Finite Element Computation of Electromechanical Micro-Switches and Micro-Pump Actuators

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Abstract
A new methodology to model the electromechanical behavior of microscale system components is presented. The original approach proposed in this paper is based on a unified FE formulation of the strong electromechanical interaction problem, in which both the electric and the mechanical fields are considered simultaneously. To this aim, finite elements based on the Mindlin shell theory are implemented. In order to illustrate the proposed methodology, numerical results are presented on the modeling and simulation of microscale electromechanical switches and microscale pump actuators.

1. Introduction

The optimal design of micro-electromechanical systems (MEMS) is currently based on experimental solution methodologies. Starting from a basic analytical design, various configurations are fabricated and tested before achieving the objective and the performance required by the customer. This heuristic methodology necessitates the achievement of numerous and costly characterization tests. The number of tests may be drastically reduced if the behavior of the microscale structure is predicted with sufficient accuracy. For this purpose, analytical models and numerical simulations have to be used, which are able to account for the strong interaction arising between various physical fields at the microscale level. In capacitive MEMS, a strong coupling arises from the simultaneous existence of an electric field and of mechanical deformations. In order to design such devices, the finite element (FE) method may be used to approximate both the electrostatic and mechanical fields. In the classical weakly coupled formulation, these fields are considered in separate FE models and many computations have to be performed to solve the coupled electromechanical problem. The original approach proposed in this paper is based on a unified FE formulation of the strong electromechanical interaction problem, in which both the electric and the mechanical fields are considered simultaneously. In order to illustrate the proposed methodology, numerical results are presented on the modeling and simulation of microscale electromechanical switches and microscale pump actuators.

2. Reference problem

The reference problem consists in a parallel plate capacitor as illustrated in figure 1. The lower electrode is fixed and the upper one is hanged by a spring. A voltage difference is applied between them so that the two plates of the capacitor attract each other.

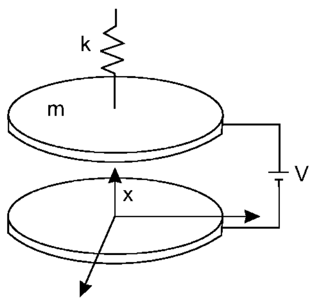

FIG. 1 – *Reference problem*

The advantage of such a reference problem is that the analytical solution is available and may be used for the validation of finite element developments. By expressing Newton's second law in the state space, the dynamic equilibrium equation of the upper plate (assumed to be rigid) is given by:

$$\left\{ \begin{array}{l} \dot{y}_1 = y_2 \\ \dot{y}_2 = -\frac{k}{m}(y_1 - x_0) - \frac{1}{2m}\varepsilon_0 \frac{V^2}{y_1^2} \end{array} \right. \tag{1}$$

where $y_1 = x$ and $y_2 = \dot{x}$ are respectively the displacement and the velocity of the upper plate; k is the stiffness of the spring, m is the mass of the upper electrode, V is the applied voltage difference, x_0 is the initial gap between the plates and ε_0 is the dielectric permittivity of the medium between the plates.

Figure 2 represents the evolution of the equilibrium position as a function of the applied voltage.

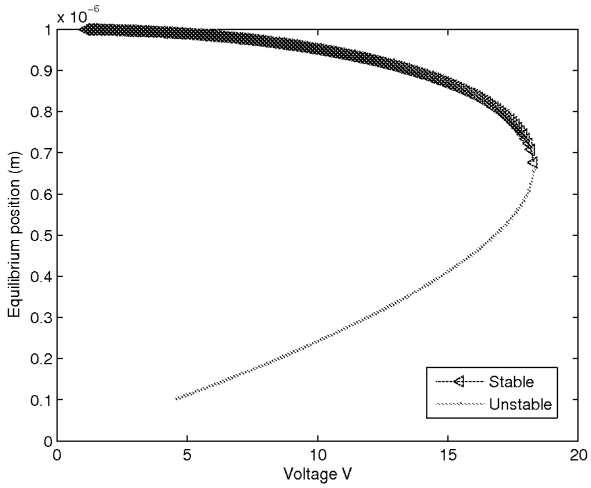

FIG. 2 – *Bifurcation diagram*

It can be observed that, if the voltage increases statically from zero to a maximum value called the pull-in voltage, the dynamic behavior of the capacitor remains stable. Beyond this point, instability occurs: the electric force becomes much higher than the spring restoring force and the two electrodes touch each other.

The natural frequency of the system may be obtained through a linearization of the equation of motion around its equilibrium position x_e:

$$m\,\ddot{x}' + k\,x' - \varepsilon_0\,\frac{V^2}{x_e^3}\,x' = -k\,(x_e - x_0) - \frac{1}{2}\,\varepsilon_0\,\frac{V^2}{x_e^2} \tag{2}$$

where $x' = x - x_e$ (x is the relative distance between the plates).
From this equation, it results that the natural frequency ω_n is a function of the applied voltage

$$\omega_n = \sqrt{\frac{k - \varepsilon_0\,\frac{V^2}{x_e^3}}{m}} \tag{3}$$

It can be noted that the electric force causes a decrease in the natural frequency of the system.

Beam finite elements have been developed in [1] to handle the problem of coupled electric and mechanical field interaction in capacitive micro-systems. Figure 3 compares the results obtained from the finite element simulation with the analytical solution of the reference problem. Perfect agreement may be observed between the results.

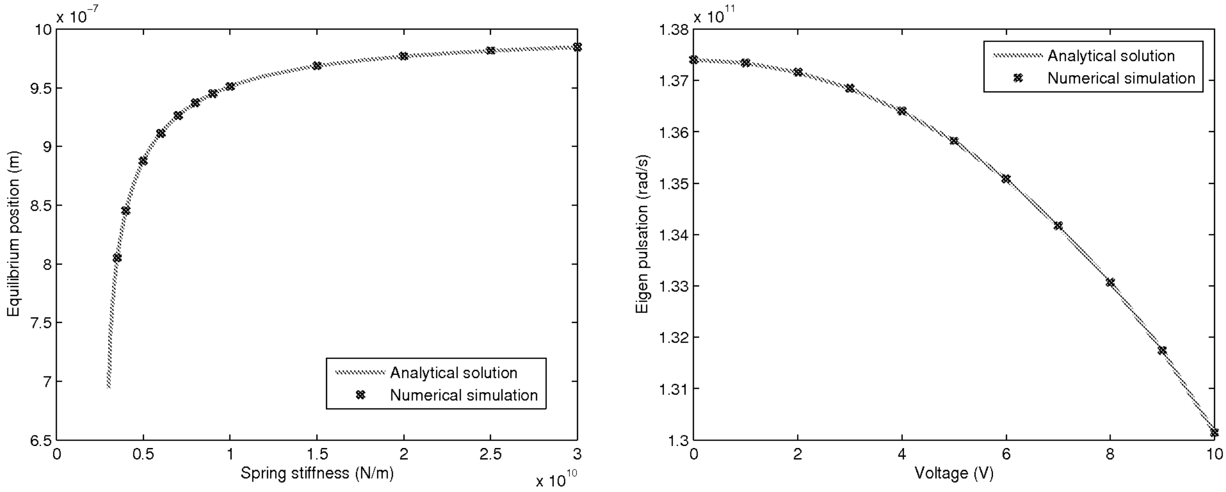

FIG. 3 – *Equilibrium position and natural frequency*

3. Finite Element Formulation of the Electro-Mechanical Problem

Predicting the dynamic behavior of a real micro-capacitor often becomes a complex problem because electric and mechanical effects are interdependent. In the approach presented here, these effects are handled using the same formulation based on variational principles.
The internal energy on a volume V is given by

$$W_{int} = \int_V \left(\frac{1}{2} \mathbf{S}^T \ \mathbf{T} - \frac{1}{2} \mathbf{D}^T \ \mathbf{E} \right) \ dV \tag{4}$$

where $\mathbf{T}$ is the stress tensor, $\mathbf{S}$ is the strain tensor, $\mathbf{D}$ is the electric displacement tensor and $\mathbf{E}$ is the electric field.
By defining $\mathbf{u}$ and ϕ as respectively the mechanical displacement vector and the electric potential, and by $\bar{\mathbf{u}}$ and $\bar{\phi}$ the displacement vector and the potential prescribed on the surface, the external energy is given by

$$W_{ext} = \int_V \mathbf{u}^T \ \bar{\mathbf{f}} \ dV + \int_{\tau_T} \mathbf{u}^T \ \bar{\mathbf{t}} \ dS - \int_V \phi \ \bar{\rho} \ dV + \int_{\tau_d} \phi \ \bar{d} \ dS \tag{5}$$

where $\bar{\mathbf{f}}$ is the vector of applied forces, $\bar{\mathbf{t}}$ is the vector of surface tractions imposed on τ_T, $\bar{\rho}$ is the imposed charge density and $\bar{d}$ the electric displacement imposed on τ_d.
Internal forces may be computed by use of the virtual work principle as developed in reference [1]. After linearization of the internal forces around a given equilibrium position, the discretized equilibrium equations are obtained in the form

$$\begin{pmatrix} \mathbf{K}_{uu}(\phi) & \mathbf{K}_{u\phi}(\phi) \\ \mathbf{K}_{\phi u}(\phi) & \mathbf{K}_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{U} \\ \Delta \phi \end{pmatrix} = \begin{pmatrix} \Delta \mathbf{F} \\ \Delta \mathbf{Q} \end{pmatrix} \tag{6}$$

where $\Delta\mathbf{Q}$ and $\Delta\mathbf{F}$ denote respectively the increments of the generalized electric loads and structural forces. Matrix $\mathbf{K}_{uu}$ may be divided into two parts: the first one corresponds to the purely mechanical stiffness matrix and the second one derives from the dependency of the electric forces to the structural displacements. Matrix $\mathbf{K}_{\phi\phi}$ is the same as the stiffness matrix resulting from the purely electric problem. Matrix $\mathbf{K}_{u\phi}$ is obtained by derivation of the mechanical internal forces with respect to the electric potential.

For the purpose of modeling micro-switches and micro-pump actuators, shell finite elements have been developed in the present work. The formulation is based on the Mindlin assumption ([2], [3]) taking into account the transverse shear effects. The shell element is a quadrangular element with 24 (electric and mechanical) degrees of freedom.

4. Specific resolution algorithm

One of the goals pursued in finite element modeling and simulation of micro-switches or micro-accelerometers is to predict design parameters such as the pull-in voltage. To this aim, the non-linear system (6) may be solved in an iterative manner by means of the Newton-Raphson method. However, when the pull-in voltage is reached, this method does not converge anymore and other specific resolution algorithms have to be used. Alternative methods are the Riks-Crisfield algorithm [4] and the Normal Flow algorithm [5]. Their principle is illustrated in figure 4.

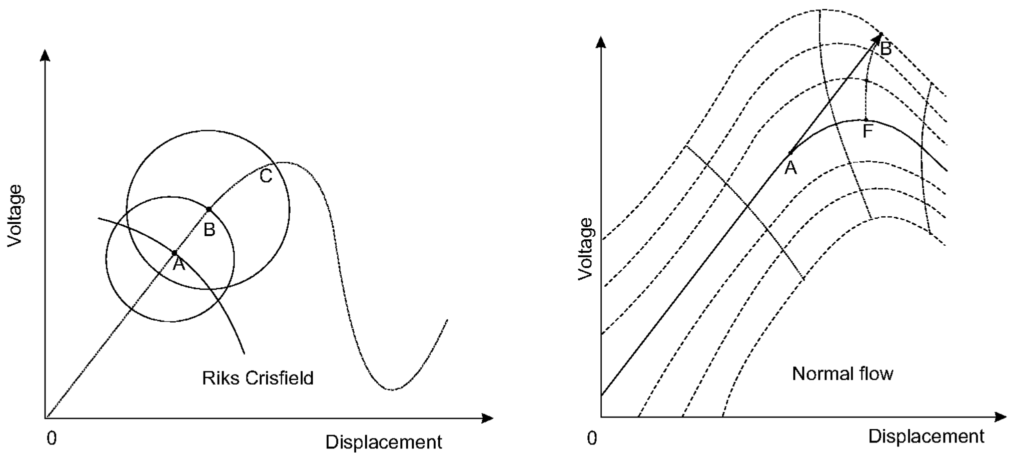

FIG. 4 – *(a) Riks-Criesfield and (b) Normal Flow algorithms*

The Riks-Crisfield algorithm consists in looking for the intersection point between an hypersphere of a given radius centered on a solution point and the voltage-displacement equilibrium path (figure 4(a)).

Referring to figure 4(b), the Normal Flow algorithm consists first to predict an estimate of the new solution (point B), starting from the converged solution at point A. Then the new solution is searched on the intersection between an homotopy map (called the Davidenko flow) and the tangent vector to the voltage-displacement curve. By successive iterations, the Normal Flow algorithm converges to the new solution on the equilibrium path along the flow normal to the Davidenko flow.

5. Examples of industrial application

5.1. Simulation of RF switches

A typical capacitive RF MEMS switch consists in a flexible metallic plate suspended over a dielectric film deposited on top of the substrate (the lower electrode) as illustrated in figure 5.

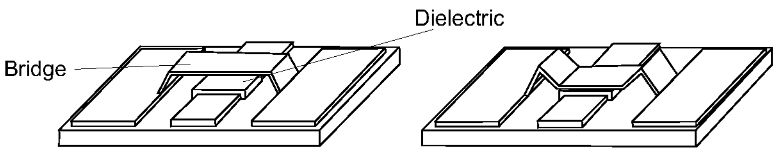

FIG. 5 – *Geometry of the bridge*

When a voltage difference is applied between the two electrodes, electrostatic forces appear and the micro-bridge is deformed. The resulting deformations induce a new distribution of the electrostatic forces, which in turn cause new structural deformations.

To solve this interaction problem, approximate analytical solutions have been proposed by Senturia in reference [1]. For example, the pull-in voltage may be roughly approximated by the relation

$$V = \sqrt{\frac{8\ k_{eff}\ d_0^3}{27\ \varepsilon_0}} \tag{7}$$

where d_0 is the initial gap between the electrodes; k_{eff} is the equivalent stiffness of the flexible plate and ε_0 is the dielectric permittivity of the vacuum.
In some cases, errors up to 20 percents have been observed between the analytical value given by equation (7) and the experimental result. More accurate results may be obtained by a so-called "macro-model" based on experiments through the following equation

$$V = \sqrt{\frac{4\ \gamma_1\ B}{\varepsilon_0\ L^4\ \gamma_2^2\ (1+\gamma_3\ d_0/\ell]}} \tag{8}$$

where γ_1, γ_2 and γ_3 are experimental coefficients; ℓ is the bridge width and L is the bridge length.

The electro-mechanical behavior of the micro-switch has been modeled using electro-mechanical and finite shell elements as described previously. The numerical solution obtained from the finite element modeling of the system is shown in figure 6, from which the pull-in voltage may be directly estimated.

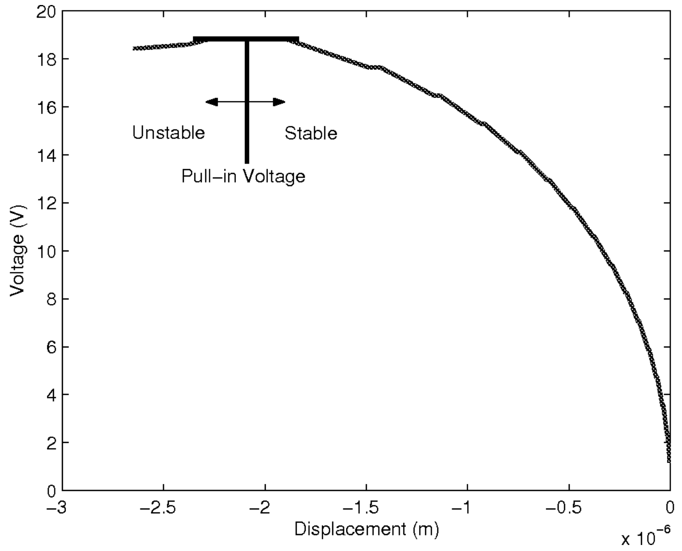

FIG. 6 – *Voltage-displacement curve predicted by the FE program*

Figure 7 gives the variation of the pull-in voltage in function of the length of the bridge. Results obtained through finite element simulations are compared to predictions given by equations (7) and (8). It can be observed that the FE prediction is closer to the "macro-model" prediction (equation 8).

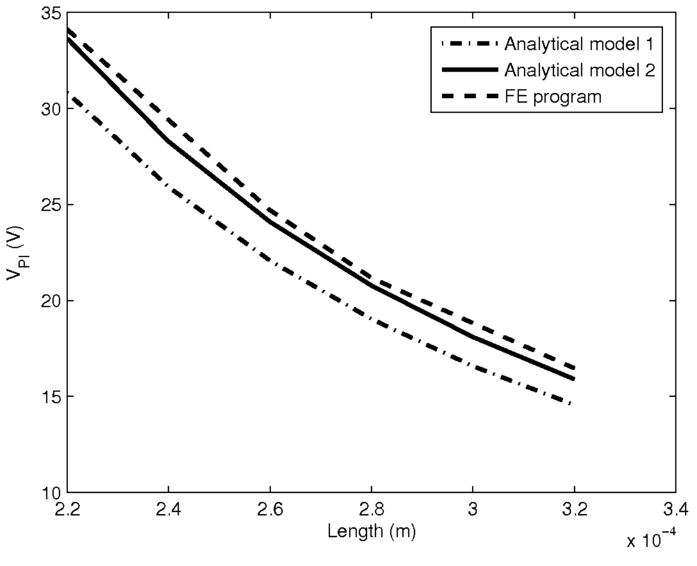

FIG. 7 – *Comparison between the numerical and analytical solutions for different lengths of the bridge*

One of the advantages of finite element modeling over "macro-models" such as (8) is that complex geometries may be simulated but also that the effect of residual stresses due micro-fabrication may be studied. Particularly, residual stresses may modify the dynamic behavior of the system and may cause buckling of the structure. In FE simulation, the effect of residual stresses is taken into account by means of a geometric stiffness matrix of the form

$$\mathbf{K}_g = \int_V \mathbf{B}^T \,\boldsymbol{\sigma}\, \mathbf{B} \, dV \tag{9}$$

where $\mathbf{B}$ is the strain interpolation matrix and $\boldsymbol{\sigma}$ is the Cauchy stress matrix.
Thus the equilibrium equation of the structure becomes

$$(\mathbf{K} + \mathbf{K}_g)\ \mathbf{U} = \mathbf{F} \tag{10}$$

where $\mathbf{K}$ is the stiffness matrix ; $\mathbf{K}_g$ is the geometric stiffness matrix and $\mathbf{F}$ is the vector of external forces.

Figure 8 shows the effect of initial stresses of respectively 20 MPa and 100 Mpa on the voltage-displacement curve of the micro-bridge. It can be observed that the equilibrium path significantly depends on the level of residual stress.

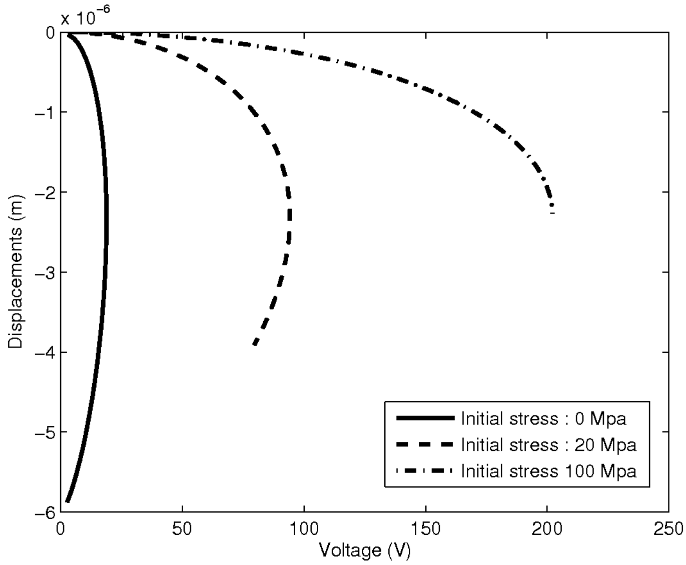

FIG. 8 – *Effect of initial stresses*

5.2. Design of micro-pump actuators

Let us now consider the case of a piezoelectric valve-less micro-pump. The device consists in a chamber connected by two input and output ports as shown in figure 9. The asymmetry of the input and output micro-diffusers insures one-way flow of the liquid through the chamber as the piezoelectric driver is excited sinusoidally.

The geometry of the diaphragm/piezoelectric driver system considered in this work is represented in figure 10.

The chamber of the micro-pump of 6 mm radius is fabricated in epoxy-resin and may be considered as rigid; the diaphragm is made of brass (Elasticity modulus : $E = 10^{11}$ Pa, Poisson coefficient : $\nu = 0.34$) and has a thickness of 100 μm. The piezoelectric driver placed on the diaphragm is of 5 mm radius. The mechanical properties of the piezoelectric actuator are summarized in Table 1.

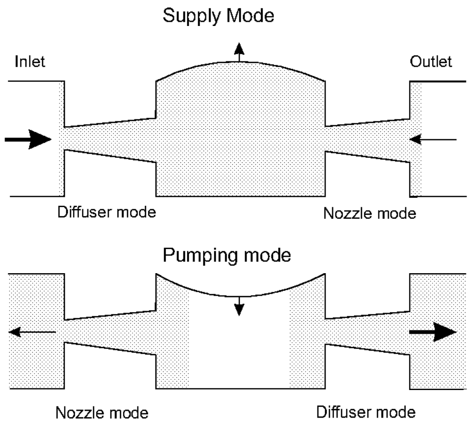

FIG. 9 – *Schematic of a valve-less micro-pump*

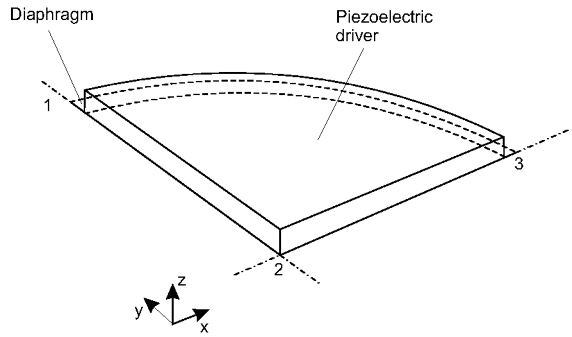

FIG. 10 – *Geometry of the diaphragm/piezoelectric driver system*

	Symbol	Value
Charge coefficient $[10^{-12}\frac{C}{N}]$	d_{33}	500
	d_{31}	270
Relative permittivity	ε	2100
Density $[\frac{kg}{m^3}]$	ρ	7800
Coupling factors	k_p	0.68
	k_{31}	0.38
Mechanical quality factor	Q	75

TAB. 1 – *Mechanical properties of the driver*

Referring to [7], the flow-rate of the micro-pump may be approximated by the formula :

$$D = \frac{\Delta V\ \omega}{\pi}\left(\frac{\mu^{\frac{1}{2}}-1}{\mu^{\frac{1}{2}}+1}\right) \tag{11}$$

where μ, is the friction loss factor ; ΔV is the variation of the volume amplitude and ω is the angular frequency of the excitation.

The diaphragm and the piezoelectric actuator were modeled using shell finite elements. Due to symmetry conditions, only a quarter structure was modeled. The total number of degrees-of-freedom is 25,432.

The results of the F.E. modeling are shown in figure 11 where the radial deformation of the diaphragm is illustrated when a voltage of 30 V is applied. It is observed that the maximum deflection appears at the center of the diaphragm-actuator system and that the deformation of the outer ring surrounding the actuator remains small. Using (11), the resulting flow-rate is estimated to 0.002 $\mu l/s$.

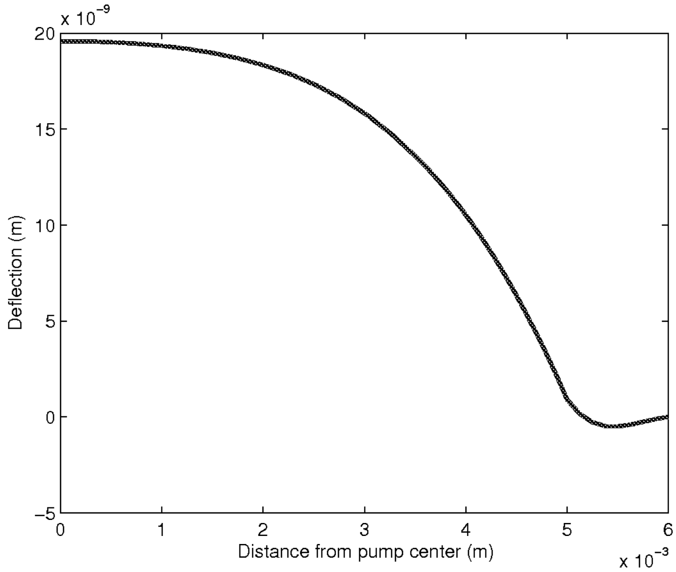

FIG. 11 – *Radial deformation of the micro-pump diaphragm*

It should be noted that the previous analysis is based on the assumption that there is no interaction between the fluid and the deformed structure. If a vibro-acoustic analysis is performed, the first natural frequency of the structure is 8,514 Hz compared to 14,266 Hz if the fluid-structure interaction is neglected. However, as illustrated in figure 12, no notable difference is found regarding to the first mode-shape of the diaphragm so that the flow-rate is not influenced.

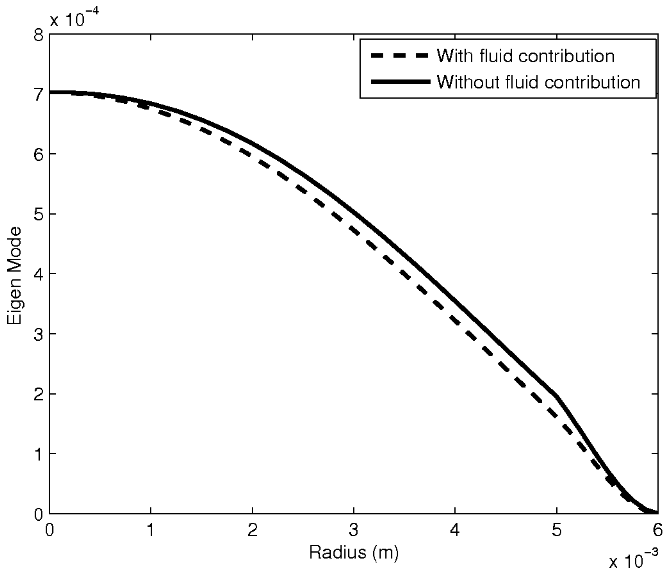

FIG. 12 – *Eigen Mode of the micro-pump diaphragm*

6. Conclusion

The aim of this work has been to develop and validate finite element modeling tools for the design and the simulation of electro-mechanical micro-systems. Even if this approach does not allow the test stage to be avoided, it allows to reduce the development times by selecting the best configurations. The simulation of capacitive systems requires specific methods to handle the problem of coupled electric and mechanical field interaction. The developed finite element program has been validated on two examples of application: a micro-switch and a valve-less micro-pump. It can now be used to model MEMS of complex geometry and to perform realistic three-dimensional simulations including edge effects and electric fringe phenomena. The proposed methodology will also enable to use multidisciplinary optimization techniques to design innovative micro-systems.

Acknowledgments

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