University of Wisconsin - Madison

DAMTP/89-14
MAD/PH/491
UH-511-676-89
April 1989

HOW WELL DO WE KNOW THE KOBAYASHI-MASKAWA MATRIX?

J.R. CUDELL

D.A.M.T.P., University of Cambridge, Cambridge CB39EW United Kingdom

F. HALZEN

Department of Physics, University of Wisconsin, Madison, WI 53706

and

S. PAKVASA

Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822

ABSTRACT

We evaluate the Kobayashi-Maskawa matrix using a maximum likelihood method which takes into account experimental errors on input data and "systematic errors" associated with theoretical approximations. We carefully reconsider the constraints from meson decays and mixings as well as CP violation. We derive values for all matrix elements with errors smaller than 0.008, show that the ratio $\frac{|U_{ub}|}{|U_{cb}|}$ must be bigger than 0.036, and that the phase angle δ of the K.M. representation is smaller than π. Surprisingly our conclusions depend very weakly on the value of the top quark mass. Any experiment violating the constraints of our fitted K.M. matrix would imply physics beyond the standard model with three generations and a minimal Higgs sector.

In the past three years our knowledge of meson mixing and CP violation has considerably progressed. On the experimental front $B_d^0\bar{B}_d^0$ mixing has been firmly established, direct CP violation (ϵ') has been observed. Various experimental and theoretical bounds limit the mass of the top quark to the range $70 \sim 200$ GeV. On the theoretical side we have witnessed progress in calculating long-distance weak interaction parameters using lattice techniques and the $1/N_c$ expansion. We reassess our knowledge of the minimal standard model parameters in the light of these developments.

The strategy that we use in our search for the structure of the Kobayashi-Maskawa (K.M.) matrix can be sketched as follows. The light quark matrix elements $|U_{ud}|$ and $|U_{us}|$ are directly determined by data on neutron and hyperon β-decay. The rest of the Cabibbo matrix, $|U_{cd}|$ and $|U_{cs}|$ will be determined to a higher precision from the unitarity of the matrix than from direct measurements. The rest of the matrix elements are determined from the following data:

$$\Gamma_B \sim |U_{cb}|^2 ,$$

$$\Delta m(B_d^0\bar{B}_d^0) \sim m_t^2 |U_{tb}U_{td}^*|^2 ,$$

$$\epsilon \sim m_t^2\, \mathcal{I}m\left[(U_{ts}U_{td}^*)^2\right] + m_c^2\, \mathcal{I}m\left[(U_{cs}U_{cd}^*)^2\right] ,$$

$$\epsilon' \sim (\ln m_t^2)[\mathcal{I}m(U_{ts}U_{td}^*)] .$$

The matrix elements are also restricted by unitarity, *e.g.* in the limit that all angles are small one can derive that

$$|U_{td}| = |U_{cb}| \left[s_1 + \frac{|U_{ub}|}{|U_{cb}|} \right] . \tag{1}$$

Here s_1 is the Cabibbo angle.

We note that theoretical calculations of the quantities listed above are becoming more reliable. The bag parameters, which gave the biggest uncertainty, seem to be under control in lattice calculations, at least for the B mesons.[1] The $1/N_c$ expansion gives similar answers.[2] One can thus, with a reasonable amount of confidence, use the method outlined above to derive limits on the K.M. matrix elements. As we want to establish reliable bounds, we shall exclude from our analysis data for which the theory is not under control.

Our main result is that the K.M. matrix elements are constrained to the following intervals:

$$U = \begin{pmatrix} 0.9748\text{-}0.9761 & 0.2173\text{-}0.2230 & 0.0013\text{-}0.008 \\ 0.2169\text{-}0.2226 & 0.9734\text{-}0.9752 & 0.039\text{-}0.062 \\ 0.004\text{-}0.020 & 0.037\text{-}0.060 & 0.9980\text{-}0.9992 \end{pmatrix} . \tag{2}$$

The intervals correspond to 2σ deviations from the fitted value. The phase δ must be less than π and $\frac{|U_{ub}|}{|U_{cb}|} > 0.036$. The latter result follows from the fact that ϵ is proportional to the K.M. angle $\sin\theta_3$. Nonvanishing values of ϵ prevent $\sin\theta_3$, and therefore $|U_{ub}|$, from vanishing. In the Wolfenstein parametrization[3] Eq. (2) corresponds to the following parameter intervals; again at the 2σ level:

$$A = 0.94 \sim 2.6\,, \quad \rho = 0.30 \sim 0.70\,, \quad \text{and} \quad \eta = 0.14 \sim 0.53\,. \tag{3}$$

Our results have implications for further study of the weak interactions of b-flavored mesons. One can obtain constraints for $B_s^0\bar{B}_s^0$ mixing $[\Delta m(B_s^0\bar{B}_s^0) \sim m_t^2|U_{tb}U_{ts}^*|^2]$ and CP violation in the B system, *i.e.* the asymmetry between like-sign dileptons from semileptonic decays of $B_0\bar{B}_0$ pairs

$$A_B = \frac{\ell^+\ell^+ - \ell^-\ell^-}{\ell^+\ell^+ + \ell^-\ell^-} \sim \frac{m_c^2 \sin\delta}{m_t^2}\,. \tag{4}$$

The observation of A_B will turn out to be extremely difficult as $\delta \simeq \pi$.

We want to stress here that our results will hold only in an SU(2)×U(1) model with three generations and a minimal Higgs sector. Slightly extended versions of the standard model, *e.g.* assuming a richer Higgs sector, could jeopardize the lower bound[4] on m_t and modify the analysis of B mixing.[5]

The structure of this paper is as follows: first, optimistically taking the theoretical formalism at face value, we shall produce bounds on the various parameters and observables that occur in decays and mixing of B-mesons and in CP violation. These limits should only be considered as best guesses. We shall refer to this approach in the following as case 1. We then consider the effect of theoretical uncertainties and produce absolute bounds, the violation of which would imply that new physics beyond the standard model is needed. This will constitute case 2. Throughout our analysis, we shall carefully define the probabilistic meaning of the produced limits. We discuss the input of our analysis next.

We use a mixing matrix which is unitary:

$$U = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 + c_2c_3e^{i\delta} \end{pmatrix} . \tag{5}$$

Given the awkwardness of the interpretation of the angles and phase, we shall later give bounds both for these and for the absolute values of the matrix elements themselves.

We firstly impose experimental bounds on the K.M. matrix elements from neutron[6] and hyperon[7] beta decay measurements, as well as from data on ν and $\bar{\nu}$ production of charm:[8] $|U_{ud}| = 0.9747 \pm 0.0011$, $|U_{us}| = 0.2197 \pm 0.019$, and $|U_{cd}| = 0.21 \pm 0.03$. In principle, one can also make use of the constraint[9]

$|U_{cs}| = 0.96 \pm 0.12$ from $D \to Ke\nu$ (case 1), but this is not model independent, the absolute limit being $|U_{cs}| > 0.66$ (case 2).

The next two observables will be related to B physics. B mesons constitute a border line case of the applicability of perturbative QCD. Nonperturbative corrections come from the fact that the quarks are bound within the meson (this is in principle measurable through the decay constant f_B) and from the possibility of bound intermediate states (this is parametrized by a bag parameter B_B). Apart from these factored corrections, which are in principle calculable through lattice calculations or by $1/N_c$ expansions, the dependence on other parameters should be well represented by perturbative calculations as momentum transfers are high.

Consider first the B meson width. Most of the theoretical complications can be avoided by considering the semileptonic decay width for which the final state interactions are negligible. This quantity can then be related to the total B-width via the measured semileptonic branching fraction $B_{s\ell}$. Thus our first constraint will be

$$\Gamma_B = \frac{1}{B_{s\ell}} \times \frac{|U_{cb}|^2 G_F^2 m_b^5}{192\,\pi^2} \times f\left(\frac{m_c}{m_b}\right) \times X_{\mathrm{QCD}} = [(1.15 \pm 0.14)\ \mathrm{ps}]^{-1}\,, \qquad (6)$$

with $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln(x)$ a phase-space factor and $X_{\mathrm{QCD}} = 1 - \frac{2\alpha_s(\Lambda_{\mathrm{QCD}};m_b^2)}{3\pi}\left(\pi^2 - \frac{25}{4}\right)$, a QCD correction factor. We use for Γ_B the value of Ref. 10, $(1.15 \pm 0.14) \times 10^{-13}$ ps. Another value has been published last year,[11] giving $\tau_B = (1.31 \pm 0.14) \times 10^{-13}$ ps. We have checked that this ambiguity hardly affects the fit, and the numbers we give are compatible with both τ_B values. The main uncertainty comes from the value of the bottom quark mass which we shall allow to be in the range 4.6-5.0 GeV. The QCD corrections introduce Λ_{QCD},

which we shall first restrict to the range 0.1-0.3 GeV (case 1), and then vary up to 0.4 GeV (case 2). The value of m_c enters in the phase-space corrections, but given the smallness of $\frac{m_c}{m_b}$ its precise value is not too important. We shall allow it to range between its current value, 1.25 GeV, and its highest constituent value, 1.8 GeV. Equation (6) also shows that the dependence on the heavy quark mass makes any constraints from the D lifetime useless for our purpose.

The second quantity which we shall study will be the $B_d^0\bar{B}_d^0$ mixing[12] parameter:

$$\frac{\delta m}{\Gamma} = \left[\frac{G_F^2 m_B}{6\pi^2 \Gamma_B}\right] \times \left[B_B f_B^2\right] \times |V_{tb}^* V_{td}|^2 \times I\left(\frac{m_c}{m_t}\right) \eta_{\mathrm{QCD}}\,. \qquad (7)$$

The first factor introduces experimentally measured quantities, the second takes long range corrections into account. The third factor depends on two unmeasured Kobayashi-Maskawa matrix elements. η_{QCD} is a QCD correction factor for which we use an exact result[13] and, finally, $I(x)$ is a phase-space factor[14] that depends quadratically on m_t for large m_t values. We take into account the new limits on the top quark mass. UA2 has derived a new upper bound on m_t of the order of 180 GeV from electroweak radiative corrections.[14] In case 1 we will assume that the bound is to 150 GeV, and take the conservative value 200 GeV in case 2. This is only to demonstrate that the exact value is not important. The new CDF/UA2 results[15] seem to indicate $m_t > 70$ GeV, a limit that we shall implement for both cases. As the dependence on m_W and m_B is small, no new parameter is introduced,[17] except the product $B_B f_B^2$. Due to the smallness of momentum transfers, this quantity is not perturbatively calculable. The calculation of these long-range constants now seems to be under control in lattice studies as a result of the discovery of the CPS switching symmetry.[1] The bag parameter and the decay constant are calculated to be $B_B = 1.011 \pm 0.19$ and $f_B = (0.105 \pm 0.035)$ GeV. This is in qualitative agreement with results from $1/N_c$ expansion[2] and from sum rule methods,[18] which give $B_B = 3/4$ and $B_B f_b^2 = (0.115 \pm 0.015\ \text{GeV})^2$ and $(0.190 \pm 0.03\ \text{GeV})^2$, respectively. We choose here to implement the lattice QCD result as it is based on a more fundamental approach. This result, however, still uses a quenched approximation and involves an extrapolation to the B meson mass. It is therefore not totally reliable. We first choose to combine the errors on f_B and B_B in quadrature and assume a gaussian probability distribution (case 1: $f_B^2 B_B = 0.011 \pm 0.0095\ \text{GeV}^2$) and subsequently use only bounds with a flat probability (case 2: $0.0027\ \text{GeV}^2 < f_B^2 B_B < 0.04\ \text{GeV}^2$).

In principle, one could include lepton distributions from semileptonic B decays. These can be used to determine the ratio $\frac{|U_{ub}|}{|U_{cb}|}$. We have shown[19] however that, in the context of the Altarelli model,[20] the theory is arbitrary enough to accommodate values for that ratio as high as 0.21 whereas the usual bounds from this model are of the order of 0.14. This analysis has recently been confirmed and extended to other models.[21] The CLEO collaboration has performed a detailed analysis of their data,[22] and their results $\frac{|U_{ub}|}{|U_{cb}|} < 0.14$, $|U_{cb}| = 0.044 \pm 0.010$ will be considered in case 1. We shall assume the conservative bound $\frac{|U_{ub}|}{|U_{cb}|} < 0.21$ in case 2. Also, bounds on $\frac{|U_{ub}|}{|U_{cb}|}$ can in principle be drawn from the non-observation of charmless B decays,[23] but the interpretation of the branching ratios is very uncertain, and bounds as high as 0.25 have been given.[24]

Finally, we introduce the CP-violation parameters of the $K^0\bar{K}^0$ system in the analysis. As in the B mixing case, the long-range contributions can be factored out into a bag parameter B_K and a decay constant f_K, which has been measured to be 0.169 GeV. The kaon bag parameter is calculable on the lattice, but the

result is much more uncertain due to the inapplicability of the CPS symmetry. The European Collaboration[25] quotes the estimate $B_K = 0.86 \pm 0.11$. Large N_c expansion gives[2] $B_K = 2/3 \pm 0.1$, and two recent sum rule estimates[26] yield $B_K = 0.50 \pm 0.22$ and $B_K = 0.58 \pm 0.16$. In case 1, we shall take the lattice estimate at face value, assuming that the probability distribution is gaussian. In case 2 we shall be much more conservative and limit the value of B_K to the interval [0.3, 1.0].

The expression for ϵ is well known:[27]

$$\epsilon \approx \left(\frac{e^{i\pi/4}}{2\sqrt{2}\,\Delta m}\right) \mathcal{I}m(\mathcal{M}_{\Delta S=2})\,. \tag{8}$$

We here assumed that the $\Delta S = 2$ box diagram dominates the amplitude $\mathcal{M}$, which is proportional to $B_K f_K^2$, and made use of the fact that the contributions from CP violation in $K \to 2\pi$ decay is experimentally negligible. The mass difference of the $K^0\bar{K}^0$ system Δm, for which non-perturbative contributions are not under control, is fortunately experimentally known to be (3.521) μeV. The expression for $\mathcal{M}$ is given in Ref. 27. It depends on m_c, m_b, m_t, and Λ_{QCD} and introduces the products $|U_{ts}U_{td}^*|^2$ and $|U_{cs}U_{cd}^*|^2$.

We then consider the direct CP-violation parameter[28] ϵ'. We limit ourselves to the leading contribution from the QCD penguin diagram and make use of the fact that for $K \to \pi\pi$ decay, the ratio $\mathcal{R}eA_2/\mathcal{R}eA_0$ is experimentally known to be $\approx 1/22$. We so obtain

$$\frac{\epsilon'}{\epsilon} = -\frac{\mathcal{R}eA_2/\mathcal{R}eA_0}{|\epsilon|}\frac{c_2 s_2 s_3 \sin(\delta)}{c_1 c_3}\,H_p\,, \tag{9}$$

with H_p the penguin height. The expression for H_p is given in Ref. 29. It depends on the same parameters as well as on an arbitrary hadronic scale μ which enters only via logarithms. We shall vary its value between 0.6 and 1 GeV. Another approach based on a $1/N_c$ expansion, is discussed in the literature. All scales are fixed here, but the strong dependence on the strange quark mass replaces the arbitrariness of our scale μ. We have checked numerically that for reasonable strange quark masses, it leads to the same results as Eq. (9).

Our various inputs, both for the parameter ranges and for the experimental values, are summarized in the first columns of Tables I and II. The problem thus amounts to fitting 10 observables q_i ($i = 1$ to 10) to theoretical expressions $t_i(p_a)$ which contain 12 parameters p_a ($a = 1$ to 12). The range of these parameters is constrained by theoretical arguments. Most previous studies have fixed some of the parameters to preferred values and then varied others to get "bounds" on them. This procedure has the advantage of simplifying the physical discussion, but the reliability of the produced bounds is questionable. We choose here to fit all the observables simultaneously and to let all the parameters vary in ranges tabulated in Table II. To get the best value for all q_i and p_a, we minimize a likelihood function. This function is normally defined by the probability density $L_\theta = \prod \exp[(q_i - t_i(p_a))/2\delta q_i]^2$. We adopt the equivalent definition

$$L_1 = \sqrt{\sum \left(\frac{q_i - t_i(p_a)}{\delta q_i} \right)^2} . \tag{10}$$

The maximum of L_1 defines the best fit. To evaluate the errors, one should in principle calculate integrals such as $P_\theta = \int_{p_1}^{p_2} dp_b \int d^{11} p_a J L_\theta$, with J a Jacobian, and from these subsequently calculate the probability for p_b to be in the interval $[p_1, p_2]$. These integrals are unfortunately extremely hard to evaluate numerically as L_θ is null in most regions of parameter space. A similar procedure should be adopted to calculate the constraints on the various q_i from the joint fit. Given these difficulties, we choose another procedure to define the various errors. We consider the quantity

$$\sigma_1 = \min_{p_a} \left(L_1 \big|_{p_b \in [p_1, p_2]} \right) , \tag{11}$$

which is the minimum of the likelihood function for p_b in the interval $[p_1, p_2]$. As the observables are independently measured, the probabilistic significance of σ_1 is the same as that of the usual standard deviation. However, this definition is not totally satisfactory as a fit with all quantities away from their central value by an amount δq_i is equivalent to a fit where one of them is off by $\sqrt{10}\,\delta q_i$. One does not want a large number of good fits to cover up a large discrepancy for a single quantity. We shall thus also use the alternative likelihood function

$$L_2 = \max_{p_a} \left(\frac{|q_i - t_i(p_a)|}{\delta q_i} \right) \tag{12}$$

and the corresponding standard deviation:

$$\sigma_2 = \min_{p_a} \left(L_2 \big|_{p_b \in [p_1, p_2]} \right) . \tag{13}$$

As a disagreement of 1 standard deviation on all q_i would not lead to a problem at the 3σ level, we shall adopt this second approach to evaluate absolute bounds (case 2). The first approach, which takes into account all the q_i simultaneously is in principle superior and will be used in case 1.

Case 1 is an elaborate version of the usual analysis proposed by many authors for the analysis of $B\bar{B}$ mixing and gives the region of parameter space where the theory would be most comfortable. Case 2 is derived from our own work on $B\bar{B}$ mixing[14,19,30] which does not lead to any significant lower bound on the top quark mass. If this kind of analysis were to disagree with experimental results, we feel that there would be a serious problem for the standard model.

The bounds for both cases are shown in Tables I and II. We stress that these limits hold individually for each quantity, *e.g.* it is not true that if all parameters but one are fixed to their best fit value, the error will be as indicated. Likewise, the best fit is given only as an indication and is not compelling in a strict statistical sense. We should mention that there is no perfect fit. The minimum number of standard deviations, σ, is always bigger than 0.8 in case 1 and 0.5 in case 2. This is due to the value of $|U_{ud}|$ which is always fitted slightly higher than its measured mean. We have already given in Eq. (2) the bounds on the K.M. matrix elements, corresponding to case 1. We want to emphasize that the smallness of the values of $\frac{|U_{ub}|}{|U_{cb}|}$ allowed at present is of the order of 0.03 and almost any value within the present experimental limits can be accommodated.

As an example of the power and use of our fit, we compare our results for $|V_{cd}/V_{cs}|^2$ with a recent Mark III result.[32] Neglecting theoretical uncertainties, they obtain $|V_{cd}/V_{cs}|^2 = 0.041 \sim 0.095$ at the 1σ level, with a central value of 0.059. Our fits give $|V_{cd}/V_{cs}|^2 = 0.049 \sim 0.052$ ($0.048 \sim 0.053$) at the 2σ level in cases 1 and 2, respectively, which are compatible with the Mark III bounds and can be used to constrain the theoretical uncertainties of their analysis.

We can translate these results into the Wolfenstein parametrization (see Eq. (3)). Our phase convention gives[31]:

$$U = \begin{pmatrix} 1-\frac{\lambda^2}{2} & -\lambda & -A\lambda^3\sqrt{\rho^2+\eta^2} \\ \lambda & 1-\frac{\lambda^2}{2} & \frac{A\lambda^2(\rho+i\eta)}{\sqrt{\rho^2+\eta^2}} \\ A\lambda^3\sqrt{(1-\rho)^2+\eta^2} & \frac{A\lambda^2(1-\rho+i\eta)}{\sqrt{(1-\rho)^2+\eta^2}} & -e^{i\delta} \end{pmatrix} . \tag{14}$$

We obtain the following formulae for λ, A, ρ, and η:

$$\lambda = s_1 , \tag{15a}$$

$$A^2 = \frac{s_2^2 + s_3^2 + 2\sqrt{1-\sin^2(\delta)}\ s_2 s_3}{s_3^4} , \tag{15b}$$

$$\rho = \frac{s_3\left(s_2\sqrt{1-\sin^2(\delta)} + s_3\right)}{s_2^2 + s_3^2 + 2\sqrt{1-\sin^2(\delta)}\ s_2 s_3} , \tag{15c}$$

$$\eta = \frac{s_3 s_2 \sin(\delta)}{s_2^2 + s_3^2 + 2\sqrt{1-\sin^2(\delta)}\ s_2 s_3} . \tag{15d}$$

Translating our fits in term of these parameters gives at the 2σ level: $A = 0.94 \sim 4.8$ ($0.94 \sim 2.6$), $\rho = 0.26 \sim 0.87$ ($0.30 \sim 0.70$), and $\eta = 0.08 \sim 0.6$ ($0.14 \sim 0.53$), with the numbers in parenthesis for case 1 and the others for case 2.

CP violation can be described by a parametrization-invariant quantity J, proportional to the area of the unitary triangles.[33] With our choice (5), this quantity becomes

$$J = s_1^2 s_2 s_3 c_1 c_2 \sin(\delta) . \tag{16}$$

We obtain $J = 1.14\times10^{-5} \sim 3.54\times10^{-5}$ ($6.97\times10^{-6} \sim 4.23\times10^{-5}$) at the 2σ level in cases 1 and 2, respectively.

We now examine the various ways through which these results might be improved. Firstly, the bounds of Table I can be used backwards, *i.e.* they give the precision which should be aimed at to improve our knowledge of (10). Accordingly, a measurement of $\frac{|U_{ub}|}{|U_{cb}|}$ at the level claimed last year by ARGUS would help. $B\bar{B}$ mixing would need to be measured at the 5% level and $\frac{\epsilon'}{\epsilon}$ at the 20%

level. Similarly, the error on ϵ should be reduced by a factor 3. As far as the parameters of the theory are concerned, very little improvement from experimental inputs can now be expected. Conversely, our determination of the K.M. matrix is insensitive to the precise value of m_t. We have checked this fact by repeating the above procedure for case 2 with the top quark mass fixed at 110 GeV. The resulting bounds are the same, showing improvement only at the 1% level, except for $|U_{td}|$ which would be restrained by the interval [0.0048, 0.0123] at the 2σ level.

We can also translate our results into definite bounds on quantities so far unobserved. For B_s mixing, we use the fact that

$$\Delta m(B_s^0\bar{B}_s^0) = \frac{|U_{ts}|^2}{|U_{td}^*|^2}\Delta m(B_d^0\bar{B}_d^0)\,. \tag{17}$$

The values we obtain are high: at the 2σ level, $\Delta m(B_s^0\bar{B}_s^0)/\Gamma = 3.6 \sim 42$ ($3.4 \sim 136$), the first interval being in case 1 and the second in case 2. These results can be modified by differences in the long-range constants B, f_B for $(b\bar{s})$ and $(b\bar{d})$ mesons.

We also consider the CP violating asymmetry in B decays:

$$\begin{aligned} A_B &= \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} \\ &= \frac{\rho - 1/\rho}{\rho + 1/\rho} \end{aligned} \tag{18}$$

with $\rho = |(M_{12}^* - i\Gamma_{12}^*/2)/(M_{12} - i\Gamma_{12}/2)|$. The formulae for M_{12} and Γ_{12} are well known[34] and lead to the following conclusions: A_B is negative, and at the 2σ level, $A_B = -1.33\times10^{-3} \sim -5.8\times10^{-5}$ $[-2.4\times10^{-3} \sim -2.0\times10^{-5}]$ for cases 1 and 2, respectively. A measurement of a positive A_B, or of any value outside the previous bounds, would endanger the standard model.

Finally, we can utilize the approach of case 2 to see if some parametrizations of the K.M. matrix are now ruled out. We illustrate this using the Fritzsch parametrization of the K.M. matrix.[35] It implies that

$$s_1 = \left|\sqrt{m_d/m_s} + e^{i\delta}\sqrt{m_u/m_c}\right|\,, \tag{19a}$$

$$s_2 = \frac{|\sqrt{m_d/m_s}|}{s_1}r\,, \tag{19b}$$

$$s_3 = \frac{|\sqrt{m_u/m_c}|}{s_1}\,, \tag{19c}$$

$$r \approx \left|\sqrt{m_c/m_t} + e^{i(\delta+\tau)}\sqrt{m_s/m_b}\right|\,, \tag{19d}$$

with τ an arbitrary phase. We fixed the ratios of quark masses as in Ref. 31 except for m_c/m_t which results from our fit. If we take these relations at face value, then the model is excluded at the 2.2σ level in case 2. If we allow Eqs. (19) to reproduce the K.M. matrix elements within 10%, then the model is allowed, and we get an upper bound $m_t < 117$ GeV at the 2σ level.[36]

In conclusion, we have presented here precise bounds within which the standard model remains valid, and thus given the precision which the experiments should aim at to reveal new physics, or to submit the electroweak theory to its most stringent tests.

ACKNOWLEDGEMENTS

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the U. S. Department of Energy under contracts DE-AC02-76ER00881 and DE-AM03-76SF00235,

REFERENCES

1. C. Bernard, T. Draper, G. Hockney, and A. Soni, Nucl. Phys. **B4** (1988) 483.
2. A. J. Buras preprint MPI-PAE/PTh 1/88 (January 1988), to appear in **CP Violation**, C. Jarlskog (Editor), World Scientific (1988).
3. L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.
4. V. Barger and R. J. N. Phillips, Phys. Lett. **201B** (1988) 553.
5. S. L. Glashow and E. E. Jenkins, Phys. Lett. **196B** (1987) 223.
6. A. Sirlin and R. Zucchini, Phys. Rev. Lett. **57** (1986) 1994; W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **56** (1986) 22.
7. H. Leutwyler and M. Ross, Zeit. Phys. **C25** (1984) 91.
8. K. Kleinknecht and B. Renk, Zeit. Phys. **C34** (1987) 209.
9. T. M. Aliev *et al.*, Yad. Fiz. **40** (1984) 823 [Sov. J. Nucl. Phys. **40** (1984) 527]; M. Bauer, B. Stech, and M. Wirbel, Zeit. Phys. **C29** (1985) 637.
10. R. A. Ong, Ph.D. Thesis, preprint SLAC-320 (September 1987).
11. Review of Particle Properties, Particle Data Group, G. P. Yost *et al.*, Phys. Lett. **B204** (1988) 220.
12. ARGUS collaboration, H. Albrecht et al., Phys. Lett. **209B** (1988) 119; CLEO collaboration, R. Fulton et al., contribution to *The XXIV International Conference on High Energy Physics*, Munich (1988).
13. A. J. Buras, W. Slominski, and H. Steger, Nucl. Phys. **B245** (1984) 369.
14. J. R. Cudell, F. Halzen, X. G. He, and S. Pakvasa, Phys. Lett. **196B** (1987) 227.

15. U. Amaldi et al., Phys. Rev. **D36** (1987) 1385; G. Costa et al., Nucl. Phys. **B297** (1988) 244.
16. D. Baden, contribution to *The 1989 Moriond Meeting on New Results in Hadronic Interactions*, Les Arcs, France, March 1989; Grunendahl, same meeting.
17. See Ref. 15. We combine the UA1-UA2 measurement with the 1987 CDF result to get $m_W = 80.7$ GeV. We also use $m_B = 5.2752$ GeV.
18. I. Bigi, preprint SLAC-PUB-4521, invited lecture at *the SLAC Summer Institute on Particle Physics*, Stanford, CA, August 10-21, 1987.
19. J. R. Cudell, in *Proceedings of the Moriond Meeting on Current Issues in Hadron Physics*, Les Arcs, France, March 13-19, 1988; p. 555 (Gif-sur-Yvette: Editions Frontière, 1988, edited by J. Tran Thanh Van).
20. G. Corbò, Nucl. Phys. **B212** (1983) 99 and references therein.
21. C. S. Kim and A. D. Martin, Durham preprint DTP/89/10.
22. J. A. Guida, Ph.D. Thesis, University of Rochester preprint UR-1007 (1987).
23. N. Mistry, in *Proceedings of the Fourth Family of Quarks and Leptons Conference*, UCLA, Los Angeles, CA (1989).
24. A. B. Kaidalov, contribution to *The 1989 Moriond Meeting on New Results in Hadronic Interactions*, Les Arcs, France, March 1989.
25. M. B. Gavela et al., Nucl. Phys. **B306** (1988) 677.
26. N. Bilic, C. A. Dominguez, and B. Guberina, preprint DESY 87-162 (1987); R. Decker, talk presented at *The Ringberg Workshop on Hadronic Matrix*

Elements and Weak Decays, April 1988, to be published in Suppl. of Nucl. Phys. B.

27. F. Halzen, C. S. Kim, and S. Pakvasa, Int. Jour. Mod. Phys. **A4** (1989) 753; C.S. Kim, Ph.D. Thesis, Madison preprint MAD/PH/416 (April 1988).
28. H. Burkhard et al., Phys. Lett. **206B** (1988) 169.
29. A. J. Buras and J. M. Gerard, Phys. Lett. **203B** (1988) 272.
30. J. R. Cudell, F. Halzen, X. G. He, and S. Pakvasa, preprint MAD/PH/376 (1987).
31. J. Ellis, J. S. Hagelin, S. Rudaz, and D.-D. Wu, preprint CERN-TH. 4816/87.
32. Mark III collaboration, J. Adler *et al.*, Phys. Rev. Lett. **62** (1989) 1821.
33. C. Jarlskog and R. Stora, Phys. Lett. **B208** (1988) 268; C. Hamzaoui, Phys. Rev. Lett. **61** (1988) 35.
34. J. S. Hagelin, Nucl. Phys. **B193** (1981) 123; E. Franco, M. Lusignoli, and A. Pugliese, Nucl. Phys. **B194** (1982) 403.
35. H. Fritzsch, Phys. Lett **73B** (1978) 317, Nucl. Phys. **B155** (1979) 189, Phys. Lett **166B** (1986) 423.
36. See also, A. Davidson, B. Margolis, and J. A. Robinson, McGill preprint 88-0760 (1988).

Table I: Allowed ranges of observables. The signification of σ is explained in the text. In the last two columns the top values refer to absolute bounds, the second line to our best estimate.

observable	input data	best fit	1σ	2σ
$\|U_{ud}\|$	0.9747 ± 0.0011	0.9754	0.9750-0.9758 0.9753-0.9756	0.9746-0.9764 0.9748-0.9761
$\|U_{us}\|$	0.2197 ± 0.0019	0.2202	0.2183-0.2216 0.2195-0.2207	0.2159-0.2235 0.2173-0.2230
$\|U_{ub}\|$	–	0.0048	< 0.015 0.0069	0.017 0.0082
$\|U_{cd}\|$	0.21 ± 0.03	0.2198	0.217-0.222 0.2191-0.2204	0.214-0.224 0.2169-0.2226
$\|U_{cs}\|$	> 0.66 0.96 ± 0.12	0.9745	0.971-0.975 0.9743-0.9747	0.969-0.975 0.9734-0.9752
$\|U_{cb}\|$	– 0.044 ± 0.010	0.044	0.042-0.081 0.041-0.050	0.039-0.097 0.039-0.062
$\|U_{td}\|$	–	0.014	0.005-0.031 0.006-0.017	0.004-0.033 < 0.020
$\|U_{ts}\|$	–	0.42	0.039-0.082 0.039-0.048	0.036-0.097 0.037-0.060
$\|U_{tb}\|$	–	0.9990	0.996-0.999 0.9987-0.9991	0.994-0.999 0.9980-0.9992
$\frac{\|U_{ub}\|}{\|U_{cb}\|}$	< 0.21 < 0.14	0.11	> 0.031 > 0.047	> 0.025 > 0.036
$\tau_B(10^{-13}\, s)$	11.5 ± 1.4 $11.5 \pm 1.4/13.1 \pm 1.4$	11.5	10.0-12.9 11.1-13.4	8.71-14.3 9.64-14.7
$\frac{\delta m}{\Gamma}(B_d^0\text{-}\bar{B}_d^0)$	0.70 ± 0.13	0.70	> 0.57 > 0.67	> 0.44 > 0.50
$\epsilon(10^{-3})$	2.275 ± 0.021	2.2750	2.264-2.290 2.2744-2.2752	2.241-2.313 2.2713-2.2756
$\frac{\epsilon'}{\epsilon}(10^{-3})$	3.1 ± 1.1	3.35	2.2-4.4 2.7-3.7	1.1-5.5 1.8-5.0

Table II: Allowed ranges of the input parameters. The signification of σ is explained in the text. The numbers in bold refer to absolute bounds, the second line to our best estimate.

parameter	range considered	best fit	1σ	2σ
s_1	0.21-0.23	0.2202	0.2188-0.2220 0.2196-0.2208	0.2158-0.2240 0.2173-0.2231
s_2	0.005-0.2	0.065	0.022-0.141 0.029-0.079	0.019-0.150 < 0.089
s_3	0.01-0.12	0.022	< 0.070 < 0.031	< 0.079 < 0.037
δ (rad)	$0\text{-}2\pi$	2.92	0.27-3.11 0.93-3.00	$0.10\text{-}\pi$ 0.75-3.06
B_{sl}	0.11 ± 0.006	0.110	0.106-0.116 0.109-0.113	0.102-0.121 0.108-0.116
Λ_{QCD} (GeV)	0.1-0.4 0.1-0.3	0.263	no improvement > 0.16	 –
B_K	0.3-1.0 0.86 ± 0.11	0.86	no improvement 0.57-0.90	 < 1.03
$B_B f_B^2$ (GeV^2)	0.0027-0.04 0.0110 ± 0.0095	0.011	no improvement > 0.009	 > 0.007
m_c (GeV)	1.25-1.80	1.41	no improvement < 1.62	 < 1.74
m_b (GeV)	4.6-5.0	5.0	no improvement no improvement	
m_t (GeV)	70-200 70-150	150	no improvement > 105	 > 70
μ (GeV)	0.6-1.0	0.61	no improvement no improvement	