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QUARK AND GLUON CONDENSATES AND THE SMALL x LIMIT OF THE NUCLEON STRUCTURE FUNCTIONS

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E R R A T U M

There are errors in the equations (2.3), (2.8), (4.1) and (4.2). They should be

$$\beta_0^2 = \frac{g_0^4}{36\pi}\int dk_T^2\, D^2(-k_T^2) \tag{2.3}$$

$$C = \frac{g_0^4}{(2\pi)^4}\int dk_T^2\, k_T^2\, D^2(-k_T^2) \tag{2.8}$$

$$xg(x) = \frac{g_0^2}{3\pi^2}\int dz\, z\, D^2(-z) \tag{4.1}$$

$$xg(x) = \frac{4}{3}\pi C\,(g_0^2/4\pi)^{-1} \tag{4.2}$$

These errors occurred because initially the equations were in a different form and went wrong in transcription. The correct form was used for the calculations, so the argument and conclusions are unaltered.

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A B S T R A C T

We calculate the non-perturbative contributions from the quark and gluon condensates to the nucleon structure functions at small x in the asymptotic region. We show that for values of Q^2 around 10 GeV^2, these contributions dominate the perturbative ones and agree reasonably with present data.

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1. - INTRODUCTION

It is well accepted that the Pomeron, whose exchange corresponds to the long-range part of the strong interaction at high energy, is generated by gluon exchange [1]. It is known that its rather simple phenomenological properties [2] are not reproduced if the gluon propagator $D(k^2)$ has a singularity at $k^2 = 0$ [3], but may readily be understood if it does not [4]. That is, the small $|k^2|$ region of the gluon propagator must be generated by non-perturbative effects.

Gluons are, of course, confined. A singularity at $k^2 = 0$ would correspond to the possibility of the gluon propagating through very long distance, so it is no surprise that in fact such a singularity is not present. If, as we assume, the gluon propagator is finite at $k^2 = 0$, the theory must contain a fixed mass or length scale, since $D(k^2)$ has dimension $(\text{mass})^{-2}$. This mass scale must be of the order of a GeV, as may be seen from the fact [2] that the observed slope α' of the Pomeron trajectory is almost exactly $(2\ \text{GeV})^{-2}$, and the strength β of the coupling of the Pomeron to a light quark is close to $(\frac{1}{2}\ \text{GeV})^{-1}$. The need for such a large mass scale provides another argument for the futility of trying to generate the Pomeron purely from perturbative QCD. Any realistic attempt [5] to do so inevitably leads to the introduction of an "infra-red" cut-off of the order of a GeV, and it is this cut-off, rather than the perturbative tail of the propagator, that generates the main part of Pomeron exchange.

There is good reason to believe [4,6] that the fixed mass scale required is that of the gluon condensate in the non-perturbative QCD vacuum. In fact there are at least two fixed mass or length scales associated with this condensate, its overall strength and a correlation length. We have shown [6] how together these determine the strength of the coupling of the Pomeron to light quarks and also how this strength decreases when one of the quark legs goes "off-shell". The strength of the coupling is directly reflected in the magnitude of hadronic total cross-sections at high energy, while the off-shell coupling is measured [7] in the exclusive deep inelastic process $\gamma^* p \to \rho p$.

In this paper, we are concerned with the small x limit of the quark and gluon structure functions of the nucleon. It has long been known [8] that this small-x limit is determined by Pomeron exchange, and that to calculate it one needs to know how the coupling of the Pomeron behaves when both parton legs go off shell. While there has recently been some suggestion that [9] the Pomeron that is involved may be different from that which controls hadronic total cross-sections, this does not seem to be supported by present data [10,11]. Maybe a different behaviour will be found when Q^2 gets very large, but for the present we can assume that the Pomeron is universal as we shall limit ourselves to values of Q^2 which are not too large, say around 10 GeV^2.

There are some old suggestions [11,12] that the non-valence part of the structure functions, which dominates in the small x limit, may be calculated by perturbative evolution alone, by assuming that at some smaller value of Q^2 the nucleon consists of valence quarks only, and using the Altarelli-Parisi equation. Our calculations do not support this idea. We find that the largest part of the structure functions at around $Q^2 = 10$ GeV^2 corresponds to non-perturbative effects. We argue that, in the case of the gluon structure function, it arises mainly from the vacuum gluon condensate, while for the quark structure function the quark condensate also is involved.

Our paper is organized as follows: the next section describes and motivates the phenomenological model that we use to describe non-perturbative exchanges of quarks and gluons. The third section concentrates on the quark structure function at small x and shows that non-perturbative contributions should be important even for Q^2 as high as 10 GeV^2. The fourth section gives our results for the gluon structure function and again emphasizes the dominance of non-perturbative exchanges.

2. - NON-PERTURBATIVE GLUON EXCHANGE

We begin by recapitulating the essential features of non-perturbative gluon exchange, and how it is related to the Pomeron. In our previous work [4,6] we made the approximation of taking the gluon to be Abelian. We now give it the proper colour, though the price we have to pay for this is to lose the previous precise relation between the strength of the Pomeron coupling and the magnitude of the gluon condensate, because of the non-linear terms that now occur in the relationship between $G^{\mu\nu}$ and A^{μ}.

We work in Feynman-like gauge, in which the gluon propagator is:

$$D^{\mu\nu}_{ab}(k) = -\delta_{ab}\, g^{\mu\nu}\, D(k^2) \qquad (2.1)$$

Because gluons do not propagate through large distance, $D(k^2)$ should not have any pole; in particular, we take it to be finite at $k^2 = 0$, and therefore dominated there by non-perturbative contributions. At large $|k^2|$, or short distance, it should be dominated by its perturbative part, that is $D(k^2) \sim 1/k^2$. If this perturbative part is subtracted off, the remaining non-perturbative part should fall off faster than $1/k^6$, in order that the gluon condensate be finite [4].

At $t = 0$, the amplitude (Fig. 1) corresponding to two gluons exchanged between a pair of quarks, in the t-channel colour-singlet configuration, is

$$i \gamma_\mu \gamma^\mu \beta_0^2 \tag{2.2}$$

with

$$\beta_0^2 = \frac{g_0^2}{36\pi} \int d\underline{k}_T^2 \; D^2(-k_T^2) \tag{2.3}$$

Here g_0 is the QCD coupling to the quarks, which here are taken to be "on shell", or nearly so. We shall argue below that $g^2{}_0/4\pi \approx 1$.

Experiment finds [2] that Pomeron exchange occurs between quarks with the γ-matrix structure (2.2), but also with an energy dependence:

$$i \gamma_\mu \gamma^\mu \beta^2 \; s^\varepsilon \tag{2.4}$$

Presumably [4] this energy dependence is generated by exchanges that are more complicated than just two gluons. However, ε is small, between 0.08 and 0.09. So it is reasonable to assume that two-gluon exchange is the most important effect, in the sense that

$$\beta^2 = \beta_0^2 + O(\varepsilon) \tag{2.5}$$

That is, β_0 is close to $(\frac{1}{2}\text{ Gev})^{-1}$, the experimental value of β.

In order that $\beta^2{}_0$ be finite, we do need $D(k^2)$ not to have a pole at $k^2 = 0$. This is necessary also [3,4] to reproduce the additive quark rule for total cross-sections, and to understand why two gluons couple more weakly to hadrons composed of heavy quarks than those made of light quarks. Notice also that the perturbative tail of the propagator contributes rather little to the integral (2.3): if we integrate it down to, say, $\underline{k}^2{}_T = 1\text{ GeV}^2$, the contribution to $\beta^2{}_0$ is only $g^2{}_0/36\pi\text{ GeV}^{-2}$, which is certainly much less than the observed value of about 4 GeV^{-2}.

So the gluons of Fig. 1 are essentially non-perturbative. The same will be true of the external quarks, in the sense that they are bound within hadrons. However, the internal quark propagators are predominantly perturbative, that is at high energy they propagate only very short distance between the interactions with the gluons. If one calculates the amplitude, this is seen directly [13]; it is the high-momentum tail of the propagator that yields the energy-independent answer (2.2). It is perhaps less obvious if one calculates the imaginary part of the amplitude, using the Cutkosky rules [13]. Then the argument is as follows. Suppose that the complete quark propagator (including the non-perturbative part) has the Lehmann representation

$$S(k^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{k^2-\sigma} \tag{2.6}$$

Then, if $S(k^2) \sim 1/k^2$ at large k^2,

$$\int_0^\infty d\sigma\ \rho(\sigma) = 1 \tag{2.7}$$

To calculate the imaginary part of Fig. 1, we first replace the propagators by their imaginary parts, that is by $\rho(\sigma_1)\delta(k^2{}_1-\sigma_1)$ and $\rho(\sigma_2)\delta(k^2{}_2-\sigma_2)$. We then calculate the high s behaviour, and the essential point is that it turns out to be independent not just of s but also of σ_1 and σ_2. So when we finally integrate over σ_1 and σ_2, this integration is trivial because of (2.7) and the answer remains independent of s. If, however, $S(k^2)$ had gone to zero at large k^2 faster than $1/k^2$, the integral (2.7) instead would vanish, and so the final integrations over σ_1 and σ_2 would remove the energy-independent term from the amplitude, leaving some quantity that goes to 0 as $s \to \infty$. That is, the non-perturbative part of the propagator does not contribute to the asymptotic behaviour of the imaginary part of the amplitude.

We define the dimensionless quantity

$$C = \frac{g_0^2}{(2\pi)^4} \int dk_T^2\ k_T^2\ D^2(-k_T^2) \tag{2.8}$$

We have shown previously [7] that this quantity enters in the calculation of the process $\gamma^* p \to \rho p$. Data for the process [14], for 2.5 GeV^2 < Q^2 < 18 GeV^2, yield the value

$$C \approx 0.15 \tag{2.9}$$

Again we see that the perturbative part of the propagator does not contribute much to (2.8). Integrating it down to $\underline{k}^2{}_T$ = 1 GeV^2 gives a contribution $\log Q^2/(2\pi)^4$ which is sizeable only when Q^2 is rather large.

3. - THE QUARK STRUCTURE FUNCTION AT SMALL XY

At small x, the quark and antiquark distributions in the nucleon are dominated by Pomeron exchange, and so become equal to each other and behave as $x^{-1-\varepsilon}$, where ε is the small parameter defined in (2.4). A satisfactory fit [10] to the CHARM collaboration data [15] for the light antiquark distributions $\bar{u}$, $\bar{d}$ is given by:

$$\bar{q}(x) = C x^{-1-\varepsilon} (1-x)^7 \tag{3.1}$$

with C = 0.16 at Q^2 = 10 GeV^2.

In this section, we show that it is likely that the main part of the contribution to the constant C is non-perturbative. Notice that the value required here for C is close to that given in (2.8) and (2.9). In our previous phenomenological work [7,10] this connection appeared to be rather natural, but in our present attempts to be more theoretical we have not been able to establish such a connection.

According to usual ideas, the quark structure function consists of a part that is of zeroth order in the perturbative QCD coupling $\alpha_s(Q^2)$, and which satisfies Bjorken scaling. The scaling violation effects then involve powers of $\alpha_s(Q^2)$. There have been some suggestions [12] that amount to saying that, for the antiquark distribution, the zeroth-order term vanishes. We regard this as unlikely and in this section argue that, although we cannot calculate it at all exactly, it most probably represents the main part of the antiquark distribution at $Q^2 = 10$ GeV^2. We consider the scaling violation in the next section.

The zeroth-order term corresponds to the familiar handbag diagram, Fig. 2. The lower part of the diagram is the imaginary part of the quark/hadron scattering amplitude, and when $x \to 0$ the squared energy associated with this scattering becomes large [8] and proportional to x^{-1}. Hence in this limit we expect the quark/hadron amplitude to be dominated by Pomeron exchange, with the result that $\bar{q}(x)$ has the small x behaviour $x^{-1-\varepsilon}$ seen in (3.1). In our model, where Pomeron exchange corresponds predominantly to two-gluon exchange, the small x limit of Fig. 2 will be Fig. 3.

The zeroth-order scaling contribution from Fig. 3 comes from substracting off not only the perturbative tail of the gluon propagator, as we discussed in the last section, but also that of the quark propagators on the vertical quark lines, that is we must use a non-perturbative quark propagator. Rather little is known about this propagator [16]. We have made a guess of its structure, consistent with what is known. Because the quark is confined, it should not have a pole. We introduce a mass m, such that the spin structure of the propagator is $(\gamma.k+m)$. The reasoning behind this is that, although the quark cannot propagate over very long distances, it behaves within the confines of its surrounding bag almost like a free particle of mass m. So we take the non-perturbative quark propagator to be

$$(\gamma.k + m)\, S(k^2) \tag{3.2}$$

where $S(k^2)$ does not have a pole, but is small unless k^2 is fairly close to m^2. We normalize $S(k^2)$ in terms of the known value of the quark condensate [16]:

$$\langle 0|\bar{\Psi}(0)\,\Psi(0)|0\rangle = -m_0^3 \tag{3.3}$$

where for the u,d quarks

$$m_0 \approx 225 \text{ MeV} \tag{3.4}$$

The quantity (3.3) is an integral over $S(k^2)$; after a Wick rotation this reads

$$\frac{3m}{4\pi^2}\int dz\; z\; S(-z) = m_0^3 \tag{3.5}$$

Although the vertical quark lines in Fig. 3 are non-perturbative, the horizontal ones are perturbative, for the reason that we discussed in Section 2. Their mass does not enter in the asymptotic contribution to $\bar{q}(x)$. We find that at small x

$$x\bar{q}(x) \sim \frac{9\beta_0^2}{8\pi^3}\int dz\; z(z+2m^2)\, S^2(-z) \tag{3.6}$$

As far as we can establish from the literature [16], m should be close to the constituent quark mass, that is for the light quarks

$$m \approx 330 \text{ MeV} \tag{3.7}$$

We assume a simple form for S(-z), chosen because it is straightforward to integrate:

$$S(-z) = \frac{4\pi^2}{3}\,\frac{m_0^3}{m^5}\, e^{-z/m^2} \tag{3.8}$$

This is for $z > 0$ and is intended as a numerical approximation to S, after the Wick rotation needed to obtain (3.5) has been performed. It gives

$$x\bar{q}(x) \sim \frac{3}{2}\pi\,\beta_0^2\,\frac{m_0^6}{m^4} \tag{3.9}$$

This is clearly rather sensitive to the values of m_0 and m. With those chosen in (3.4) and (3.7), it is 0.2. The fact that this is of the same order of magnitude as the experimental value [10] 0.16 of the coefficient of $x^{-\varepsilon}$ in $x\bar{q}(x)$ at small x and $Q^2 = 10\ \text{GeV}^2$ is the basis of our belief that, at this value of Q^2, the zeroth-order term represents a large part of $\bar{q}(x)$. Of course this conclusion is only qualitative. To make it more quantitative, one would need a better knowledge of the shape of the quark propagator and of the parameters such as m_0 and m on which it depends. A complete discussion would also have to pay attention to problems of gauge invariance, which we have had to ignore in this first attempt to calculate $\bar{q}(x)$.

4. - THE GLUON STRUCTURE FUNCTION AND SCALING VIOLATION

If in Fig. 3 we make all the quark propagators perturbative, the contribution from the diagram to the quark structure function no longer scales. In a conventional analysis of the scaling violation, one would consider a diagram such as Fig. 3, with a simple quark loop at the top but with a complete gluon structure function at the bottom. By comparing the two calculations, we therefore obtain a model for the gluon structure function at small x. Like our calculation of $\bar{q}(x)$ in the last section, it is a zeroth-order model, in which there is no scaling violation. We find that, for small x, the gluon structure function of a quark is

$$x\, g(x) = \frac{3 g_0^2}{2\pi^2} \int dz\, z\, D^2(-z) \qquad (4.1)$$

where D is the gluon propagator with the perturbative tail subtracted off. If we use (2.8), this is

$$x\, g(x) = 6C\left(g_0^2/4\pi\right)^{-1} \qquad (4.2)$$

As for $\bar{q}(x)$, we expect that more complicated diagrams will generate for g(x) the behaviour $x^{-1-\varepsilon}$ characteristic of Pomeron exchange. In a similar way to that in which we argued in Section 2, the coefficient of $x^{-1-\varepsilon}$ should be close to the result (4.2). This is for the gluon structure function of the quark; for the nucleon we must multiply by 3 because it has three valence quarks.

Comparison with experiment still leaves considerable uncertainty [17] about the shape and magnitude of the gluon structure function of the nucleon at small x. However, if we take the coupling g_0 of the gluon to "near shell" quarks to be such that $g^2{}_0/4\pi$ is of order 1, our calculated magnitude is in agreement with what is known. Such a value for $g^2{}_0/4\pi$ seems sensible: it corresponds to the perturbative $\alpha_s(Q^2)$ freezing at its value near $Q^2 = 4\Lambda^2$ when Q^2 is decreased to small values. This is around a GeV^2, and this is indeed the characteristic scale of non-perturbative strong interactions, which must cause the freezing.

Our conclusion, then, is that as with $\bar{q}(x)$, while we cannot perform an accurate calculation we seem to have a good understanding in semi-quantitative terms of the largest part of the gluon structure function at moderate Q^2 values.

The scaling violation, to lowest order in $\alpha_s(Q^2)$, comes from Figs. 2 and 4. It is the perturbative tails of the quark propagators that lead to the leading log scaling violation. If we include also the perturbative tails of the gluon propagators, we obtain a contribution that is higher order in the perturbative coupling. (Including a perturbative tail for the gluons but not for the quarks does not cause any scaling violation; it just gives a small correction to the calculation of Section 3.)

The calculation of the scaling violation effects corresponds to the standard diagrammatic analysis of the Altarelli-Parisi evolution [18], with the particular model that we have for the gluon structure function. Our gluons are "near shell" rather than strictly "on shell", but this does not affect the results of the analysis. Notice, however, that we will recover the standard Altarelli-Parisi evolution only for $Q^2 >> (1\ GeV)^2$, rather than merely $Q^2 >> \Lambda^2$. This is because we maintain that the quark propagator does not behave like $1/k^2$ until k^2 is much greater than the scale characteristic of non-perturbative strong interactions, which is $(1\ GeV)^2$.

5. - CONCLUSION

We have extended the original Landshoff-Nachtmann model to take partially into account the colour structure of the full theory. The present model has most of the properties that non-perturbative QCD should exhibit. Although we had to neglect questions of gauge invariance, the main uncertainty comes from the rudimentary knowledge that we have of the quark and gluon propagators in the non-perturbative region. Nevertheless, our calculations show that at small x and for $Q^2 = 10\ GeV^2$ it is very likely that the nucleon structure functions are dominated by non-perturbative exchanges which are zeroth order in the perturbative coupling $\alpha_s(Q^2)$. The full perturbative evolution, corresponding to the Altarelli-Parisi equations, is valid only for Q^2 well above the scale characteristic of non-perturbative effects, that is $Q^2 >> 1\ GeV^2$.

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FIGURE CAPTIONS

Fig. 1 Two-gluon exchange between quarks.

Fig. 2 Zeroth-order (scaling) contribution to quark and antiquark distributions. The vertical quark propagators are non-perturbative.

Fig. 3 Zeroth-order contribution to the quark and gluon distributions. The gluon lines are non-perturbative.

Fig. 4 A scale-breaking contribution to the quark and antiquark distributions. The quark lines are perturbative, and the gluon lines non-perturbative.

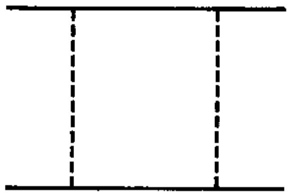
Fig. 1

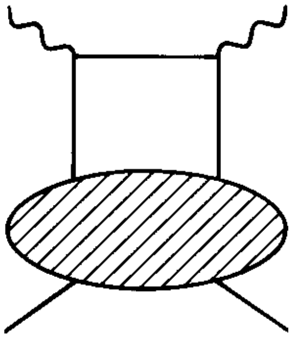
Fig. 2

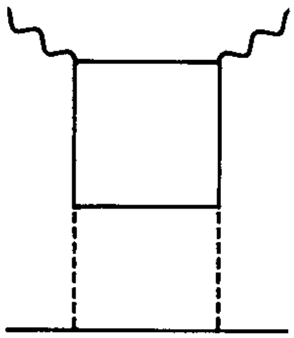
Fig. 3

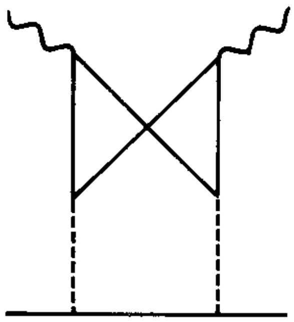
Fig. 4