The Stiffness Model of revised Annex J of Eurocode 3

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Abstract

In 1994 a revised draft of Annex J of Eurocode 3 entiteld 'joints in building frames' was approved by CEN. For this Annex a new model for the determination of the rotational stiffness was developed. This paper provides backgrounds to this new stiffness model and shows comparisons with test results.

1. INTRODUCTION

A major technical improvement in the revised Annex J of Eurocode 3 [1] (hereafter: Annex J) is the new model for the determination of the rotational response of joints. The objective of this paper is to provide backgrounds to this model. These are given in the first part of this paper. In the second part comparisons are made with test results.

In general, the moment rotation characteristic (M-ϕ curve) of joints is non-linear. Although Annex J can be used to determine a simplified linear, bi-linear or multi-linear M-ϕ curve, this paper will focus on its potential to predict a full non-linear curve. This is to enable a direct comparison between model and test results.

The test results are taken from the databank SERICON [2]. This databank forms a collection of M-ϕ data from different laboratories all over Europe. The databank contains results for different types of joints (e.g. welded joints, joints with extended end

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plates, joints with flush end plates and cleated joints) and for different joint configurations (e.g. single sided, double sided).

Key differences between the model in the new Annex J and the old Annex J [3] are the following:

1) In the old Annex the calculated deformations of the components were those corresponding to the design resistance of these components (see chapter 3 for the definition of the word "component"). The elastic deformations were calculated back from these deformations by dividing with a factor 2,25. In the new Annex, the elastic deformations are calculated directly.
2) Unlike in the old Annex, these elastic deformations are now only dependent on the lay-out of the joint and the Young modulus and not any more on strength properties or safety factors.
3) The calculation of the full non-linear curve in the new Annex is simplified compared to the old one.
4) In the old Annex, the stiffness prediction of a stiffened end plated joint could be below the prediction of an unstiffened one. This problem is now resolved.

2. THE GENERAL MODEL

Provided that the non-linear M-ϕ curve of the new Annex J is not limited by the rotational capacity (ϕ_{Cd}), this curve consists of 3 parts, see figure 1. Up to a level of 2/3 of the design moment resistance ($M_{j,Rd}$), the curve is assumed to be linear elastic. The corresponding stiffness is the so-called initial stiffness $S_{j,ini}$. Between $2/3 \cdot M_{j,Rd}$ and $M_{j,Rd}$, the curve is non-linear. After the moment in the joint reaches $M_{j,Rd}$, a yield plateau could appear. The end of this M-ϕ curve indicates the rotational capacity (ϕ_{Cd}) of the joint. Since the determination of $M_{j,Rd}$ and the rotational capacity in the new Annex J is not significantly different from the old Annex J and backgrounds are well documented [4, 5], this paper will not focus on these aspects.

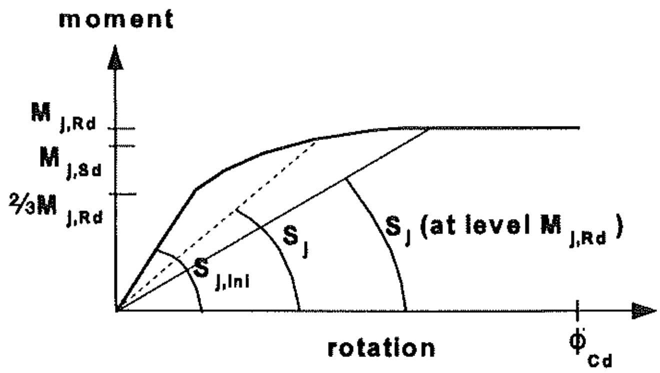


Figure 1: Non-linear M-ϕ curve according to Annex J.

The model assumes a fixed ratio between the initial stiffness $S_{j,ini}$ and the secant stiffness at the intersection between the non-linear part and the yield plateau (S_j at level $M_{j,Rd}$). For end plated and welded joints, this ratio is equal to 3. For flange cleated joints, this ratio is 3,5, see figure 1. These values are simplified values and result from numerous parameter studies and test observations [5].

The shape of the non-linear part for $M_{j,Sd}$ between $2/3 \cdot M_{j,Rd}$ and $M_{j,Rd}$ can be found with the following interpolation formula:

$$S_j = \frac{S_{j.ini}}{\left(\frac{1{,}5\ M_{j.Sd}}{M_{j.Rd}}\right)^{\psi}} \qquad (1)$$

where ψ = 2,7 for end plated and welded joints and 3,1 for flange cleated joints.

In this interpolation formula, the value of S_j is dependent on $M_{j,Sd}$.

3. DETERMINATION OF THE INITIAL STIFFNESS $S_{j,ini}$

The Annex J stiffness model utilizes the so called "component method". The essence of this method is that the rotational response of the joint is determined based on the mechanical properties of the different components in the joint. The advantage of this method is that an engineer is able to calculate the mechanical properties of any joint by decomposing the joint into relevant components. Annex J gives direct guidance for end plated, welded and flange cleated joints for this decomposition. Table 1 shows an overview of components to be taken into account when calculating the initial stiffness for these types of joints.

Table 1: Overview of components for different joints

Component	Number	End plated	Welded	Flange cleated
Column web panel in shear	1	x	x	x
Column web in compression	2	x	x	x
Column flange in bending	3	x		x
Column web in tension	4	x	x	x
end plate in bending	5	x		
flange cleat in bending	6			x
bolts in tension	7	x		x
bolts in shear	8			x
bolts in bearing	9			x

In the model it is assumed that the deformations of the following components: a) beam flange and web in compression, b) beam web in tension and c) plate in tension or compression are included in the deformations of the beam in bending. Consequently they are not assumed to contribute to the flexibility of the joint.

The initial stiffness $S_{j,ini}$ is derived from the elastic stiffnesses of the components. The elastic behaviour of each component is represented by a spring. The force-deformation relationship of this spring is given by:

$$F_i = k_i \cdot E \cdot \Delta_i \qquad (2)$$

where F_i = the force in the spring i,
k_i = the stiffness coefficient of the component i,
E = the Young modulus and
Δ_i = the spring deformation i.

Chapter 4 gives backgrounds of the formulae to determine k_i.

The spring components in a joint are combined into a spring model. Figure 2 shows for example the spring model for an unstiffened welded beam-to-column joint.

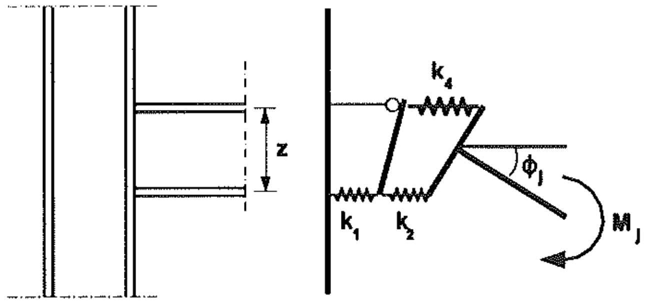


Figure 2: Spring model for an unstiffened welded joint.

The force in each spring is equal to F. The moment M_j acting in the spring model is equal to $F \cdot z$, where z is distance between the centre of tension (for welded joints located in the centre of the upper beam flange) and the centre of compression (for welded joints located in the centre of the lower beam flange). The rotation ϕ_j in the joint is equal to $(\Delta_1 + \Delta_2 + \Delta_4) / z$. In other words:

$$S_{j.ini} = \frac{M_j}{\phi_j} = \frac{Fz}{\frac{\Sigma \Delta_i}{z}} = \frac{Fz^2}{\frac{F}{E}\Sigma \frac{1}{k_i}} = \frac{Ez^2}{\Sigma \frac{1}{k_i}} \qquad (3)$$

For an end plated joint with only one bolt row in tension and for a flange cleated joint the same formula yields. However, components to be taken into account are different, see table 1.

Figure 3a shows the spring model adopted for end plated joints with two or more bolt rows in tension. It is assumed that the bolt row deformations for all rows are proportional to the distance to the point of compression, but that the elastic forces in each row are dependent on the stiffness of the components. Figure 3b shows how the deformations per bolt row of components 3, 4, 5 and 7 are added to an effective spring per bolt row, with an effective stiffness coefficient $k_{eff,r}$ (r is the index of the row number). In figure 3c is indicated how these effective springs per bolt row are replaced by an equivalent spring acting at a lever arm z. The stiffness coefficient of this effective spring is k_{eq}. The effective stiffness coefficient k_{eq} can directly be applied in formula 3. The formulae to determine $k_{eff,r}$, k_{eq} and z as given in Annex J can directly be derived from the sketches of figure 3. The bases for these formulae is that the moment-rotation behaviour of each of the systems in figure 3a, 3b and 3c is equal. An additional condition is that the compressive force in the lower rigid bar is equal in each of these systems.

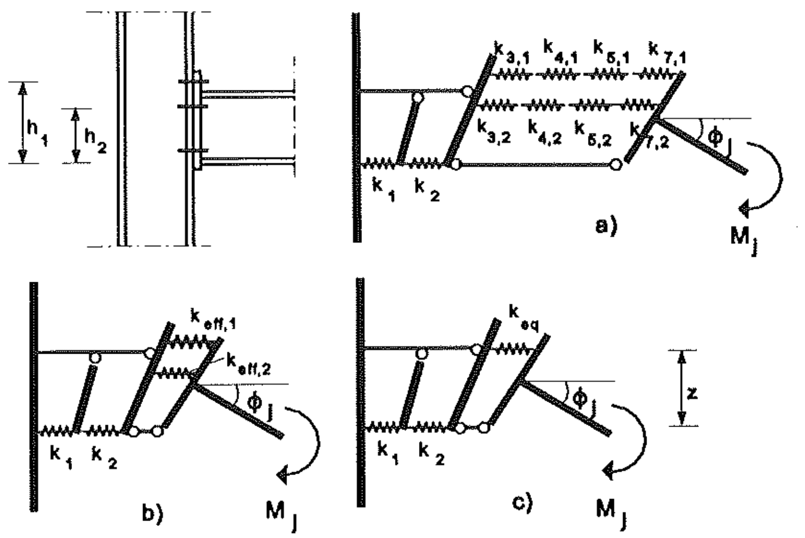


Figure 3: Spring model for a beam-to-column end plated joint with two bolt rows in tension.

4. DETERMINATION OF THE STIFFNESS COEFFICIENTS k

4.1 Plates in bending and bolts in tension

In the procedure for strength calculation included in Annex J, the three following components: (i) column flange in bending, (ii) end plate in bending and (iii) flange cleat in bending are idealized as T-stubs (see figure 4). These ones are assumed to be connected by means of bolts to an infinitely rigid foundation (figure 5 and 6.a). Their so-called "effective length l_{eff}" is such that the failure modes and the corresponding collapse loads are similar to those of the actual joint components. The concept of "equivalent T-stubs" for strength is easy to use and allows to cover the calculations of all the plated components with the same set of formulae.

The "T-stub concept" may also be referred to for stiffness calculation as shown in [5] and [6]. The equivalence between the actual component and the equivalent T-stub in the elastic range of behaviour (initial stiffness) is however to be expressed in a different way then at collapse and requires the definition of a new effective length $l_{eff,ini}$. In view of the determination of the related stiffness coefficients k, two problems have to be investigated:

- the response of the T-stub in the elastic range of behaviour;
- the determination of $l_{eff,ini}$.

These two points are successively addressed hereunder.

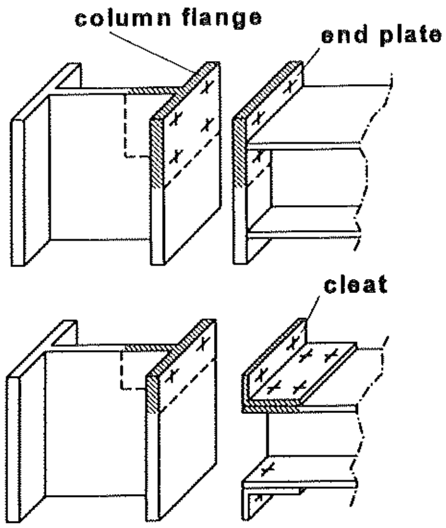


Figure 4: T-stub idealizations

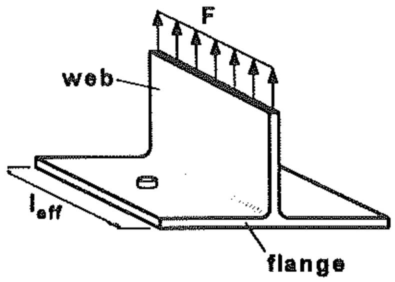


Figure 5: T-stub on rigid foundation

a) T-stub response

When subjected to tension forces, the flange of the T-stub deforms in bending and the bolts mainly in tension (figure 6.a). The elastic response of this system has been studied first by YEE and MELCHERS [6]. A slight refinement related to the location of the prying effect has been proposed later in [5]. The corresponding expressions are rather long to apply so simplifications have been introduced by the authors:

- *to simplify the formulae*: n is considered as equal to 1,25 m (m and n are indicated in figure 6.a)
- *to dissociate the bolt deformability* (figure 6.c) *from that of the T-stub* (figure 6.b).

Under these assumptions, it can be shown that:

- for the T-stub (figure 6.b):

$$k_{3,5,6} = \frac{l_{eff,ini}\ t^3}{m^3} \quad (4)$$

- for the bolts (figure 6.c):

$$k_7 = 1,6\ \frac{A_s}{L_b} \quad (5)$$

where A_s = bolt reduced area, L_b = bolt length including half thickness of the bolt head and of the nut and t = T-stub flange thickness. The indexes of the k coefficients relate to the component numbers listed in table 1. In equation (5), a factor 2,0 instead of 1,6 would be expected at first sight; in reality, the value 1,6 is defined in such a way that the prying effect is taken into consideration.

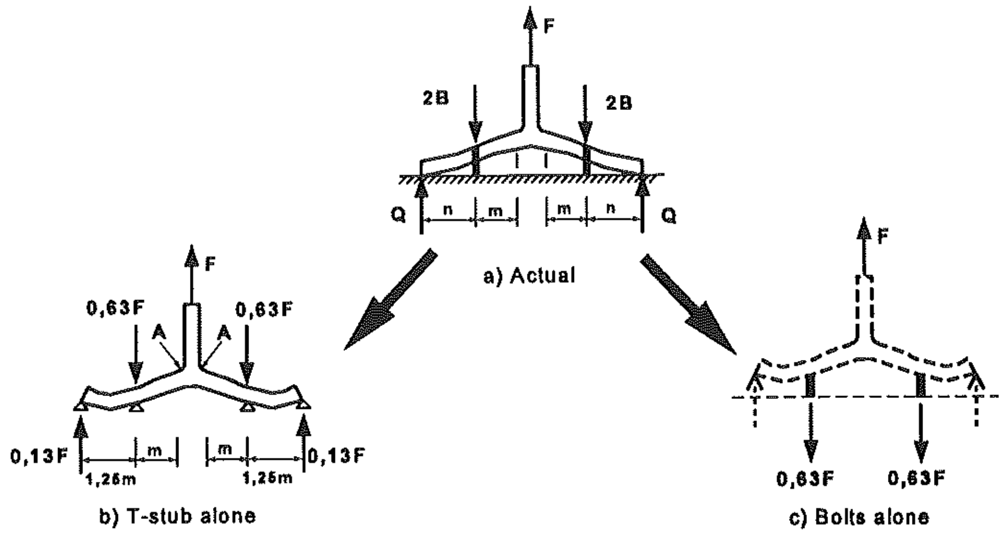


Figure 6: Elastic deformation of the T-stub

b) Definition of $l_{eff,ini}$

In figure 6.b, the maximum bending moment in the T-stub flange (points A) is expressed as $M_{max} = 0{,}322 \cdot F \cdot m$. Based on this expression, the maximum elastic load F_{el} (first plastic hinges in the T-stub at points A) to be applied to the T-stub can be derived:

$$F_{el} = \frac{4\ l_{eff,ini}}{1{,}288\ m}\ \frac{t^2\ f_y}{4} = \frac{l_{eff,ini}\ t^2}{1{,}288\ m}\ f_y \quad (6)$$

In Annex J, the ratio between the design resistance and the maximum elastic resistance of each of the components is taken as equal to 3/2 so:

$$F_{Rd} = \frac{3}{2}\ F_{el} = \frac{l_{eff,ini}\ t^2}{0{,}859\ m}\ f_y \quad (7)$$

As, in figure 6.b, the T-stub flange is supported at the bolt level, the only possible collapse mode of the T-stub is the development of a plastic mechanism in the flange. The associated collapse load is given by Annex J as:

$$F_{Rd} = \frac{l_{eff}\ t^2\ f_y}{m} \quad (8)$$

where l_{eff} is the effective length of the T-stub for strength calculation.
By identification of expressions (7) and (8), $l_{eff,ini}$ may be derived:

$$l_{eff,ini} = 0{,}859\ l_{eff} \approx 0{,}85\ l_{eff} \quad (9)$$

Finally, by introducing equation (9) in the expression (4) giving the value of $k_{3,5,6}$:

$$k_{3,5,6} = \frac{0{,}85\ l_{eff}\ t^3}{m^3} \quad (10)$$

4.2 Column web panel in shear

In beam-to-column joints, column web panels are subjected to high shear forces V (see figure 7). The shear force V can be expressed as β·F (F forces are statically equivalent to the applied moment M, see figure 7). β values are dependent on the joint configuration and loading; related values are given in Annex J.

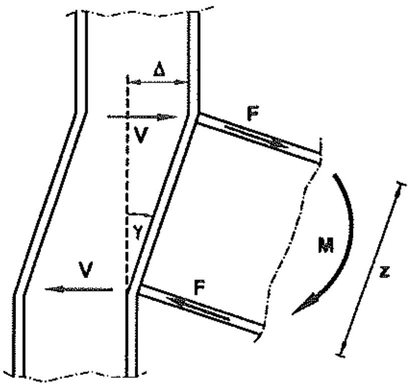


Figure 7: Column web panel in shear

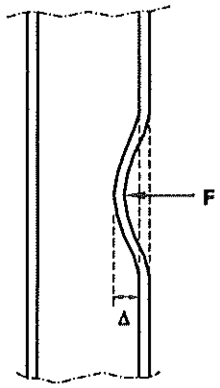


Figure 8: Column web in tension or compression

Through experimental and numerical research works e.g. [5], it has been shown that shear stresses τ in column web panels are more or less uniformly distributed. The corresponding deformation γ is therefore such that $\tau = G \cdot \gamma$. V can be expressed as $A_{vc} \cdot \tau$ and γ as Δ/z so:

$$F = \frac{V}{\beta} = \frac{A_{vc}\,\tau}{\beta} = \frac{A_{vc}\,G}{\beta\,z}\,\Delta \qquad (11)$$

As $G = E/[2(1+\upsilon)]$ and $\upsilon = 0{,}3$, the following expression of k_1 can be simply derived:

$$k_1 = \frac{A_{vc}}{2(1+\upsilon)\,\beta\,z} \approx 0{,}38\,\frac{A_{vc}}{\beta\,z} \qquad (12)$$

4.3 Column web in tension or compression

In [5], the elastic linear relationship between the tension or compression force F transversally applied to the column and the corresponding elongation or shortening Δ of the web (see figure 8) is expressed as:

$$F = \frac{E\,t_{wc}}{d_c}\,\xi\,\Delta \qquad (13)$$

d_c is defined as the clear depth of the column web. The coefficient ξ depends on the relative stiffness of the column flange in bending and the column web in tension or compression; its expression - which differs for welded and bolted joints - is rather complicated so simplifications have been brought by the authors. These simplifications are based, as for the T-stub in section 4.1.b, on the ratio (= 3/2) between the design resistance of the web defined in Annex J as:

$$F_{Rd} = b_{eff}\, t_{wc}\, f_y \tag{14}$$

and the maximum elastic resistance of the web expressed (from equation 13) as:

$$F_{el} = \xi\, t_{wc}\, E\, \frac{\Delta}{d_c} = \xi\, t_{wc}\, E\, \epsilon_{el} = \xi\, t_{wc}\, E\, \frac{f_y}{E} = \xi\, t_{wc}\, f_y \tag{15}$$

From this ratio, an approximated value of ξ is derived: $\xi = 2/3\ b_{eff}$. By introducing this value in equation (13), the following expression of the stiffness coefficient is obtained:

$$k_{2,4} = \frac{0{,}667\ b_{eff}\ t_{wc}}{d_c} \approx \frac{0{,}7\ b_{eff}\ t_{wc}}{d_c} \tag{16}$$

4.4 Bolts in shear and bolts in bearing

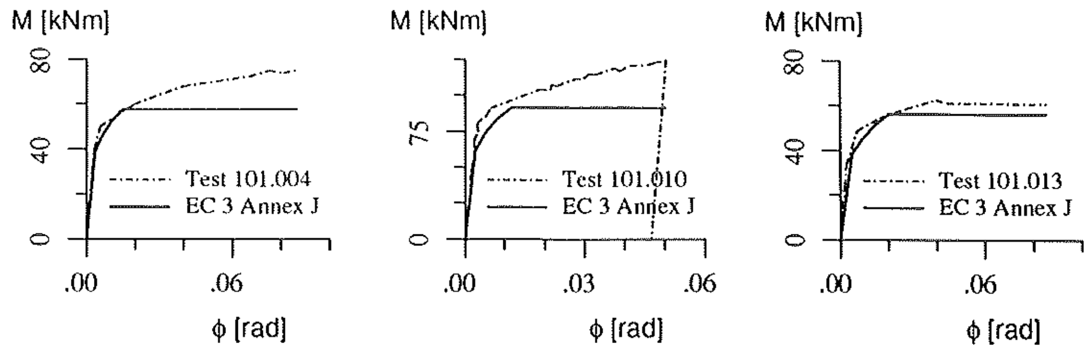
Formulae for stiffness prediction are proposed in [5]; they are based on previous works by PAVLOV and KARMALIN and are validated by comparisons with test results. Limited modifications (to avoid the use of specific units) have been brought to these formulae in Annex J. The reader is therefore asked to refer to [5] for background information.

5. COMPARISON WITH TEST RESULTS

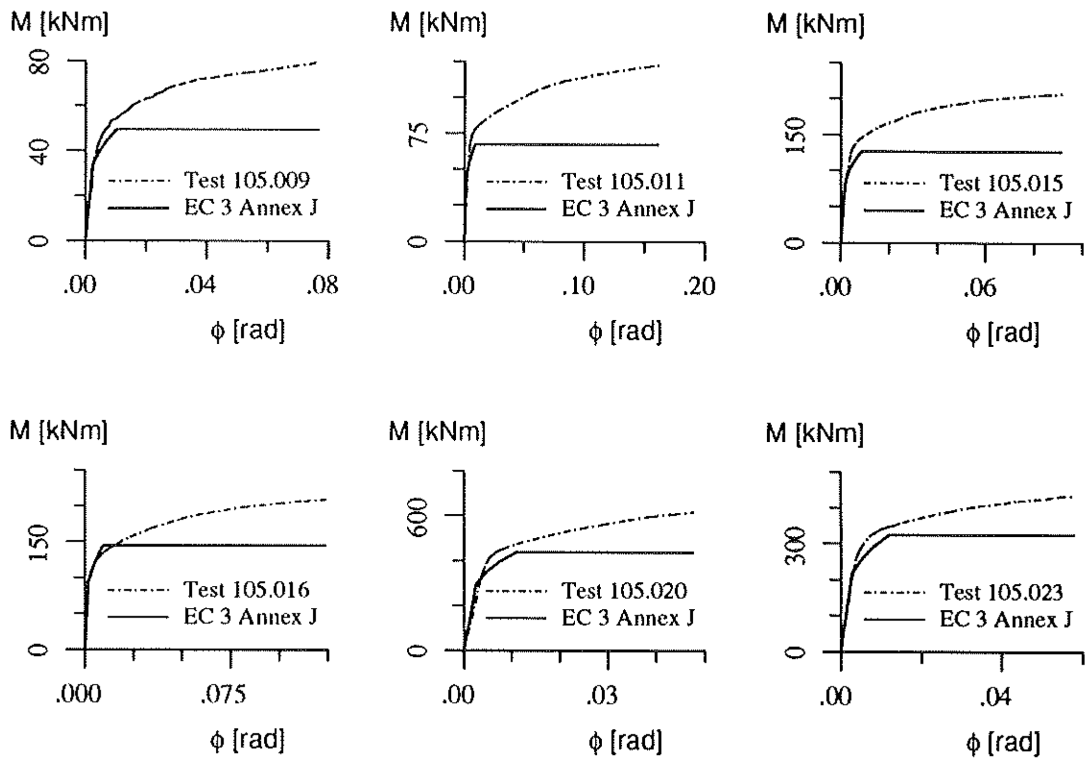
This section shows comparisons of the presented stiffness model with test results. The test data are taken from the databank SERICON [2]. In order to enable comparisons of the complete stiffness model with test results it is necessary to show the full non-linear curves which are obtained by using the application rules of Annex J.

For the determination of the joint properties, i.e initial stiffness and design resistance, measured material and geometrical data obtained from tests are used. The value of the moment resistance $M_{j,Rd}$ is calculated with safety factor γ=1,0. The moment resistance is determined according to the most accurate model of Annex J, e.g. the alternative method to determine the resistance of the T-stub is used for joints with bolted end plates. Both the rotational stiffness and the moment resistance are calculated by taking into account the actual forces in the shear panel of the column web through the exact values of the β-coefficients.

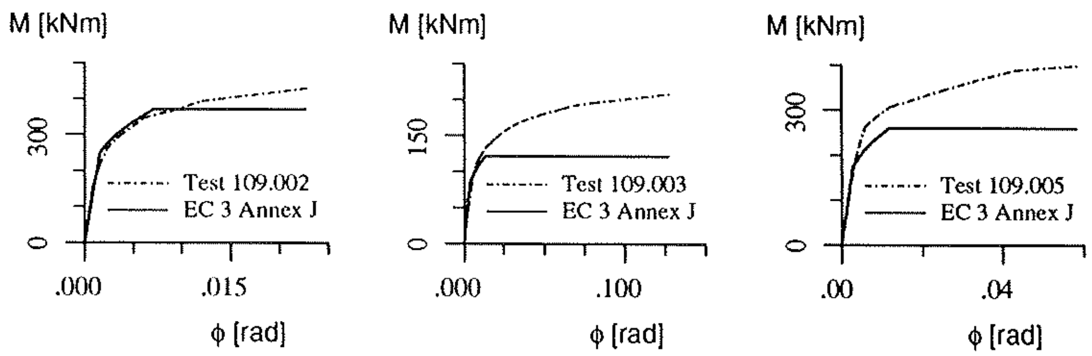
It can be seen from figure 9 that the prediction of the joint stiffness and resistance is in good agreement with the actual behaviour. The differences in the resistance are due to strain hardening and membrane effects which are not taken into account in the design rules of Annex J. In the stiffness model it is assumed that a joint remains elastic up to a level of 2/3 of $M_{j,Rd}$. This assumption is confirmed by the curves.



a) Liège, single sided, end plated joints

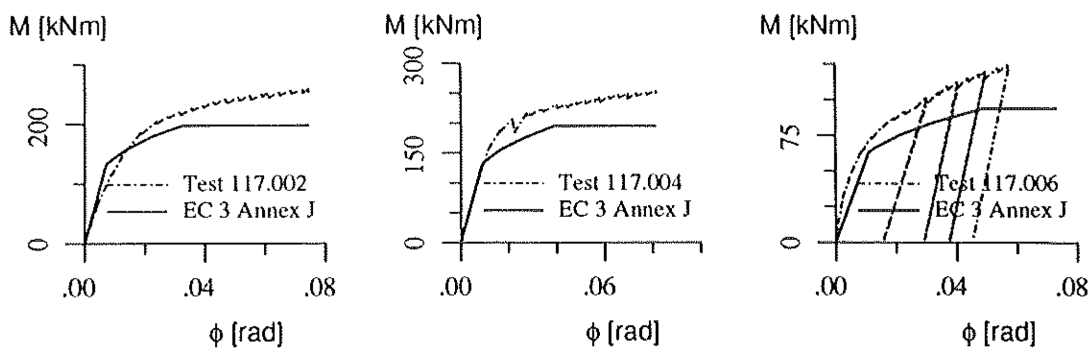


b) Innsbruck, single sided, welded joints

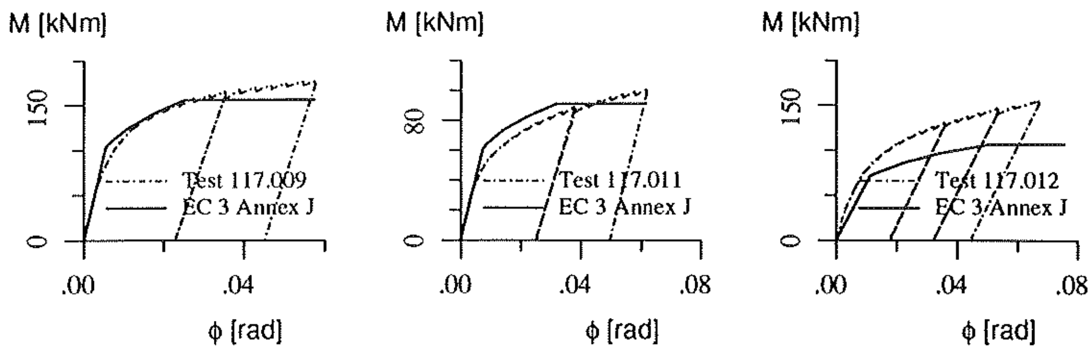


c) Innsbruck, double/single sided, end plated joints

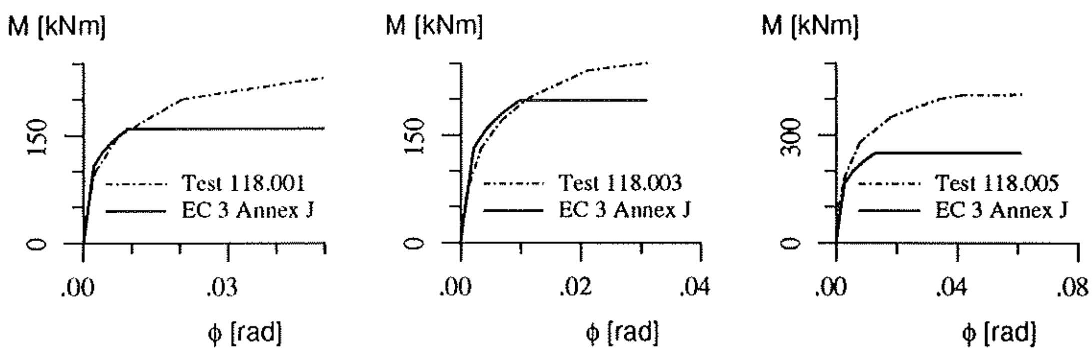
Figure 9: Comparison with test results



d) Leipzig/Aachen, single sided, end plated joints



e) Leipzig/Aachen, double sided, end plated joints



f) Dundee, double/single sided, end plated joints

Figure 9: Comparison with test results (continued)

6. CONCLUSION

The new stiffness model of the revised Annex J allows for the determination of the initial (elastic) stiffness of a joint independently of a strength calculation. It also gives design rules for the determination of a full non-linear M-ϕ curve. Comparisons with test results show a good agreement between the predicted curves and the real ones obtained from tests.

The model for the stiffness calculation is based on the so-called component method. Therefore it can be applied for many types of joints and joint configurations. Moreover it can be easily extended to new types of joints as for example composite joints provided that on one side the deformation behaviour is known for all components (i.e. stiffness coefficients k) and on the other side that the contribution of the different sources of deformability can be taken into consideration with the presented model by means of a set of springs.

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