

**PSEUDOSCALAR VERTEX, GOLDSTONE BOSON
AND QUARK MASSES ON THE LATTICE.**

Jean-René Cudell^a, Alain Le Yaouanc^b and Carlotta Pittori^a

^a Institut de Physique, Université de Liège au Sart Tilman
B-4000 Liège, Belgique.

^b L.P.T.H.E.¹, Université de Paris Sud, Centre d'Orsay,
91405 Orsay, France.

Abstract

We analyse the Structure Function collaboration data on the quark pseudoscalar vertex and extract the Goldstone boson pole contribution, in $1/p^2$. The strength of the pole is found to be quite large at presently accessible scales. We draw the important consequences of this finding for the various definitions of quark masses (short distance and Georgi-Politzer), and point out problems with the operator product expansion and with the non-perturbative renormalisation method.

1. Continuum model for the quark pseudoscalar vertex

It is well known that the quark pseudoscalar (PS) vertex contains a non-perturbative contribution from the Goldstone boson, in the continuum. On the lattice, the use of a non perturbative renormalisation scheme [2] makes this contribution manifest, although it should go to zero for large momentum transfers. The purpose of this letter is to extract it from lattice simulation data, and to show that it is not negligible for presently accessible scales. In particular, it must be subtracted when evaluating the short distance quark masses from the lattice through the non perturbative method of ref. [1]. This method uses the off-shell axial Ward identity (AWI) and renormalises the mass in the momentum subtraction (MOM) scheme of ref. [2], which can afterwards be related to the $\overline{\text{MS}}$ scheme. The MOM renormalisation involves the pseudoscalar vertex, whence the necessity of the subtraction of the Goldstone contribution to extract the short distance quantity². For physical u, d quarks, the Goldstone contribution becomes very large, larger than the perturbative part; this corresponds to a very large dynamical u, d mass, larger than the usual current mass at the scales accessible to standard lattice calculations.

The expected behaviour of the pseudoscalar vertex in the continuum has been described as follows in the 70's in the works of Lane and Pagels [4, 5] and others. Near the chiral limit, the one-particle-irreducible PS quark vertex Λ_5 can be described through a perturbative contribution plus a non-perturbative Goldstone boson contribution.

The perturbative contribution is of course $1 \times \gamma_5$, with QCD radiative corrections, which according to the renormalisation group lead to a logarithmic behaviour $[\alpha_s(p^2)]^{4/11}$ for $N_F = 0$.

As to the non-perturbative contribution, firstly, according to PCAC, the Goldstone boson must dominate other pole contributions in the PS vertex near the chiral limit: the coupling of the pion to the PS vertex indeed gives a pole $\sim 1/(q^2 + m_\pi^2)$, where q is the momentum transferred at the vertex; other poles (radial excitations) contributions are suppressed in the chiral limit. At $q = 0$, in terms of the quark mass m , this gives a $1/m$ pole contribution. We emphasize that this pole contribution explodes at $q = 0$ in the chiral limit, *i.e.* it is singular

¹Laboratoire associe au CNRS-URA D00063.

² The problem would be quite different if other methods are used to extract quark masses, see *e.g.* ref. [3].

on the border of the “physical” region of momenta. This well-known fact seems to have been underestimated, especially in its consequence for the renormalisation of mass on the lattice near the chiral limit (see below, section 3).

Secondly, according to the Wilson operator product expansion (OPE), the non-perturbative contribution must be power behaved, *i.e.* at large p^2 it must drop as $1/p^2$ up to logs. This power behavior, as shown by Lane, is dictated by one of the pion Bethe-Salpeter amplitude in the OPE. In the latter, the dominant operator is the vacuum to pion matrix element of the pseudoscalar density. This leads to $1/p^2$ by the canonical dimensions and further to logs, $[\alpha_s(p^2)]^{7/11}$ at $N_F = 0$ ³.

Finally, a very important relation, emphasized by Lane and Pagels and derived from the Ward identity, connects directly the forward pseudoscalar vertex to the scalar part of the propagator (they are essentially proportional)⁴. Hence the study of propagator OPE by Politzer [6] closely parallels the above considerations on the PS vertex, with the dominant non-perturbative (power) contribution corresponding to the quark condensate, which corresponds to the Goldstone pole contribution in the PS vertex. We postpone the discussion of the propagator lattice data, which involves delicate problems of improvement, to another paper [8]. Nevertheless, in the last part of the letter, using only the vertex lattice data, we draw important consequences for the propagator thanks to this Ward identity.

In addition, we use the Ward identity to borrow the two-loop corrections to the above logarithmic factors, from those calculated in the case of the propagator by the Rome group [1], and by Pascual and de Rafael [9].

In the following, we will show that the lattice data behave as expected from the continuum theoretical expectations. To relate lattice to physical numbers, we will use the lattice unit $a^{-1} = 1.9$ GeV at $\beta = 6.0$ and $a\Lambda_{\overline{MS}} = 0.174$ for the QCD scale parameter in the quenched approximation. We must stress that nothing in our discussion depends critically on the precise values of these parameters: this work is not oriented towards accurate numerical determinations, but rather towards questions of principle.

2. Fit of lattice data for Γ_5 as function of the hopping parameter κ

Recently [10], Paul Rakow presented, on the behalf of the Structure Function (QCDSF) collaboration, the bare vertex Γ_5 data at $\beta = 6.0$ (Fig. 1 of [10]) through the product

$$am_q \tilde{\Gamma}_5(p^2) \equiv C \times am_q \Gamma_5(q = 0, p^2) \quad (1)$$

as a function of $a^2 p^2$ at three κ values, with:

$$\Gamma_5 \equiv \frac{Tr(\gamma_5 \Lambda_5)}{4} \quad (2)$$

This is the only contribution that survives at $q = 0$ in the continuum limit. By construction, C is a constant with the value [10]:

$$C = 0.75, \quad (3)$$

³In the work of Lane, as well as in some subsequent works, one quotes another power of logs: $[\alpha_s(p^2)]^{-4/11}$. This latter power corresponds to the anomalous dimension of the local operator $\bar{\psi}\gamma_5\psi$. But one requires a gluon exchange at lowest order, which gives an additional α_s factor. This was shown by Politzer [6] for the corresponding $\bar{\psi}\psi$ contribution to the propagator

⁴Let us also recall that in the context of the lattice, this relation has been discussed and used by the Rome group to fix Z_A on quark states [7].

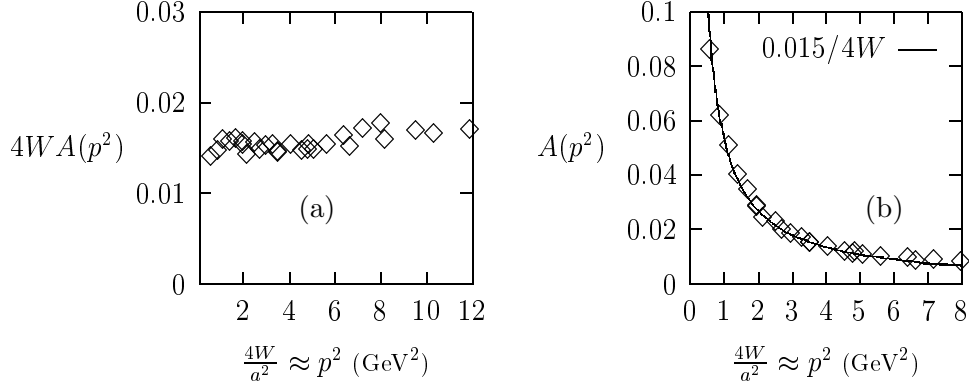


Figure 1: (a) The value of the coefficient of $a^2 p^2 A(p^2)$ in eq. (6) from the lattice data extrapolated at κ_c ; (b) Our fit to the extrapolated data.

defined in [10] so that the numbers in r.h.s. of eq. (1) equate approximately the lattice data for the scalar part of the bare improved propagator (same Fig. 1 of [10]) on some range of momenta, increasing with κ . This approximate equality corresponds to the Ward identity to be discussed in section 4, but we are not concerned with it, as we rely only on the vertex data. The QCDSF data were obtained with the SW improved fermionic action [11] at $c_{SW} = 1.769$, $N_F = 0$, in the Landau gauge, and p^2 denotes the following lattice definition of the momentum squared,

$$a^2 p^2 \equiv 4W \equiv 4 \sum_{\lambda} \sin^2 \left(\frac{ap_{\lambda}}{2} \right) \quad (4)$$

where a is the lattice unit. This notation will be used herefrom, and identified with the continuum p^2 . We take $a^{-1} = 1.9 \pm 0.1$ GeV at $\beta = 6.0$ [12]. We also use the standard lattice definition of the bare mass:

$$am_q = \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \quad (5)$$

Let us write

$$am_q \tilde{\Gamma}_5(p^2) = A(p^2) + am_q B(p^2) \quad (6)$$

We shall first perform an extrapolation linear in m_q of the three datasets to the chiral limit at $\kappa_c = 0.1352$, for each value of p^2 . This fit gives us both $A(p^2)$ and $B(p^2)$, as shown in Figs. 1 and 2.

As $\Gamma_5(q=0, p^2) \propto [A(p^2)/am_q + B(p^2)]$, it seems reasonable to identify the $A(p^2)$ term as the Goldstone contribution (*i.e.* as the pole in am_q), and the second one as the perturbative contribution, if we are sufficiently close to the chiral limit. The continuum Ward identity enables us to borrow the two-loop renormalisation group improved factors, from those calculated in the case of the propagator by the Rome group [1] for the perturbative part:

$$B(p^2) = B_0 \times [\alpha_s(p^2)]^{4/11} \left[1 + 8.1 \frac{\alpha_s(p^2)}{4\pi} \right] \quad (7)$$

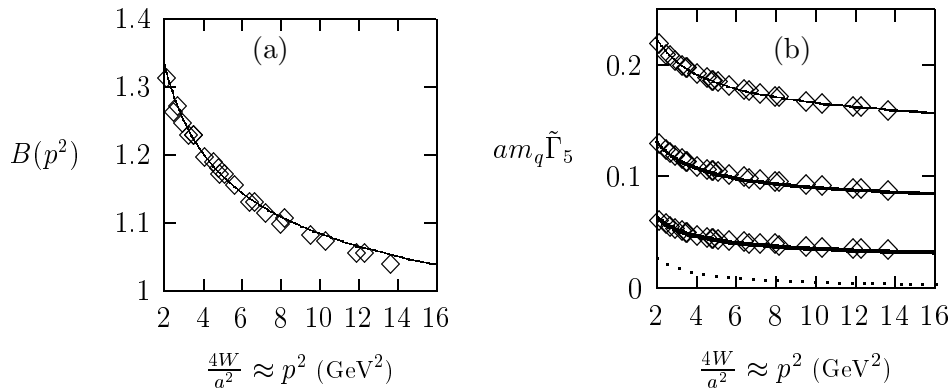


Figure 2: (a) The value of $B(p^2)$ in eq. (6) from our extrapolation at κ_c , compared with eq. (7); (b) Our fit to the lattice data in the region of high p^2 for $B_0 = 1.735$. The plain curves correspond from top to bottom respectively to $am_q = 0.148$, 0.078 , and 0.028 , and the dashed one to the Goldstone boson contribution.

and by Pascual and de Rafael [9] for the non-perturbative part:

$$A(p^2) = A_0 \times \frac{[\alpha_s(p^2)]^{7/11}}{a^2 p^2} \left[1 + 22.0 \frac{\alpha_s(p^2)}{4\pi} \right] \quad (8)$$

with A_0 and B_0 some constants. These two-loop corrections are valid for the MOM renormalisation scheme, in the Landau gauge, but with α_s taken as the \overline{MS} coupling constant⁵. As already mentioned, we take $a\Lambda_{\overline{MS}} = 0.174$ at $N_F = 0$ from the three gluon coupling measurement with asymmetric momenta [13, 14]. Note that, a priori, the evolution formulae properly apply to the renormalised propagator, which is the bare one divided by $Z_\psi(p^2)$ ⁶; but, at least theoretically and in the continuum, $Z_\psi(p^2)$ should evolve very slowly, since $\gamma_\psi = 0$ at one loop in the Landau gauge, and the two-loop correction seems also to have very little effect [15]. Lattice data [16, 17] confirm this perturbative argument. Then, we conclude that the two-loop corrections should be obeyed by the bare PS vertex with good accuracy.

The lattice data turn out to be quite close to the continuum theoretical expectations near the chiral limit, confirming the above interpretation of A and B . Indeed one finds the following:

- $A(p^2)$ is behaving remarkably close to $1/p^2$ over a large interval of p^2 , see Fig. 1(a). From the numerical analysis, the Goldstone contribution appears to be very large. Indeed,

$$a^2 p^2 A(p^2) \simeq 0.015 \quad (9)$$

from the lowest point $a^2 p^2 = 0.16$.

On the other hand, we do not see the log factors expected from the perturbative calculation, eq.(8), which remains to be understood; we shall discuss further problems in the perturbative evaluation of the Wilson coefficient at the end of the paper.

⁵for which we use the expression $4\pi/\alpha_s(q^2) = 11 \log\left(\frac{q^2}{\Lambda_{\overline{MS}}^2}\right) + \frac{102}{11} \log\left(\log\left(\frac{q^2}{\Lambda_{\overline{MS}}^2}\right)\right)$.

⁶In this letter, we use the convention of ref.[2] for the Z 's.

• $B(p^2)$ is found to evolve closely to $\ln(p^2/\Lambda_{QCD}^2)^{-4/11}$, more precisely it is evolving in good conformity with the two-loop MOM renormalisation formula quoted above. We obtain

$$B_0 = 1.735 \quad (10)$$

which provides a very good fit to the data for p^2 larger than 2 GeV², see Fig. 2(b).

The Goldstone contribution is felt already at rather large quark masses and, for physical u, d quarks it is in fact very large: $A(p^2)$ is larger than the perturbative part $am_q B(p^2)$ even at rather large p^2 , as shown in Fig. 3(a) and further discussed in section 4.

3. Consequences on the short-distance mass from the lattice

Let us then recall that the pseudoscalar vertex is an important ingredient in the method first developed by the Rome group [1] to determine the short distance quark masses. One starts from the “axial” MOM renormalised quark mass given by:

$$am_{AWI}^{Landau}(\mu^2) = \rho \frac{Z_A}{Z_P(\mu^2)} \quad (11)$$

where ρ is a dimensionless parameter determined from a ratio of matrix elements involving the pion and the bare axial and pseudoscalar bilinear operators. Z_A is the standard renormalisation of the axial current, determined from the axial Ward identity or approximated through the MOM renormalisation non-perturbative method, or from one-loop perturbation theory. Finally $Z_P(\mu^2)$ is defined through the MOM renormalisation condition for the vertex and is determined by the same non-perturbative method from the PS quark vertex at $q = 0$, or again approximated from one-loop perturbation theory⁷. Then, from $m_{AWI}^{Landau}(\mu^2)$, one can deduce the standard short distance masses, for instance the \overline{MS} mass, but this requires to work in the perturbative, short distance, region.

Then our findings for the PS vertex have important consequences. Let us note that, in principle, the fact that at $q = 0$, the PS vertex not only is influenced by a large Goldstone contribution (as noted in [2], [18]), but really explodes in the chiral limit, is crucial for the MOM procedure of renormalisation, since the renormalisation constant Z_P is defined as:

$$Z_P^{MOM}(\mu^2) = \frac{Z_\psi(\mu^2)}{\Gamma_5(q=0, p^2=\mu^2)} \quad (12)$$

This definition ensures that

$$\Gamma_5^R(q=0, p^2) = 1 \text{ at } p^2 = \mu^2 \quad (13)$$

and one concludes from it that Z_P has a trivial chiral limit at fixed p^2 . Indeed, near κ_c , since $m_\pi^2 \propto m_q$ and $\Gamma_5(q=0, p^2) \propto 1/m_q$, Z_P^{MOM} tends to zero, or its inverse goes to infinity.

3.1. Calculation of Z_P

One can translate the above fit of Γ_5 into an expression for Z_P^{MOM} , or rather its inverse which is more directly physical:

$$\frac{1}{Z_P^{MOM}(p^2)} = \frac{\Gamma_5(p^2)}{Z_\psi(p^2)} = \frac{A_Z(p^2)}{am_q} + B_Z(p^2) \quad (14)$$

⁷Of course, using the PS quark vertex for normalisation is a particular way to define the MOM scheme, which, in the non-perturbative regime, is not necessarily compatible with *e.g.* the use of the scalar vertex. See remarks below.

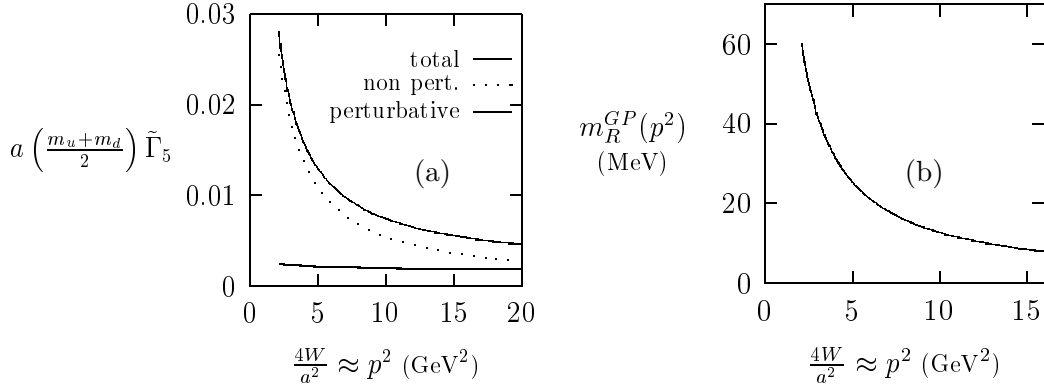


Figure 3: (a) The value of $am_q \tilde{\Gamma}_5(q=0, p^2)$ for light quarks; (b) The value of the dynamical u, d masses.

where $A_Z(p^2) = A(p^2)/[CZ_\psi(p^2)]$ and $B_Z(p^2) = B(p^2)/[CZ_\psi(p^2)]$.

This is to be contrasted with usual fits, which assume that Z_P is linear in am_q of eq. (5)⁸. One notes that since $Z_\psi(p^2)$ is weakly dependent on p^2 and on κ , and close to 1, the expression is quite similar to the preceding one: it consists in a first term which is approximately in $1/m_q$, and the second one which is approximately constant⁹. Just as for the Γ_5 vertex, the former corresponds to the non perturbative Goldstone contribution while the latter, B_Z , corresponds to the short distance contribution, as measured by the lattice numerical simulation, including all the orders of perturbation theory by a non perturbative method. To give numbers, we need now values for Z_ψ , which we borrow from the data of the Rome group [16], with an improvement procedure trying to parallel as much as possible the one followed by Rakow for the scalar part. However, we would like to emphasize that all the qualitative conclusions of this paper are independent of precise values of Z_ψ as long as they stay around 1.

At $a^2 p^2 = 1$ which is close to the standard reference point $p = 2 \text{ GeV}$, and at $\kappa = 0.1342$, which is the κ closest to the chiral limit, we estimate:

$$Z_\psi \simeq 0.85 \quad (15)$$

One has then numerically:

$$A_Z \simeq 0.023, \quad B_Z \simeq 1.88 \quad (16)$$

We emphasize that this identification of the two contributions does not rely on Boosted Perturbation Theory (BPT), but purely on numerical simulations. We shall refer to B_Z as the “short distance contribution” not to be confused with its one-loop perturbative estimate. Now, we shall compare to BPT estimates here and in the following for illustrative purposes only.

⁸D. Becirevic [19] has now done a fit along the above lines with the data of the Orsay-Rome group and found roughly similar conclusions.

⁹One could be worried by the fact that the dependence of Z_ψ on κ could generate from $A_Z/[(1/\kappa - 1/\kappa_c)/2]$ an additional contribution to Z_P^{-1} independent of κ . However, it can be seen that since A_Z decreases rapidly, this contribution is not large with respect to the one coming from B_Z .

B_Z is indeed very close to the one-loop standard BPT evaluation $Z_P^{-1}(a^2p^2 = 1) = 1.7$ (from $Z_P = 0.59$) in the chiral limit, with $g^2 = 1.6822$, whereas the non-perturbative term contributes around $0.023/am_q \sim 0.8$ on a total of 2.7. Therefore, the departure of Z_P from its one-loop perturbative evaluation seems to be essentially due to the Goldstone boson contribution. Higher-order radiative corrections do not seem to be very large¹⁰. However, these conclusions could be sensitive to details of the data or of the fits.

3.2. Calculation of the \overline{MS} mass

We can now convert our results for the renormalised mass m_{AWI}^{Landau} into a calculation of the \overline{MS} mass. In ref. [20], it is suggested that the use of the MOM non-perturbative determination of Z_P (with linear extrapolation in κ) in the AWI method improves the results for the \overline{MS} mass with respect to previous determinations of Z_P by one-loop perturbative calculations.

However, in principle, to make the conversion to a short distance mass, we must still make sure that we work in the perturbative regime. Now, we have isolated the Goldstone boson contribution which is *essentially* non-perturbative as it does *not* correspond to higher order contributions but rather to power corrections. The fact that the non-perturbative estimate of the full Z_P differs sizeably from the short distance B_Z already at the measured kappas, is a signal that it is not presently possible to work at p^2 high enough for the Goldstone contribution to be negligible. Hence, we must first subtract it from Z_P^{-1} .

$$\left[Z_P^{Subtr}(p^2)\right]^{-1} = \left[Z_P^{-1}(p^2)\right] - \frac{AZ(p^2)}{am_q} = B_Z(p^2) \quad (17)$$

Numerically, the remaining short distance B_Z is in fact close to the one-loop BPT estimate of Z_P , as we have just seen. Indeed, at $a^2p^2 = 1$, with this subtraction and using again $Z_\psi = 0.85$ near the chiral limit, we find $Z_P^{Subtr} = 1/1.88 = 0.53$ which corresponds to the fully resummed short-distance contribution determined directly from the lattice data¹¹. To get an estimate of the consequences on the light quark masses, we use [22] (which uses the notation $a\tilde{m}$ for ρ) where one finds $\rho \sim am_q$, with $am_{u,d} = 0.001836$, and we take $Z_A = 0.79$ [23]. We then find:

$$a m_{u,d}^{Landau} \sim 0.0027 \quad (18)$$

Converted into the \overline{MS} scheme through [1]

$$m_q^{\overline{MS}} = m_q^{Landau} \left(1 - \frac{16}{3} \frac{\alpha_s}{4\pi}\right) \quad (19)$$

with $\alpha_s(a^2p^2 = 1) = 0.25$ at two loops, this gives:

$$a m_{u,d}^{\overline{MS}} \sim 0.0024 \quad (20)$$

therefore about 4.6 MeV at $N_F = 0$. One would obtain about 6 MeV if one used the full MOM non-perturbative estimate of Z_P linearly extrapolated to the chiral limit and 4.2 MeV from one-loop BPT. Note that these numbers are only indicative; in view of the many uncertainties in the subtraction procedure, we do not try to discuss the other sources of error necessary to give

¹⁰These two possible explanations were suggested in ref. [2].

¹¹We take Z_P^{Subtr} approximatively independent of mass, except for a small variation of Z_ψ . One would otherwise require a fit of $m_q\Gamma_5$ with one more term in m_q .

a real determination of the mass. Our aim is only to underline the necessity of the subtraction of the Goldstone contribution.

Despite the fact that in this case the two methods lead to comparable results, the subtraction method just described, which does not rely on perturbation theory but which rather uses the lattice data directly, is in general superior to that based on the BPT estimate, and knowledge of the full Z_P^{MOM} is in general necessary even if we aim at measuring short distance quantities.

Indeed, the BPT method has the following drawbacks. Firstly, the unknown higher-order perturbative corrections may be large. Secondly, the one-loop perturbative evaluation can be tadpole-improved in many ways, potentially leading to very different estimates. Finally, the one-loop estimate, even if tadpole-improved, does not automatically follow the behavior dictated by the renormalisation group; the problem is then cured by taking the one-loop estimate at some momentum, and then imposing the renormalisation group evolution for the other momenta, but this is obviously presenting a rather arbitrary choice of a privileged point.

On the other hand, the subtraction method used here avoids these problems. It amounts in general to a non-perturbative measurement of the Z 's, followed by the evaluation and removal of the pole contributions. As shown above in Fig. 2(b), this procedure leads to a result which evolves per se according to the renormalisation group.

The evaluation of the pole contribution from the pion is especially easy, because of its particular singular nature at $m_q = 0$. Other pole contributions are expected to be smaller, because they are regular in the chiral limit. If one were to subtract them, one could only rely on the expected power behavior of the particle vertex function, and their extraction would thus be more difficult.

4. Consequences for the renormalised mass à la Georgi-Politzer

In this section, we show that the Goldstone boson contribution to the PS vertex, which is only parasitical in the calculation of $\overline{\text{MS}}$ masses, and has to be subtracted as shown in the previous sections, retains an important physical meaning, as can be seen through the use of other definitions of renormalised quark masses.

4.1. Physical relevance of the full renormalised axial mass

One should remember that the full axial MOM renormalised mass, which is calculated through:

$$am_{AWI}^{Landau}(p^2) = \rho Z_A [Z_P(p^2)]^{-1} \quad (21)$$

with $Z_P(p^2)$ not submitted to the above subtraction, retains a physical significance by itself, since it is the mass defined through the divergence of the axial current, with the corresponding natural renormalisation condition which consists in setting the pseudoscalar vertex Γ_5^R to 1 on quark states at the renormalisation scale¹². We stress that in contrast to the standard $\overline{\text{MS}}$ current mass, it does not vanish in the chiral limit, because the chiral limit $\rho \rightarrow 0$ is compensated by the pole in Z_P^{-1} . The meaning of this last result will be further developed in the next subsection. Of course, it is unpleasant to have renormalised quantities with a rather queer behavior in the

¹²Recall then that the scalar vertex should be consistently normalised through the Ward identities, as repeatedly emphasized by the Rome group, therefrom one sees that Z_S/Z_P must be independent of p^2 , in obvious contradiction with imposing simultaneously a similar MOM condition for the scalar vertex.

chiral limit. Indeed, for instance, hadronic matrix elements of the renormalised pseudoscalar density which do not have a pion pole should tend to zero in the chiral limit- and therefore also, the ones of the scalar density defined in accordance with the Ward identity. But as we hope to have shown, this is an unavoidable consequence of the non-perturbative method as applied to the pseudoscalar density. Of course, one could avoid this by preferring the corresponding MOM normalisation condition for the scalar vertex. But then, one must remember another aspect of the axial MOM renormalised mass, which gives it another important physical significance, and which we shall now discuss.

4.2. Relation with the mass as defined by the propagator

It can indeed be easily shown that the “axial” renormalised mass is also essentially identical to a standard renormalised mass defined through the scalar part of the propagator, as becomes obvious through the renormalised axial Ward identity at zero transfer (this has been recalled in the talk of Rakow); in fact it is essentially identical to the Georgi-Politzer mass. Let us indeed write the Ward identity¹³:

$$m_{AWI}^{Landau}(\mu^2)\Gamma_5^R(q=0, p^2, \mu^2) = \frac{Tr[S_R^{-1}(p, \mu^2)]}{4} \quad (22)$$

Here, it is assumed of course that the propagator is also calculated in the Landau gauge, and that the renormalisation of the vertex and of the propagator are both consistently performed according to the MOM scheme. Therefore the propagator is normalised in the Euclidean region according to¹⁴:

$$S_R^{-1}(p, \mu^2) = i \not{p} + m_R^{GP}(\mu^2) \quad \text{at} \quad p^2 = \mu^2 \quad (23)$$

which is nothing else than the Georgi-Politzer renormalisation condition.

Setting $p^2 = \mu^2$, one obtains from eqs. (13), (22) and (23):

$$m_{AWI}^{Landau}(\mu^2) = \frac{Tr[S_R^{-1}(p, \mu^2)]}{4} \Big|_{p^2=\mu^2} = m_R^{GP}(\mu^2) \quad (24)$$

therefore the announced identity of $m_{AWI}^{Landau}(\mu^2)$ and the Georgi-Politzer mass function $m_R^{GP}(\mu^2)$ is derived.

Through eq. (24), the $1/p^2$ power contribution in Z_P^{-1} corresponding to the Goldstone boson is related to a similar contribution in the scalar part of the propagator, which represents a dynamically generated mass for light quarks, though off-shell, gauge-dependent and Euclidean. This is a well-known [24] signal of the spontaneous breakdown of the chiral symmetry. We can then translate our knowledge of the PS vertex into information on this dynamical mass.

4.3. Physical consequences

Numerically, for the non-perturbative contribution, which corresponds to the chiral limit of m_{AWI}^{Landau} , one finds at $a^2 p^2 = 1$, from eq. (21), with, as before, Z_P from eq. (14), $Z_A = 0.79$ and

¹³Here we use eq. (22) as a constraint allowing to calculate the r.h.s. from the l.h.s. We do not require the independent input of the scalar part of the propagator data. As a side remark, as a check of the validity of eq. (22) on the lattice QCDSF propagator data, we note that it would imply for the C defined above $C = Z_A \rho / am_q$, which is not far from their $C = 0.75$.

¹⁴We disregard here the difference between this standard MOM condition for Z_ψ and the derivative MOM condition derived from the vector Ward identity by the Rome group [2], which seems to lead numerically to very small differences.

$\rho \sim am_q$ [22],

$$a m_R^{GP}(a^2 p^2 = 1) \sim 0.018 \quad (25)$$

therefore around 34 MeV at $p = 1.9$ GeV.

We have obtained analogous results directly from the scalar part of the bare improved propagator as given in [10]. Extended considerations and estimates on the propagator will be given in a forthcoming paper [8].

At large p , we want to stress that the non-perturbative contribution to the mass remains of the order of the u, d perturbative masses and even larger, see Fig. 3(a). It must be emphasized that this result is rather safe, since it does not depend critically on improvement procedures.

The sign is as expected, but the magnitude is much larger than what one would expect from the estimate by the $q\bar{q}$ condensate and a perturbative calculation of the Wilson coefficient[9]:

$$m_R^{GP}(p^2) = -\frac{4}{3}\pi\alpha_s \frac{\langle 0|\bar{q}q|0 \rangle}{p^2} \quad (26)$$

Indeed, with $\alpha_s^{\overline{MS}}(a^2 p^2 = 1) \sim 0.3$ (one-loop), and with a standard \overline{MS} renormalised evaluation of the $\bar{q}q$ condensate of $-(225 \text{ MeV})^3$, at $\mu = 1 \text{ GeV}$, rescaled at $a^2 p^2 = 1$ by a factor 1.18, one would find an answer lower by a factor ten. However, a large value is inevitable if we admit as usual that at moderately low p^2 the mass is of the order of a constituent mass, i.e. several tens of MeV, and if we take into account that the decrease is only in $1/p^2$, as found with remarkable accuracy on the lattice data of the QCDSF group.

That a value of several tens of MeV is needed at low p^2 is confirmed by direct lattice calculations of the propagator, to be compared with our estimate of Fig. 3(b):

- in configuration space, the propagator in time $S(t)$ has been found exponential over a large range of t , with a coefficient of the exponential around 300 MeV [17, 26];
- in momentum space, at the lowest points, corresponding to $p^2 \rightarrow 0$, where it should be insensitive to the improvement, the scalar part of the inverse propagator is found to be around similar values of 200 – 400 MeV [10, 16], for the lowest mass $\kappa = 0.1342$. A large value for the coefficient of $1/p^2$ was also found in a phenomenology of the pion as a Goldstone boson [27], based on an assumption:

$$m_R^{GP}(p^2) = \frac{4(m_D)^3}{p^2} \quad (27)$$

where m_D is a free parameter. The phenomenology seems to require $m_D \approx 300$ MeV, $m_R^{GP}(p^2) \sim 27$ MeV at $p = 2$ GeV, in agreement with the above estimate.

It must be emphasized that this large non-perturbative contribution does not contradict directly the sum rule calculation of correlators, which uses a perturbative evaluation of the quark propagator. Indeed, as in the calculation of propagator by Politzer, in sum rule calculations non-perturbative contributions are consistently added through condensates. Moreover, as emphasized by Pascual and de Rafael [9], the condensate contributions are quite different for the quark propagator and the correlators, therefore our finding for the full quark propagator does not have a direct impact on sum rules.

Of course, the discrepancy with the naïve perturbative estimate of the Wilson coefficient is nevertheless worrying and deserves further reflexion since the latter is known to work in general.

One could imagine that the perturbative calculation of the Wilson coefficient is not valid for some particular reason. In this direction, one must observe that the two-loop correction is very large, around 50% of the first order at $a^2 p^2 = 1$. It may be therefore that the perturbative expansion happens to fail.

Conclusion

The lattice numerical calculations can be seen as the triumph of the general theoretical predictions of the 70's for the quark pseudoscalar vertex. Nevertheless, a striking and unexpected feature of the lattice data is the very large size of the Goldstone boson contribution to the pseudoscalar vertex. This corresponds, through the Ward identity, to a very large non-perturbative contribution to the renormalised mass function of Georgi and Politzer, by far larger than what is expected from the quark condensate and a perturbative evaluation of the Wilson coefficient, but in agreement with other physical expectations.

These large non-perturbative contributions then give a warning that there may be a possible problem with the use of lowest order perturbation theory in the estimate of the Wilson coefficient of condensates. This question requires more investigation.

The Goldstone contribution must be subtracted from the pseudoscalar vertex to calculate the short distance mass from a normalisation of this pseudoscalar vertex. This has important numerical consequences. The short distance Z_P to be used should be sizeably larger than the one measured directly on the lattice.

Finally, it must not be forgotten that lattice artefacts may still be large at $\beta = 6$, , and that the vertex has not been improved with rotations as an off-shell Green function, therefore one can expect large uncertainties on the quantitative estimate of the effect at large p^2 .

Note added in proof: When completing this paper, we became aware of the work of the JLQCD collaboration, presented at Denver conference, [28] where certain parallel conclusions on Z_P have been drawn from lattice staggered fermion data. Paul Rakow has also drawn our attention to ref. [29] , where connected observations on PS vertex and the scheme of Pagels are made.

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