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QUARK CONDENSATES, POMERON STRUCTURE AND DISTRIBUTION FUNCTIONS*

J.R. Cudell
D.A.M.T.P., University of Cambridge
Cambridge CB3 9EW United Kingdom

ABSTRACT

We give a brief description of the Landshoff-Nachtmann (LN) model, which incorporates vacuum condensates and pomeron behavior in a QCD-inspired description of soft hadronic interactions. We discuss its predictions for quark and gluon distribution functions.

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1. Introduction:

Long-range strong interactions at high energy are very well decribed by an amplitude corresponding to Pomeron exchange[1]:

$$A(s,t) \sim s^{\alpha(t)} \tag{1}$$

with $\alpha(0) \approx 1$. The imaginary part of that amplitude at $t = 0$ gives the total cross-section through the optical theorem and obeys the additive quark rule, i.e. the cross-section for the interaction of two hadrons containing n_A and n_B valence quarks is proportional to $n_A \times n_B$.

Difficulties arise when one tries to get such a behavior from QCD. The simplest object with the right quantum numbers that can be exchanged between quarks to reproduce (1) consists of a pair of gluons. The additive quark rule is normally violated by such an exchange. This can be traced back to the fact that diagram 1.a, where the two gluons are attached to different quarks, contributes as much as diagram 1.b, where they are attached to the same one. Through special assumptions about the hadronic wave function, one can approximately recover the additive quark rule, but the infrared singularities cannot be cancelled for nonzero t[2]. These problems can all be traced back to the presence of the $k^2 = 0$ pole of the perturbative gluon propagator.

To remedy this, Landshoff and Nachtmann proposed a model[3] in which the $k^2 = 0$ singularity of the gluon propagator was removed. More precisely, it was assumed that at small distances, the gluon propagator would be given by perturbation theory, whereas for long distances, nonperturbative contributions would change it into $D(k^2)$, a non-singular function for all k^2. The quark counting rule is then automatically recovered if $D(0)$ is finite. This introduces a length scale, a, which can be thought of as a correlation length and will turn out to be of the order of 0.2 fm. For small k^2, the propagator then becomes $D(k^2) = a^2\ F(a^2\ k^2)$ with $F(0) = 1$, and diagram 1.a dominates diagram 1.b by a factor R^2/a^2, with R a typical hadronic radius. The total hadronic cross-section can then be reproduced if the following constraint is satisfied:

$$2\pi \int_0^\infty dK^2\ \alpha_n^2 D^2(-K^2) = \beta_c^2 \tag{2}$$

where $\beta_c = 18$ GeV^2 is the pomeron coupling constant multiplied by a colour factor and α_n the nonperturbative coupling constant.

One can constraint the propagator further using information on the nonperturbative QCD vacuum from QCD sum rules. Nonperturbative effects induce a finite gluon condensate:
$4\pi < 0|\alpha_n G_{\mu\nu}(0)G^{\mu\nu}(0)|0 >= M_c^4 \approx 1$ GeV^4. One can relate this, in an abelian model, to a moment of the gluon propagator[3]:

$$\int_0^\infty dK^2\ K^4 \alpha_n D(-K^2) = \frac{4\pi}{27} M_c^4 \tag{3}$$

Thus the part of the propagator generated by nonperturbative effects should fall off faster than $1/k^6$ at large $|k^2|$. One futher constraint can be obtained from the study of ρ_0 diffractive production in deep-inelastic scattering[4]:

$$4\pi \int_0^\infty dK^2 K^2 \alpha_n^2 D^2(-K^2) = \mu_0^2 \beta_c^2 \tag{4}$$

with $\mu_0 = 1.1$ GeV. The constraints 2 to 4 can be met by a very simple choice of propagator, to which we will restrict ourselves in the following:

$$D(k^2) = \frac{\beta_c}{\sqrt{\pi}\alpha_n \mu_0} exp(\frac{-|k^2|}{\mu_0^2}) \tag{5}$$

The nonperturbative part of the propagator is assumed to dominate up to a scale Q_0 and then to join smoothly with the perturbative answer. The continuity of α_n D gives $Q_0^2 = 3$ to 6 GeV^2 and the nonperturbative coupling is then $\alpha_n = 0.3$ to 0.7.

2. The gluon structure function at small x:

We are now in a position to evaluate the gluon structure function to zeroth order in the perturbative coupling[5]. We limit ourselves to the small x region, where pomeron exchange dominates. The leading diagram for deep inelastic scattering through gluons is given in figure 2.a. The usual parton picture introduces a gluon structure function which can be defined by:

$$d\sigma = g(\xi) d\xi d\hat{\sigma} \tag{6}$$

with $d\hat{\sigma} \sim W_{\mu\mu}$ the cross-section at the parton level, and $g(\xi)$ the gluon structure function. $W_{\mu\nu}$ is the usual tensor giving the hadronic contribution to the hard scattering. The answer of the LN model is given by the diagram of figure 2.b:

$$d\sigma_{LN} = T.W4\pi\alpha_n D^2 dPS \tag{7}$$

with T the trace along the lower fermion line and dPS the phase-space volume element. Matching the two equations, and taking into account the fact that from current conservation, parity conservation and finiteness one knows $W_{\mu\nu}$ up to a constant function which can be cancelled between the two equations, one gets:

$$\begin{aligned} g(x)|_{x\to 0} &= [\frac{4}{3}\,\frac{1+(1-x)^2}{x}]\,\frac{2\int_0^\infty dK^2 K^2 \alpha_n^2 D^2(-K^2)}{(4\pi\alpha_n)} \\ &\approx \frac{1}{x}\,\frac{\beta_c^2\mu_0^2}{\pi^2\alpha_n} \approx \frac{3.2 \text{ to } 7.4}{\text{x}} \quad \text{with } \alpha_n = 0.3 \text{ to } 0.7 \end{aligned} \tag{8}$$

This prediction, expected to become valid for $x \leq 0.01$ seems in good agreement with the latest BCDMS data[6] ($xg(x) \approx 3.7$ to 4.9 for $x \to 0$ at $Q^2 = 5$ GeV^2 and from data at $x \geq 0.05$) and with other recent estimates (2.8 to 2.9[7]; 2.1 to 2.9[8] at 10 GeV^2).

3. The Quark Structure Function at small x:

In order to calculate the sea quark structure function, we need to look inside the tensor $W_{\mu\nu}$.

The lowest order contribution is that of figure 3. The dominant contribution is expected to come from a region where both the gluons and the quarks are soft, i.e. where both are subject to nonperturbative effects. We are then faced with the problem of guessing what the nonperturbative part of the quark propagator should be. We assume that the quark is free within a hadronic bag, thus getting the spin structure of its propagator: $(\gamma.k + m)S(k^2)$ with $m \approx 0.33$ GeV. We further assume that it has no pole because of confinement and that it fulfills the condensate constraint $< 0|\bar{\psi}(0)\psi(0)|0 > = -m_0^3 \approx (0.225 \text{ to } 0.2 \text{ GeV})^3$ which implies:

$$\int_0^\infty dK^2 K^2 S(-K^2) = \frac{4\pi^2}{3}\frac{m_0^3}{m} \tag{9}$$

As no other constraint is available at present, we assume a structure for $S(k^2)$ similar to (5) and satisfying (9):

$$S(k^2) = \frac{4\pi^2}{3}\frac{m_0^3}{m^5} \exp(-\frac{|k^2|}{m^2}) \tag{10}$$

The evaluation of diagram 3 then gives[6]:

$$\begin{aligned} xq(x)|_{x\to 0} &= \frac{\beta_c^2}{4\pi^3}\int_0^\infty dK^2 K^2 (K^2 + 2m^2) S^2(-K^2) \\ &= \frac{\pi}{3}\frac{m_0^6}{m^4}\beta_c^2 \approx 0.1 \text{ to } 0.2 \end{aligned} \tag{11}$$

We have evaluated the contribution of the same diagram where both quarks are perturbative, but the gluons still nonperturbative. Assuming that the scale at which quarks become described by perturbation theory is the same as for gluons, we get the following fit to a numerical evaluation of the answer:

$$xq(x)|_{x\to 0} \approx \frac{\beta_c^2\mu_0^2}{4\pi^3}\,\frac{12\pi}{27\alpha_n}\,[0.32\log(1.5\log\frac{2Q^2}{Q_0^2})] \approx \frac{0.07}{\alpha_n} \tag{12}$$

at $Q^2 = 10$ GeV^2 and for $Q_0^2 = 4$ GeV^2. CHARM data give $xq(x) \approx 0.16$ for the same Q^2[9]. Again, within the big uncertainties related to this calculation, connected with quark masses and gauge invariance, we see that the LN model provides us with a reasonable answer.

4. Conclusion:

The LN model, constructed to reproduce pomeron behaviour and vacuum condensate results, seems to be very successful in predicting results for soft physics with only a handful of parameters. The main uncertainties are associated with the quark sector, but an educated guess can provide a reasonable answer. This should not shadow the fact that many things remain to be learned in its context, the main one being how to implement correctly gauge invariance.

References:

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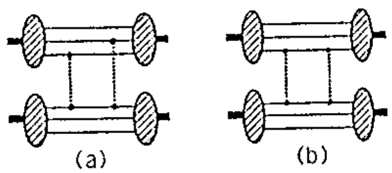

Figure 1: Two-gluon exchange between protons.

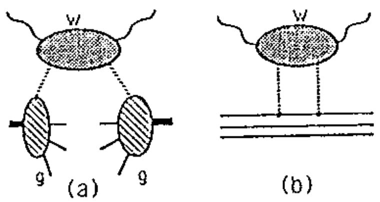

Figure 2: The gluon structure function (a) and the lowest order (zeroth order in the perturbative coupling) contribution to it in the Landshoff-Nachtmann model(b).

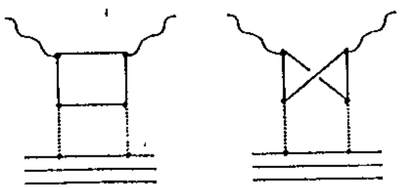

Figure 3: The leading order contribution to the quark and antiquark distributions at low x.