Enhanced proof strength after cold process of fabrication of non-linear metallic profiles – Comparison of two predictive models for hollow sections

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ABSTRACT: The mechanical properties of hollow sections are known to be greatly affected by the cold forming process, the strain hardening leading to increased resistance compare to a resistance assessment based on nominal properties. It is thus necessary to accurately determine the mechanical properties after the cold process of fabrication. In the present paper, a theory-based formula evaluating the enhanced 0.2% proof stress in flats and corners of hollow sections using the virgin sheet material properties and the final geometry is presented. A comparison is made to the recent predictive model developed by Cruise for the evaluation of the enhanced mechanical properties in the corners and flats of stainless steel hollow sections. The new model is not restricted to a single alloy but is valid for non-linear metallic materials. In contrast to the traditional empirical formula found in the literature, its main thrust lies in its axiomatic roots. The predictions are validated against experimental data collected in the literature for steel and stainless steel sections.

1 INTRODUCTION

1.1 *Production route*

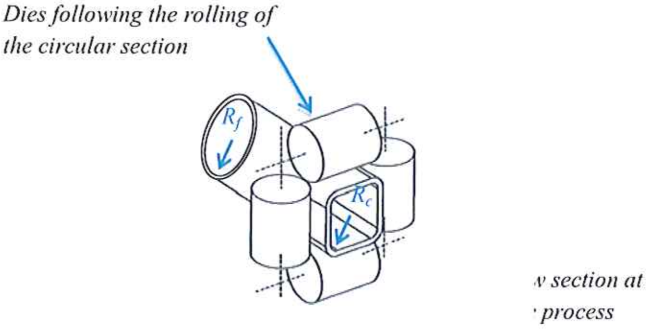

Figure 1. Process of fabrication of roll-formed tubes. The circular section is crushed into a square.

The production route that is considered herein is the well-known automated process of cold-rolling. During cold-rolling, the flattened sheet of metal is gradually deformed by passing through successive rollers in order to reach the desired cross-section shape (angle, channel, lipped channel, square, rectangular, oval…).

In case of tubular sections, the flat sheet is first rolled into a circular tube that is subsequently deformed into a square or rectangle by means of dies (see Figure 1). The tube's cross-section at the entrance is circular (radius R_f) whereas the cross-section at the exit of the process is a square/rectangle with round corners (radius R_c). At the end of the process, the member's lips are automatically welded at the top of the member.

Finite element models including complex contact problems allow the user to reproduce this continuous process but are quite difficult to achieve and time consuming even if very sophisticated tools are used. Moreover, it is easy to understand that the complexity of the process of fabrication of hollow sections cannot be summed up with a small number of steps such as it could be done for press-braking for instance (quite successfully modeled as pure-bending). The problem of tube roll-forming is thus uneasy to theoretically implement.

1.2 *Importance of work-hardening*

It is not new to assess the strength enhancement induced during the process of fabrication of cold-formed sections. The strength enhancement in the corner regions of cold-formed carbon steel sections due to plastic deformation was first observed by Karren (Karren 1967). A power model providing the strength increase in terms of the yield stress of the virgin material and the internal radius of the corner R_c to thickness t ratio was proposed. The author suggested that since the corner regions can represent between 5% and 30% of the total cross-sectional area, the influence of the enhanced strength should be included in structural calculations.

Since then, numerous researches were carried out on that subject, demonstrating the importance of work hardening and its implications on the global stability of members.

Karren's expression was modified by Van Den Berg and Van Der Merwe (Van Den Berg & Van Den Merwe 1992) on the basis of new experimental data for stainless steel sections and more recently by Ashraf (Ashraf et al. 2005, Ashraf 2006) who provided empirical predictive models (power model as well) for the evaluation of the corners enhanced properties in stainless steel sections. The methods employ the 0.2% proof stress and the ultimate stress of the virgin sheet material and, still, the geometrical R_c/t ratio. For cold-rolled hollow sections, Gardner and Nethercot (Gardner & Nethercot 2004a) proposed a linear relationship between the 0.2% proof stress of the corners and the ultimate stress of the flat faces $\sigma_{ult,f}$. Later, Cruise (Cruise 2008) reused this expression and refitted it with all available experimental data. She also proposed a new model predicting the enhanced proof stress of the flats of stainless steel hollow sections.

In the current context of sustainable use of the resources, it is of great interest to focus the researches to a better comprehension of the increase in strength induced during cold forming. Especially when materials exhibit a non-linear stress-strain curve with high strain hardening such as stainless steels or high-strength steels. Indeed, due to the difference in cost as compared with carbon steel equivalents, it is important that such effects be incorporated in the design rules. Authors (Rasmussen & Hancock 1993, Van Den Berg 2000, Young & Liu 2003, Gardner & Nethercot 2004b, Ellobody & Young 2005, Ashraf & Gardner 2006, Rossi 2008…) showed that the material behavior is inseparable of the structural response of members and that numerical analyses fails to properly model the behavior of stainless steel members if the work hardening is not well taken into account. In (Ashraf et al. 2006a & 2006b), the authors recommend incorporating the higher strength of the corners regions into the recent Continuous Strength Method in order to better evaluate the average increases in global resistance offered by the member.

In the present paper, two methods predicting the enhanced mechanical properties using the material properties of the unformed sheet and the geometry of the final cross-section are presented. One of them was developed by Cruise in 2007 on the basis of a set of experimental data concerning stainless steel hollow sections (see (Cruise 2007) for the different references to the set of data included in the study). The second one is a theoretical model developed by Rossi in 2008, which was also verified against the same set of experimental data (Rossi 2008).

The accuracy of the predictions is illustrated and the methods are compared in terms of more qualitative aspects.

2 PREDICTIVE MODELS

2.1 *Introduction*

On the basis of experimental measurements made on hollow stainless steel sections combined with literature review, Cruise (Cruise 2007) developed an empirical model predicting the enhanced proof stress distribution along the cross-section of cold-rolled stainless steel hollow sections. In the model, flats and corners are studied separately and the regions where the material strength is influenced by corner forming are also identified.

In (Rossi 2009), the author examines the through-thickness residual stresses distribution and strength enhancement induced during cold forming of sections made of non-linear metallic materials. During the study, the complexity of the forming process is summarized in four steps: (A and B) coiling-uncoiling, (C) forming into a circular section and (D) subsequent deforming into a rectangular section (just as in Figure 2); and that in order to be able to analytically implement the problem. The flats of cold-rolled hollow sections were thus supposed to undergo coiling-uncoiling and then bending-unbending in the direction perpendicular to the rolling direction.

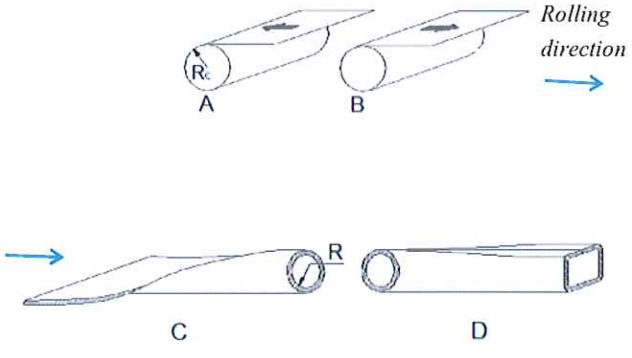

Figure 2. Roll-forming of hollow sections. Four steps were considered for the modeling of the process of fabrication.

It was thus possible to theoretically assess the influence of each step of the process of fabrication. As an illustration of it, Figure 3 depicts the percentage of enhancement (obtained with the theoretical study) in the flats of 21 hollow sections after each step of the process for three types of sections studied in (Talja & Salmi 1995), (Gardner & Nethercot 2004a) and (Hyttinen 1994).

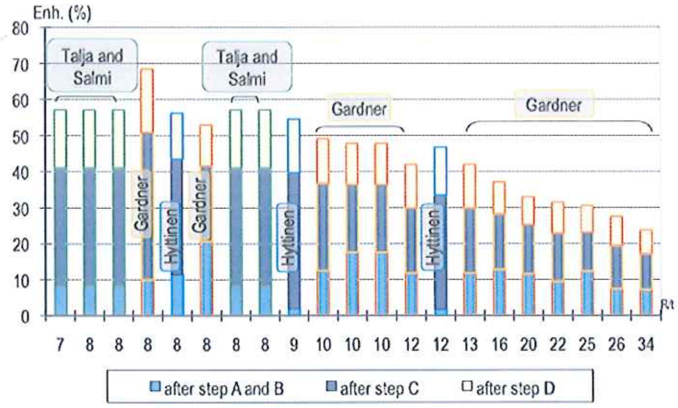

Figure 3. Percentage of enhancement in flats of roll-formed tubes after each step of the process considered in the study of (Rossi 2008). Three sets of data are included in this study.

Consequently, knowing the material properties of the virgin sheet and the final geometry of the cross-section from the concerned research studies, it was possible to compare the theoretical relative enhancement (Figure 3) to the experimental measurements of it (obtained with tensile tests on coupon taken from the faces of the formed hollow sections).

Afterwards, using the inversion of the stress-strain curve's mathematical expression developed by Abdella in (Abdella 2006), an analytical expression providing the enhanced 0.2% proof stress was established. Assuming pure bending during forming and using the simplified engineering strain expression in Abdella's analytical formula, Rossi's predictive model was obtained. This formula does not distinguish flats and corners and is based on the conclusions of the previous theoretical study: (1) for *flats*, the second part of the process (step C: forming into a circle of radius R_f) had the greatest influence and should thus be considered as the most important one, (2) for *corners*, the higher curvature leads to increased strain hardening and R_c should be employed.

By way of clarification, in Rossi's formula, R indicates the corner's radius R_c or the radius of curvature R_f that was undergone by the section if the user's interest goes to the flats.

In both models, if the cross-section is rectangular, the curvature experienced by the section face during the forming can be written in terms of the section geometry using $2\pi R_f = 2(B+D)$ (see Figure 4, the insignificant R_c terms are not considered).

2.1 *Cruise's predictive models for flats and corners*

Firstly, Cruise provided an empirical formula for the strength increase in the central 50% of the faces of hollow sections after cold-rolling, see Equation (1).

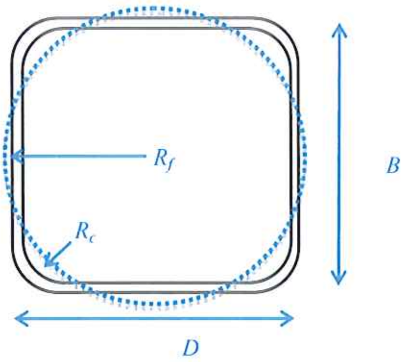

Figure 4. Hollow section geometry. The curvature undergone by the faces can be expressed in terms of the section geometry.

In the procedure of curve fitting, she included data from her test program and all published data at that time. The formula includes the mill certificate 0.2% proof stress namely $\sigma_{0.2,\text{mill}}$ and the geometrical parameter equivalent to an R/t ratio.

$$\sigma_{0.2,f} = \frac{0{,}85\sigma_{0.2,mill}}{-0{,}19 + \dfrac{1}{12{,}42\left(\dfrac{\pi t}{2(B+D)}\right) + 0{,}83}} \tag{1}$$

where $\sigma_{0.2,mill}$ = 0.2% mill certificate proof stress; t, B and D = respectively, the section thickness and the section faces widths (Figure 4).

Secondly, she also refitted Gardner's model for the corners of hollow sections and provided the Equation (2) for the strength increase after forming.

$$\sigma_{0.2,c} = 0{,}83\sigma_{ult,f} \tag{2}$$

where $\sigma_{ult,f}$ = predicted ultimate stress of the faces of cold-rolled hollow sections,

$$\sigma_{ult,f} = \sigma_{ult,mill}\left(0{,}19\frac{\sigma_{0.2,f}}{\sigma_{0.2,mill}} + 0{,}85\right) \tag{3}$$

and $\sigma_{ult,mill}$ = mill certificate ultimate stress.

Finally, the simple power model provided by Ashraf in (Ashraf 2006), despite its slightly higher scatter of predictions, was also refitted for the corners of press-braked sections. This model is used in a more extensive comparative study including press-braked sections but will not be discussed herein.

$$\sigma_{0.2,c,PB} = \frac{1{,}673\sigma_{0.2,mill}}{\left(\dfrac{R_c}{t}\right)^{0{,}126}} \tag{4}$$

2.2 *Rossi's predictive model (for flats and corners)*

The strength increases in cold-rolled sections is provided by Equation (5) with the assumed simplified strain expression provided in (Rossi 2008).

$$\frac{\sigma_{ult,mill}}{\sigma_{0.2,f\,or\,c}-\sigma_{0.2,mill}}=C_1\left(\frac{R}{t/2}\right)+C_2\left(\frac{R}{t/2}\right)^{\alpha} \tag{5}$$

where the parameters C_1, C_2 and α are provided in the Equations (6), (7) and (8) and depend on the material parameters included in the modified Ramberg-Osgood's expression for the stress-strain curve (Rasmussen 2003, Abdella 2006). Such that the proposed model is thus different if another material is considered.

$$C_1=\frac{\varepsilon_{0.2}}{r_2}\frac{\sigma_{ult}}{\sigma_{0.2,mill}} \tag{6}$$

$$C_2=\frac{\left(r^*-1\right)\varepsilon_{0.2}}{r_2\left(\varepsilon_u-\varepsilon_{0.2}\right)^{p^*}}\frac{\sigma_{ult}}{\sigma_{0.2,mill}} \tag{7}$$

$$\alpha=\left(1-p^*\right) \tag{8}$$

In the Equation (4), $\sigma_{0.2,\,f\,or\,c}$ is the predicted quantity. The formula is used for flats ($\sigma_{0.2,f}$) or corners ($\sigma_{0.2,c}$) with the respective radius of curvature $R_f=(B+D)/\pi$ and R_c.

3 VALIDATION

Table 1. Average and Coefficient of Variation for the $\sigma_{0.2,f\,(predicted)}$ to $\sigma_{0.2,f\,(measured)}$ ratio in flats of hollow sections.

Model	Average	CoV
Cruise	1.09	0.31
Rossi	1.03	0.21

The reference researches (Talja & Salmi 1995), (Gardner & Nethercot 2004a), (Hyttinen 1994), (Cruise 2007), (Gardner et al. 2006), (Rasmussen 993), (Key & Hancock 1985), (Niemi & Rinnevalli 1990), (Nip et al. 2010), (Guo et al. 2007) were used for the comparison of the two models predictions. In the present contribution, the four last ones have been added to the previous set of data such that stainless steel and steel sections are included.

The results shown herein only concern the strength enhancement induced in the flats of hollow sections. A more extensive comparison is currently undertaken. In this study, flats and corners strength enhancement are collected for cold-formed sections (not only hollow sections) made of steel, high strength steel and stainless steel (austenitic and ferritic).

For each section described, the geometry, mill certificate data ($\sigma_{0.2,mill}$) and measured strength namely $\sigma_{0.2,f\,(measured)}$ were collected. The increased strength namely $\sigma_{0.2,f\,(predicted)}$ was evaluated using the models.

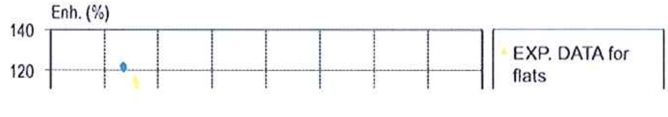

Figure 5. Relative enhancement *Enh.(%)* in flats of roll-formed tubes, all available data are represented. The experimental measurements and the two models are included.

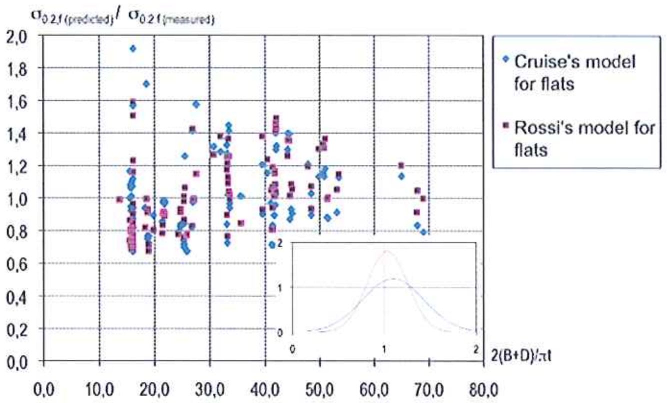

Figure 6. $\sigma_{0.2,f\,(predicted)}$ to $\sigma_{0.2,f\,(measured)}$ ratio in flats of roll-formed tubes, all available data are represented. The two models are included.

The relative enhancement *Enh.(%)* in the flats of the sections was then calculated (Equation 9) allowing the comparison to be done with the measured enhancements.

$$Enh.(\%)=\frac{\sigma_{0.2,f\,(measured\,or\,predicted)}-\sigma_{0.2,mill}}{\sigma_{0.2,mill}} \tag{9}$$

The comparison shows that very similar results can be obtained with a slightly better agreement for Rossi's model in terms of the average value of the $\sigma_{0.2,f\,(predicted)}$ to $\sigma_{0.2,f\,(measured)}$ ratio (see Table 1). Moreover, the Standard Deviation to Average ratio (CoV) is smaller if Rossi's model is considered.

Both models provide very accurate results and the difference between them is not pronounced as shown in Figure 6 depicting the $\sigma_{0.2,f\,(predicted)}$ to $\sigma_{0.2,f\,(measured)}$ ratio.

The same study is currently been done for corners although less data were available in the literature.

4 QUALITATIVE DIFFERENCES AND CRITICS

4.1 *Difficulty*

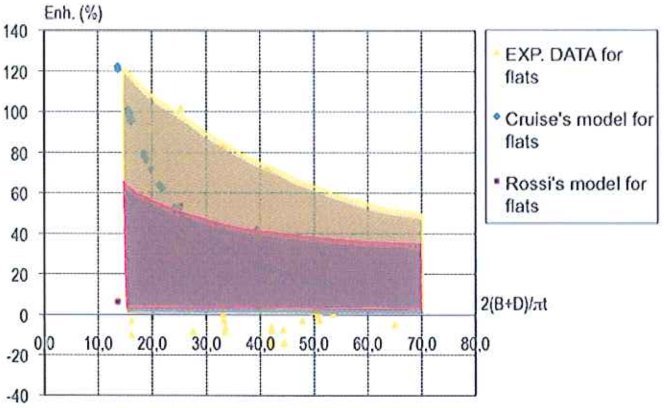

Figure 7. Percentage of enhancement *Enh.(%)* in flats of roll-formed tubes. Trends of the models. Color coding: Light orange area for the surface including the experimental points and light magenta for the surface covering the points obtained with Rossi's model. For Cruise's model, the surface becomes a curve (dark blue points).

The first difference that could be noticed is the mathematical form taken by the models. Cruise's model requires less detailed information about the virgin sheet material and is therefore much easier to employ.

4.2 *Empirical VS Theoretical*

Cruise's model is an empirical model, the expression of which was based on (1) previous models for corners (Ashraf's power model and Gardner's linear model for corners of cold-rolled sections) and (2) careful observations of the level of cold work (the increase in 0.2% proof stress and ultimate stress) that has occurred during the forming of the section faces.

The main thrust of the second model lies in its axiomatic roots. The analytical form was established using the inverted Ramberg-Osgood's mathematical expression of the stress-strain curve without introducing empirical parameters. To obtain the 0.2% proof stress after forming, the user needs several material parameters that can be obtained from the material stress-strain curve. As a consequence of it, it can be supposed that if another set of data is added, this model should provide at least as good results as before.

4.3 *Trends of the models*

If a closer look is given to Figure 7, it can be noticed that the trend of *Enh.(%)* (the relative enhancement of the proof stress) is perfectly represented by a power model if Cruise's model is used since $(\sigma_{0.2,f} - \sigma_{0.2,mill})/\sigma_{0.2,mill}$ is not dependent upon $\sigma_{0.2,mill}$ (see Equation 1). In other words, for a given value of $2(B+D)/\pi t$, the model provides the same relative enhancement whatever the 0.2% proof stress of the material considered.

Whereas, for the experimental data and for the second model, a relatively high dispersion of the data is observed – see the two colored areas covering the data in Figure 7.

4.4 *Mathematical form*

The last difference is related to the previous one but leads to a major comment against the first model.

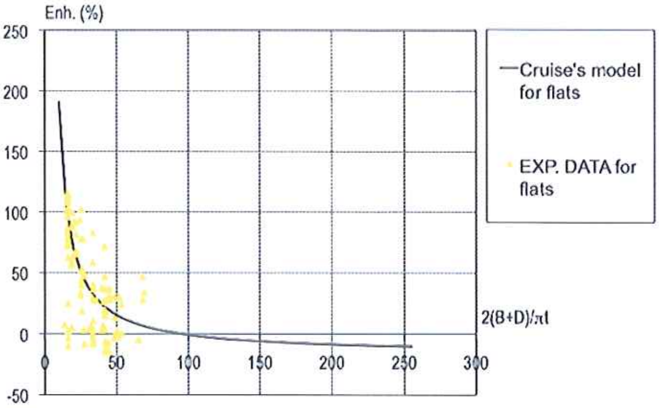

Figure 8. $\sigma_{0.2,f}$ predicted using Cruise's model. The predictions go below zero for a ratio $2(B+D)/\pi t$ equal to 94,4.

If we assume that $\sigma_{0.2,f}$ cannot be smaller than $\sigma_{0.2,mill}$ (the process of fabrication leading to increased resistance), then the first model's analytical expression is not acceptable. Indeed, negative value of $\sigma_{0.2,f}$ - $\sigma_{0.2,mill}$ appear for values of $2(B+D)/\pi t$ higher than 94,4 (Equation 10).

$$\frac{2(B+D)}{\pi t} > \frac{12,42}{\left(\frac{1}{0,85+0,19}-0,83\right)} = 94,4 \qquad (10)$$

If one looks closer to the yellow points (EXP. DATA for flats in Figure 8), no such case appears. Although, in Table 2, an example of geometry for which the enhancement is negative, is provided. Such combination of dimensions could be of practical interest.

Table 2. Dimensions of a square hollow section verifying the Equation 10.

Section's dimensions		B/t or D/t
mm		
B	370	
D	370	94,4*π/4
t	<5	

5 CONCLUSIONS

In the present paper, two models predicting the enhanced proof stress after the process of fabrication in flats of hollow sections are compared. One of the models was designed to be employed for sections made of stainless steel. It is an empirical model that was calibrated against a great amount of experimental data (Cruise 2008). The other one is a theoretical model that was established for material presenting a non-linear stress strain curve. It wasn't calibrated against experimental data (theoretical model) although the accuracy of the predictions was verified for the same set of data (Rossi 2009).

Both models are found to give excellent predictions, conclusion that was expectable since only a few other data for steel hollow sections were added to the set of data used in (Cruise 2008). The difference between the models, in terms of Average and Standard Deviation is not pronounced (see Table 1).

The models are then compared in terms of more qualitative aspects: difficulty, trends, mathematical form…One of these has nevertheless a more physical meaning. Indeed, although Cruise's formula provides excellent predictions, it is underlined that its mathematical form is not acceptable. Assuming an increase in strength after the process, the 0.2% proof stress in the faces of hollow sections should remain higher than the 0.2% proof stress of the unformed material; a relative enhancement tending to be zero with a radius of curvature to thickness ratio increasing. It is shown that it is not the case for every cross-section geometry (see Figure 8) and that the domain of validity of this formula should be revised.

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