Analytic parametrizations of the non-perturbative Pomeron and QCD-inspired models[1]

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Abstract

We consider several classes of analytic parametrizations of hadronic scattering amplitudes, and compare their predictions to all available forward data ($pp, \bar{p}p, \pi p, Kp, \gamma p, \gamma\gamma, \Sigma p$). Although these parametrizations are very close for $\sqrt{s} \geq 9$ GeV, it turns out that they differ markedly at low energy, where a universal Pomeron term $\sim \ln^2 s$ enables one to extend the fit down to $\sqrt{s} = 4$ GeV.

1 The COMPETE project

Analytic parametrizations of forward ($t = 0$) hadron scattering amplitudes is a well-established domain in strong interactions. The basic idea is to implement as much as possible general principles - analyticity, unitarity, crossing -symmetry and positivity of total cross-sections - supplemented by some other general properties like the connection between Regge poles and resonance masses. In the absence of an explicit form of the forward amplitude derived from QCD, such an approach could provide a useful tool for studying non-perturbative physics.

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However, in the past, the phenomenology of forward scattering had quite a high degree of arbitrariness.

Some of the arbitrariness in question are :

i) An excessive focus on pp and $\bar{p}p$ scattering. When other reactions were studied they were often analyzed one by one. Of course, the extraction of the free parameters and their physical interpretation lead to contradictory conclusions.

ii) Important physical constraints are often mixed with less general or even ad-hoc properties.

iii) The cut-off in energy, defining the region of applicability of the high-energy models, differs from one author to the other.

iv) The set of data considered by different authors is sometimes not the same, even if the cut-off in energy is the same : arbitrary exclusions of experimental data are performed.

v) No rigorous connection is made between the number of parameters and the number of data points. The requirement of the smallest possible number of free parameters does not necessarily signify the physical validity of the model.

vi) No attention was paid to the necessity of the stability of parameter values when different blocks of data or different sets of observables were used.

vii) The experiments were performed in the past in quite a chaotic way : huge gaps are sometimes present between low-energy and high-energy domains or inside the high-energy domain itself. As the data at low energies are often quite precise we are in the paradoxical situation of jumping to conclusions concerning the high-energy structure of scattering amplitudes on the basis of information concentrated at low energies.

This is only a partial list and is of course not exhaustive.

The COMPETE (<u>CO</u>mputerized <u>M</u>odels and <u>P</u>arameter <u>E</u>valuation for <u>T</u>heory and <u>E</u>xperiment) project [1, 2] tries to cure as much as possible the above discussed arbitrariness.

First of all, we include in our analysis [1] all the existing forward data ($pp, \bar{p}p$,
$\pi p, Kp, \gamma p, \gamma\gamma, \Sigma p$) for the total cross-sections $\sigma(s)$ and the ρ-parameter

$$\rho(s) = \left.\frac{ReF(s,t)}{ImF(s,t)}\right|_{t=0} \tag{1}$$

by developing the procedure initiated in Refs. [3] and [4], thus taking problem i) into account.

The problem ii) is cured by studying a large variety of variants of a given model, each variant corresponding to a given set of physical properties.

The problems iii)-vi) are treated by defining appropriate numerical indicators [1, 2]. The χ^2/dof criterium is not able, by itself, to cure the difficulties iii)-vi): new indicators have to be defined.

Once these indicators are defined, an appropriate sum of their numerical values is proposed in order to establish the **rank** of the model under study : the highest the numerical value of the rank the better the model under consideration.

The problem vii) is not yet solved: theory or phenomenology can not replace the experiment itself. We very much hope that, in the future, experiments (e.g. at RHIC) will scan the available region of energies in small energy steps.

The final aim of the COMPETE project is to provide our community with a periodic cross assessments of data and models via computer-readable files on the Web [5].

2 The form of the forward scattering amplitudes

We consider the following exemplar cases of the imaginary part of the scattering amplitudes :

$$ImF^{ab} = s\sigma_{ab}(s) = P_1^{ab}(s) + P_2^{ab}(s) + R_+^{ab}(s) \pm R_-^{ab}(s) \tag{2}$$

where :
- the $\pm$ sign in formula 2 corresponds to antiparticle(resp. particle) - particle scattering amplitude.

- $R_\pm$ signify the effective secondary-Reggeon $((f, a_2),\ (\rho, \omega))$ contributions to the even(odd)-under-crossing amplitude

$$R_\pm(s) = Y_\pm \left(\frac{s}{s_1}\right)^{\alpha_\pm} \tag{3}$$

Y being a constant residue, α - the reggeon intercept and s_1 - a scale factor fixed at 1 GeV^2 ;

- $P_1(s)$ is the contribution of the Pomeron Regge pole

$$P_1^{ab}(s) = C_1^{ab} \left(\frac{s}{s_1}\right)^{\alpha_{P_1}}, \tag{4}$$

α_{P_1} is the Pomeron intercept $\alpha_{P_1} = 1$, and C^{ab} are constant residues.

- $P_2^{ab}(s)$ is the second component of the Pomeron corresponding to three different J-plane singularities :

a) a Regge simple - pole contribution

$$P_2^{ab}(s) = C_2^{ab} \left(\frac{s}{s_1}\right)^{\alpha_{P_2}}, \text{ with } \alpha_{P_2} = 1+\epsilon,\ \epsilon > 0, \text{ and } C_2^{ab} \text{ const. ;} \tag{5}$$

b) a Regge double-pole contribution

$$P_2^{ab}(s) = s\left[A^{ab} + B^{ab} \ln\left(\frac{s}{s_1}\right)\right], \text{with } A^{ab} \text{ and } B^{ab} \text{ const. ;} \tag{6}$$

c) a Regge triple-pole contribution

$$P_2^{ab}(s) = s\left[A^{ab} + B^{ab} \ln^2\left(\frac{s}{s_0}\right)\right], \tag{7}$$

where A^{ab} and B^{ab} are constants and s_0 is an arbitrary scale factor.

The real part of the F^{ab} amplitude is obtained from formula 2 via the well-known substitution rule $s \to se^{-i\pi/2}$ or, in an equivalent way, from the derivative relations [6].

In other words, (2) with (4),(5) and (7) represents three exemplar cases for the asymptotic behaviour of total cross-sections :

$$\sigma \underset{s\to\infty}{\longrightarrow} \text{const.},\ \ln s,\ \ln^2 s, \tag{8}$$

while (2) with (5) is the special case

$$\sigma \underset{s\to\infty}{\longrightarrow} s^{1+\epsilon} \tag{9}$$

corresponding to the violation of the Froissart axiomatic bound [7, 8, 9]. It however proved to be useful for studying the data at non-asymptotic energies [10] and therefore we will include it as well, and assume that unitarity corrections will restore this bound. In fact, the three exemplar cases (4), (6) and (7) are representatives of an infinite class of analytic parametrizations leading to a $(\ln s)^{\beta_+}$ $(0 \leq \beta_+ \leq 2)$ behaviour of the total cross-sections [6], so our study is quite general.

As can be seen from formulae (2)-(7), we consider that the Pomeron has two soft components , a property in agreement with perturbative QCD : recently, Bartels, Lipatov and Vacca [11] discovered that there are, in fact, two types of Pomeron in LLA : besides the well-known BFKL pomeron associated with 2-gluon exchanges, and with an intercept bigger than 1, there is a second one associated with $C = +1$ three-gluon exchanges and having an intercept precisely located at 1. The case (4)-(5) is directly inspired by this result, while the cases (4) with (6) and (7) are subtler because both the components of the Pomeron satisfy the unitarity constraint via Regge singularities all located at $J = 1$ [12, 13].

3 Results of the fits

We consider all the existing forward data for $pp, \bar{p}p, \pi p, Kp, \gamma\gamma$ and Σp scatterings [5]. The number of data points is : 904, 742, 648, 569, 498, 453, 397, 329 when the cut-off in energy is 3, 4, 5, 6, 7, 8, 9, 10 GeV respectively. When only σ data are considered the number of points is reduced to 726, 581, 507, 434, 369, 331, 285, 230 respectively.

A large number of variants were studied and classified [1]. All definitions and numerical details can be found in Ref.[1].

In order to describe our main results let us first introduce some notations.

The 2-component Pomeron classes of models are RRPE, RRPL and RRPL2, where by RR we denote the two effective secondary-reggeon contributions, by P - the contribution of the Pomeron Regge-pole located at $J = 1$, by E - the contribution of the Pomeron Regge-pole located at $J = 1+\epsilon$, by L - the contribution of the component of the Pomeron, located at $J = 1$ (double pole) and leading to a $\ln s$ behaviour of σ, and by L2 - the contribution of the component of the Pomeron located at $J = 1$ (triple pole) and leading to a $\ln^2 s$ behaviour of σ.

We also studied the 1-component Pomeron classes of models RRE, RRL and RRL2.

Our two main conclusions are the following :

1. The familiar RRE model is rejected at the 98% C.L. when models which achieve a χ^2/dof less than 1 for $\sqrt{s} \geq 5$ GeV are considered.

2. The best fits are given by models that contain a triple pole at $J = 1$, which then produce $\log^2 s$, $\log s$ and constant terms in the total cross section.

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In his seminal paper, published in 1952 [7], Heisenberg was the first to introduce the $\ln^2 s$ dependence of the hadronic total cross-sections, by simple and familiar arguments. His main assumption was that the fraction of energy which goes into the meson field (whose lower limit is given by the minimal energy for the creation of at least a pair of pions) is proportional to the overlap of the meson fields in the nucleon. Interestingly enough, the Heisenberg result is a **finite-energy** one : his total cross-section is a quadratic form in $\ln s$. Only a decade later, a rigorous proof of the Heisenberg result, but only in the case of asymptotic energies, was given [8, 9].

A striking result, directly seen from the data, is the **universality** of the $\ln^2(s/s_0)$ terms in eq. 7 : B^{ab} and s_0^{ab} are independent of the hadrons involved in the scattering [12, 13]. A general theoretical proof of this property does not exist yet. It is tempting to speculate that, after unitarization is performed in the gluon sector, the BFKL Pomeron would finally lead to a universal Heisenberg-type Pomeron, exclusively connected with the gluon sector.

As a typical example of high-rank RRPL2 models let us consider in more detail the RRPL2$_u$ (21) model (where the u index denotes the above discussed universality property and the number between parenthesis denotes the number of the free parameters.

The explicit forms of the observables are the following :

$$\sigma_{pp} = Z_{pp} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{pp}s^{-\eta_+} - Y_-^{pp}s^{-\eta_-}, \tag{10}$$

$$\sigma_{\bar{p}p} = Z_{pp} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{pp}s^{-\eta_+} + Y_{2-}^{pp}s^{-\eta_-}, \tag{11}$$

$$\sigma_{\pi^+p} = Z_{\pi p} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{\pi p}s^{-\eta_+} - Y_-^{\pi p}s^{-\eta_-}, \tag{12}$$

$$\sigma_{\pi^-p} = Z_{\pi p} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{\pi p}s^{-\eta_+} + Y_-^{\pi p}s^{-\eta_-}, \tag{13}$$

$$\sigma_{K^+p} = Z_{Kp} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{Kp}s^{-\eta_+} - Y_-^{Kp}s^{-\eta_-}, \tag{14}$$

$$\sigma_{K^-p} = Z_{Kp} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{Kp}s^{-\eta_+} + Y_-^{Kp}s^{-\eta_-}, \tag{15}$$

$$\sigma_{\gamma p} = Z_{\gamma p} + \delta B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{\gamma p}s^{-\eta_+}, \tag{16}$$

$$\sigma_{\gamma\gamma} = Z_{\gamma\gamma} + \delta^2 B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{\gamma\gamma}s^{-\eta_+}, \tag{17}$$

$$\sigma_{\Sigma^-p} = Z_{\Sigma p} + B\ln^2\left(\frac{s}{s_0}\right) + Y_+^{\Sigma p}s^{-\eta_+} - Y_-^{\Sigma p}s^{-\eta_-}, \tag{18}$$

$$\rho_{pp}\sigma_{pp} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{pp}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} - \frac{Y_-^{pp}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{19}$$

$$\rho_{\bar{p}p}\sigma_{\bar{p}p} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{pp}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} + \frac{Y_-^{pp}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{20}$$

$$\rho_{\pi^+p}\sigma_{\pi^+p} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{\pi p}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} - \frac{Y_-^{\pi p}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{21}$$

$$\rho_{\pi^-p}\sigma_{\pi^-p} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{\pi p}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} + \frac{Y_-^{\pi p}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{22}$$

$$\rho_{K^+p}\sigma_{K^+p} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{Kp}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} - \frac{Y_-^{Kp}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{23}$$

$$\rho_{K^-p}\sigma_{K^-p} = \pi B\ln\left(\frac{s}{s_0}\right) - \frac{Y_+^{Kp}s^{-\eta_+}}{\tan\left[\frac{1-\eta_+}{2}\pi\right]} + \frac{Y_-^{Kp}s^{-\eta_-}}{\cot\left[\frac{1-\eta_-}{2}\pi\right]}, \tag{24}$$

$$\text{where} \quad \eta_\pm = 1 - \alpha_\pm \text{ and } Z_{ab} \equiv C_1^{ab} + A^{ab}. \tag{25}$$

The numerical values of the free parameters are given in Table 1.

This model gives a very good $\chi^2/dof = 0.973$ and has a high rank point with a total indicator value $P^M = 222$. It describes well the data for $\sqrt{s} \geq 5$ GeV (648 data points), respecting all our numerical criteria for the selection of acceptable models.

In order to understand the content of this empirically-found universality property let us consider the most general form of the two - component PL2 Pomeron as given by (4) and (7) with Z_{ab} defined by (25) :

$$\frac{1}{s}\left(P_1^{ab}+P_2^{ab}\right)=Z_{ab}+B^{ab}\ln^2\left(\frac{s}{s_0^{ab}}\right). \tag{26}$$

This form contains 18 free parameters :
$Z_{pp}, Z_{\pi p}, Z_{Kp}, Z_{\Sigma p}, Z_{\gamma p}, Z_{\gamma\gamma}$; $B^{pp}, B^{\pi p}, B^{Kp}, B^{\Sigma p}, B^{\gamma p}, B^{\gamma\gamma}$; $s_0^{pp}, s_0^{\pi p}, s_0^{Kp}, s_0^{\Sigma p}, s_0^{\gamma p}, s_0^{\gamma\gamma}$.

The content of our empirical universality property is threefold :

1) a **finite energy** property : all scale factors s_0 are the same :

$$s_0 \equiv s_0^{pp}=s_0^{\pi p}=s_0^{Kp}=s_0^{\Sigma p}=s_0^{\gamma p}=s_0^{\gamma\gamma}; \tag{27}$$

2) an **asymptotic** property : all hadron-hadron cross-sections are the same at infinite energies :

$$\sigma_{ab}\underset{s\to\infty}{\longrightarrow} B\cdot\ln^2 s, \tag{28}$$

where

$$B\equiv B^{pp}=B^{\pi p}=B^{Kp}=B^{\Sigma p}; \tag{29}$$

3) a second (factorization) **asymptotic** property :

$$\sigma_{\gamma p}\underset{s\to\infty}{\longrightarrow}\sqrt{\sigma_{\gamma\gamma}}\cdot\sqrt{\sigma_{pp}}=\delta B\ln^2 s \tag{30}$$

The nine constraints resulting from (27)-(30) reduce the number of free parameters of the PL2 Pomeron from 18 to 9.

The secondary-reggeon contributions involve 12 more parameters : $Y_\pm^{pp}$, $Y_\pm^{\pi p}, Y_\pm^{Kp}, Y_\pm^{\Sigma p}, Y_+^{\gamma p}, Y_+^{\gamma\gamma}$; α_+, α_-.

Hence we have a total of 21 parameters for the RRPL2$_u$ (21) model. It is interesting to note that the high-energy (Pomeron) part is more constrained than the low-energy (secondary-reggeon) part. At the RHIC, LHC and cosmic-ray energies the RRPL2$_u$ model has only 9 free parameters. The cosmic-ray experimental data (not included in fits) are very well described by our high-rank model.

Model	$RRPL2_u$	
χ^2/dof	0.973	
CL[%]	67.98	
Parameter	Mean	Uncertainty
s_0	34.0	5.4
B	0.3152	0.0095
α_+	0.533	0.015
α_-	0.4602	0.0064
Z_{pp}	35.83	0.40
$Z_{\pi p}$	21.23	0.33
Z_{Kp}	18.23	0.30
$Z_{\Sigma p}$	35.6	1.4
$Z_{\gamma p}$	0.109	0.021
$Z_{\gamma\gamma}$	0.075	0.026
Y_+^{pp}	42.1	1.3
Y_-^{pp}	32.19	0.94
$Y_+^{\pi p}$	17.8	1.1
$Y_-^{\pi p}$	5.72	0.16
Y_+^{Kp}	5.72	1.40
Y_-^{Kp}	13.13	0.38
$Y_+^{\Sigma p}$	-250.	130.
$Y_-^{\Sigma p}$	-320.	150.
$Y_+^{\gamma p}$	0.0339	0.0079
$Y_+^{\gamma\gamma}$	0.00028	0.00015
δ	0.00371	0.00035

Table 1. The numerical values of the free parameters, χ^2/dof and the confidence level CL in the case of the $RRP2_u$ (21) model ($\sqrt{s} \geq 5$ GeV).

	Region of validity	Numerical value of the rank points	$\alpha_+ - \alpha_-$	Remarks
$RRPL2_u(21)$	$\sqrt{s} \geq 5$ GeV	222	$\simeq 0.07$	-
RRPL(21)	$\sqrt{s} \geq 5$ GeV	173	$\simeq 0.33$	$Z_{pp}, Z_{\pi p}, Z_{Kp} < 0$
$RRPE_u(19)$	$\sqrt{s} \geq 8$ GeV	158	$\simeq 0.20$	$Z_{\pi p}, Z_{Kp} < 0$

Table 2. Comparison between 3 high-rank representative models.

In Table 2 we compare three representative models for each class of 2-component Pomeron : RRPL2$_u$ (21), RRPL (21) and RRPE$_u$ (19).

It is seen from Table 2 that the RRPL2$_u$ (21) model respects the Regge exchange-degeneracy to a very good approximation, in contrast with the RRPE$_u$ (19) and RRPL (21) models. The latter model leads to a huge violation, incompatible with the masses of the resonances on the respective Regge trajectories. This difficulty of the RRPE$_u$ (19) and RRPL (21) models is algebraically correlated with the negativity of the Z parameters (see Table 1), a suspect feature which could lead - via factorization of Regge residues - to negative cross-sections in the case of the scattering involving s, c, b and t quarks.

This point reinforces our conclusion that the solution RRPL2$_u$ (21) is the best one, currently meeting all theoretical, phenomenological and numerical requirements.

4 Future prospects

One problem remaining in the analysis of the forward data is the difficulty in adequately fitting the data for the ρ parameter in pp and in π^+p reactions. The extraction of the ρ data from the measurements of the differential cross sections data at small t is a delicate problem. A reanalysis of these data may be needed, but it will call for simultaneous fits to the total cross-section data and to the elastic differential cross-sections in the Coulomb-nuclear interference region and in the diffractive cones, hence an extension of the parametrization considered here to the non-forward region.

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