Detuning effects on the mazer

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A short review of the theoretical studies of the cold atom micromaser (mazer) is presented. Existing models are then improved by considering more general working conditions. Especially, the mazer physics is investigated in the situation where a detuning between the cavity mode and the atomic transition frequency is present. Interesting new effects are pointed out. Especially, it is shown that the cavity may slow down or speed up the atoms according to the sign of the detuning and that the induced emission process may be completely blocked by use of a positive detuning. The transmission probability of ultracold atoms through a micromaser is also studied and we generalize previous results established in the resonant case. In particular, it is shown that the velocity selection of cold atoms passing through the micromaser can be very easily tuned and enhanced using a nonresonant field inside the cavity. This manuscript is a summary of Refs. [1, 2, 3].

Introduction

Laser cooling of atoms is a rapidly developing field in quantum optics. Cold and ultracold atoms (temperature of the order of or less than 1 μK) introduce new regimes in atomic physics often not considered in the past. In particular, Englert *et al.* [4] have demonstrated new interesting properties in the interaction of cold atoms with a micromaser field (see Fig. 1). They have shown that excited atoms incident upon the entrance port of a maser cavity will be reflected half of the time, if the atoms are slow enough, even when the maser field is in its vacuum state. This happens because the interaction strength, between the atom and the maser field, changes strongly when passing from the exterior to the interior of the cavity. More recently, Scully *et al.* [5] have shown that a new kind of induced emission occurs when a micromaser is pumped by ultracold atoms, requiring a quantum-mechanical treatment of the center-of-mass motion. To insist on the importance of this quantization usually defined along the z axis, the system was called mazer (for microwave amplification via z-motion-induced emission of radiation). The complete quantum theory of the mazer has been first described in a series of three papers by Scully and coworkers [6, 7, 8]. The theory was written for two-level atoms interacting with a single mode of a high-Q cavity. In particular it was shown that the induced emission properties are strongly dependent on the cavity mode profile. Results were presented for the mesa, sech^2 and sinusoidal modes. Retamal *et al.* [9] later refined these results in the special case of the sinusoidal mode, and a numerical method was proposed by Bastin and Solano [10] for efficiently computing the mazer properties with arbitrary cavity field modes. Löffler *et al.* [11] also demonstrated that the mazer can act as a velocity selection device for an atomic beam and Bastin and Solano [12] studied the trapping state properties of the system. The mazer concept was extended by Zhang *et al.* [13, 14, 15], who considered two-photon transitions [13], three-level atoms interacting with a single cavity [14] and with two cavities [15]. Collapse and revival patterns with a mazer have been computed by Du *et al.* [16]. Arun *et al.* [17, 18] studied

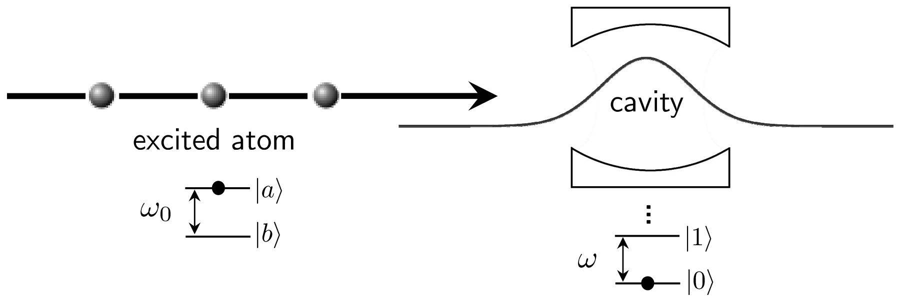

FIG. 1 – Micromaser pumped by two-level atoms.

the mazer with bimodal cavities and Agarwal and Arun [19] demonstrated resonant tunneling of cold atoms through two mazer cavities.

In all these previous studies, the mazer properties were always presented in the resonant case where the cavity mode frequency ω is equal to the atomic transition frequency ω_0. In this paper (see also Refs. [2, 1, 3]), we remove this restriction and present the properties of the mazer in the nonresonant case ($\omega \neq \omega_0$).

The paper is organized as follows. We first give a short review of the maser action. Next we describe the mazer in the resonant case ($\omega = \omega_0$). We then discuss our results in the nonresonant case ($\omega \neq \omega_0$). The photon emission process inside the cavity and the transmission properties of the mazer are especially investigated.

Maser action

The interaction of a two-level atom with a single mode of an electromagnetic field is well described by the Jaynes and Cummings Hamiltonian [20]

$$\hat{H}_{JC} = \underbrace{\hbar\omega\hat{a}^\dagger\hat{a}}_{\hat{H}_{\text{field}}} + \underbrace{\hbar\omega_0\hat{\sigma}^\dagger\hat{\sigma}}_{\hat{H}_{\text{atom}}} + \underbrace{\hbar g\big(\hat{\sigma}\hat{a}^\dagger + \hat{a}\hat{\sigma}^\dagger\big)}_{\hat{H}_{\text{interaction}}} \tag{1}$$

where ω_0 is the atomic transition frequency, ω the cavity field mode frequency, $\hat{\sigma} = |b\rangle\langle a|$ ($|a\rangle$ and $|b\rangle$ are respectively the upper and lower levels of the two-level atom), $\hat{a}$ and $\hat{a}^\dagger$ are respectively the annihilation and creation operators of the cavity radiation field, and g is the atom-field coupling strength.

The behavior of this system is well-known. If the atom is initially in the excited state $|a\rangle$ and the cavity field in the Fock state $|n\rangle$, then the probability to find the atom in the lower state $|b\rangle$ at a later time t is given by

$$\mathcal{P}_{ab}(t) = \sin^2 2\theta_n \sin^2 \frac{\sqrt{\delta^2 + \Omega_n^2}\, t}{2} \tag{2}$$

where δ is the detuning ω - ω_0, Ω_n the Rabi frequency $2g\sqrt{n+1}$ and θ_n the angle defined by

$$\cot 2\theta_n = -\frac{\delta}{\Omega_n} \tag{3}$$

The atom carries out oscillations between the upper and the lower energy states. This cycle of emission-absorption is called a Rabi oscillation.

When thermal atoms, initially prepared in the excited state $|a\rangle$, travel one by one through a maser cavity, Eq. (2) gives the probability to find the atom at the exit of the cavity in the state $|b\rangle$ provided t represents the atom-field interaction time, that is $t = L/v$ with L the cavity length and v the velocity of the atom.

Mazer action

If colder and colder (*i.e.* slower and slower) atoms are injected in the maser cavity, a quantized description of the center-of-mass motion needs to be done as soon as the atomic kinetic energy becomes of the order of or lower than the interaction energy $\hbar g$. In this case, the Hamiltonian that has to be considered reads

$$\hat{H} = \frac{\hat{p}_z^2}{2m} + \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\hat{\sigma}^\dagger\hat{\sigma} + \hbar g\, u(\hat{z})\big(\hat{\sigma}\hat{a}^\dagger + \hat{a}\hat{\sigma}^\dagger\big) \tag{4}$$

where p_z is the atomic center-of-mass momentum along the z axis, m the atomic mass and $u(z)$ the cavity field mode function modelling the spatial variations of the atom-field interaction. The system described by the Hamiltonian (4) is called *mazer* [5].

Resonant case : $\omega = \omega_0$

In the resonant case, the system dynamics governed by the Hamiltonian (4) may be easily studied in the atomic dressed state basis $|\gamma_n^\pm\rangle = \frac{1}{\sqrt{2}}\,(|a,n\rangle \pm |b,n+1\rangle)$. In this basis, we show that the global wavefunction components $\psi_n^\pm(z,t) = \langle z,\gamma_n^\pm|\Psi(t)\rangle$ (where $|\Psi(t)\rangle$ is the global wavefunction of the system) obey a Schrödinger equation describing an elementary one-dimensional scattering process upon the well defined potentials $V_n^\pm = \pm\hbar g\sqrt{n+1}\; u(z)$ (see Ref. [4]). If initially the incoming atoms upon the cavity are in the excited state $|a\rangle$ and the field contains n photons, the atom-field state has initially two non-vanishing components in the dressed state basis (the $\psi_n^\pm(z,t)$ components) and each of them is scattered differently. This process results in a possible reflection or transmission of the atoms by or through the cavity. These processes are accompanied by a probability of finding the atom at the end in the lower state $|b\rangle$, raising the photon number inside the cavity by one unity (photon emission process). An analytical calculation of these probabilities has been obtained in the particular cases where the cavity field mode function is either given by the mesa function ($u(z) = 1$ inside the cavity, 0 elsewhere) either by the sech^2 function which approximates the gaussian mode (see Refs. [6, 7]). An efficient numerical method has been proposed by Bastin and Solano [10] to compute the mazer properties for an arbitrary field mode.

If we denote by k the wave number of the incoming atom and by κ the particular wave number for which the atomic kinetic energy $\hbar^2\kappa^2/2m$ equals the vacuum coupling energy $\hbar g$, we distinguish 3 regimes : the hot atom regime ($k \gg \kappa$), the intermediary regime ($k \simeq \kappa$) and the cold atom regime ($k \ll \kappa$).

In the hot atom regime, the quantization of the atomic motion does not play any role in the system dynamics (the scatterer potentials are too small compared to the atomic kinetic energy). This is well illustrated in Fig. 2 which displays the photon emission probability computed both from the mazer Hamiltonian (4) and from the Jaynes-Cummings one.

Scully *et al.* [6, 7, 8] remarkably showed that, in the cold atom regime, the wave properties of the atoms become important and the results predicted on the basis of the mazer Hamiltonian

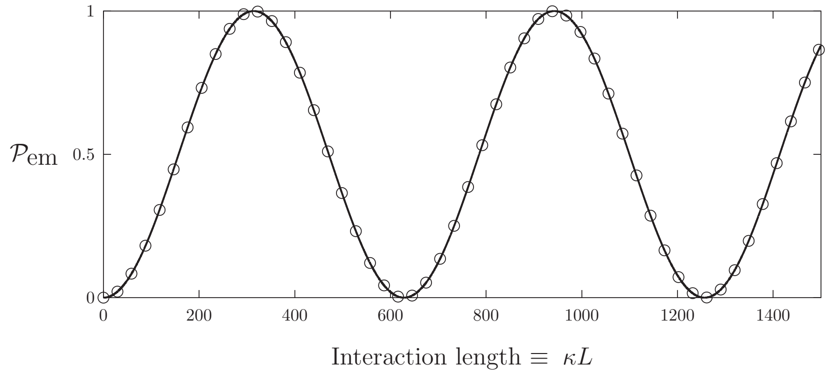

FIG. 2 – Photon emission probability in the hot atom regime ($k/\kappa = 100$). Plain curve : mazer Hamiltonian. Dots : J-C Hamiltonian.

(4) completely differ from those predicted with the Jaynes-Cummings one. In particular, the behavior of the photon emission probability $\mathcal{P}_{\rm em}$ changes completely compared to the hot atom regime. For $k/\kappa \ll 1$ and the mesa mode function, $\mathcal{P}_{\rm em}$ shows very sharp resonances versus the interaction length κL as shown in Fig. 3.

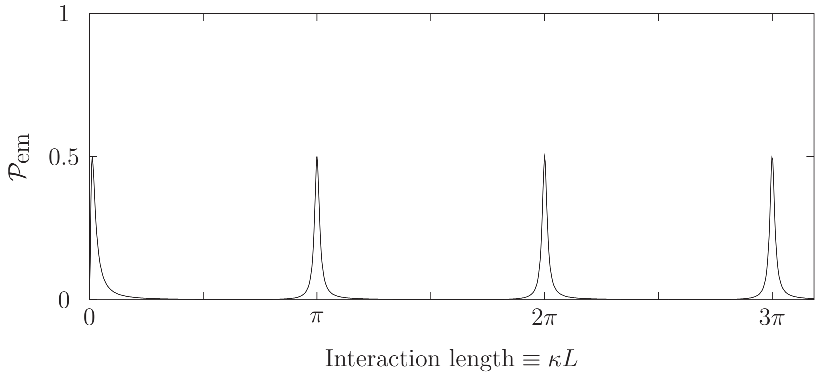

FIG. 3 – Photon emission probability in the cold atom regime ($k/\kappa = 0.01$) for a mesa mode function.

Nonresonant case : $\omega \neq \omega_0$

In the nonresonant case, the equations verified by the wavefunction components take a much more complicated form than in the resonant case. It has however been possible to obtain analytical expressions of the transmission and the photon emission probabilities for a mesa mode function (see Ref. [2] for more details). In this case, interesting new effects have been obtained compared to the resonant case. Especially, the cavity may slow down or speed up the atoms, according to the sign of the detuning, and the induced emission probability may be completely forbidden for positive detunings. These new effects are easily understandable by considering the energy conservation depicted in Fig. 4. When, after leaving the cavity region, the atom is passed from the excited state $|a\rangle$ to the lower state $|b\rangle$, the photon number has increased by one unit in the cavity and the internal energy of the atom-field system has varied by the quantity $\hbar\omega - \hbar\omega_0 = \hbar\delta$. This variation needs to be exactly counterbalanced by the external energy of the system, i.e. the atomic kinetic energy. In this sense, when a photon is emitted inside the

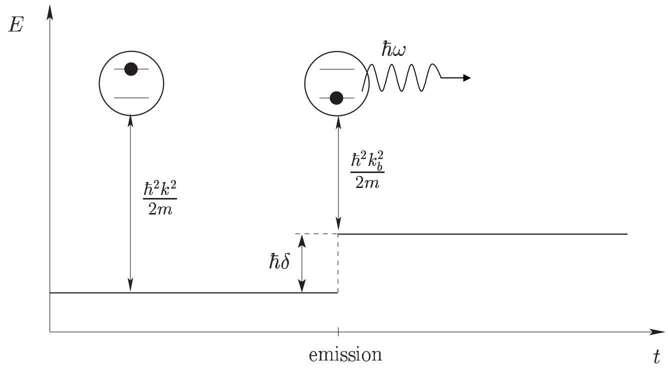

FIG. 4 – Potential step effect of the cavity when a photon is emitted by the atom. E represents the total energy of the atom-field system.

cavity by the atom, the cavity acts as a potential step $\hbar\delta$ (see Fig. 4). We denote by k_b the wave number of the atom after emission of a photon. The atomic transition $|a\rangle \to |b\rangle$ induced by the cavity is therefore responsible for a change of the atomic kinetic energy. According to the sign of the detuning, the cavity will either speed up the atom (for $\delta < 0$) or slow it down (for $\delta > 0$).

The use of positive detunings in the atom-field interaction defines a well-controlled cooling mechanism. A single excitation exchange between the atom and the field inside the cavity is sufficient to cool the atom to a desired temperature $T = \hbar^2 k_b^2/2mk_B$ (k_B is the Boltzmann constant) which may be in principle as low as imaginable. However, if the initial atomic kinetic energy $\hbar^2 k^2/2m$ is lower than $\hbar\delta$ i.e. if $k/\kappa < \sqrt{\delta/g}$), the transition $|a, n\rangle \to |b, n+1\rangle$ cannot take place (as it would remove $\hbar\delta$ from the kinetic energy) and no photon can be emitted inside the cavity. In this case the emission process is completely blocked.

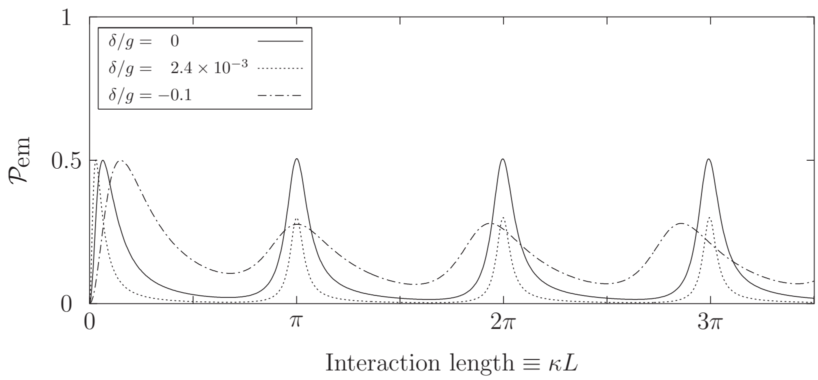

FIG. 5 – Photon emission probability in the cold atom regime ($k/\kappa = 0.05$) for different detuning values and in the case of a mesa mode function.

Figure 5 illustrates the induced emission probability with respect to the interaction length κL for various values of the detuning and in the cold atom regime. Like the resonant case, the curves present a series of peaks where the induced emission probability is optimum. However, the detuning strongly affects the peak position, amplitude and width.

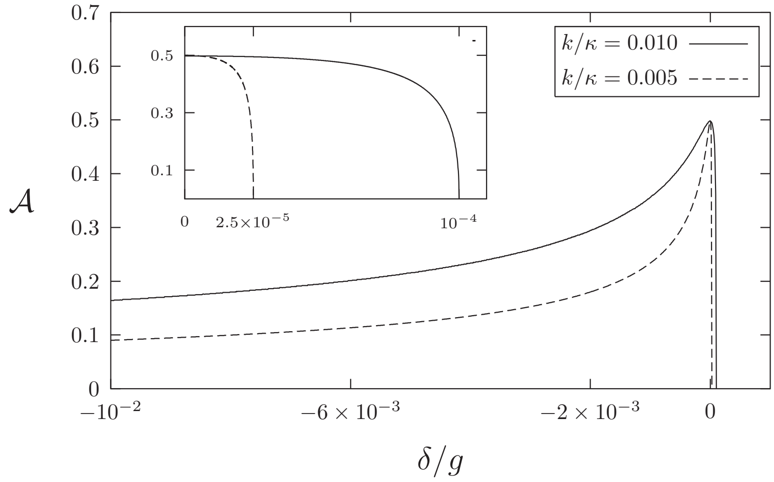

FIG. 6 – Amplitude $\mathcal{A}$ of the resonances with respect to δ/g for 2 values of k/κ in the cold atom regime.

We illustrate in Fig. 6 the amplitude $\mathcal{A}$ of the resonance peaks as a function of the detuning. Contrary to the hot atom regime where the amplitude of the Rabi oscillations is given by the factor $\sin^2 2\theta_n$ (see Eq. (2)) which is insensitive to the sign of the detuning, the curves in Fig. 6 present a strong asymmetry with respect to the sign of δ. This results from the potential step $\hbar\delta$ felt by the atoms when they emit a photon. For cold atoms whose energy is similar or less than the step height, the sign of the step is a crucial parameter. The induced emission probability drops very rapidly down to zero for positive detunings, in contrast to what happens for negative detunings. It is very interesting to note that the peak amplitude is equal to the amplitude at resonance (1/2) times the transmission factor of a particle of momentum $\hbar k$ through a potential step $\hbar\delta$ $((4k_b/k)/(1+k_b/k)^2)$. This is an additional argument to say that the use of a detuning adds a potential step effect for the atoms emitting a photon inside the cavity (see Fig. 4).

Transmission properties of the mazer

Löffler *et al.* [11] have proposed recently to use the mazer for narrowing the velocity distribution of an ultracold atomic beam. This could be very useful to define long coherence lengths. We investigated the effects of a detuning on the transmission probability of an atom through the mazer and on the velocity selection process (see Ref. [3]). We found that the atomic transmission probability through the cavity shows with respect to the detuning fine resonances that could be very useful to define extremely accurate atomic clocks. It also turns out that the velocity selection in an atomic beam could be significantly enhanced and easily tuned by use of a positive detuning.

Transmission probability

Figure 7 illustrates the atomic transmission probability with respect to the detuning (in the ultracold regime). The curve obtained presents very sharp resonances. For realistic experimental parameters (see discussion in [7]), these resonances may even become extremely narrow. Their width amounts only 10^{-2} Hz for $\kappa L = 10^5$, $g = 100$ kHz and $k/\kappa = 0.01$. This could define very useful metrology devices (atomic clocks for example) based on a single cavity passage and with better performances than what is usually obtained in the well known Ramsey configuration with two cavities or two passages through the same cavity [21].

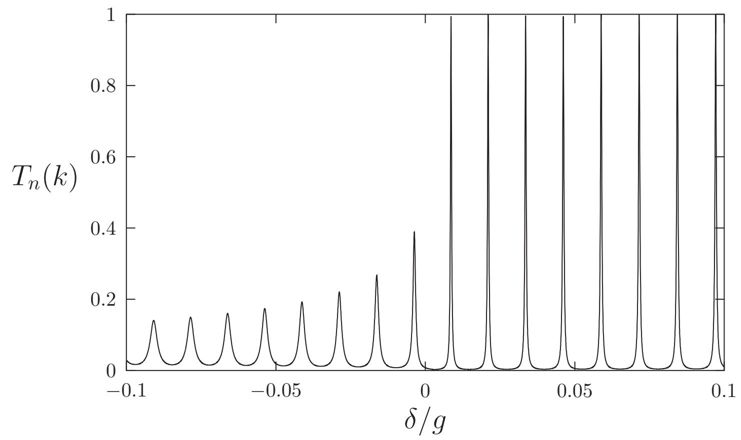

FIG. 7 – Transmission probability of an excited atom through the mazer with respect to the detuning ($k/\kappa = 0.05$, $\kappa L = 1000$, $n = 0$)

Velocity selection

We show in Figs. 8 how a Maxwell-Boltzmann distribution (with $k_0/\kappa = 0.05$ where k_0 is the most probable wave number) is affected when the atoms are sent through the cavity. The cavity parameters have been taken identical to those considered in [11] to underline the detuning effects. We see from these figures that the final distributions are dominated by a narrow single peak whose position depends significantly on the detuning value. This could define a very convenient way to select any desired velocity from an initial broad distribution. Also, notice from the $\mathcal{P}_f$ scale that a positive detuning significantly enhances the selection process.

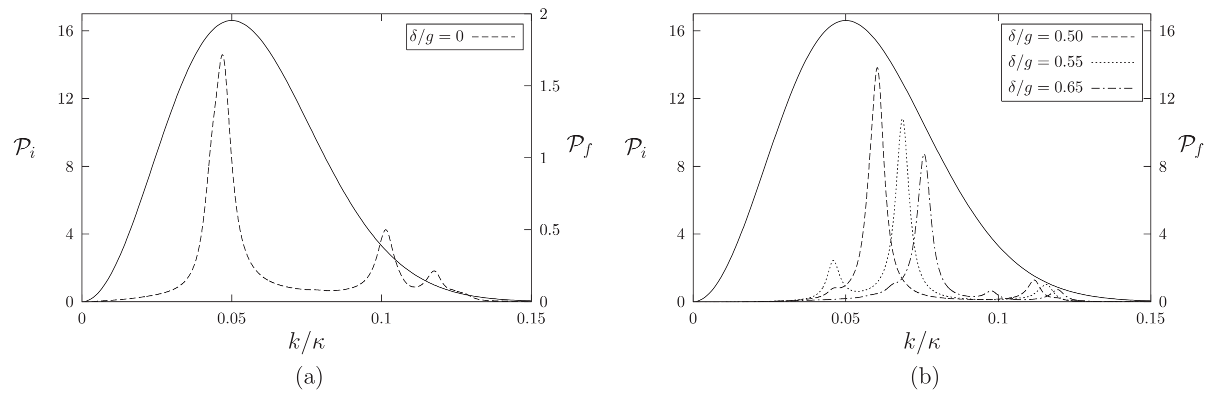

FIG. 8 – Initial (plain curve) and final (dashed curves) velocity distributions (a) at resonance and (b) for various detuning values.

References

[1] J. Martin, *Master thesis* (Université de Liège - Belgium, 2002).

[2] T. Bastin, J. Martin, Phys. Rev. A **67**, 053804 (2003).

[3] J. Martin, T. Bastin, Eur. Phys. J. D **29**, 133 (2004).

[4] B.G. Englert, J. Schwinger, A.O. Barut, M.O. Scully, Europhys. Lett. **14**, 25 (1991).

[5] M.O. Scully, G.M. Meyer, H. Walther, Phys. Rev. Lett. **76**, 4144 (1996).

[6] G.M. Meyer, M.O. Scully, H. Walther, Phys. Rev. A **56**, 4142 (1997).

[7] M. Löffler, G.M. Meyer, M. Schröder, M.O. Scully, H. Walther, Phys. Rev. A **56**, 4153 (1997).

[8] M. Schröder, K. Vogel, W.P. Schleich, M.O. Scully, H. Walther, Phys. Rev. A **56**, 4164 (1997).

[9] J.C. Retamal, E. Solano, N. Zagury, Opt. Commun. **154**, 28 (1998).

[10] T. Bastin, E. Solano, Comp. Phys. Commun. **124**, 197 (2000).

[11] M. Löffler, G.M. Meyer, H. Walther, Europhys. Lett. **41**, 593 (1998).

[12] T. Bastin, E. Solano, Opt. Commun. **217**, 239 (2003).

[13] Z.M. Zhang, Z.Y. Lu, L.S. He, Phys. Rev. A **59**, 808 (1999).

[14] Z.M. Zhang, L.S. He, Opt. Commun. **157**, 77 (1998).

[15] Z.M. Zhang, S.W. Xie, Y.L. Chen, Y.X. Xia, S.K. Zhou, Phys. Rev. A **60**, 3321 (1999).

[16] Si-de Du, Lu-wei Zhou, Shang-qing Gong, Zhi-zhan Xu, Jing Lu, J. Phys. B : At. Mol. Opt. Phys. **32**, 5645 (1999).

[17] R. Arun, G.S. Agarwal, M.O. Scully, H. Walther, Phys. Rev. A **62**, 023809 (2000).

[18] R. Arun, G.S. Agarwal, Phys. Rev. A **66**, 043812 (2002).

[19] G.S. Agarwal, R. Arun, Phys. Rev. Lett. **84**, 5098 (2000).

[20] E.T. Jaynes, F.W. Cummings, Proc. IEEE **51**, 89 (1963).

[21] A. Clairon, C. Salomon, S. Guellati, W.D. Phillips, Europhys. Lett. **16**, 165 (1991).