to e\nu)/\Gamma_W}{\Gamma(Z \to e^+e^-)/\Gamma_Z}\right] \times \left[\frac{\sigma_W}{\sigma_Z} \equiv \mathcal{R}_\sigma\right] \tag{1}$$

The first factor, giving the ratio of the branching fractions, is entirely determined within the minimal standard model. Via the W and Z total widths Γ_W and Γ_Z, it depends on the number of light neutrinos (which we set equal to 3 in the following) and on m_t. The second factor, representing the ratio of the W and Z production cross sections, contains all the theoretical uncertainties. At the parton level, $\mathcal{R}_\sigma$ is well determined: the Feynman diagrams entering the calculation are identical to order α_S^2, thus guaranteeing the absence of big corrections in the ratio, such as K factors. The order α_S corrections have been estimated1) to be at most 3% and have been included in the present analysis. The main uncertainty in the calculation comes from the embedding of the parton process into the hadronic structure. The first attempts2) used standard sets of structure functions. Figure 1 shows that such a procedure produces a bound $m_t \leq m_W$ at the 1.5 σ level. It was then noticed3) that $\mathcal{R}_\sigma$ depended effectively on the ratio u/d ($x = m_W/\sqrt{s}$) only and that this ratio had a negligible Q^2 dependence. One can thus directly input experimental measurements of this ratio to evaluate $\mathcal{R}_\sigma$. A compilation of the estimates from most of the available data sets is given in Fig. 2 and leads to the average:

$$< \mathcal{R}_\sigma > = 3.42 \pm 0.01 \pm 0.02 \pm 0.04 \pm \epsilon \tag{2}$$

The first uncertainty comes from averaging the results of Fig. 2, the other uncertainties originate respectively from the errors on m_W, $\sin(\theta_W)$, and from the charm contribution to the process. As we are considering a variety of independent data sets, we have assumed that it is safe to neglect the systematic uncertainties. Comparing the resulting $\mathcal{R}$ value with the combined measurements of UA1 and UA2 leads to an upper bound $m_t \leq m_W$ at the 96% C.L. In this approach, each individual data set has little weight of its own, so that the inclusion of the new BCDMS data4) in our analysis modifies $< \mathcal{R}_\sigma >$ by less than 1%.

II. Lower Limit on m_t from $B_d^0\bar{B}_d^0$ Mixing:

The ARGUS collaboration measurement of B_d^0 $\bar{B}_d^0$ mixing5), combined with well-known box-diagram calculations6), gives:

$$\frac{|m_{B_d^0} - m_{\bar{B}_d^0}|}{\Gamma_B} \equiv \frac{|\delta m|}{\Gamma_B} = 0.73 \pm 0.18 = \left[G_F^2\ |U_{tb}^* U_{td}|^2\right]\left[f_B^2 m_B B_B\right]\left[I_{loop}\ \eta_{QCD}\ \tau_B\right] \tag{3}$$

The main theoretical uncertainties are as follows: the first factor contains the K.M. matrix element $|U_{td}|$ which can be accessed only via the measurements of the other K.M. matrix elements and the imposition of unitarity; the second factor introduces the B decay constant f_B (coming from the vacuum insertion) and the B bag parameter B_B (from the insertion of higher order intermediate states), both of which can only be approximately estimated via lattice QCD, QCD sum rules or large N expansions. Furthermore, the other factors (i.e the loop integral $I_{loop} \approx m_t^2$ and the QCD radiative corrections η_{QCD}) depend on quark masses and Λ_{QCD}. To establish a real bound on m_t, one needs to let all the parameters of the problem vary in a reasonable range and to fit all the data constraining $|U_{td}|$. The present analysis[7] thus simultaneously fits i) the K.M matrix elements $|U_{ud}|$, $|U_{uc}|$, $|U_{us}|$, $|U_{cs}|$; ii) the ϵ parameter of the $K^0\bar{K}^0$ system; iii) the B lifetime τ_B; iv) $|\delta m|/\Gamma_B$. These quantities depend on the following parameters, which we vary: i) the K.M. angles and phase; ii) the quark masses m_c and m_b; iii) the bag parameters B_K and B_B, as well as the decay constant f_B; iv) the QCD parameter Λ_{QCD}. We further assume that $|U_{ub}/U_{cb}| \leq 0.2$ and use a QCD model for the B semileptonic width, from which we extract τ_B using the measured semileptonic branching fraction. Using a procedure close to the maximum likelihood method, we fit all the above data at the 1.5 σ level for $m_t \geq 25$ GeV (if $f_B^2\ B_B \leq 0.04\ \mathrm{GeV}^2$) and for $m_t \geq 30$ GeV (if $f_B^2\ B_B \leq 0.03\ \mathrm{GeV}^2$).

All the parameters are fitted to conventional values, except the matrix element $|U_{cb}|$ and the ratio $r_u \equiv |U_{ub}|/|U_{cb}|$ which respectively take the values 0.07 and 0.2 for $m_t \approx$ 30 GeV. This might seem at first sight to contradict the CLEO analysis[8] which gives bounds $|U_{cb}| \leq 0.05$ and $r_u \leq 0.09$ for QCD-based models. We thus reconsider their analysis in the explicit case of the Altarelli model[9] of B semileptonic decay, which we extend somewhat. The original model differed from previous ones by the inclusion of first-order QCD corrections and more importantly by the elimination of the m_b^5 dependence of the rate. To achieve this, it assumed that the produced light spectator quark had a momentum distribution $\Phi(p)$ (taken to be gaussian) and that the b quark was off-shell by an amount determined by exact kinematics. We keep exactly the same assumptions, except that we allow the distribution $\Phi(p)$ to be undetermined. We show in Fig. 3 the spectra resulting from rectangular spikes in $p^2\Phi(p)$ of width 100 MeV centred around p_0. We clearly fit both the semi-leptonic width and the endpoint spectrum if we assume a momentum distribution peaked around 0.7 GeV. This would mean that the b quark would have an effective mass close to 4.6 GeV which is the best value of m_b determined by our simultaneous fit. Finally, the bounds that one got from considering the D* polarization in $B \rightarrow D^* l\nu$ exclusive decay have to be reconsidered in view of the new ARGUS analysis[10], which contradicts the original CLEO results and favours QCD-based models for B decay.

III. Conclusion:

Although large theoretical uncertainties affect the bounds one can get on m_t, we conclude that from $\mathcal{R}$ and $|\delta m|/\Gamma_B$, one can extract: 30 GeV $\leq m_t \leq$ 63GeV (90% C.L.) and 25 GeV $\leq m_t \leq m_W$ (95% C.L.).

References:

1. A. Datta, M. Drees, R.M. Godbole and X. Tata, Wisconsin preprint MAD/PH/373 (1987).
2. F. Halzen, Phys. Lett. **182B**, 388 (1986).
3. F. Halzen, C.S. Kim and S. Willenbrock, Wisconsin preprint MAD/PH/342 (1987); J.R. Cudell, F. Halzen and C.S. Kim, Wisconsin preprint MAD/PH/374 (1987).
4. A. Milsztajn, talk at the Moriond Meeting, Les Arcs 1600, March 13-19, 1988.
5. ARGUS Collaboration, H. Albrecht et al., DESY preprint DESY-87-029 (1987).
6. See for instance A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. **B245**, 369 (1984).
7. J.R. Cudell, F. Halzen, S. Pakvasa and X.-G. He, Phys. Lett. **196B**, 227 (1987) and Wisconsin preprint MAD/PH/376 (1987).
8. CLEO Collaboration, Phys. Rev. Lett. 59, 407 (1987).
9. G. Altarelli, N. Cabibbo, G. Corbò, L. Maiani and G. Martinelli, Nucl. Phys. **B208**, 365 (1987).
10. N. Kwak, talk at the Moriond Meeting, Les Arcs 1600, March 13-19, 1988.

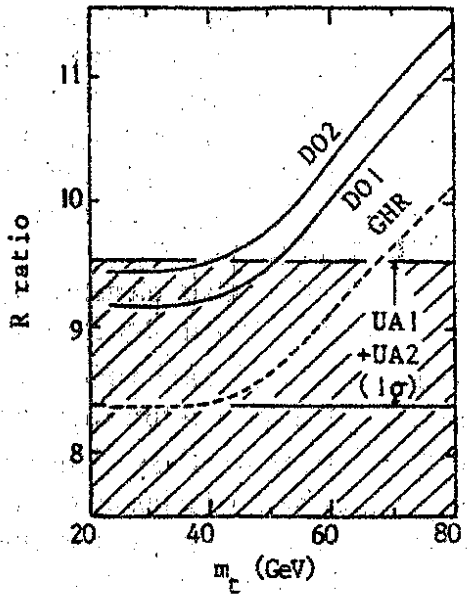

Fig. 1: Dependence of $\mathcal{R}$ on m_t assuming 3 generations of quarks and leptons. The calculation$^{2)}$, using standard sets of structure functions, is compared with the measured value obtained by combining UA1 and UA2 results.

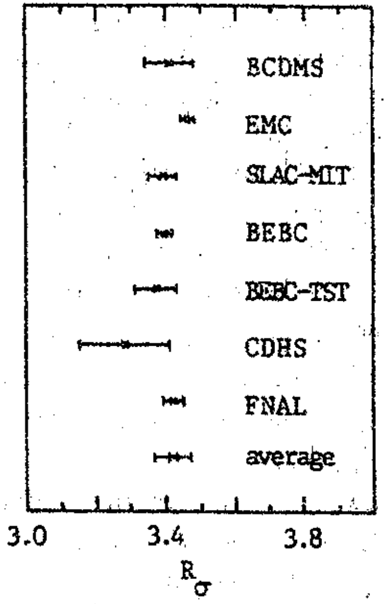

Fig. 2: Values of $\mathcal{R}_\sigma$ are calculated$^{3)}$ by extracting the ratio u/d from a series of experiments. We assumed $m_W = 80.1$ GeV and fixed m_Z from $\sin^2\theta_W = 0.232$. After including the errors on these quantities we obtain $< \mathcal{R}_\sigma >= 3.42 \pm 0.05$.

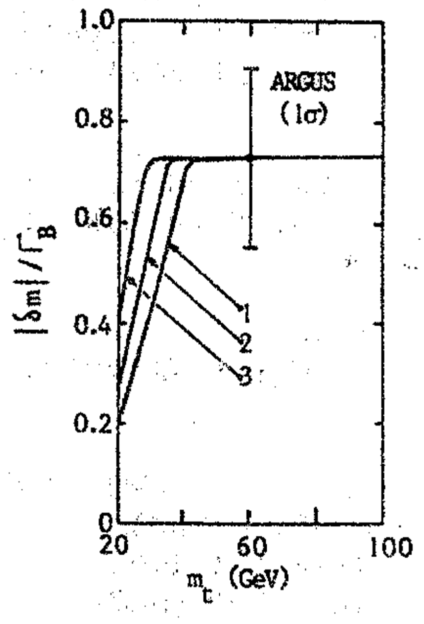

Fig. 3: The values of $|\delta m|/\Gamma_B$ resulting from our fit are compared with the ARGUS measurement for $f_B^2 B_B \leq$ 0.03 (1), 0.04 (2) and 0.06 (3) GeV^2.

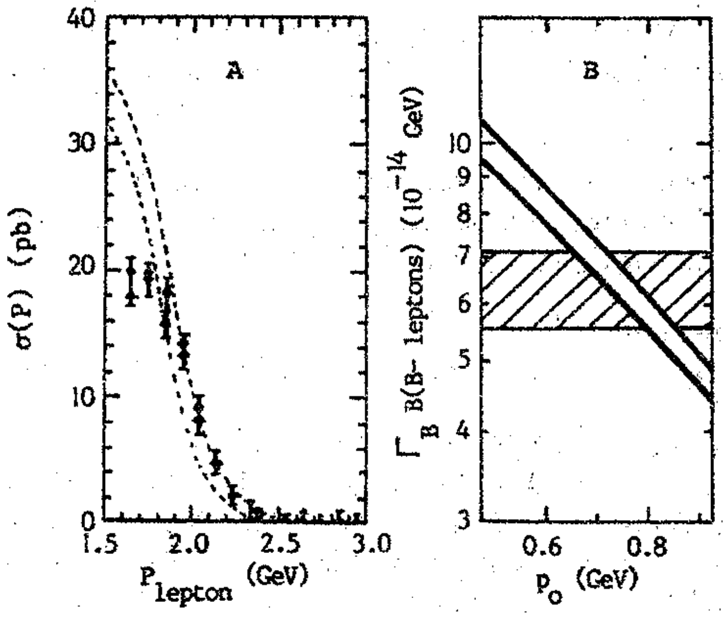

Fig. 4: Simultaneous fit to the endpoint spectrum of B semileptonic decay$^{8)}$ (A) and to the B semileptonic width (B) for $|U_{cb}| = 0.07$, $|U_{ub}| = 0.014$, $m_b = 4.6$ GeV, $m_c = 1.8$ GeV and $0 \leq m_u \leq 300$ MeV. p_0 is the momentum of the light spectator quark (see text).