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SIMPLE MODEL FOR TOTAL CROSS SECTIONS¹

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Abstract

Adopting the philosophy *à la* Donnachie and Landshoff that simple pole exchanges could account for all data of total, elastic and diffractive scattering cross sections to present energies, we show that such simple pole fits to pp and $\bar{p}p$ total cross sections are indeed very successful. We assess the uncertainties of the various parameters by making careful statistical analysis of the data and their correlations. In particular, the pomeron intercept which controls total cross sections and the real part of the elastic amplitude at high energies is shown to lie anywhere between 1.07 and 1.11, with a preferred value 1.096.

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It is well-known that total cross sections rise at high energy. The simplest object responsible for the rising behavior would be a Regge trajectory, the pomeron, with an intercept somewhat greater than 1. This object has been hypothesized a long time ago [1], and presumably arises from the gauge sector of QCD. Its intercept and slope are then fundamental numbers characterizing the pure gauge sector of Yang-Mills SU(3).

There has been a renewed interest in the pomeron after the observation of rapidity gaps in deep inelastic ep scattering at HERA [2] and the suggestion that such cross sections might be used for the detection of new physics [3]. One can view the emergence of gaps as resulting from the emission of a pomeron by the colliding proton (which then remains in a color-singlet state) followed by a pomeron-photon collision. The validity of this picture, as well as the measurement of the pomeron intercept and structure function is thus a central issue for HERA, and will no doubt have a bearing on the extrapolation to the physics at the LHC energies. Hence we propose to re-evaluate the pomeron intercept from the simple-pole model fits to the total cross sections as carefully as possible and to estimate the errors on the various parameters. The information on the soft pomeron intercept comes from the high energy behavior of the total cross sections. Because of the presence of the sub-leading Regge poles at low energy and because the unitarity effect due to multipomeron exchanges at high energy, one must not only determine the best parameterization but also the range of validity of the model. It is clear that a simple minded χ^2 test cannot be sufficient, primarily because the cross section data contain many points that are inconsistent with their neighboring points. We therefore must invent a reasonable method to filter the data sets independently of any underlying theoretical model or prejudice. A reasonable criterion is that a given data point should not deviate by more than 1 or 2σ from the average of all data in a bin of ± 1 GeV centered around it and yet the central values of the parameters and their errors should not depend too sensitively on the filtering itself. The stability of the parameter values is more reasonable criterion than the value of χ^2_{min} in our opinion. In the following, we use two strategies to evaluate the best central values and their errors. The first is to use all the data available [4, 8]. This gives us the central values. However, some of the data points are incompatible at the 2σ level or more so that the value of χ^2 will be artificially inflated. In fact the best fits [9, 10] that one can produce have a $\chi^2/d.o.f.$ of 1.3 or more. We then use the data sets that are filtered by using the proposed criterion, which will give

us stable central values of the parameters and their uncertainties. The best selection criterion would be based on physics arguments to separate wrong experimental results. Unfortunately, this would prove infeasible for old (ISR) data, where most of the incompatibilities lie. We give in Table 1 the number of data points that are kept before and after filtering. The full data sets are available at <http://nuclth02.phys.ulg.ac.be/Data.html>.

data set	σ_{tot}^{pp} (mb)	$\sigma_{tot}^{\bar{p}p}$ (mb)	ρ^{pp}	$\rho^{\bar{p}p}$
P.D.G. [4]	94	28	-	-
P.D.G. [4] - 2σ	84	28	-	-
P.D.G. [4] - 1σ	65	20	-	-
Ref. [8]	66	29	41	13
Ref. [8] - 2σ	60	28	38	13
Ref. [8] - 1σ	53	19	31	13

Table 1: The number of points kept after data selection, for $\sqrt{s} > 10$ GeV.

Note that the 2σ selection includes both CDF and E710 points, whereas the 1σ one rejects them both. This procedure is not unlike the one followed by UA4/2 in [7]. As the value of $\chi^2 - \chi^2_{min}$ is distributed as a χ^2 with N parameters of the model, the $\Delta\chi^2$ corresponding to 70% confidence level(C.L.) is 6.06 [11, 12] in the DL case with 5 parameters. The errors we quote in this paper are to this χ^2 -interval, to be contrasted to those quoted in the Particle Data Group(PDG) [10] who simply renormalized the χ^2 to $\chi^2/d.o.f. = 1$ and let the new χ^2 change from the minimum by one unit.

Donnachie and Landshoff (DL) have proposed[5] to fit the pp and $\bar{p}p$ cross section using a minimal number of trajectories: the leading meson trajectories of the degenerate a/f ($C=+1$) and ρ/ω ($C=-1$), plus the pomeron trajectory. They fit data for $s > 100$ GeV², as lower trajectories would then contribute less than 1%, which is less than the errors on the data. The result of their fit [6] is a pomeron intercept of 1.0808, for which they did not quote a χ^2 or error bars and said that the χ^2 was very flat near the minimum. We show in Table 2 our results for such a fit. We use the usual definition

$$\chi^2 = \sum_i \left(\frac{d_i - s_i^{-1} \mathcal{I}m\mathcal{A}(s_i, 0)}{e_i} \right)^2 \quad (1)$$

parameter	all data	filtered data (2σ)	filtered data (1σ)
χ^2	410.8	80.3	32.4
χ^2 per d.o.f.	3.16	0.62	0.25
pomeron intercept-1	$0.0912^{+0.0077}_{-0.0070}$	$0.0887^{+0.0079}_{-0.0071}$	$0.0863^{+0.0096}_{-0.0084}$
pomeron coupling (mb)	$19.3^{+1.5}_{-1.7}$	$19.8^{+1.6}_{-1.7}$	$20.4^{+1.8}_{-2.1}$
$\rho/\omega/a/f$ intercept-1	$-0.382^{+0.065}_{-0.071}$	$-0.373^{+0.067}_{-0.073}$	$-0.398^{+0.083}_{-0.090}$
ρ/ω coupling $C_-(pp)$ (mb)	$-13.2^{+4.1}_{-6.5}$	$-13.3^{+4.3}_{-6.9}$	$-15.3^{+5.7}_{-10.1}$
a/f coupling (mb)	69^{+20}_{-13}	62^{+19}_{-12}	67^{+25}_{-16}

Table 2: Simple pole fits to total pp and $\bar{p}p$ cross sections, assuming degenerate $C = +1$ and $C = -1$ exchanges.

with $d_i \pm e_i$ for the measured pp or $\bar{p}p$ total cross section at energy $\sqrt{s_i}$, and

$$\mathcal{I}m\mathcal{A}(s, 0) = C_- s^{\alpha_m} + C_+ s^{\alpha_m} + C_P s^{\alpha_P} \quad (2)$$

where C_- flips sign when going from pp to $\bar{p}p$. We use the same data set as DL [4] to determine the central value, and use the selected data sets at the 1- or 2- σ level to determine the errors. We see that the fit to all data gives a totally unacceptable value, $\chi^2 = 410$ for 135 data points with 5 parameters, corresponding to a C.L. of 2×10^{-36} ! There are two possible outcomes to such a high value of the χ^2 : either the model is to be rejected, or some of the data are wrong. As we already mentioned, there are a few obviously wrong points within the data. Hence before rejecting the model, let us eliminate those points. Table 2 shows that the central values and their errors indeed do not depend too much on the filtering itself, while filtering the data does change the value of χ^2_{min} drastically so that the model becomes perfectly acceptable. Also we see that the pomeron intercept is determined to be about 1.090, and that it could be as high as 1.096. We think the stability of the parameter values is a more important than the value of χ^2_{min} itself.

As for the energy range of validity of the model, we require the two basic requirements: that the $\chi^2/d.o.f.$ be of the order of 1, and that the determination of the intercept be stable. We show in Fig. 1 the result of varying the energy range. Clearly, the lower trajectories seem to matter for $\sqrt{s_{min}} < 10$ GeV, whereas the upper energy does not seem to modify the results (in other words, there is no sign of the onset of unitarisation). Hence we adopt $\sqrt{s_{min}} = 10$ GeV as the lowest energy at which the model is correct.

This happens to be the point at which χ_{min}^2 is lowest. This dependence on the lower energy cut explains why both the PDG [10] and Bueno and Velasco [13] obtain an wrong value for the intercept, much lower than ours.

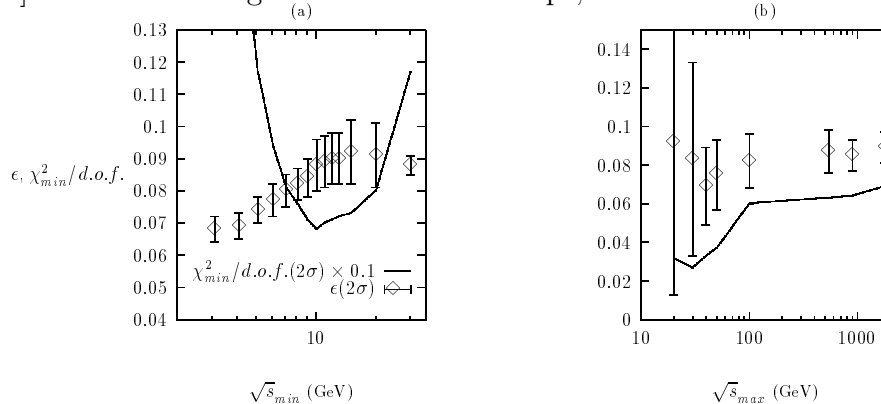


Figure 1: DL intercept-1 as a function of the lower (a) and upper (b) energy cuts on the data. The curve shows the $\chi^2/dof.$ for data filtered at the 2σ level.

One must wonder if it is possible to get a better determination of the soft pomeron intercept by using more data. The PDG [10] obtained very narrow determinations of the pomeron intercept from the other hadronic reactions. We believe that their conclusions are wrong, and illustrate this in the case of the $\pi^\pm p$ total cross sections, for which they use $\sqrt{s_{min}} \approx 4$ GeV and obtain an intercept of 1.079 ± 0.003 . We show in Fig. 2 our results for such a fit: for the data set filtered at the 2σ level (92 points), and for $\sqrt{s_{min}} = 4$ GeV, we obtain $\alpha_0 = 1.115^{+0.030}_{-0.023}$. Fig. 2(b) shows our best fit together with that of the PDG [10]. Although according to their estimate our central value for the intercept is 10 standard deviations from theirs, we see that the two fits are indistinguishable. Hence we believe that both their standard deviations and their central values are wrong. These conclusions are not affected by the use of the full data set instead of the one filtered at the 2σ level. The above value of the intercept in fact gives a slightly smaller $\chi^2/d.o.f.$ than the one quoted in the PDG [10]. Note that this intercept is consistent with the one we got from the analysis of pp and $\bar{p}p$ total cross sections. The conclusion from this exercise is that the errors from the low-energy hadronic data are large, especially if we use a low-energy cutoff of the order of 10 GeV. Hence we want to limit ourselves to pp and $\bar{p}p$ amplitudes at $t = 0$.

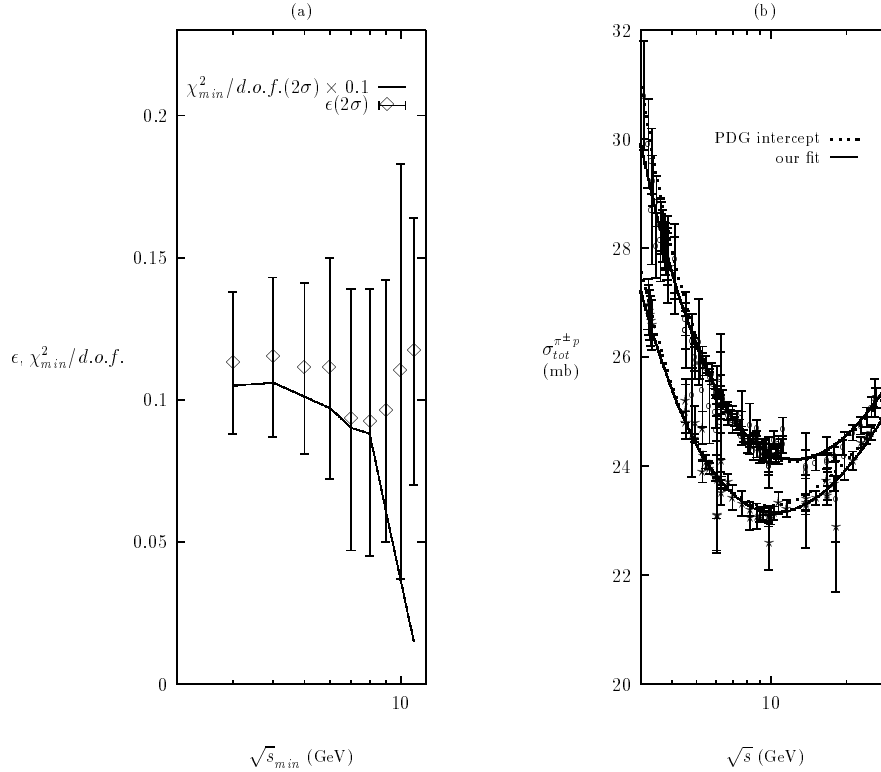


Figure 2: (a) shows the pomeron intercept from πp data as the lower energy cut on the data is changed; (b) shows our best fit to the data set filtered at the 2σ level together with that of the Particle Data Group.

One more piece of pp and $\bar{p}p$ data can be used however: the knowledge of the intercept is sufficient to determine the value of the real part of the amplitude, using crossing symmetry, and hence the measurements of the ρ parameter provide an extra constraint. We use the data collected in Ref. [8], and obtain a somewhat worse fit, as shown in Table 3, even when filtering data at the 1 or 2σ level. For the 2σ filtering of the data, the C.L. goes from 99.4% to 36%. We show the curves corresponding to the second column of Table 3 in Fig. 3 with dotted lines. Whether one should worry about the change of χ^2 and the change of central value for the parameters, is a matter of taste. But it is this small change of central values, combined with the effect of too low an energy cut, that lead Bueno and Velasco [13] to conclude that simple-pole parametrisations were disfavored.

parameter	all data	filtered data (2σ)	filtered data (1σ)
χ^2	561.3	168.3	94.9
χ^2 per d.o.f.	3.28	1.07	0.77
pomeron intercept-1	0.0840 ± 0.0050	$0.0817^{+0.0055}_{-0.0053}$	$0.0804^{+0.0064}_{-0.0061}$
pomeron coupling (mb)	20.8 ± 1.1	21.4 ± 1.1	21.8 ± 1.3
$\rho/\omega/a/f$ intercept-1	$-0.408^{+0.032}_{-0.033}$	$-0.421^{+0.034}_{-0.036}$	$-0.431^{+0.037}_{-0.040}$
ρ/ω coupling $C_-(pp)$ (mb)	$-14.0^{+2.6}_{-3.3}$	$-16.5^{+3.3}_{-4.2}$	$-17.7^{+3.7}_{-4.9}$
a/f coupling (mb)	$67.0^{+7.6}_{-6.7}$	$66.6^{+8.3}_{-7.2}$	$67.6^{+9.0}_{-7.8}$

Table 3: Simple pole fit to total pp and $\bar{p}p$ cross sections, and to the ρ parameter, assuming degenerate $C = +1$ and $C = -1$ meson exchanges.

Before concluding on the best value of the intercept, we need to examine the influence of low energy cut on the determination of the intercept. Although the energy cut \sqrt{s}_{min} eliminates sub-leading meson trajectories, there is still an ambiguity in the treatment of the leading meson trajectories. In fact, a slightly different treatment to that of DL leads to a better χ^2 and to more stable parameters. Indeed, there is neither theoretical nor experimental reason to assume that the ρ , ω , f and a trajectories are degenerate. On the other hand, the data are not constraining enough to determine the effective intercepts of the four meson trajectories together with the pomeron intercept. We adopt an intermediate approach, which is to assume the exchange of separate $+$ and $-$ trajectories with independent intercepts:

$$Im\mathcal{A}(s, 0) = C_-s^{\alpha_-} + C_+s^{\alpha_+} + C_Ps^{\alpha_P} \quad (3)$$

The resulting numbers are shown in Table 4, and are plotted in Fig. 3 with plain lines. The χ^2 is smaller and the parameters are more stable than in the previous case. The bounds on the soft pomeron intercept hardly depend on the criterion used to filter the data, and intercepts as large as 1.108 are allowed. In order to better understand the treatment of the errors, we give in Table 5 the result of a fit to the data of Ref. [8]. We see that the results are very stable, especially those for the pomeron intercept, independently of the data filtering.

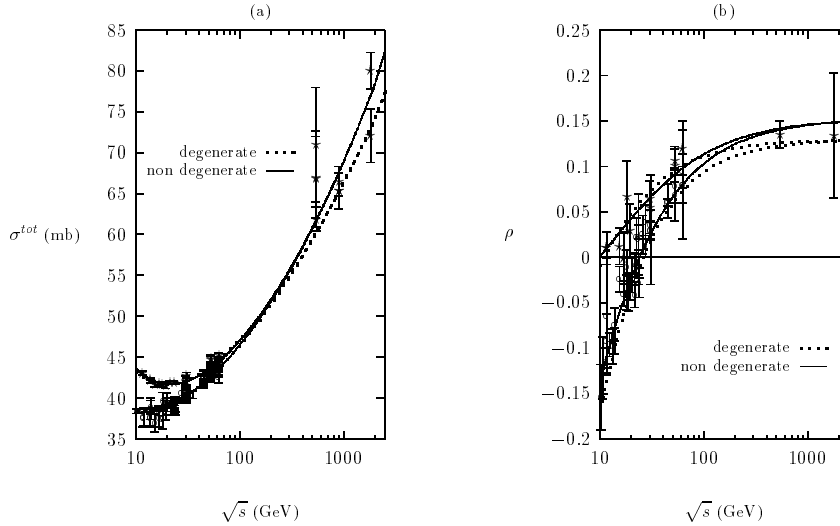


Figure 3: Best fits to 2σ filtered data. The dotted lines correspond to the original DL model given in Eq. (2), whereas the plain ones correspond to a model where the degeneracy of the lower trajectories is lifted, as in Eq. (3). The data points are the PDG data filtered at the 2σ level.

parameter	all data	filtered data (2σ)	filtered data (1σ)
χ^2	505.4	119.6	57.6
χ^2 per d.o.f.	2.99	0.77	0.47
pomeron intercept-1	$0.0990^{+0.0099}_{-0.0088}$	$0.0964^{+0.0115}_{-0.0091}$	$0.095^{+0.013}_{-0.010}$
pomeron coupling (mb)	$17.5^{+1.9}_{-2.0}$	$18.0^{+2.0}_{-2.2}$	$18.2^{+2.3}_{-2.6}$
ρ/ω intercept-1	$-0.494^{+0.056}_{-0.066}$	$-0.498^{+0.057}_{-0.067}$	$-0.510^{+0.064}_{-0.077}$
ρ/ω coupling $C_-(pp)$ (mb)	$-24.0^{+6.8}_{-10.9}$	$-26.5^{+7.7}_{-12.5}$	$-28.2^{+8.9}_{-15.4}$
a/f intercept-1	$-0.312^{+0.051}_{-0.052}$	-0.315 ± 0.058	-0.324 ± 0.066
a/f coupling (mb)	$56.8^{+8.1}_{-6.7}$	$54.9^{+9.0}_{-7.2}$	$56.2^{+9.9}_{-7.8}$
$\sigma_{tot}(1.8 \text{ TeV})$ (mb)	$77.6^{+2.5}_{-2.7}$	$76.8^{+2.9}_{-2.7}$	$76.4^{+3.4}_{-3.1}$
$\sigma_{tot}(10 \text{ TeV})$ (mb)	$108.4^{+7.0}_{-6.7}$	$106.4^{+7.9}_{-6.7}$	$105.4^{+9.2}_{-7.5}$
$\sigma_{tot}(14 \text{ TeV})$ (mb)	$115.8^{+8.3}_{-7.7}$	$113.5^{+9.3}_{-7.8}$	$112.3^{+10.8}_{-8.6}$

Table 4: Simple pole fit to total pp and $\bar{p}p$ cross sections, and to the ρ parameter, with non-degenerate $C = +1$ and $C = -1$ meson exchanges.

parameter	all data	filtered data (2σ)	filtered data (1σ)
χ^2	197.9	107.7	56.3
χ^2 per d.o.f.	1.39	0.82	0.52
pomeron intercept-1	$0.0955^{+0.0097}_{-0.0083}$	$0.0940^{+0.0092}_{-0.0079}$	$0.095^{+0.013}_{-0.010}$
pomeron coupling (mb)	$18.4^{+1.8}_{-2.0}$	$18.8^{+1.7}_{-2.0}$	$18.5^{+2.1}_{-2.6}$
ρ/ω intercept-1	$-0.535^{+0.051}_{-0.059}$	$-0.518^{+0.050}_{-0.058}$	$-0.540^{+0.059}_{-0.067}$
ρ/ω coupling $C_-(pp)$ (mb)	$-31.6^{+7.6}_{-11.5}$	$-28.9^{+6.8}_{-10.4}$	$-32.5^{+8.8}_{-13.9}$
a/f intercept-1	$-0.338^{+0.054}_{-0.055}$	$-0.355^{+0.056}_{-0.057}$	$-0.346^{+0.067}_{-0.066}$
a/f coupling (mb)	$58.8^{+8.7}_{-6.8}$	$61.5^{+9.8}_{-7.7}$	$60.4^{+10.5}_{-7.9}$
$\sigma_{tot}(1.8 \text{ TeV})$ (mb)	$77.3^{+2.6}_{-2.7}$	77.2 ± 2.6	$77.2^{+3.6}_{-3.3}$
$\sigma_{tot}(10 \text{ TeV})$ (mb)	$106.8^{+7.1}_{-6.3}$	$106.3^{+6.8}_{-6.2}$	$106.5^{+9.7}_{-7.8}$
$\sigma_{tot}(14 \text{ TeV})$ (mb)	$113.9^{+8.3}_{-7.4}$	$113.2^{+8.0}_{-7.1}$	$113.5^{+11.4}_{-9.0}$

Table 5: Simple pole fit to total pp and $\bar{p}p$ cross sections, and to the ρ parameter, with non-degenerate $C = +1$ and $C = -1$ meson exchanges, and using the alternative data set of Ref. [8].

Our best estimate for the pomeron intercept is then:

$$\alpha_P = 1.0964^{+0.0115}_{-0.0091} \quad (4)$$

based on the 2σ -filtered PDG data, in the non-degenerate case.

At this point, the only additional piece of data might be the direct observation of the pomeron, *i.e.* of a 2^{++} glueball. Using the observed X(1900) mass as confirmed by the WA91 collaboration [14], 1918 ± 12 MeV, for a $I^G J^{PC} = 0^+ 2^{++}$ state, $f_2(1900)$, and using $\alpha' = 0.250 \text{ GeV}^{-2}$ [5], we obtain $\alpha_P = 1.0803 \pm 0.012$. This is the value of the intercept for 1-pomeron exchange. The intercepts that we obtained in Tables 2, 3, 4 and 5 from scattering data cannot be directly compared with this value, as they include the effect of multiple exchanges, of pomerons and reggeons. But the values we have derived are certainly compatible with the WA91 measurement. Note however that an intercept of 1.094 would be in perfect agreement with the WA91 observation for a slope $\alpha' = 0.246 \text{ GeV}^{-2}$. Hence it would be dangerous to mix this piece of information with the t -channel information.

Finally, we can place constraints on physics beyond one-pomeron exchange. Using 2σ -filtered data, we obtain an upper bound on the ratio of the 2-pomeron coupling to that of the pomeron to be 4.7% at 70% C.L., and

including such a contribution would bring the best value for the intercept of the 1-pomeron exchange term to $1.126^{+0.051}_{-0.082}$. Also at the 70% C.L., the ratio of the coupling of an odderon to that of a pomeron is smaller than 0.1% (the best odderon intercept would then be 1.105 and the pomeron intercept become 1.099). This would correspond to 0.08 mb at the Tevatron. As for the “hard pomeron”, there is no trace of it in the data. Constraining its intercept to being larger than 1.3 leads to an upper bound on the ratio of its coupling to that of the pomeron of 0.9% (the soft pomeron intercept then becomes 1.065). This would correspond to a maximum hard contribution of 19 mb at the Tevatron. Consequently, the simple pole model fits to total cross sections are very successful.

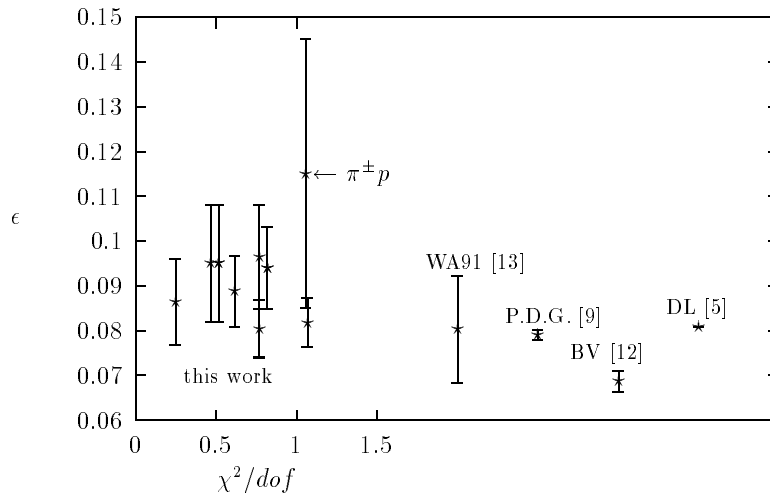


Figure 4: Our results for the pomeron intercept, compared with others in the literature. The values of the $\chi^2/d.o.f.$ are indicated for the points of this work only.

We show in Fig. 4 the results obtained in this paper together with other estimates present in the literature. All the points from this work have an acceptable χ^2 , and the main difference between them is either the filtering of data or the physics of lower trajectories. Since all these estimates are acceptable, we conclude that the pomeron intercepts as high as 1.11, and as low as 1.07, are possible. When comparing with other works in the literature, we have explained that the use of a small energy cutoff leads to smaller intercepts, and reflects the fact that sub-leading meson trajectories are to be

included. Note however that the original DL fit [6] used the same cutoff as ours, but used a different definition of χ^2 [15].

Our errors are much larger than those of other estimates because we fully take into account the correlation of the various parameters, and because our statistical analysis of the data is much more careful than previous ones. Though these results depend only on pp and $\bar{p}p$ data, we have argued that little could be learned from other hadronic reactions, given that they are measured at low energy. In particular, we point out that our fit to total cross sections, as shown in Fig. 3, is indistinguishable from the DL fit for $\sqrt{s} < 300$ GeV, and hence the parametrisation we propose is expected to fit well the total γp cross sections, as well as the πp and Kp data.

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