PREDICTION OF THE SEMI-RIGID AND PARTIAL-STRENGTH PROPERTIES OF STRUCTURAL JOINTS

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1. Abstract

In the present paper, a general analytical procedure for the prediction of the behaviour of joints is introduced. This so-called component method applies to any type of steel and composite joints whatever is the geometrical configuration (single-sided or double-sided beam-to-column joint, beam or column splices, ...), the type of loading (axial force and/or bending moment, ...) and the type of member profiles. Different levels of refinement and therefore of complexity may be envisaged when applying the procedure, depending on the persons to whom it is devoted, scientist or practitioner.

The method is then applied to a specific joint commonly used in practice : the steel beam-to-column joint with extended end plate (4 bolts in tension zone) subjected to bending moment, the beam and column members of which are hot-rolled H or I ones. Lastly the component method is applied to joints tested experimentally in different European laboratories to demonstrate its accuracy.

2. Introduction to the component method

2.1. Principles of the method

The component method may be presented as the application of the well-known finite element method to the calculation of structural joints.

A joint is generally considered as a whole and is studied accordingly ; the originality of the component method is to consider any joint as a set of « individual basic components ». In the particular case of figure 1 (joint with extended end plate connection subject to bending), the relevant components are the following :

- compression zone :
  - column web in compression ;
  - beam flange in compression ;

- tension zone :
  - column web in tension ;
  - column flange in bending ;
  - bolts in tension ;
  - end plate in bending ;
  - beam web in tension ;

- in shear zone :
  - column web panel in shear.

Each of these basic components possesses its own level of strength and stiffness in tension, compression or shear. The coexistence of several components within the same joint element - for instance, the column web which is simultaneously subjected to compression (or tension) and shear - can obviously lead to stress interactions that are likely to decrease the strength and the stiffness of each individual basic component ; this interaction affects the shape of the deformability curve of the related components but does not call the principles of the component method in question again.

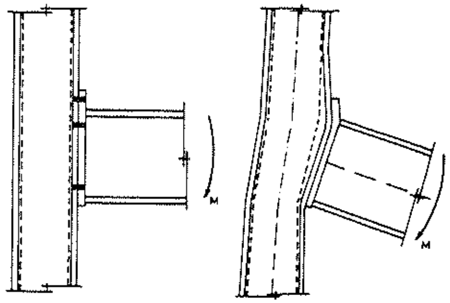


Figure 1 - Joint in bending with extended end plate

The application of the component method requires the following steps :

a) listing of the « activated » components for the studied joint ;
b) evaluation of the stiffness and/or strength characteristics of each individual basic component (specific characteristics - initial stiffness, design strength, ... - or whole deformability curve) ;
c) « assembling » of the components in view of the evaluation of the stiffness and/or strength characteristics of the whole joint (specific characteristics - initial stiffness, design resistance, ... - or whole deformability curve).

As specified here above, the parallelism with the finite element method is obvious. To « component » and « joint » may then be substituted the words « finite element » and « structure ».

The « assembling » is based on a distribution of the internal forces within the joint. As a matter of fact, the external loads applied to the joint distribute, at each loading step, between the individual components according to the instantaneous stiffness and resistance of each component. Distributions of internal forces may be obtained through different ways : an analytical one is presented in section 3 of the present paper while a mechanical one has been described in [1].

The application of the component method requires a sufficient knowledge of the behaviour of the basic components. Research works carried out in the last years at the University of Liège

allows to predict analytically the stiffness and strength characteristics of the following components [2, 3] :

- column web panel in shear ;
- column web in compression ;
- beam flange in compression ;
- column flange in bending ;
- column web in tension ;
- end plate in bending ;
- beam web in tension ;
- flange cleat in bending ;
- plate in bearing ;
- bolt in tension ;
- bolt in shear ;
- plate in tension or compression ;
- concrete reinforcement in tension ;
- concrete slab in tension or compression.

The combination of these components allows to cover a wide range of joint configurations, what should largely be sufficient to satisfy the needs of practitioners as far as beam-to-column joints and beam splices in bending are concerned.

Some fields of application are however not yet covered nowadays :

- Joints subject to bending moment (and shear) and axial forces have been less studied and, in particular, the way to distribute the internal forces for stiffness and strength calculations (the stiffness and strength component properties remain unchanged whatever is the loading type).
- Column bases are subjected to combinations of bending moments and axial forces and possess specific components for which a limited knowledge is available. For instance :
  - concrete block in compression ;
  - end plates with specific geometries ;
  - anchorages in tension ;
  - contact between soil and foundation ;
  - ...

Research works are presently in progress in Liège ; they should bring answers to these last questions which prevent now the general use of the component method to any structural joint.

2.2. Levels of refinement

The framework of the component method is sufficiently general to allow the use of various techniques of component characterization and joint « assembling ».

In particular, the stiffness and strength characteristics of the components may result from experimentations in laboratory, numerical simulations by means of finite element programs or analytical models based on theory. At Liège, experimentations and numerical simulations have been performed and used as references when developing and validating analytical models. These ones may be developed with different levels of sophistication according to the persons to whom they are devoted :

- the expressions presented in [2] including those presented in section 3 of the present paper are able to cover the influence of all the parameters which affect significantly the component behaviour (strain hardening, bolt head and nut dimensions, bolt prestressing, ...) from the beginning of the loading to collapse and fit therefore well with a scientific publication ;

- the rules which have been introduced, for instance, in Annex J of Eurocode 3 [4] (annex devoted to joint design) and in its revised version [5] to which the University of Liège has largely contributed with the University of Aachen (D) and TNO Delft (NL) are far more simple and are therefore more suitable for practical use.

Similar levels of sophistication exist also, as those presented in [2], [4] and [5], for what regards the joint « assembling ».

3. Application of the component method to joints with extended endplates

In this section, the component method is applied to one of the most commonly used type of joints : the single-sided beam-to-column unstiffened joint in bending with extended end-plate connection - 2 bolt-rows in the tension zone and H or I hot-rolled profiles for the beam and the column - (see figure 1).

The analytical rules to characterize the components and to assemble them are extracted from

[2].

3.1. General model

The global response of a joint in bending may be presented in the form of a M-ϕ curve where M and ϕ represent respectively the bending moment to which the joint is subjected and the resultant relative rotation between the connected members.

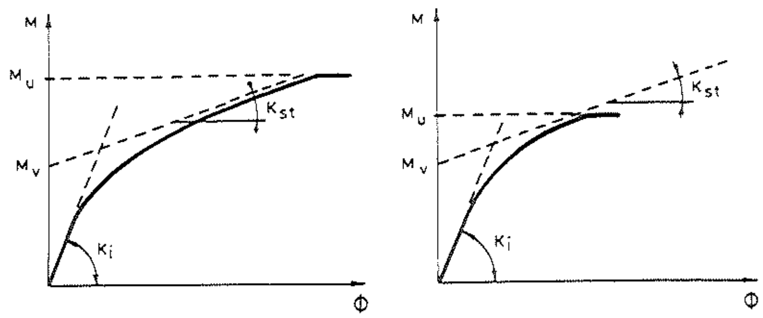
In the case of welded and bolted joints, the shape of the M-ϕ curve is approximately bi-linear ; it can therefore be characterized by four parameters [6, 2] (see figure 2.a.) :

- an initial stiffness K_i ;
- a design resistance M_v ;
- a strain-hardening stiffness K_{st} ;
- an ultimate resistance M_u.

When no instability occurs in the joint at ultimate state, M_u differs significantly from M_v and the bi-linear shape is well marked ; when instability occurs - for instance in the column web or the beam flange in compression - M_u comes closer to M_v, what tends to give a more or less round shape to the M-ϕ curve (see figure 2.b.).

The following mathematical expression allows a rather good approximation of the actual M-ϕ curves :

$$M = \frac{(K_i - K_{st})\phi}{\left(1 + \left[\frac{(K_i - K_{st})\phi}{M_v}\right]^c\right)^{1/c}} + K_{st}\phi \le M_u \qquad (1)$$



a. $M_u >> M_v$ b. $M_u > M_v$

Figure 2 - General model for M-ϕ curves

This expression satisfies the following conditions (see figure 2.a.) :

- $\frac{dM}{d\phi} = K_i$ when $\phi \to 0$ (2.a.)

The M-ϕ curve is tangent to the initial stiffness at the beginning of the loading ;

- $M = M_v + K_{st}\phi$ when $\phi \to \infty$ (2.b.)

The M-ϕ curve is asymptotic to the « strain-hardening line » when ϕ is increasing.

- $M \le M_u$ (2.c.)

The maximum transferred moment is equal to M_u.

The coefficient c has to be considered as a shape factor influencing the bi-linear character of the curve and depending on the type of joint ; the adjustment of its value through the comparisons of the expression (1) with experimental tests results leads to the definition of :

$$c = 1{,}5 \qquad (3)$$

in the case of joints with end-plates. Values of c are given in [2] for other types of joints.

The ultimate rotation of the joint (rotation corresponding to a bending moment $M = M_u$) may be derived from expression (1) ; this rotation is of particular importance when performing an elasto-plastic frame design.

3.2. Analytical evaluation of the four main parameters

Reference is made here to the above described component method to derive the values of the initial stiffness (K_i), the design resistance (M_v), the strain-hardening stiffness (K_{st}) and the ultimate resistance (M_u) of the joint.

3.2.1. Initial stiffness K_i

The initial stiffness K_i results from the elastic deformation Δ of the joint basic components :

- column web panel in shear (Δ_{wp} due to shear force) ;

- column web in compression ($\Delta_{wc.c}$ due to compression force) ;
- column web in tension ($\Delta_{wc.t}$ due to tension force) ;
- column flange in bending + bolts in tension (Δ_{cf} due to tension force) ;
- end plate in bending + bolts in tension (Δ_e due to tension force).

The other joint basic components (beam flange in compression and beam web in tension) do not contribute to the joint deformability and are therefore not taken into consideration in the stiffness calculation.

For each component, the relationship between the applied force F and the resultant deformability Δ is as follows :

$$F = Ek\,\Delta \qquad (4)$$

where k is the stiffness coefficient of the component ($k = k_i$ in the elastic domain) and E, the Young modulus.

When assembling the components in view of the determination of the joint initial stiffness, the following idealization is made (see figure 3) [6] :

$$\phi = \frac{\Delta_c + \Delta_t + \Delta_s}{h} \qquad (5)$$

where $\Delta_c = \Delta_{wc.c}$
= total joint deformation in the compression zone

$\Delta_t = \Delta_{wc.t} + \Delta_{cf} + \Delta_e$
= total joint deformation in the tension zone

$\Delta_s = \Delta_{wp}$
= shear deformation of the column web panel

h = level arm of forces statically equivalent to the applied bending moment

As : $M = Fh$ (6)
and : $M = K_i\phi$ (7)
the following expression of K_i can be derived :

$$K_i = \frac{Eh^2}{\sum \frac{1}{k_i}} \qquad (8)$$

where

$$\sum \frac{1}{k_i} = \frac{1}{k_{i,wp}} + \frac{1}{k_{i,wc.c}} + \frac{1}{k_{i,wc.t}} + \frac{1}{k_{i,cf}} + \frac{1}{k_{i,e}} \qquad (9)$$

The evaluation of K_i requires therefore the prediction of the stiffness coefficient for all the relevant components.

a. Column web in compression or tension

In [2], the stiffness coefficient of a column subjected to compression or tension forces results from the study (see figure 4.a.) of an « elastic beam » (the column flange) lying on an « elastic foundation » (the column web). This idealization allows to simulate the flexural behaviour of the column flange and the « axial » behaviour of the column web supporting the flange.

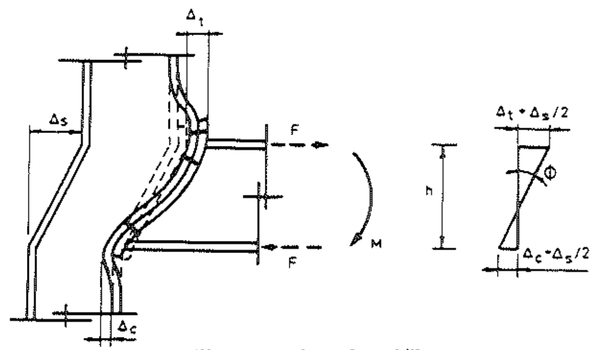


Figure 3 - Joint deformability

From such a so simplified model, the stiffness coefficients $k_{i,wc.c}$ and $k_{i,wc.t}$ may be easily derived [2] :

$$k_{i,wc.c} = \frac{2t_{cw}}{h_{cw}} \frac{\mu_c}{\lambda} \qquad (10.a.)$$

$$k_{i,wc.t} = \frac{2t_{cw}}{h_{cw}} \frac{\mu_t}{\lambda} \qquad (10.b.)$$

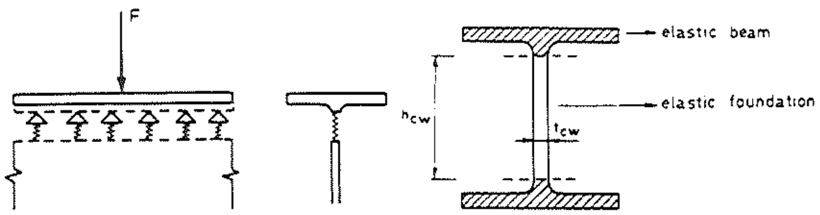
where t_{cw} = column web thickness (see figure 4.b.);
h_{cw} = depth of the column web (see figure 4.b.);
$\lambda = \sqrt[4]{t_{cw} / 4h_{cw}I_{cf}}$ (11)
with I_{cf} = moment of inertia of the « elastic beam » defined in figure 4.b.

The μ value (μ_c in compression and μ_t in tension) expresses the importance of the diffusion of the applied load through the column flange and radius of fillet. The μ value differs obviously in the tension zone (diffusion through the end plate, the bolts, the flange in bending) and in the compression zone (contact between end plate and flange). The following μ expression is proposed in [2] :

$$\mu = \frac{\xi}{1 - e^{-\xi}\cos\xi} \qquad (12)$$

where $\xi = b / 2L$
with $L = 1/\lambda$
$b = t_{bf} + 2a_f\sqrt{2} + 2t_e$ (compression)
$= t_{bf} + 2w_e$ (tension)

where t_{bf} = thickness of the beam flange, a_f = throat dimension of the welds connecting the beam flanges to the end plate, t_e = thickness of the endplate and w_e = length of the extended part of the end plate in the tension zone.



a. Idealization   b. Geometrical characteristics

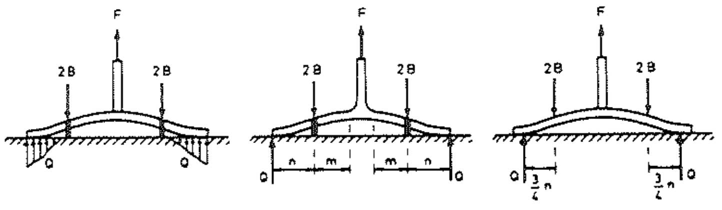
Figure 4 - Elastic beam on an elastic foundation

b. Column flange and end plate in bending

For these components, it is referred to the model presented some years ago by YEE and MELCHERS [6] who consider the column flange and end plate in bending as two « T-stubs » connected together (see figure 6).

The expressions of $k_{i,cf}$ and $k_{i,e}$ differ however from those which could be derived from the original publicaton of YEE and MELCHERS for the two following reasons :

- some corrections have been made by the authors, in [7], to the original formulae given by YEE and MELCHERS;
- the location of the prying force Q (see figure 5.b.) considered by YEE and MELCHERS is not optimum; several researches have shown that, in the elastic range of behaviour, the prying force may be located closer to the bolts; so it is referred in [2] to the proposal made by DOUTY and McGUIRE in [8] (see figure 5.c.).



a. Actual distribution   b. Idealization by YEE and MELCHERS   c. Idealization by DOUTY and Mc GUIRE

Figure 5 - Location of the prying forces

The following expressions are therefore considered :

$$k_{i,cf}=\left[Z_{cf}\left(\frac{1}{8}-\frac{1}{4}q\alpha_{cf}\right)\right]^{-1} \quad (13.a.)$$

$$k_{i,e}=\left[Z_{e}\left(\frac{1}{8}-\frac{1}{4}q\alpha_{c}\right)\right]^{-1} \quad (13.b.)$$

For connections with non prestressed bolts :

$$q=\frac{Z_e\alpha_{e1}+Z_{cf}\alpha_{cf1}}{Z_e\alpha_{e2}+Z_{cf}\alpha_{cf2}+\dfrac{k_1+2k_4}{2A_b}} \quad (14.a.)$$

For connections with prestressed bolts :

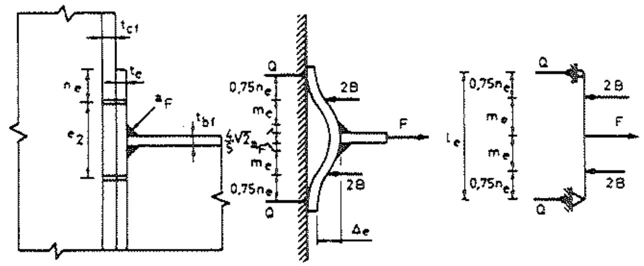
$$q=\frac{Z_e\alpha_{e1}+Z_{cf}\alpha_{cf1}}{Z_e\alpha_{e2}+Z_{cf}\alpha_{cf2}+\dfrac{k_2k_3^{*}}{2A_b(k_2+k_3^{*})}} \quad (14.b.)$$

In these formulae :

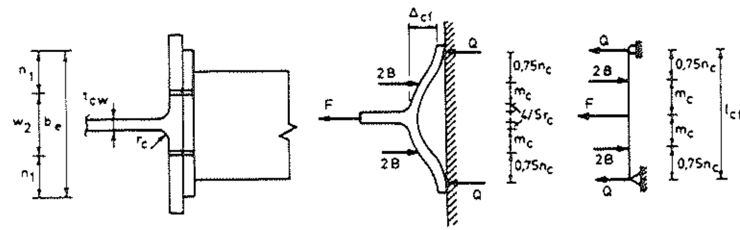
$Z_e=l_e^3/w_{ep}t_e^3$	$Z_{cf}=l_{cf}^3/w_{cf}t_{cf}^3$
$\alpha_{e1}=1{,}5\alpha_e-2\alpha_e^3$	$\alpha_{cf1}=1{,}5\alpha_{cf}-2\alpha_{cf}^3$
$\alpha_{e2}=6\alpha_e^2-8\alpha_e^3$	$\alpha_{cf2}=6\alpha_{cf}^2-8\alpha_{cf}^3$
$l_e=2(m_e+0{,}75n_e)$	$l_{cf}=2(m_c+0{,}75n_1)$
$2w_{ep}=b_e$	$2w_{cf}=e_2+1{,}5n_e$
$\alpha_e=0{,}75n_e/l_e$	$\alpha_{cf}=0{,}75n_1/l_{cf}$

All the geometrical properties are defined in figure 6. A_b is the bolt shaft area.
The other terms characterizing the bolt deformability are extracted from AGERSKOV's works [9] (see figure 6) :

$$k_1=\ell_s+1{,}43\,\ell_t+0{,}71\,\ell_n$$
$$k_2=\ell_s+1{,}43\,\ell_t+0{,}91\,\ell_n+0{,}4\,\ell_w$$
$$k_3^{*}=\frac{t_e+t_{cf}}{5}$$
$$k_4=0{,}1\,\ell_n+0{,}2\,\ell_w$$



a. End plate (Δ_e)



b. Column flange (Δ_{cf})

Figure 6 - End plate and column flange deformability

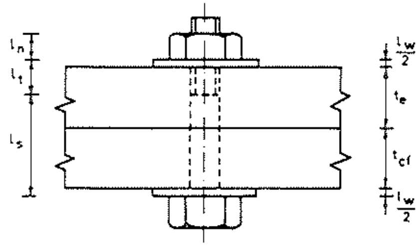


Figure 7 - Bolt geometrical properties

c. Column web panel in shear

According to [2] :

$$k_{i,wp} = A_{v,c} / [2(1+\nu)\zeta h] \tag{15}$$

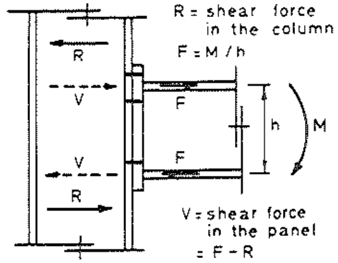
in which :

- $A_{v,c}$ = shear area of the column web panel (figure 8.b.);
- ν = Poisson's coefficient;
- ζ = ratio between the shear force V in the web panel and the force F carried over from the beam to the column at the beam flange level (figure 8.a.)

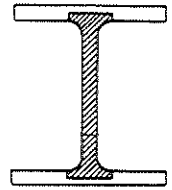
$k_{i,wp}$ may be simply derived from expression (4) and from the well-known following one :

$$V = \frac{E}{2(1+\nu)} A_{v,c} \cdot \gamma \tag{16}$$

in which $\gamma = \Delta / h$, where γ represents the beam-to-column relative rotation due to the shear deformability of the panel.



a. Shear force V



b. Shear area $A_{v,c}$

Figure 8 - Shear deformability of the web panel

3.2.2. Design moment resistance M_V

The design moment resistance of the joint corresponds to that of the weakest individual basic component.

The way to evaluate the design resistance of each of the individual components as well as the procedure for « joint assembling » is described in [2]. Due to the limited number of pages, this will not be presented here, but references are made to other already published documents :

- Design resistance of a column web in tension or compression : references [10];
- Design resistance of an end plate and a column flange in bending : reference [11];
- Design web panel in shear : reference [12].

For the other components, it may be referred to the revised Annex J of Eurocode 3 [5] where the recommended « assembling » procedure is the same than in [2].

It has to be mentioned that the rules provided in this revised Annex J for the evaluation of the design resistance of the here above considered components have been derived from those proposed in the indicated references.

3.2.3. Strain-hardening stiffness K_{st}

The component method may also be applied to the calculation of the strain-hardening stiffness K_{st}. As explained hereafter, it requires, in a first step, to be able to evaluate the strain-hardening stiffness coefficient for each basic component. Studies of numerous test results on components [2] have led to the following definition of this value :

$$k_{st} = \frac{E_{st}}{E} k_i \tag{17.a.}$$

for : column webs in compression and tension;
column flanges and end plates in bending;

$$k_{st} = \frac{2(1+\nu)}{3} \frac{E_{st}}{E} k_i \tag{17.b.}$$

for column web panels in shear

where : k_{st} = strain-hardening stiffness coefficient;
k_i = initial stiffness coefficient;
E_{st} = strain-hardening modulus in the steel σ-ε curve;

The « assembling » step in itself is highly dependent on the relative importance of the design moment resistance $M_{v,comp}$ of each individual basic component - $M_{v,comp}$ is calculated by considering temporaly the component as the single one in the joint - in comparison with the joint design moment resistance M_v evaluated in 3.2.2.

For instance, let us assume a joint in which one of the components is much more weaker than the others. K_{st} will result, in such a case, from the combination of the strain-hardening stiffness of the weak component and the initial stiffness of the others; as a matter of fact, these last ones remain in the elastic range of behaviour for applied moments higher than M_v.
In more usual joints, the successive apparition of yielding in the different components during the joint loading beyond M_v leads to a progressive decrease of the actual strain-hardening stiffness in comparison with the previous case. The complexity of the problem has been overcome in [2] as explained here below.

Each component which possesses a high design moment resistance in comparsion with M_v will contribute in an elastic way to K_{st} which, in fact, should probably be better called « post-limit » stiffness. In the contrary, a component, the design resistance of which is closer to M_v, will experience strain-hardening and will affect more significantly K_{st}. The simplified evaluation of K_{st} consists therefore in the classification of the components according to their design resistance $M_{v,comp}$ in order to distinguish those which will contribute to K_{st} by means of their initial stiffness coefficient k_i from those which will contribute by means of the strain-hardening coefficient k_{st}. Deep study of experimental tests on joints with endplates has allowed to determine the boundary value of the moment capacity :

$$M_{v,up} = 1{,}65\, M_v \qquad (18)$$

which allows to classify the components (elastic contribution to K_{st} if $M_{v,comp} > M_{v,up}$; strain-hardening contribution if $M_{v,comp} \le M_{v,u}$).

The strain-hardening stiffness of the joint may therefore be evaluated in the following way :

$$K_{st} = \frac{Eh^2}{\sum \frac{1}{k^*}} \qquad (19)$$

where :

$$\sum \frac{1}{k^*} = \sum_j \left(\frac{1}{k_{i,j}}\right)_{M_{v,j} > M_{v,up}} + \sum_k \left(\frac{1}{k_{st,k}}\right)_{M_{v,k} \le M_{v,up}} \qquad (20)$$

k and j are component indices.

3.2.4. Ultimate moment resistance M_u

A good estimation of the ultimate moment resistance M_u of the joint may simply be obtained by substituting :
- the yield stress of the steel material f_y by the ultimate stress f_u;
- the design resistance of the bolt in tension by the ultimate resistance of the bolt in tension;

in the formulae to which it is referred in 3.2.2. for the evaluation of the joint design moment resistance M_V.

The risks of instability of the column web in compression and of the beam flange in compression have however not to be forgotten.
Specific rules to quantify the related instability loads are respectively given in [10] and [5].

As for M_v, the ultimate moment resistance M_u is associated to the ultimate resistance of the weakest component.

3.2.5. Comparisons of full M-ϕ curves with test results

The combined use of the mathematical expression of the M-ϕ curves - formula(1) - and of the expressions of K_i - formula(8) -, M_v - section 3.2.2., - K_{st} - formula (19) - and M_u - section 3.2.4. - enables the analytical prediction of the characteristic M-ϕ curve of joints with extended end-plates. Comparisons between this model and tests carried out in different European laboratories in Belgium, Italy and the Netherlands are presented in [2] and the good agreement obtained between the tests and the model has allowed to validate the analytical procedure. Obviously, the fact that series of tests performed by different persons in different laboratories have been considered in this comparative study is likely to increase the confidence in the model.

Examples of such comparisons are given in figure 9.

4. References

[1] JASPART, J.P., MAQUOI, R., ALTMANN, R. and SCHLEICH, J.B.
Experimental and Theoretical Study of Composite Connections.
IABSE Symposium on Mixed Structures including New Materials, Brussels, Belgium, 1990, pp. 407-412.

[2] JASPART, J.P.
Study of the Semi-Rigid Behaviour of Beam-to-Column Joints and of its Influence on the Stability and Strength of Steel Building Frames (in French).
Ph. D. Thesis, MSM Department, University of Liège, Belgium, 1990.

[3] Semi-Rigid Action in Steel Frame Structures.
Final Report of the ECSC Research Program n° 7210/SA-507, Commission of the European Communities, 1990.

[4] Eurocode 3 : Design of Steel Structures. Part 1.1. : General Rules and Rules for Buildings, European Prestandard - ENV 1993-1-1, February 1992.

[5] Eurocode 3 : Revised Annex J. Joints in Building Frames. CEN/TC250/SC3-PT9, March 1994.

[6] YEE, Y.L. and MELCHERS, R.E.
Moment-Rotation Curves for Bolted Connections.
Journal of Structural Engineering, vol. 112, n° 3, March 1986, pp. 615-635.

[7] MAQUOI, R. and JASPART, J.P.
Moment-Rotation Curves for Bolted Connections.
Discussion of the paper par YEE, Y.L. and MELCHERS, R.E., Journal of the Structural Division, ASCE, vol. 113, ST10, October 1987, pp. 2324-2329.

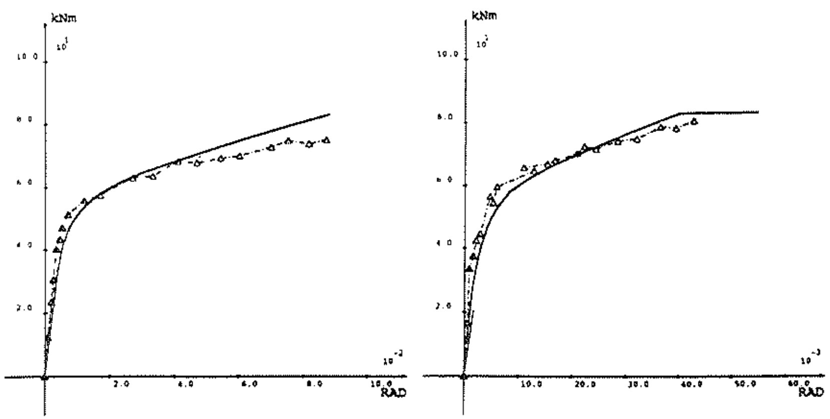
[8] DOUTY, R.T. and Mc GUIRE, W.
High Strength Bolted Moment Connections.
Journal of the Structural Division, ASCE, vol. 91, n° ST2, April 1965, pp. 101-128.

[9] AGERSKOV, H.
High-Strength Bolted Connections Subject to Prying.
Journal of the Structural Division, ASCE, vol. 102, n° ST1, January 1976, pp. 161-175.

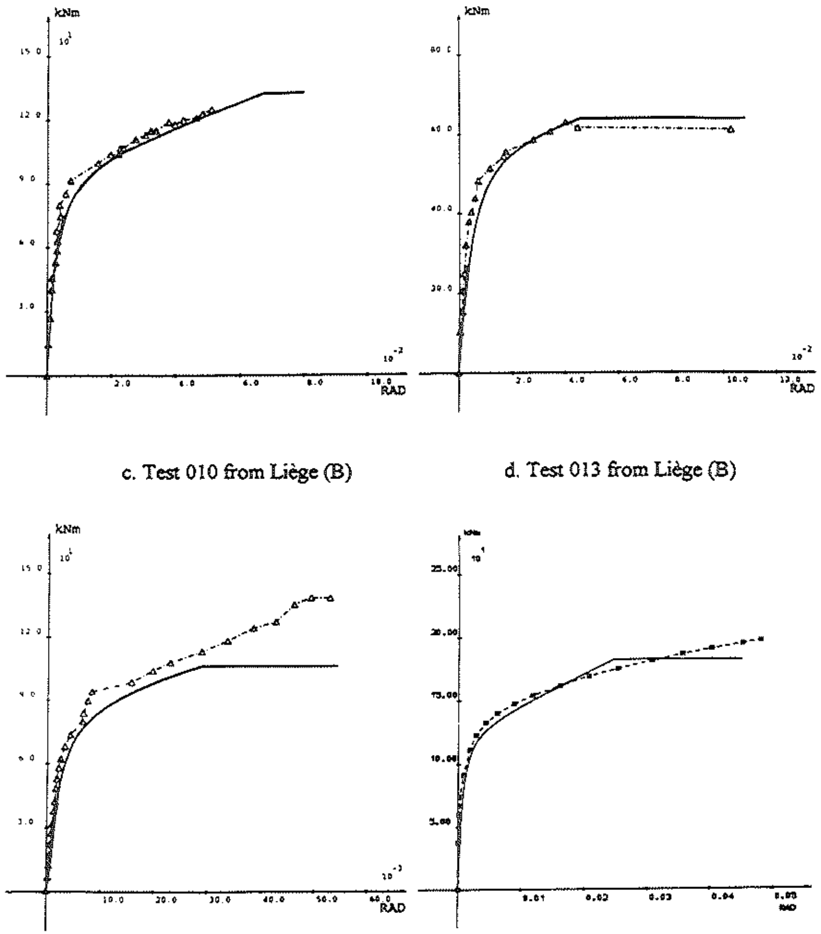
[10] MAQUOI, R. and JASPART, J.P.
Study of the Load-Introduction Deformability of Column Webs in Strong Axis Beam-to-Column Joints.
Proceedings of the IUTAM Symposium on Contact Loading and Local Effects in Thin-Walled Plated and Shell Structures, Prague, Czechoslovakia, September 4-7, 1990.

[11] JASPART, J.P. and MAQUOI, R.
Plastic Capacity of End-Plate and Flange Cleated Connections. Prediction and Design Rules. Proceedings of the Second International Workshop on Connections in Steel Structures : Behaviour, Strength and Design, Pittsburgh, U.S.A., April 10-12, 1991.

[12] JASPART, J.P. and MAQUOI, R.
Study of the Shear Deformability of Column Web Panels in Strong Axis Beam-to-Column Joints.
Proceedings of the SSRC Annual Technical Session, St-Louis, U.S.A., April 9-11, 1990.



a. Test 04 from Liège (B) b. Test 07 from Liège (B)



c. Test 010 from Liège (B) d. Test 013 from Liège (B)

e. Test T9 from Delft (NL) f. Test EP1-2 from Trento (I)

Figure 9 - Comparisons between model and tests