

THE BFKL POMERON: CAN IT BE DETECTED?

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Abstract

An estimate is derived for the absolute magnitude of BFKL pomeron exchange at $t = 0$. The analysis takes account of energy conservation and of the need realistically to model nonperturbative contributions to the BFKL integral from infrared regions. Experiment finds that there is little or no room for a significant BFKL term in soft processes, and this constrains its magnitude in hard and semihard processes, so that it is unlikely to be detectable.

1 Introduction

Total cross-sections for hadron-hadron and photon-hadron collisions all seem to increase^[1] at high energy as the same very-slowly varying power of the energy, $s^{0.08}$. This is said to be characteristic of soft pomeron exchange and is an inherently nonperturbative phenomenon. While experiment finds^[2] that the soft pomeron is exchanged also in diffractive electroproduction at quite high Q^2 , there are other high- Q^2 data in which a somewhat more rapid variation with energy is found. These are data for the small- x behavior of νW_2 , which seems^[3] to be more like $f(Q^2)(W^2)^{0.3}$ than $f(Q^2)(W^2)^{0.08}$, and exclusive J/ψ photoproduction and ρ electroproduction, which again seem to behave^[4] more like $[f(Q^2)(W^2)^{0.3}]^2$. It is not yet clear what is the cause of this more violent variation with energy.

A candidate explanation is that the perturbative BFKL pomeron is responsible^[5]. In this paper we argue that this explanation is unlikely to be correct: while the power of W predicted by the BFKL equation can fit the observed behaviour, the magnitude of the constant that multiplies it is almost certainly much too small.

A clean calculation of this magnitude is not possible, because one cannot cleanly separate the perturbative and nonperturbative effects. This problem arises already in lowest order. The crudest model^[6] for pomeron exchange is simple two-gluon exchange between quarks, figure 1. At large s this gives a constant cross-section:

$$\sigma_0 = \frac{8}{9} \int d^2 k_T \frac{\alpha_s^2}{k_T^4} \quad (1.1)$$

Here k_T is the transverse momentum of the internal quark lines, which correspond to final-state jets. It is unsafe to use this perturbative formula for the production of quark jets with too low a transverse momentum, $k_T^2 < 1 \text{ GeV}^2$, say. This is because the squared gluon 4-momentum asymptotically is just $k^2 \sim -k_T^2$, and it is illegal to extend the integration into the nonperturbative region. If we exclude this nonperturbative region from the integration in (1.1), then for fixed α_s we obtain a quark-quark cross-section equal to $1.1 \alpha_s^2 \text{ mb}$, which for any reasonable perturbative value of α_s is considerably

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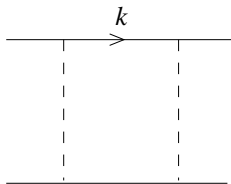


Figure 1: Exchange of two gluons between a pair of quarks.

less than the observed cross-section of a few mb: if the lowest-order calculation is a good guide most of the cross-section comes from the nonperturbative region^[7]. We argue in this paper that a similar result holds when we include higher orders, and that it applies not only to purely soft processes like total hadron-hadron cross sections, but also to semihard ones such as exclusive vector production or the small- x behaviour of structure functions.

We begin in the next section by exploring further the exchange of the perturbative pomeron between a pair of quarks at $t = 0$. This is generated by the BFKL equation. In its simplest form, the BFKL equation describes asymptotically large energies, where energy conservation constraints have become unimportant. Previous attempts^{[8][9]} to impose energy conservation have been unsatisfactory in two respects. The BFKL equation takes an input amplitude $\text{Im } T_0(s)$ and modifies it by real and virtual gluon corrections. These two types of correction need to be handled differently. Energy conservation restricts the *sum* of the transverse energies of all the real gluons to be less than \sqrt{s} , while previous work has imposed this constraint on just their individual energies and has applied it also to the virtual gluons, which is not correct.

Energy conservation imposes a cut-off at the high-momentum end of the loop integrations in the BFKL equation. As we have already indicated, the low-momentum end also needs attention, since the BFKL equation works with perturbative gluon propagators. Because of confinement effects, at small k^2 the gluon propagator receives very significant nonperturbative corrections^[10] so that, even if the BFKL equation has a finite solution with a purely perturbative propagator, this solution makes no physical sense. There have been several attempts to take this into account, none of them very satisfactory^{[8][9][11]}. They either simply exclude the low- k^2 part of the loop integration, or they try to use a nonperturbative propagator at low k^2 , or they use a nonperturbative input amplitude T_0 , which can be only part of the solution. In this paper we attempt to improve on this, though inevitably we cannot deal with two issues that arise: that of gauge invariance, and whether the BFKL equation itself, and not just the gluon propagator, must not also be modified.

In section 3, we initially impose a lower cut-off μ on the transverse momenta of the real gluons. That is, at first we calculate only a small part $\sigma(K_T > \mu)$ of the total cross-section for quark-quark scattering, arising from events where the final state consists only of any number of partons of transverse momentum greater than μ . The question arises: what is the minimum choice for μ such that the perturbative calculation of $\sigma(K_T > \mu)$ is likely to be trustworthy? By comparing our calculation with pp and $\bar{p}p$ total cross section data, we show that it is unsafe to take μ to be less than 2 GeV.

In section 4 we take account of the fact that it is extremely rare that *all* the partons will have $K_T > \mu$. In a general event, we may group the final-state partons according to their rapidities. As there is no transverse-momentum ordering, their transverse momentum is not correlated with their rapidity. So as we scan the rapidity range we find groups of partons all having transverse momentum greater than μ , with each such group separated by a group in which none of the partons has transverse momentum greater than μ . This we show in figure 2a, where the heavy lines have transverse momentum $K_T > \mu$, while the light lines have $K_T < \mu$. When we sum over all possible numbers of lines in a group with $K_T > \mu$ we obtain the hard pomeron \mathbb{P}_H which we calculate in section 3, while a group with $K_T < \mu$ sums to a soft exchange \mathbb{P}_S . So the result is figure 2b. When we sum over all final states, we obtain

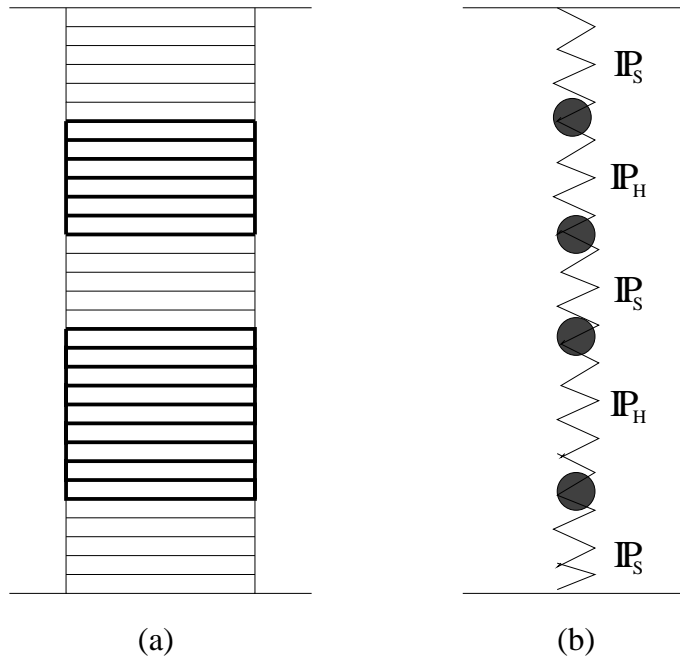


Figure 2: (a) alternating groups of partons with low and high K_T , with (b) their sum giving alternating soft and hard pomerons.

terms

$$\mathbb{P}_S + \mathbb{P}_H + \mathbb{P}_S \otimes \mathbb{P}_H + \mathbb{P}_H \otimes \mathbb{P}_S + \mathbb{P}_S \otimes \mathbb{P}_H \otimes \mathbb{P}_S + \dots \quad (1.2)$$

The separate terms here each depend on the value chosen for μ , but of course the sum must not. As we explain in section 4, this means that \mathbb{P}_S is not exactly the contribution from soft pomeron exchange, only nearly so when μ is large enough. In section 4 we analyse whether we may expect to obtain an enhancement of the fiercely-varying part of the cross-section by including in this way also final states where only a subset of the partons have transverse momentum greater than μ . That is, we ask whether mixing in contributions from soft interactions can very significantly enhance the magnitude of the contribution from the hard ones. Our conclusion is that such enhancement is at the very most an order of magnitude.

In section 5 we take a first look at a semisoft process, using $\gamma^* p \rightarrow \rho p$ as an example. While we do not expect that a “soft” process such as the total cross-section for quark-quark scattering should receive most of its contribution from states containing only partons having transverse momentum greater than 2 GeV, for semihard processes things might be different^[12]. We find that, although as Q^2 increases it is true that high-transverse-momentum partons become relatively more likely, the hard-pomeron-exchange contribution is tiny at HERA energies. Section 6 is devoted to a first look at a purely hard process, $\gamma^* \gamma^* \rightarrow \rho \rho$ at large Q^2 . Again we find that the perturbative contribution to the amplitude is tiny, but we verify that in such a hard process the “diffusion” ideas^[12] about the magnitude of the parton transverse momenta are valid.

Finally, in section 7 we discuss various points. Our discussion throughout is confined to $t = 0$, where hard-pomeron exchange has to compete with the large contribution from soft exchange.

2 The BFKL cross-section

We consider purely-gluon exchange between a pair of zero-mass quarks at $t = 0$ in 3-colour QCD. We begin by recapitulating the calculation of the lowest-order graph, figure 1. Change integration variables from k to (x, y, k_T) , where

$$k = xp + yp' + k_T \quad (2.1)$$

We need the imaginary part of the amplitude, for which the two internal quarks K_1 and K_2 are on shell. The δ -functions that put them on shell give

$$x = -y = \frac{1}{2}(-1 + R)$$

$$R = \sqrt{1 - 4k_T^2/s} \quad (2.2)$$

Because, for large \sqrt{s} , the main contribution to the integral comes from values of k_T much less than \sqrt{s} , we may approximate

$$x = -y \approx k_T^2/s$$

$$k^2 = xys - k_T^2 \approx -k_T^2 \quad (2.3)$$

These approximations then give the cross-section

$$\sigma_0 = \int \frac{d^2 k_T}{k_T^4} t_0(k_T^2)$$

$$t_0(k_T^2) = \frac{8\alpha_s^2}{9} \quad (2.4)$$

Because we want to calculate the cross-section for production of a pair of partons each having transverse momentum greater than some fixed value μ , we confine the integration to $|k_T^2| > \mu^2$. This will also prevent the integration extending into the region where the two propagators that carry momentum k are nonperturbative. In any case, we cannot simply integrate (2.4) down to $k_T = 0$; not only would this give an infrared divergence, it would also not provide any means of giving the cross-section its correct dimensiona.

It is evident from (2.2) that there must also be an upper limit, in order that R be real:

$$2k_T < \sqrt{s} \quad (2.5)$$

This is just the condition that the total transverse energy of the real partons is no more than \sqrt{s} . Of course, an exact calculation would not merely impose this kinematic constraint on the asymptotic form of the integrand associated with a given graph; it would also include nonasymptotic terms in the integrand. However, this is almost impossible to achieve beyond the lowest order, and so we shall be content with simply imposing the kinematic constraints. When $\sqrt{s} \gg \mu$ the upper limit (2.5) has little effect, but when we consider the production of a large number of partons, it becomes important. The study of this is one subject of our paper.

Cheng and Wu^[14] have calculated the sum of the order α_s^3 graphs. Their result may be written in a form that makes contact with the BFKL equation, as follows. Write the imaginary part of the α_s^{n+2} contribution to the cross-section as

$$\sigma_n(s) = \frac{(\log \bar{s})^n}{n!} \int \frac{d^2 k_n}{k_n^4} t_n(k_n)$$

a Other authors^[13] introduce a dimension into hadron-hadron scattering (though not quark-quark scattering) through a form factor, which is another way of bringing in a nonperturbative effect. In the following, we shall cut off the transverse momentum of the real emission at a scale $\mu > 1 \text{ GeV} \gg 1/1 \text{ fm}$. This means that the quark momenta entering the form factor will be very different, unless the gluons are coupled to the same quark. Hence the extra terms which lead to an infrared stable answer for $\mu = 0$, and which involve different quarks within the hadrons, are negligible in the cut-off case. This is of course not true at high- $Q^2 > \mu^2$, and we shall take them into account when we consider semihard and hard processes.

$$\bar{s} = \frac{s}{\mu^2} \quad (2.6)$$

It is just a guess that the appropriate scale for s here is μ^2 ; to check this would require an almost-impossible nonleading calculation. But it does seem the most reasonable guess. Then the imaginary part of the sum of the order α_s^3 graphs at high s may be written in the form

$$t_1(k_1) = K \otimes t_0 \quad (2.7a)$$

where

$$K \otimes t_0 = \int d^2 k_0 K(k_1, k_0) \{t_0(k_0) - \frac{1}{2}t_0(k_1)\} \quad (2.7b)$$

with^[14]

$$K(k_1, k_0) = \frac{3}{\pi^2} \frac{\alpha_s k_1^2}{k_0^2 (k_0 - k_1)^2} \quad (2.7c)$$

Written in this way, the relation (2.7) between the $n = 1$ and $n = 0$ terms is just the BFKL relation: The first term in the curly bracket is associated with real gluons, and the second with virtual. The virtual-gluon term may be written in a more familiar form^[15], by using the simple identity

$$\frac{1}{k_0^2 (k_0 - k_1)^2} = \frac{1}{k_0^2 [k_0^2 + (k_0 - k_1)^2]} + \frac{1}{(k_0 - k_1)^2 [k_0^2 + (k_0 - k_1)^2]}$$

When we subject this to the integration in (2.7), the last two terms contribute equally, as can be seen by the change of integration variable

$$k_0 \rightarrow k_1 - k_0$$

Hence we can write (2.7) in a form that has become more familiar^[8]

$$t_1(k_1) = \frac{3}{\pi^2} \int d^2 k_0 \frac{\alpha_s k_1^2}{(k_1 - k_0)^2} \left\{ \frac{t_0(k_0)}{k_0^2} - \frac{t_0(k_1)}{k_0^2 + (k_0 - k_1)^2} \right\} \quad (2.8)$$

We shall assume that (2.7) generalises to higher values of n :

$$t_n(k_n) = \int d^2 k_{n-1} K(k_n, k_{n-1}) \{t_{n-1}(k_{n-1}) - \frac{1}{2}t_{n-1}(k_n)\} \quad (2.9a)$$

with K given in (2.7c), so that

$$t_n = K \otimes K \otimes \dots \otimes t_0 \quad (2.9b)$$

Then, in the absence of any cut-offs on the k integrations and with fixed coupling α_s , the infinite sum of σ_n over n yields^[5,8] just the familiar power of s

$$\frac{12\alpha_s}{\pi} \log 2 \quad (2.10)$$

3 The cut-off BFKL equation

We may write (2.9a) as

$$t_n(k_n) = \int d^2 k_{n-1} K(k_n, k_{n-1}) t_{n-1}(k_{n-1}) - \frac{1}{2} \phi(k_n^2) t_{n-1}(k_n) \quad (3.1)$$

where

$$\phi(k^2) = \frac{3}{\pi^2} \int d^2 q \frac{\alpha_s k^2}{q^2 (k - q)^2} \quad (3.2a)$$

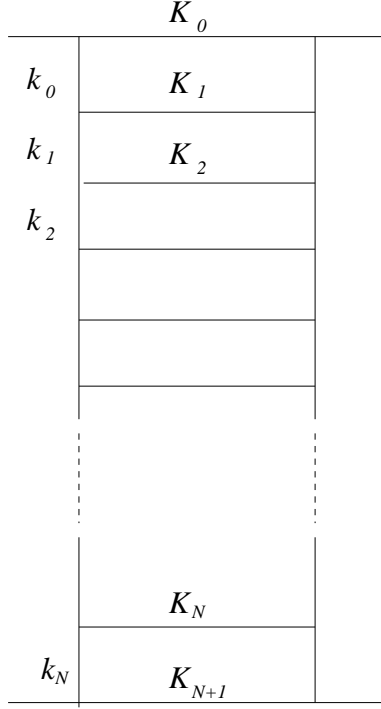


Figure 3: Kinematics of the BFKL ladder

In the absence of an infrared cut-off, each term in (3.1) is separately divergent, but the divergences cancel between them. As we have explained, the fact that this cancellation of divergences occurs does not imply that it is meaningful, because without an infrared cut-off the integration extends illegally into the nonperturbative domain.

The function ϕ represents the virtual-gluon insertions. When the divergence has been regulated somehow, we may resum these insertions. In order to do this, first write

$$\frac{(\log \bar{s})^n}{n!} = \frac{1}{2\pi i} \int \frac{dj}{j^{n+1}} \bar{s}^j \quad (3.3)$$

Then the cross-section for the production of N gluon partons plus 2 quark partons is

$$\begin{aligned} \sigma^{(N)}(s) &= \frac{1}{2\pi i} \int dj \sigma^{(N)}(j) \bar{s}^j \\ \sigma^{(N)}(j) &= \frac{8}{9} \left(\frac{3}{\pi^2} \right)^N \int d^2 k_0 d^2 k_1 \dots d^2 k_N \\ &\quad \Psi_j(k_0^2) \Psi_j(k_1^2) \dots \Psi_j(k_N^2) \frac{\alpha_s}{k_0^2} \frac{\alpha_s}{(k_0 - k_1)^2} \frac{\alpha_s}{(k_1 - k_2)^2} \dots \frac{\alpha_s}{(k_{N-1} - k_N)^2} \frac{\alpha_s}{k_N^2} \end{aligned} \quad (3.4a)$$

with

$$\Psi_j(k^2) = [j + \frac{1}{2}\phi(k^2)]^{-1} \quad (3.4b)$$

Introduce the variables

$$K_0 = k_0, \quad K_1 = k_0 - k_1, \quad K_2 = k_1 - k_2, \quad \dots, \quad K_N = k_{N-1} - k_N, \quad K_{N+1} = k_N \quad (3.5)$$

which are just the transverse momenta of the partons – see the kinematics shown in figure 3. We impose the conditions that each parton has transverse momentum at least equal to μ , and that the total transverse energy of the partons is no more than \sqrt{s} :

$$K_r^2 > \mu^2 \quad r = 0, 1, 2 \dots N + 1$$

$$|K_0| + |K_1| + |K_2| + \dots + |K_{N+1}| < \sqrt{s} \quad (3.6)$$

Because ϕ represents virtual-gluon insertions, the integration (3.2) should not have an upper limit. Nor should it have a lower limit: it does not make sense simply to remove the nonperturbative region from the integration. We have to decide what to take for the argument of α_s ; once this is done, the large- k^2 behaviour of $\phi(k^2)$ is determined, independently of how one handles the nonperturbative region. We choose to make α_s run with k^2 - the scale of α_s can only be determined by a nonleading calculation, hence we can use the most convenient choice; then for large k^2 we find that ϕ becomes constant:

$$\phi(k^2) \rightarrow 2C = \frac{72}{33 - 2N_f} \quad (3.7)$$

A well-motivated way to handle the nonperturbative region is that of Cornwall^[16] who deduced by solving Schwinger-Dyson equations that the gluon propagator $D(q^2)$ and the running coupling should be well approximated by

$$D^{-1}(q^2) = q^2 + m^2(q^2)$$

$$\alpha_s(k^2) = \frac{12\pi}{(33 - 2N_f) \log \left[\frac{k^2 + 4m^2(k^2)}{\Lambda^2} \right]} \quad (3.8a)$$

where the running gluon mass is given by

$$m^2(q^2) = m^2 \left[\frac{\log \frac{q^2 + 4m^2}{\Lambda^2}}{\log \frac{4m^2}{\Lambda^2}} \right]^{-12/11} \quad (3.8b)$$

The fixed mass m is determined^{[17][18]} from the condition that the simple exchange of a pair of gluons between quarks is the zeroth-order approximation to soft pomeron exchange at $t = 0$. This requires that the integral

$$\beta_0^2 = \frac{4}{9} \int d^2k [\alpha_s(k^2) D(k^2)]^2$$

be about 4 GeV^{-2} . With the choice $\Lambda = 200 \text{ MeV}$ this gives $m = 340 \text{ MeV}$.

A consequence of m being quite small is that $\phi(k^2)$, which is now given by

$$\phi(k^2) = \frac{3}{\pi^2} \int d^2q \alpha_s(k^2) k^2 D(q^2) D((k+q)^2) \quad (3.2b)$$

risks rapidly from its value 0 at $k^2 = 0$ and is already close to its asymptotic value by $k^2 = 10 \text{ GeV}^2$. We show this in figure 4, where we have taken 4 flavours, so the asymptotic value (3.8b) is 2.88.

The infrared cut-offs (3.6) on the variables K will tend to suppress contributions from small values of the variables k also. Hence it should be a good numerical approximation to assign $\phi(k^2)$ a constant value $2 C_{\text{eff}}$ somewhere in the range 1.5 to 2.88. We discuss this in section 7; without such an approximation, further calculation is very difficult. The larger the value of C_{eff} , the smaller the output, so if we are trying to estimate an upper bound to the amplitude we should take a fairly small value for C_{eff} . We shall work with $C_{\text{eff}} = 1.0$. We discuss this choice in section 7. With constant C_{eff} ,

$$\sigma^{(N)}(j) = \frac{8\pi^2}{27} \left(\frac{3}{\pi^2(j + C_{\text{eff}})} \right)^{N+1} \int d^2K_0 d^2K_1 \dots d^2K_{N+1}$$

-
- a This is very similar to the treatment found in ref. 5, which introduces a gluon mass to regulate the infrared divergences at intermediate steps of the calculation. In our case, the introduction of a small mass has no effect on the real emissions since $m \ll \mu$, but does matter in the virtual terms, which do depend on the details of the infrared region.
- b Note that the value of this mass is an intrinsic QCD parameter, which comes from the structure of the vacuum, and that it is in no way related to μ .

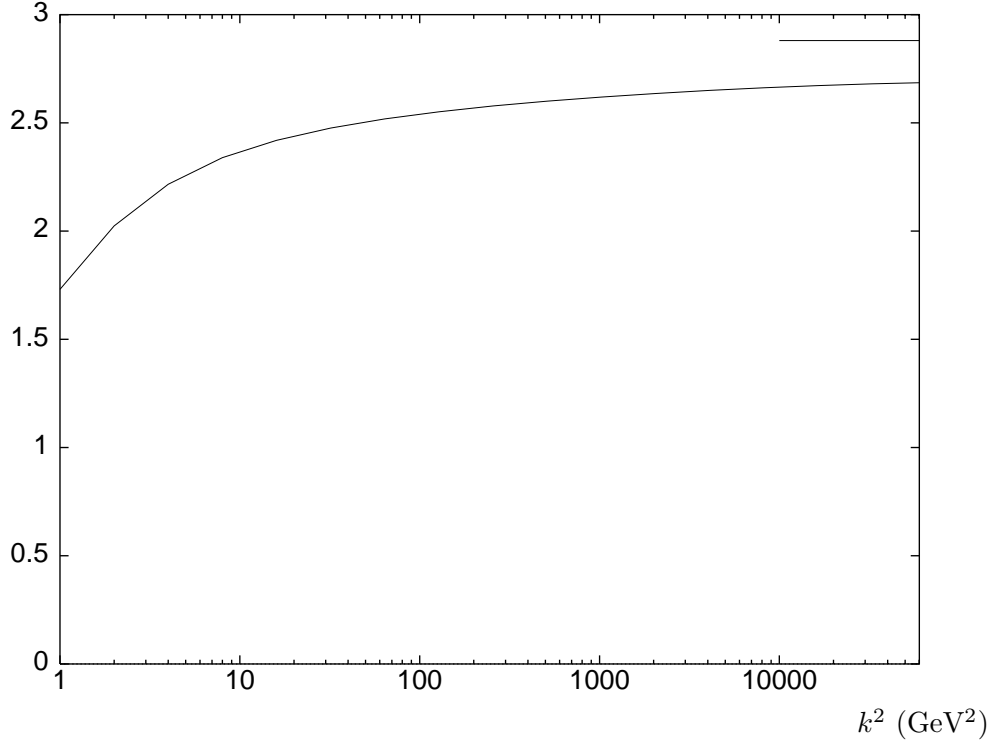


Figure 4: The function ϕ ; the horizontal line denotes the asymptotic value

$$\frac{\alpha_s}{K_0^2} \frac{\alpha_s}{K_1^2} \dots \frac{\alpha_s}{K_{N+1}^2} \delta^2(K_0 + K_1 + \dots + K_{N+1}) \theta(\sqrt{s} - |K_0| - |K_1| - \dots - |K_{N+1}|) \quad (3.9)$$

Introduce the representations

$$\begin{aligned} \delta^2(\kappa) &= \frac{1}{4\pi^2} \int d^2b e^{ib \cdot \kappa} \\ \theta(\sqrt{s} - E) \theta(E) &= \frac{i}{2\pi} \int dc e^{icE} \frac{e^{-ic\sqrt{s}} - 1}{c} \end{aligned} \quad (3.10)$$

We can then sum over N , with the result that the part of the cross-section where the final state contains only partons with transverse momentum greater than μ is

$$\sigma(s|K_T > \mu) = \frac{i\pi}{81} \bar{s}^{-C_{\text{eff}}} \int dc d^2b \frac{e^{-ic\sqrt{s}} - 1}{c} [I(b, c)]^2 \bar{s}^{I(b, c)} \quad (3.11a)$$

with

$$\begin{aligned} I(b, c) &= \frac{3}{\pi^2} \int d^2K \frac{\alpha_s(K)}{K^2} e^{i(b \cdot K + c|K|)} \\ &= C \int_{\mu}^E dK \frac{1}{K \log(K/\Lambda)} J_0(bK) e^{icK} \end{aligned} \quad (3.11b)$$

with C as in (3.7). Here, somewhat arbitrarily, we have chosen to make α_s run with K . We have also introduced the cut-offs of (3.6). The upper limit E on the K integration in (3.11b) can be any value not less than \sqrt{s} ; the θ -function in (3.9) ensures that values of K greater than \sqrt{s} will not contribute.

Numerical integration of (3.11) shows that, not surprisingly, the result is very sensitive to the value chosen for μ . This is partly because the running coupling is largest at the lower end of the K integrations. We use the lowest-order α_s , so that $\alpha_s(\mu) = 0.33$ at $\mu = 2$ and 0.47 at $\mu = 1$. We believe that the latter value, at least, is too large for a perturbative calculation of the cross-section to be valid: for reasonable safety, we should choose μ to be at least 2. (We recall that the usual evaluation of the simple answer (2.10) for the Lipatov power chooses α_s to be just less than 0.2, making the power 0.5.)

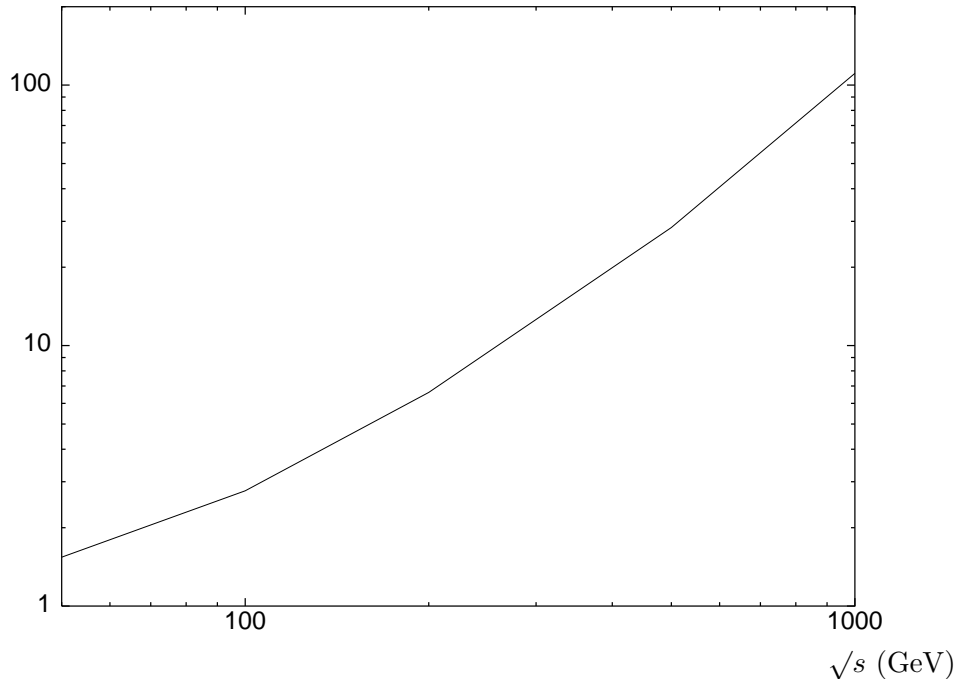


Figure 5: $\sigma_{qq}(K_T > \mu)$ in microbarns for $\mu = 2$ GeV

Although the BFKL pomeron is not supposed to be the dominant term in total cross sections, we can certainly calculate the perturbative contribution to pp and $\bar{p}p$ once we impose that the intermediate state lies in the perturbative region. Surprisingly, we find that we have to go to rather large values of μ to ensure compatibility with the data.

The curve in figure 5 shows the output for $\sigma(s|K_T > \mu)$ with $\mu = 2$ GeV. The output shown is for quark-quark scattering; according to the additive-quark rule we should multiply it by 9 to obtain the contribution to the pp or $\bar{p}p$ total cross-section. However, the variable \sqrt{s} is the centre-of-mass energy of the quark pair, which is approximately 1/3 that of the pp or $\bar{p}p$ system. (We are assuming that the quark additivity that is valid^[1] for soft pomeron exchange is also applicable to hard-pomeron exchange. This means that we are neglecting a possible shadowing suppression associated with the gluons coupling to different quarks in a hadron.)

We show in the next section that $\sigma(s|K_T > \mu)$ must be multiplied by a factor which may approach an order of magnitude, to account for nonperturbative effects. We may conclude from pp or $\bar{p}p$ total cross-section data that the value 2 GeV we have used for μ is the minimum safe value for the perturbative calculation. This is because there is little or no room in the existing pp or $\bar{p}p$ total cross-section data for anything that rises with s more rapidly than soft pomeron exchange^[19]. We recall^[1] that soft-pomeron exchange describes the rising component of the cross sections extremely well over a huge range of s , from $\sqrt{s} = 5$ GeV or less, to 1800 GeV. There perhaps is some room in the data for a hard-pomeron component in addition, depending on which of the two conflicting Tevatron experiments^[20] one believes. If one accepts the CDF result, there could be a hard-pomeron contribution that has reached as much as 10 mb at Tevatron energy. Because it would fall rapidly with decreasing \sqrt{s} , this would not cause a problem with the fit to the data at ISR energies and below. 10 mb is approximately the value we deduce from figure 5 when we allow for nonperturbative corrections. If we changed from $\mu = 2$ GeV to 1 GeV, we would obtain at qq energy $\sqrt{s} = 600$ GeV an increase of 3 orders of magnitude, which is certainly excluded. If the hard-pomeron-exchange contribution at the Tevatron is actually somewhat less than 10 mb, then the message is that the “safe” value for μ is higher than 2 GeV.

Before we discuss the nonperturbative corrections to the perturbative calculation in the next section, we point out the importance of the energy-conservation constraint. Without this constraint, (3.11)

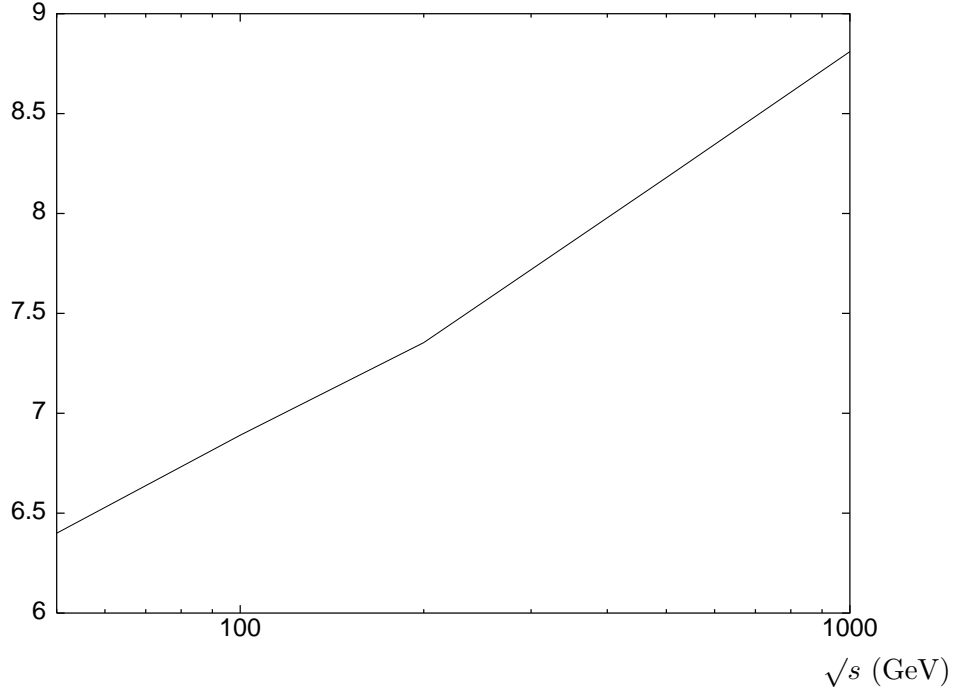


Figure 6: Effect of energy conservation: ratio of (4.1) to (3.11a) for the case $\mu = 2$ GeV

becomes

$$\sigma(s|K_T > \mu) = \frac{2\pi^2}{81} \bar{s}^{-C_{\text{eff}}} \int d^2b [I(b, 0)]^2 \bar{s}^{I(b, 0)} \quad (3.12a)$$

with

$$I(b, 0) = C \int_{\mu}^{\infty} dK J_0(bK) \frac{1}{K \log(K/\Lambda)} = C \int_{b\mu}^{\infty} dz J_0(z) \frac{1}{z \log(z/b\Lambda)} \quad (3.12b)$$

Figure 6 shows the ratio of (4.1) to (3.11a) for the case $\mu = 2$ GeV.

4 Complete total cross-section

In section 3 we derived $\sigma_{qq}(K_T > \mu)$, the contribution to the total quark-quark cross-section from events where the final state contains only partons with transverse momentum greater than μ . We showed that, while this has the expected fierce variation with energy, it is numerically quite small at reasonable energies. In this section we explore whether the fiercely-varying contribution is significantly enhanced by the inclusion of final states where there are also partons whose transverse momentum is less than μ . Our calculation is for a “quenched” pomeron: we do not include quark loops, though they may well be important^[21].

Define $\sigma'_{qq}(s)$ by

$$\sigma_{qq} = \sigma'_{qq} + \sigma_{qq}(K_T > \mu) \quad (4.1)$$

The choice of μ is arbitrary, except we want to make it large enough for us to be able to calculate $\sigma_{qq}(K_T > \mu)$ purely perturbatively. Because σ_{qq} is the total cross-section, it does not depend on μ , so σ'_{qq} must have a variation with μ that compensates that of $\sigma_{qq}(K_T > \mu)$. However, this variation is a tiny fraction of the total σ'_{qq} , because $\sigma_{qq}(K_T > \mu) \ll \sigma_{qq}$. Thus, while by definition $\sigma_{qq}(K_T > \mu)$ is the result of the exchange of the perturbative BFKL pomeron \mathbb{P}_H , σ'_{qq} essentially corresponds to the observed exchange of the soft pomeron \mathbb{P}_S . In what follows, we also need $\sigma_{qq}(K_T < \mu)$, the contribution to the cross-section from events where there are no partons having transverse momentum greater than μ . Evidently $\sigma_{qq}(K_T < \mu) \leq \sigma'_{qq}$ and therefore $\sigma_{qq}(K_T < \mu) \approx \sigma_{qq}$ provides a good estimate of the largest possible value for $\sigma_{qq}(K_T < \mu)$. Of course, at reasonably high energy it is

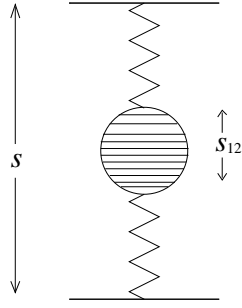


Figure 7: Hard exchange sandwiched between two soft exchanges

rather likely that at least one parton with transverse momentum greater than μ will be produced, so that actually $\sigma_{qq}(K_T < \mu)$ will be considerably less than σ_{qq} .

Exactly similar statements apply to the cross-sections for quark-gluon and gluon-gluon scattering. Also, these cross-sections are related by factorisation to those for quark-quark scattering. In the case of those cross-sections that correspond to the exchange of the soft pomeron \mathbb{P}_S , in particular our upper estimate for $\sigma(K_T < \mu)$, this factorisation is the result of the pomeron apparently being a simple pole in the complex angular momentum plane; it has recently been well tested^[2] at HERA. In the case of the exchange of the BFKL pomeron \mathbb{P}_H , the factorisation results from the factorisation of the leading part of the lowest-order contribution: the coupling to a gluon, averaged over spins and colours, is 9/4 times that to a quark^[22].

We explained in section 1 how, in a general event, we may group the final-state partons according to their rapidities, and that summing over final states leads to the series (1.2). As we have explained above, by using the full soft pomeron \mathbb{P}_S we over-estimate $\sigma(K_T < \mu)$ and then have upper bounds for the terms in (1.2). The question we explore now is whether, by coupling \mathbb{P}_H to quarks through \mathbb{P}_S instead of directly, we increase the effective strength of its coupling. First, we investigate whether the term $\mathbb{P}_S \otimes \mathbb{P}_H \otimes \mathbb{P}_S$ is significantly larger than \mathbb{P}_H . We show this term in figure 7. In a previous paper^[23], we have calculated the term in the approximation where \mathbb{P}_H is replaced with its lowest-order contribution. It is a simple matter to use instead the whole of \mathbb{P}_H :

$$\mathbb{P}_S \otimes \mathbb{P}_H \otimes \mathbb{P}_S = 32\pi^2 \beta_0^2 g^2 \int_{4\mu^2}^s \frac{ds_{12}}{s_{12}} \left(\frac{s}{s_{12}} \right)^{\epsilon_0} \sigma_{gg}(s_{12}|K_T > \mu) \log \left(\frac{s}{s_{12}} \right) \quad (4.2)$$

This is for quark-quark scattering. Here, $(1 + \epsilon_0)$ with $\epsilon_0 \approx 0.08$ is the intercept of the soft pomeron, $\beta_0 \approx 2 \text{ GeV}^{-1}$ is the strength of its coupling to a quark, and $g \approx 15 \text{ MeV}$ its coupling to a gluon. (We defined^[23] β_0 and g in somewhat different ways, such that they even have different dimensions, so the values we have given for them do not directly reflect the relative strengths of the two couplings). We fit the upper energy part of the curve in figure 5 with an effective power,

$$\sigma_{qq}(s|K_T > \mu) \sim 2\beta_H^2 (s/\mu^2)^\lambda \quad (4.3a)$$

This gives

$$\begin{aligned} \lambda &\approx 0.86 \\ \beta_H^2 &\approx 3.3 \times 10^{-6} \text{ GeV}^{-2} \end{aligned} \quad (4.3b)$$

Then $\sigma_{gg}(s_{12}|K_T > \mu)$ is 81/16 times this, We find that (4.2) is approximately

$$\frac{324\pi^2 \beta_0^2 g^2 \beta_H^2}{(\lambda - \epsilon_0)^2} (s/\mu^2)^\lambda \approx \frac{1.5}{\lambda^2} \sigma_{qq}(s|K_T > \mu) \quad (4.4)$$

This is a few times $\sigma_{qq}(s|K_T > \mu)$, but remember that it is an upper bound.

Consider now the term $\mathbb{P}_S \otimes \mathbb{P}_H \otimes \mathbb{P}_S \otimes \mathbb{P}_H \otimes \mathbb{P}_S$, where the final state contains two groups of high-transverse-momentum partons. This term is^[23]

$$512\pi^4 \beta_0^2 g^4 \int_{4\mu^2}^s \frac{ds_{12} ds_{34}}{s_{12} s_{34}} \left(\frac{s}{\alpha' s_{12} s_{34}} \right)^{\epsilon_0} \sigma_{gg}(s_{12}|K_T > \mu) \sigma_{gg}(s_{34}|K_T > \mu) \log^2 L \theta(L - 1)$$

$$L = \frac{\mu_0^2 s}{s_{12} s_{34}} \quad (4.5)$$

Here, the upper limit on the integrations is the surface $L < 1$ and μ_0 is a nonperturbative scale, expected^[23] to be about 1 GeV, associated with the coupling of the soft pomeron to gluons. Using again the effective-power description (4.4) for $\sigma_{gg}(K_T > |\mu)$, we find that (4.5) gives (4.4) times

$$\frac{324\pi^2 g^2 \beta_H^2}{(\lambda - \epsilon_0)} (\mu^2 / \mu_0^2)^\lambda \log \frac{\mu_0^2 s}{16\mu^4} \quad (4.6a)$$

which is approximately (4.4) times

$$\frac{2 \times 10^{-6}}{\lambda} (\mu^2 / \mu_0^2)^\lambda \log \frac{\mu_0^2 s}{16\mu^4} \quad (4.6b)$$

This is much less than (4.4) until the energy is very high indeed.

We must consider also the terms $\mathbb{P}_S \otimes \mathbb{P}_H$ and $\mathbb{P}_H \otimes \mathbb{P}_S$. They are equal, and their sum works out to be

$$36\pi g \beta_0 \sigma_{qq}(s|K_T > \mu) \approx 3\sigma_{qq}(s|K_T > \mu) \quad (4.7)$$

Again this is an upper bound.

Our conclusion is that the inclusion of the nonperturbative contributions multiplies the rapidly-rising component of the cross section by a number that is at most an order of magnitude when the lower limit μ of the perturbative calculation is chosen to be 2 GeV. As we explained in the last section, this leads us to conclude that it is unsafe to choose μ to be significantly lower than this, because it would conflict with data for the $\bar{p}p$ total cross section.

5 Semihard processes

As a first look at a semihard process, we consider $\gamma^* q \rightarrow \rho q$. At high Q^2 , the dominant polarisations for the γ^* and ρ are longitudinal^{[24][25][17]}. In lowest order, the amplitude is given by the graph of figure 8. We consider forward scattering in the zero-mass limit. The amplitude is

$$a_0(Q^2) = i \int \frac{d^2 k_T}{k_T^4} u_0(Q^2, k_T^2)$$

$$u_0(Q^2, k_T^2) = \frac{16ef_\rho}{3\sqrt{3}Q} \frac{\alpha_s^2 k_T^2}{(k_T^2 + \frac{1}{4}Q^2)} \quad (5.1)$$

We have used a nonrelativistic wave function for the ρ . We have used perturbative gluon propagators, in place of the nonperturbative ones of reference 17, because here we wish to consider the exchange of the perturbative pomeron rather than the nonperturbative. The similarity with (2.3) is evident, so that we may immediately write down the result of making all the higher-order insertions as in sections 2 and 3. Instead of (3.11a),

$$a(s, Q^2 | K_T > \mu) = \frac{2\pi ef_\rho}{27\sqrt{3}Q} x^{C_{\text{eff}}} \int dc d^2 b \frac{e^{-ic\sqrt{s}} - 1}{c} I(b, c) J(b, c, Q^2) x^{-I(b, c)} \quad (5.2)$$

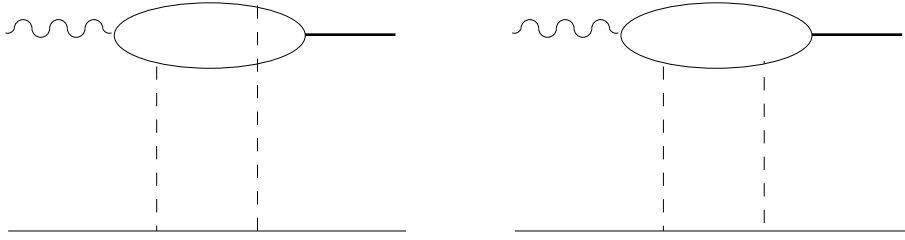


Figure 8: Lowest-order graphs for $\gamma^* q \rightarrow \rho q$

with

$$J(b, c, Q^2) = \frac{3}{\pi^2} \int d^2 K \frac{\alpha_s(K)}{K^2 + \frac{1}{4}Q^2} e^{i(b \cdot K + c|K|)} \quad (5.3)$$

We assume that it is now appropriate to replace \bar{s} with $x^{-1} = s/Q^2$. The forward-scattering differential cross-section is obtained by squaring the amplitude (5.2), and dividing by $16\pi^2$. We should have to multiply by 3 if we were to change from a quark target to a proton.

It seems that, for large enough Q^2 , the amplitude (5.2) at fixed x varies as $1/Q^3$. However, it turns out that the integral varies quite slowly with Q^2 until Q^2 becomes extremely large, and so until then the fall-off with increasing Q^2 of the differential cross-section is much slower than $1/Q^6$. This conclusion contrasts with that of Brodsky and collaborators^[26], who guess that the asymptotic Q^2 -dependence of perturbative exchange is achieved quite early.

At NMC energies, it is soft pomeron exchange rather than perturbative exchange that describes the data^{[24][27]}. This has been tested out to the $Q^2 \approx 20 \text{ GeV}^2$, by which time the soft exchange has already achieved the $1/Q^6$ fall-off. But at HERA energies for the same Q^2 values there seems to be a more rapid rise^[4] with energy than is expected from soft pomeron exchange. One might seek to explain this by supposing that the BFKL contribution is fairly small at NMC energies but, with its more rapid rise than the soft pomeron term, it has become important at HERA energies. Our calculations suggest that this is unlikely.

The soft-pomeron-exchange amplitude is^[24]

$$a_{\text{soft}}(s, Q^2) = \left(\frac{s}{s_0}\right)^{\epsilon_0} \int \frac{d^2 k_T}{(k_T^2 + \frac{1}{4}Q^2)} k_T^2 D^2(-k_T^2) u_0(Q^2, k_T^2) \quad (5.4a)$$

where D is the nonperturbative gluon propagator^[7] and $\epsilon_0 \approx 0.08$ is the intercept of the soft pomeron trajectory, with $s_0 \approx 4 \text{ GeV}^2$. For large Q^2 the soft amplitude may be written as^[24]

$$a_{\text{soft}}(s, Q^2) = \frac{8\sqrt{3}ief_\rho}{Q^3} \beta_0^2 \mu_0^2 \left(\frac{s}{s_0}\right)^{\epsilon_0} \quad (5.4b)$$

where β_0 is the coupling of the soft pomeron to a light quark and μ_0 is a mass scale which experiment finds^[24] to be about $2/\beta_0$.

In figure 9a we plot the hard and soft amplitudes at $\gamma^* q$ energy $\sqrt{s} = 50 \text{ GeV}$. We have used $\mu = 2 \text{ GeV}$. Since the soft amplitude is of the same order of magnitude as the data for $Q^2 < 20 \text{ GeV}^2$, the hard amplitude is obviously unimportant. Even if we allow for the nonperturbative corrections of the type we have discussed in section 4, at $Q^2 = 100 \text{ GeV}^2$ it is at least an order of magnitude smaller than the soft amplitude. Figure 9b shows the Q^2 variation of the hard amplitude at fixed x . At $Q^2 = 1000 \text{ GeV}^2$ the x -dependence, averaged between $1/x = 100$ and $1/x = 10000$, is $(1/x)^{0.58}$, but at smaller Q^2 the variation is slower — $(1/x)^{0.44}$ at $Q^2 = 10 \text{ GeV}^2$. Notice that at fixed x the fall-off of the amplitude with Q^2 is very much slower than $1/Q^3$.

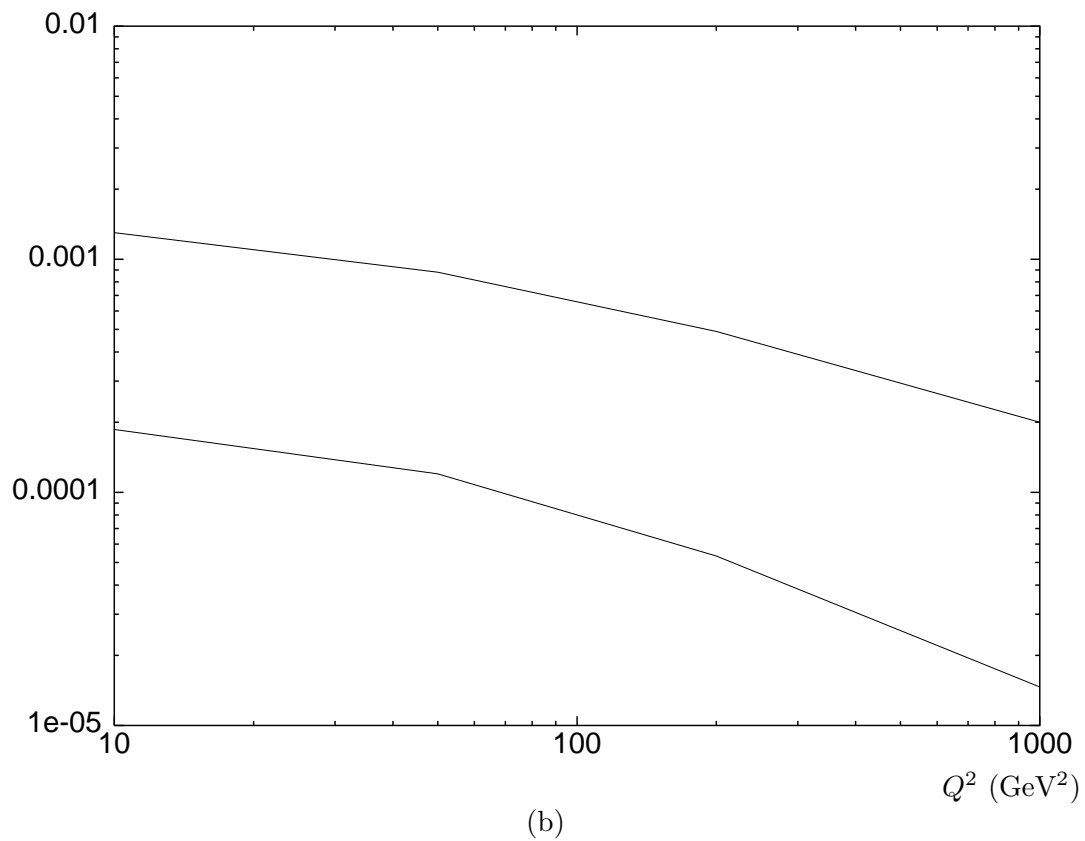
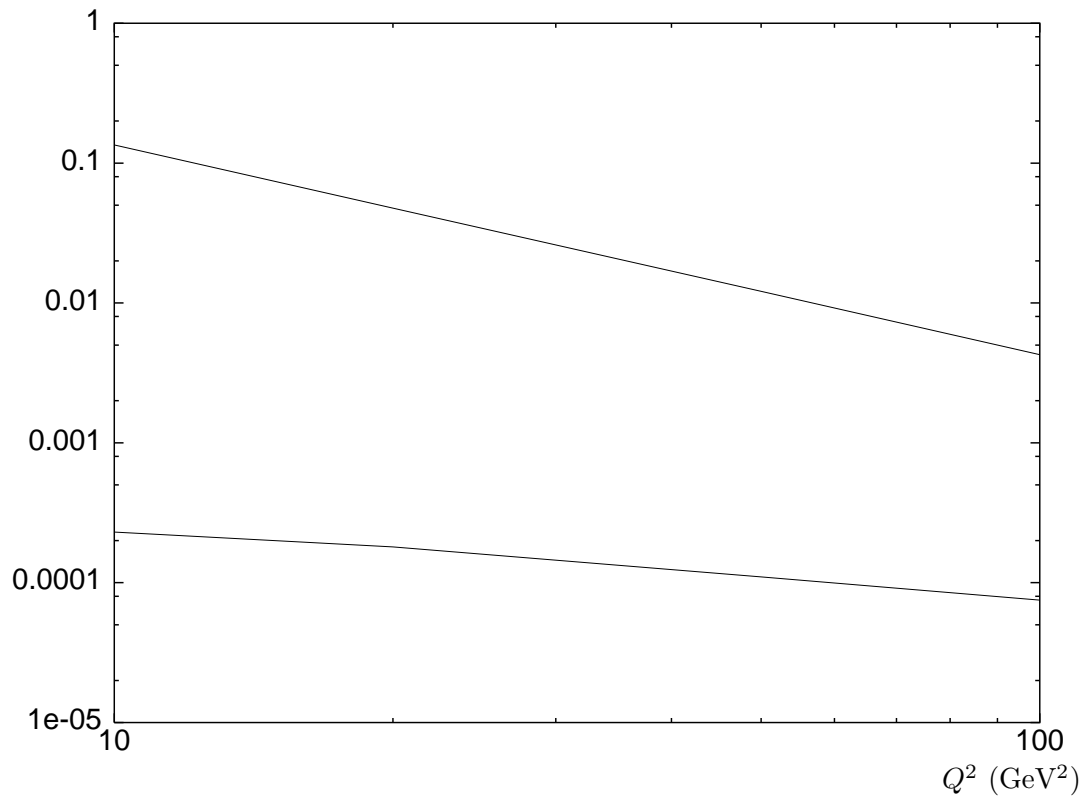


Figure 9: the amplitude for $\gamma^*q \rightarrow \rho q$ in GeV units (a) at $\sqrt{s} = 50$ GeV, hard exchange (lower curve) and soft exchange (upper curve) (b) hard exchange at $x = 0.01$ (lower curve) and $x = 10^{-4}$ (upper curve)

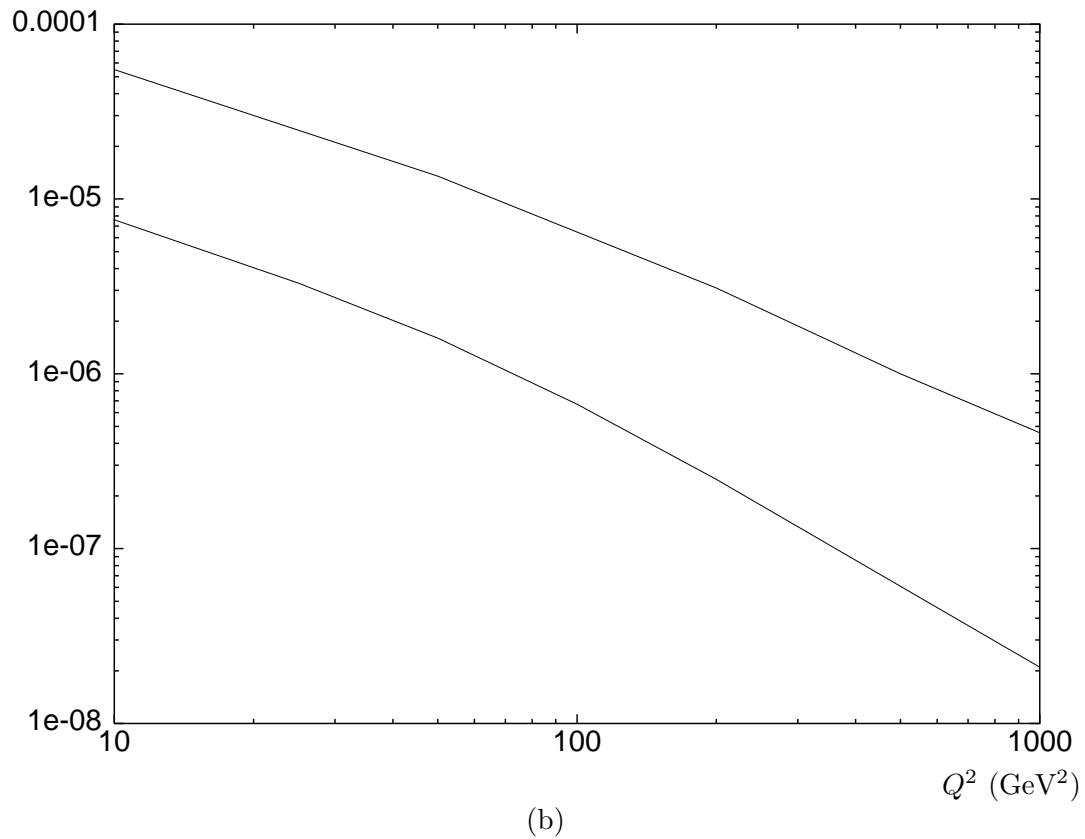
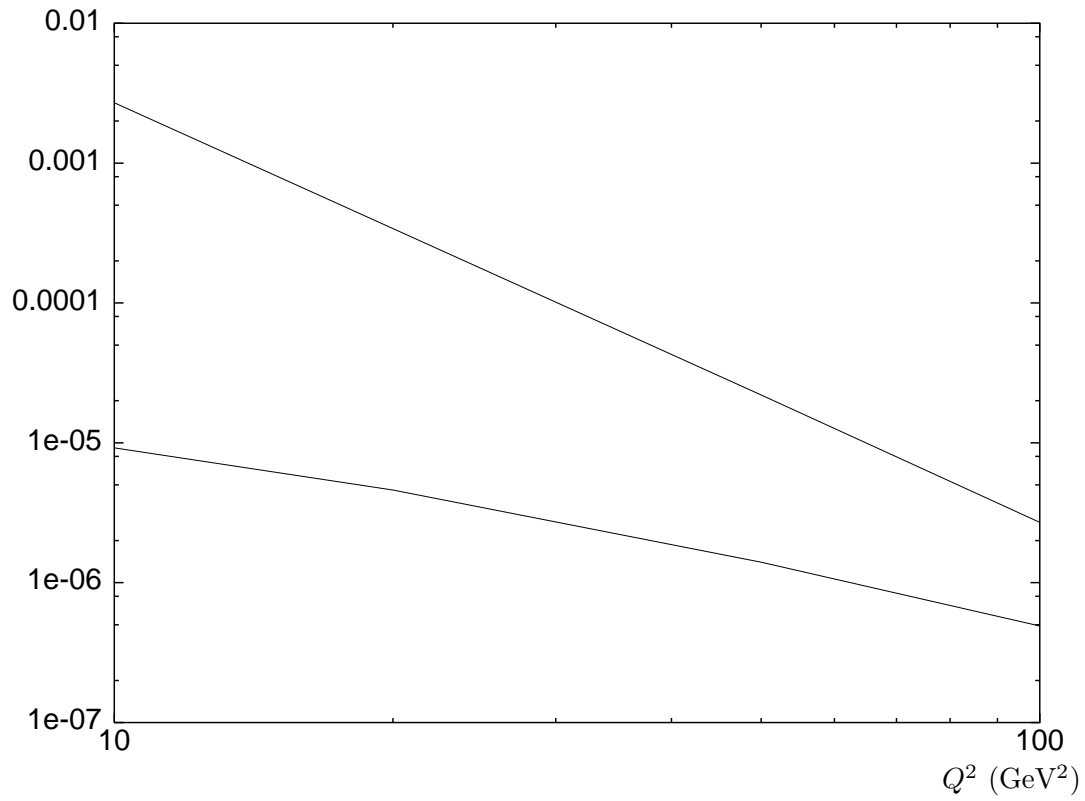


Figure 10: the amplitude for $\gamma^*\gamma^* \rightarrow \rho\rho$ in GeV units (a) at $\sqrt{s} = 50$ GeV, hard exchange (lower curve) and soft exchange (upper curve) (b) hard exchange at $x = 0.01$ (lower curve) and $x = 10^{-4}$ (upper curve)

In this semihard process the dependence on μ is found to be much less than for the purely soft process. At $\sqrt{s} = 100$ GeV, changing from $\mu = 2$ GeV to 1 GeV increases the $qq \rightarrow qq$ amplitude by two orders of magnitude, but for the semihard $\gamma^*q \rightarrow \rho q$ at the same \sqrt{s} this factor has reduced to 5 by $Q^2 = 1000$ GeV².

6 Hard process

For an initial look at a purely hard process we choose $\gamma^*\gamma^* \rightarrow \rho\rho$. We take the two photons to have the same virtuality Q^2 and again consider forward scattering and longitudinal polarisations. The lowest-order amplitude is

$$a_0(Q^2, Q^2) = i \int \frac{d^2k_T}{k_T^4} v_0(Q^2, Q^2, k_T^2) \quad (6.1)$$

There is k_T -factorisation^[28], so that

$$v_0(Q^2, Q^2, k_T^2) = \frac{u_0^2(Q^2, k_T^2)}{t_0(k_T^2)} \quad (6.2)$$

and (5.2) becomes

$$a(s, Q^2, Q^2 | K_T > \mu) = \frac{4\pi e^2 f_\rho^2}{27Q^2} x^{C_{\text{eff}}} \int dc d^2b \frac{e^{-ic\sqrt{s}} - 1}{c} J^2(b, c, Q^2) x^{-I(b, c)} \quad (6.3)$$

Superficially, at fixed x the Q^2 dependence at large Q^2 of this amplitude is $1/Q^6$, but Q^2 needs to be extremely large to get anywhere near this. It is uncertain how soft-pomeron exchange contributes to this process, but if we assume that it factorises we obtain the amplitude

$$a_{\text{soft}}(s, Q^2, Q^2) = \frac{48ie^2 f_\rho^2}{Q^6} \beta_0^4 \mu_0^4 \left(\frac{s}{s_0} \right)^{\epsilon_0} \quad (6.4)$$

In figure 10a we plot the hard and soft amplitudes against Q^2 , for $\sqrt{s} = 50$ GeV. If we include the nonperturbative corrections of the type we discussed in section 4, it may be that the hard amplitude becomes comparable with the soft by $Q^2 = 100$ GeV², but by then the cross-section is very small. In figure 10b we plot the hard amplitude at $x = 10^{-2}$ and 10^{-4} .

We said at the end of the last section that for the semihard process $\gamma^*q \rightarrow \rho q$ at $\sqrt{s} = 100$ GeV and $Q^2 = 1000$ GeV² the effect of decreasing μ from 2 to 1 GeV is to increase the amplitude by a factor of 5, which is very much less than for the purely soft process $qq \rightarrow qq$ at the same energy. For the hard process at the same \sqrt{s} and Q^2 the factor is further reduced, to about 3. Also, while for the soft process at this energy relaxing the energy-conservation constraint, as in (3.12), increases the amplitude by a factor close to 7, for the hard process this factor is only about 5. Further, while for $Q^2 = 10$ GeV² the x -dependence averaged between $1/x = 100$ and $1/x = 10000$ is almost the same as for the semihard process, at $Q^2 = 1000$ GeV² it is rather fiercer: $(1/x)^{0.69}$ instead of $(1/x)^{0.58}$. These conclusions are in line with general expectations about the ‘‘diffusion’’ of transverse momentum^[12] at high Q^2 , though the effect is perhaps not as dramatic as might have been hoped.

7 Discussion

We have argued in this paper that the BFKL pomeron is not detectable, at least at $t = 0$. This means that some other explanation must be found for the rapid rise in the HERA data for ρ electroproduction and presumably also for F_2 at small x , for example^[29] the onset of perturbative Altarelli-Parisi evolution at a smaller value of Q^2 than many people expected.

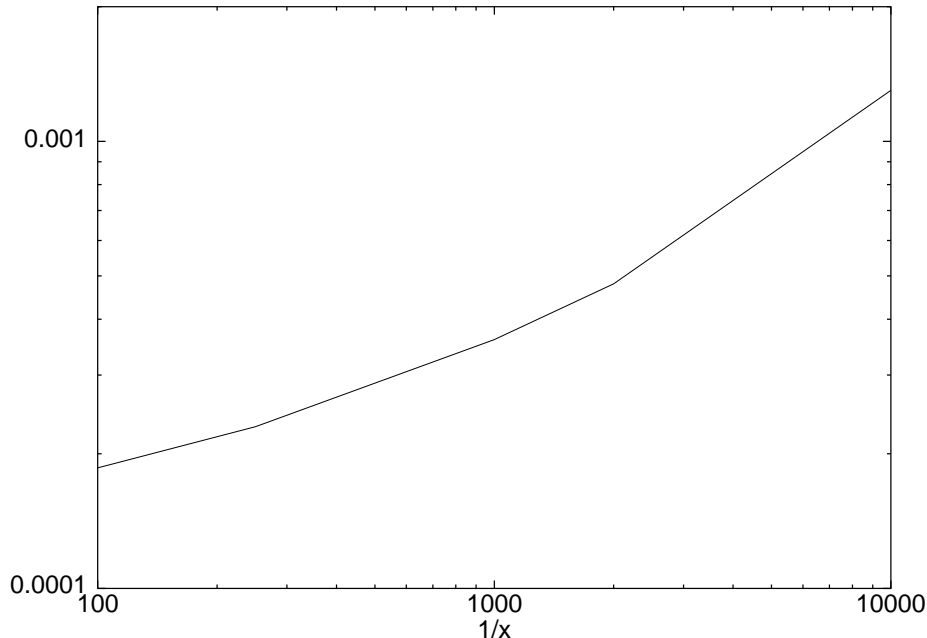


Figure 11: the hard amplitude for $\gamma^* q \rightarrow \rho q$ at $Q^2 = 10 \text{ GeV}^2$

We have considered only the BFKL pomeron at $t = 0$. It remains possible that at large t the situation will be different since then, even if the BFKL contribution is small, there is much less competition from nonperturbative mechanisms. We intend examining this in a future paper.

As the discussion of the BFKL pomeron inevitably requires consideration of nonperturbative contributions, it must involve considerable uncertainty. Our approach, when faced with decisions where there is theoretical uncertainty, has been to maximise the cross section, within the constraints coming from HERA and the Tevatron. One issue is the value we have chosen for C_{eff} . We have chosen it to be 1.0, and found in particular that the perturbative contribution to $\gamma^* q \rightarrow \rho q$ is totally negligible, even if we allow for the type of corrections discussed in section 4. To make it comparable with the soft contribution (and therefore with the data) we see from figure 9a that we should need to reduce C_{eff} to 0. This is not reasonable, because it would correspond to an absence of virtual corrections. More importantly, its x dependence would be totally wrong. As we show in figure 11, the choice $C_{\text{eff}} = 1$ already gives an effective power $(1/x)^{0.3}$; changing C_{eff} to 0 would make this $(1/x)^{1.3}$.

An alternative way to increase the size of the perturbative contribution is to reduce the value of μ (though again this would give the wrong x dependence). Also, one might guess that perhaps C_{eff} should vary with \sqrt{s} , being smaller than our chosen value 1.0 at the left of figure 5, and larger at the right. However, we have explained that the value of the cross-section at qq energy $\sqrt{s} \approx 600 \text{ GeV}$ is constrained by the Tevatron data to be no larger than is shown in the figure. To bring the hard-exchange curve in figure 9a near to the level of the soft-exchange curve, we need an increase of at least 2 orders of magnitude at the left in figure 5. If C_{eff} is made energy-dependent in such a way to achieve this, while keeping the curve at $\sqrt{s} \approx 600 \text{ GeV}$ at the same level, the whole curve would become so flat that it would no longer be distinguishable from soft exchange.

In applying the analysis to hard and semihard processes, we have used experimental information from soft processes: that the pp and $\bar{p}p$ total cross sections have at most only a very small very-rapidly-rising component. Although nobody expects the perturbative contribution to such soft processes to form a significant fraction of the total, it should nevertheless be present and it is important to use experimental information about how large it can be to pin down some of the uncertainties about the corresponding contributions to semi-hard and hard processes. We have said that the pp and $\bar{p}p$ total-cross-section data show that any hard pomeron contribution is no more than 10% at $\sqrt{s} = 1800 \text{ GeV}$;

because it falls so rapidly as \sqrt{s} is decreased, it becomes negligibly small at HERA energies, even in hard or semihard processes.

In our calculations, we have had to decide how to choose the arguments of the couplings α_s . The choices we made were those that enabled us to calculate most easily. In order to investigate how our choices influence the output, we consider the simpler integral (3.12) in which the energy-conservation constraint is removed. We investigate its high-energy behaviour. (We should have preferred to discuss the constrained cross-section (3.11), but we have found this to be too difficult.) The asymptotic behaviour of (3.12) is controlled by the behaviour of the integrand for small b . So we need the small- b behaviour of $I(b, 0)$, which comes from the small- z region of the integration in (4.2), that is z less than some fixed z_0 . In this region, we may replace the Bessel function with unity. Then simple integration gives

$$I(b, 0) \sim C \log \left(\frac{\log(z_0/b\Lambda)}{\log(\mu/\Lambda)} \right) \quad (7.1)$$

and so the asymptotic behaviour of the contribution from values of $|\mathbf{b}|$ less than some fixed b_0 in the integral (4.1) is

$$\begin{aligned} \frac{2\pi^3}{81} \bar{s}^{-C_{\text{eff}}} \left(\frac{\partial}{\partial \log \bar{s}} \right)^2 \left\{ [\log(\mu^2/\Lambda^2)]^{-C_{\text{eff}} \log \bar{s}} \int_0^{b_0^2} db^2 [\log(z_0^2/b^2\Lambda^2)]^{C_{\text{eff}} \log \bar{s}} \right\} \\ \sim \text{const } \bar{s}^{C_{\text{eff}} \log \log \bar{s}} \end{aligned} \quad (7.2)$$

This rise, faster than any power of s , is perhaps unexpected, though we know of no basic reason to reject it. In any case, the inclusion of the energy-conservation constraint certainly slows the rise, and ultimately, of course, it would be moderated by shadowing corrections. However, it is sensitive to how we make the coupling α_s run, in particular how we play off the running α_s in the real-gluon BFKL insertions against that in the virtuals. As we have already mentioned, there is a delicate cancellation between infrared real and virtual contributions, but the same is true of the ultraviolet. The key point is that, although when we change from the usual fixed α_s to a running one, both the real and virtual terms are suppressed, they enter with opposite signs. Hence, by suppressing the virtuals less than the reals we can actually increase the total output. In the function $\phi(k^2)$ of (3.2a), which describes the virtual corrections, we chose to make α_s run with k^2 , leading to ϕ becoming constant at large k^2 ; see (3.7). If we were to choose instead to make α_s in the integration (3.2a) run with q^2 , then instead of $\phi(k^2) \rightarrow 2C$ for large k^2 we find $\phi(k^2) \rightarrow 2C \log \log k^2$. Since each k^2 cannot be much larger than s , this is likely to reflect itself in the fixed power $\bar{s}^{-C_{\text{eff}}}$ outside the integral (3.12a) effectively being changed to $\bar{s}^{-C \log \log \bar{s}}$. This would then damp the $\bar{s}^{C \log \log \bar{s}}$ that comes from the integral, and perhaps leave something that in total looks much more like a fixed power. To decide how to make α_s run in the different contributions would need a nonleading calculation, which cannot be achieved at present, so the best that we can say is that our calculation, which perhaps rises faster with energy than it really should, already gives an output that is too small to be relevant to experiment.

Finally, we note that we have talked throughout about final-state “partons” rather than “minijets” because an observed minijet can achieve some of its high transverse momentum by including fairly soft partons. We have not attempted to separate this out from our calculations: it is buried within the contributions we have called $\mathcal{P}_S \otimes \mathcal{P}_H$ etc — see (1.2).

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