COMPARATIVE STUDY OF BEAM-COLUMN INTERACTION FORMULAE

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1. Introduction

Steel building frames are mainly constituted of beams subject to bending and beam-columns subject to combined bending and compression.

Several researches in the past have been devoted to the evaluation of the carrying capacity of beam-columns; they have led to the proposal of numerous interaction formulae which have been progressively introduced in national (LRFD, DIN, ...) and international codes.

The application of these different formulae to reference cases shows unfortunately a large discrepancy between the calculated values of the ultimate load, on one hand, and between the calculated values and results of numerical simulations, on the other hand.

Such a comparative study has been recently performed at the University of Liège; four interaction formulae have been considered : ECCS (1984), DIN 18800 (1988), LRFD (1986) and Eurocode 3 (1993). The present paper is aimed at presently the results and the conclusions of this comparative study.

2. Beam-columns

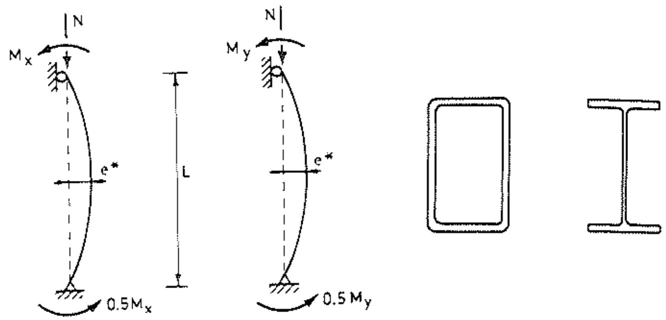
The beam-columns considered in the present study are constituted of H or rectangular tubular cross-sections (figure 1.a). They are bent either uniaxially, about strong or weak axis, or biaxially. For simplicity, only plastic and compact cross-sections are considered, so avoiding local instability and coupling with global member instability.

The first order distribution of bending moments along the beam-column is linear and characterized by M and 0.5 M values at the member extremities (see figure 1.b), whatever be the axis of bending.
As in braced frames, the transversal displacements of both beam-column extremities are assumed to be prevented.

Lastly, the buckling load of the beam-columns under pure compression is assumed to be lower about the weak axis than about the strong one (the beam-column extremities are hinged about X and Y axes). This last assumption may be expressed mathematically in the following way :

$$\bar{N}_y < \bar{N}_x \qquad (1)$$

where $\bar{N}$ represents the reduced compression normal force, defined as the ratio between the ultimate buckling resistance N_u of the column under compression and its squash load N_p.



a. Loading and support conditions b. Cross-sections

Figure 1 - Beam-columns considered

3. Numerical simulations

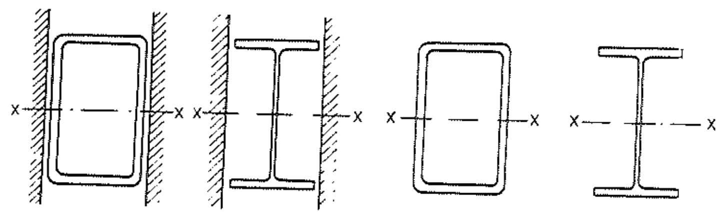
The numerical simulations of the behaviour till collapse of the beam-columns have been performed by means of the materially and geometrically non-linear finite element program FINELG which has been developed at the University of Liège and in which factors such as residual stresses and initial deformations may be accounted for.
Two different cross-sections are dealt with in this numerical study :

- a hot-rolled 200x100x6.3 rectangular tubular cross-section;
- a hot-rolled HE160B wide flange H section.

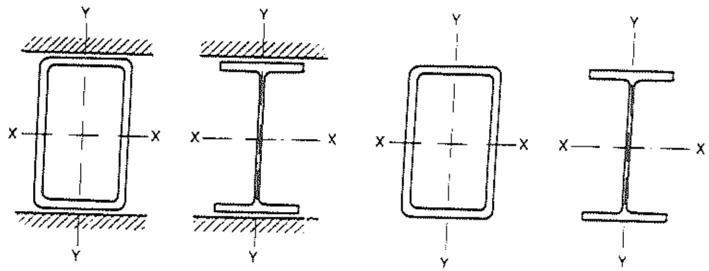
They are representative of the common practice in Europe. Both beam-columns are characterized by a reduced slenderness $\bar{\lambda}$ equal to 1.5 about the weak axis and about 0.9 about the strong one. Mild steel (yield stress f_y = 235 MPa) is used. A sinusoïdal imperfection (e^* = L/1000 in figure 1.a.) is assumed in the X and Y directions.

The numerical simulations performed for each profile are aimed at describing the evolution of the collapse load versus the N/M (normal force/bending moment) ratio in each of the four following situations :

- uniaxial bending about strong axis; lateral displacements prevented by an appropriate bracing system (figure 2.a.);
- uniaxial bending about strong axis; lateral displacements not prevented (figure 2.b.);
- uniaxial bending about weak axis (figure 2.c.);
- biaxial bending: bending moments M_x and M_y are such that $M_x/M_{px} = M_y/M_{py}$ with M_{px} and M_{py} defined as the plastic moment resistances of the cross-section respectively about strong and weak axis (figure 2.d.).



a. Strong axis; no lateral displacement b. Strong axis; lateral displacement allowed



c. Weak axis d. Strong and weak axes

Figure 2 - Loading and support conditions for beam-columns under compression and uniaxial or biaxial bending.

4. Interaction formulae

The collapse mode of a beam-column is highly dependent on the here above described loading and support conditions, as specified in figure 3.

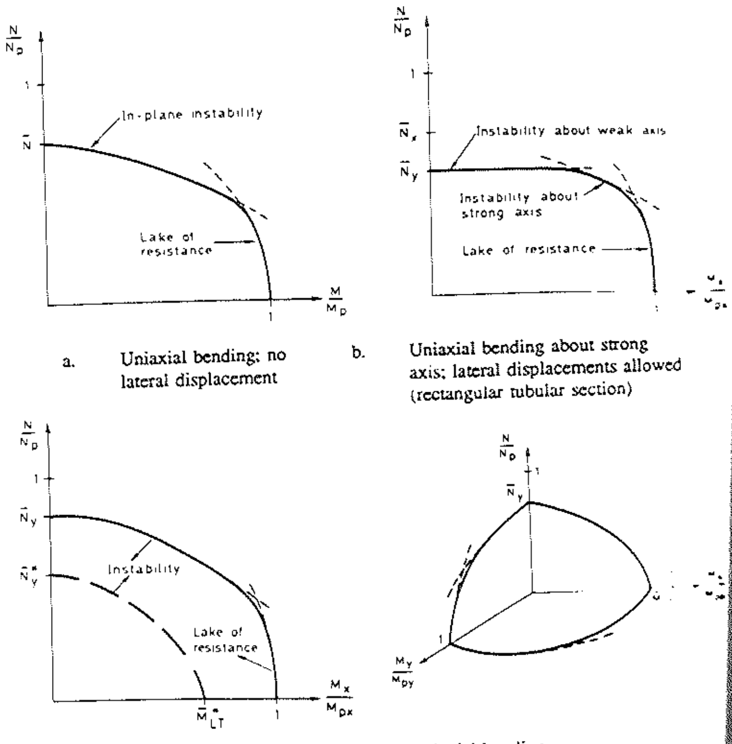
Figure 3.a. is representative of a beam-column subject to uniaxial bending about weak axis or strong axis, with lateral displacements prevented. The in-plane instability under compression and bending is controlling for high and intermediate compression forces while the lake of resistance of the cross-section is predominant for low compression forces.

For beam-columns uniaxially bent about strong axis, the lateral displacements of which are not prevented, a supplementary collapse mode is controlling for low bending moments : the buckling instability about weak axis under pure compression (figure 3.b. for rectangular tubular cross-sections). For H sections, the lateral torsional buckling may obviously become predominant for high bending moments; for slender members, the instability controls then the collapse of the beam-column, whatever be the N/M ratio (dashed line in figure 3.c. where

$\overline{M}_{LT} = M_{LT} / M_p$ with M_{LT} defined as the resistance moment for lateral torsional buckling in bending).

The collapse mode of beam-columns under biaxial bending is also seen to be highly dependent on N/M_x, N/M_y and M_x/M_y ratios characterizing the loading path (see figure 3.d.).

The diagrams reported in figure 3 are indicative ones. Depending on the flexural slenderness of the considered member, some collapse modes may be more or less predominant; the collapse of a very short column for instance will always be associated to the lake of resistance of the cross-section, whatever be the loading conditions.



a. Uniaxial bending; no lateral displacement

b. Uniaxial bending about strong axis; lateral displacements allowed (rectangular tubular section)

c. Unilateral bending about strong axis; lateral displacements allowed (H section)

d. Biaxial bending

Figure 3 - Collapse modes of beam-columns

These different possible collapse modes have to be covered by the interaction formulae developed for design practice. The so-called ECCS [1], DIN 18800 [2], Eurocode 3 [3] and LRFD [4] ones are studied in the present paper; they correspond to four different ways to approach the problem, as clearly shown in table 1.

The 1985 ECCS proposal suggests the use of a specific formula for each possible collapse mode while in the DIN code, the stability and resistance checks are integrated into a single formula, except for beam columns subject to uniaxial bending about the strong axis and for which lateral torsional buckling is not likely to appear; for such beam-columns, the out-of-plane flexural buckling under pure compression has to be independently checked.
The EC3 European prestandard proposes a higher level of simplification by only distinguishing two cases according as the lateral torsional buckling is or not likely to occur.
Lastly, the LRFD code suggests a single interaction curve which is aimed at covering all the possible collapse modes under compression and uniaxial or biaxial bending.

The interested reader is kindly asked to refer to original references for a more detailed description of each of these four design approaches.

5. Comparative study

The four design approaches described in table 1 are compared in figures 4.a. to 4.c. to the results of the numerical simulations on beam-columns constituted of a 200x100x6.3 rectangular tubular profile in which the instability by lateral torsional buckling is not likely to occur.

The following conclusions may be drawn :

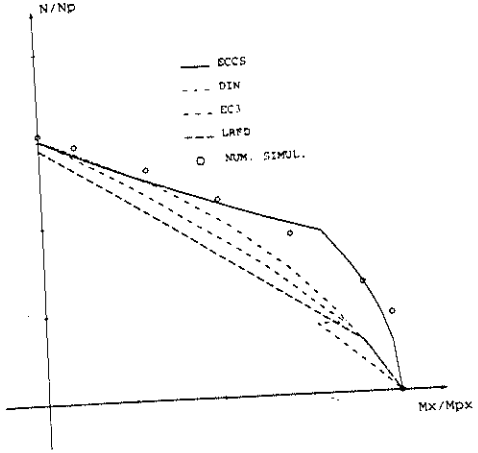
- ECCS, DIN, EC3 and LRFD interaction formulae predict in a reasonable way the collapse load of beam-columns subject to uniaxial bending about strong axis and prevented to deform in the plane perpendicular to that of loading (figure 4.a.). For low N/M ratios where the resistance is controlling, DIN, EC3 and LRFD approaches leads generally to particularly conservative assessments of the ultimate resistance. This clearly results from the way in which the related formulae are "forced" to include the resistance verification. The ECCS approach provides the designer with accurate estimations of the collapse load, except in the transition zone from "instability mode" to "resistance mode" where the results are slightly overestimated. These conclusions may be extended to beam-columns subject to uniaxial bending about weak axis (figure 4.c.).

- LRFD and EC3 interaction formulae understimate dramatically the ultimate resistance of the rectangular tubular beam-columns in case of uniaxial bending about strong axis, with allowed transversal displacements (figure 4.b.). The high level of interaction between the in-plane and out-of-plane instabilities predicted by these three formulae does not correspond at all to the actual behaviour in which the moment amplifications about strong and weak axis seem to be independent, except in a limited zone of transition from one collapse mode (buckling about weak axis) to another one (instability about strong axis or lake of resistance). In ECCS and DIN formulae, on the other hand, no coupling is considered; this explains the slightly unconservative results given by the ECCS approach in the transition zone.

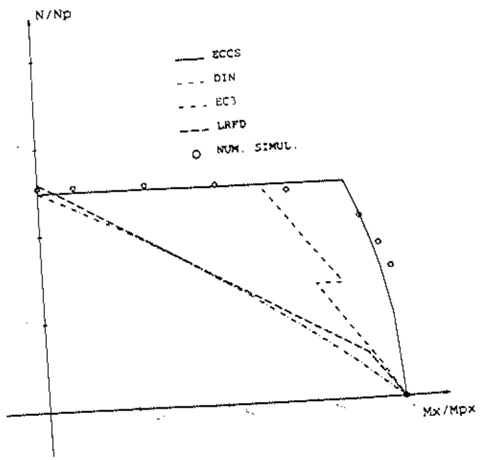
- DIN, EC3 and LRFD design approaches provide similar ultimate carrying capacities over the whole range of normal forces and bending moments in case of combined

	UNIAXIAL BENDING			BIAXIAL BENDING	
	Stability check in the plane of loading (∥)	Resistance check for the end-sections	Stability check in the plane ⊥ to the plane of loading	Stability check	Resistance check
ECCS	Lateral torsional buckling not likely to appear $\frac{N}{N_{u\parallel}} + \frac{1}{1-\frac{N}{N_{u\parallel}}(Nu_\parallel\bar{\lambda}_\parallel)^2}\frac{C_{m}M_\parallel}{M_{p\parallel}}K_{u\parallel} \leq 1$	$f(\frac{N}{N_p};\frac{M_\parallel}{M_{p\parallel}}) \leq 1$	$N < N_{u\perp}$	$\frac{C_{m\parallel}M_\parallel}{M_{u\parallel}} + \frac{C_{m\parallel}M_\perp}{M_{u\perp}} \leq 1$	$f(\frac{N}{N_p};\frac{M_\parallel}{M_{p\parallel}};\frac{M_\perp}{M_{p\perp}}) \leq 1$
	Lateral torsional buckling likely to appear $\frac{N}{N_{u\perp}} + \frac{1}{1-\frac{N}{N_{u\parallel}}(\bar{N}_{u\parallel}\bar{\lambda}_\parallel)^2}\frac{C_{m}M_\parallel}{M_{u\parallel}} \leq 1$	$f(\frac{N}{N_p};\frac{M_\parallel}{M_{p\parallel}}) \leq 1$	-		
DIN	Lateral torsional buckling not likely to appear $\frac{N}{N_{u\parallel}} + \frac{\beta_{m\parallel}M_\parallel}{M_p\parallel} \leq 1-\Delta n$		$N < N_{u\perp}$	$\frac{N}{\min(N_{u\parallel};N_{u\perp})} + \frac{\beta_{m\parallel}M_\parallel}{M_{p\parallel}}k_\parallel + \frac{\beta_{m\perp}M_\perp}{M_p\perp}k_\perp \leq 1-\Delta_n$	
	Lateral torsional buckling likely to appear $\frac{N}{N_{u\perp}} + \frac{M_\parallel}{\kappa_{m\parallel}M_{p\parallel}}k_\parallel \leq 1$		-	$\frac{N}{N_{u\perp}} + \frac{M_\parallel}{\kappa_{m\parallel}M_{p\parallel}}k_\parallel + \frac{M_\perp}{M_{p\perp}}k_\perp \leq 1$	
EC3	Lateral torsional buckling not likely to appear $\frac{N}{\min(N_{u\parallel};N_{u\perp})} + \frac{k'_\parallel M_\parallel}{M_{p\parallel}} + \frac{k'_\perp M_\perp}{M_{p\perp}} \leq 1$				
	Lateral torsional buckling likely to appear $\frac{N}{N_{u\perp}} + \frac{k'_{LT}M_\parallel}{\chi_{LT}M_{p\parallel}} + \frac{k'_\perp M_\perp}{M_{p\perp}} \leq 1$				
LRFD	$\frac{N}{2\min(N_{u\parallel};N_{u\perp})} + \frac{M_\parallel^*}{M_{u\parallel}} + \frac{M_\perp^*}{M_{p\perp}} \leq 1$ for $< \frac{N}{\min(N_{u\parallel};N_{u\perp})} < 0.2$ $\frac{N}{\min(N_{u\parallel};N_{u\perp})} + \frac{8}{9}(\frac{M_\parallel^*}{M_{u\parallel}} + \frac{M_\perp^*}{M_{p\perp}}) \leq 1$ for $\frac{N}{\min(N_{u\parallel};P_{u\perp}})) \geq 0.2$				

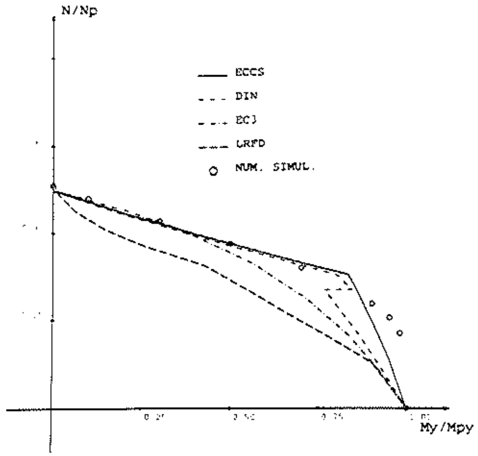
Table 1 - Four considered design approach



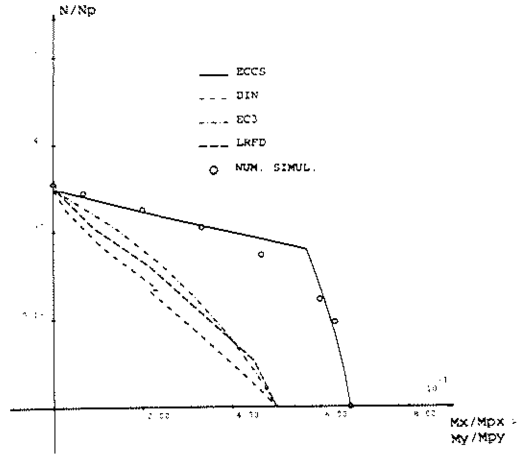
a. Uniaxial bending about strong axis; no lateral displacement



b. Uniaxial bending about strong axis; lateral displacements allowed



c. Uniaxial bending about weak axis



d. Biaxial bending

Figure 4 - Comparisons between numerical simulations and four considered design approaches for beam-columns constituted of a 200x100x6.3 rectangular tubular profile.

compression and biaxial bending. Unfortunately, these collapse loads are quite different from the actual ones, especially for intermediate and high bending moments (figure 4.d.). In these three approaches, the interaction formula for biaxial bending reduce to the following expression when the normal compression force in the column vanishes :

$$\frac{M_x}{M_{px}} + \frac{M_y}{M_{py}} \leq 1 \tag{2}$$

This linear expression differs considerably (see figure 5) from the following design criteria which is recommended in [5] for the evaluation of the plastic resistance of rectangular tubular cross-sections subject to biaxial bending :

$$\frac{3}{4}\left(\frac{M_x}{M_{px}}\right)^2 + \frac{M_y}{M_{py}} \leq 1 \text{ for } \frac{M_x}{M_{px}} \leq \frac{2}{3} \tag{3.a.}$$

$$\frac{3}{4}\left(\frac{M_y}{M_{py}}\right)^2 + \left(\frac{M_x}{M_{px}}\right) \leq 1 \text{ for } \frac{M_x}{M_{px}} > \frac{2}{3} \tag{3.b.}$$

The separate verification of the column stability and of the resistance in the most stressed cross-sections, on the other hand, allows a far better assessment of the maximum carrying capacity, as demonstrated by ECCS approach (figure 4.d.)

Similar conclusions may be drawn from the comparisons on beam-columns constituted of a hot-rolled HE160B profile : only the comparisons relative to beam-columns bent about strong axis, with lateral displacements allowed, are reported in this paper (figure 6). They present two particular features :

- The actual interaction between the out-of-plane flexural buckling of the beam-column and the lateral torsional buckling is seen to be more significant than the similar interaction between out-of-plane flexural buckling and in-plane instability in case of rectangular tubular cross-sections.

- The ECCS formula, as the DIN, EC3 and LRFD ones, underestimate, but less than the others, the actual carrying capacity. As a matter of fact, and contrary to what has been explained for rectangular tubular sections in which the lateral buckling is not likely to occur, the ECCS formula takes the coupling between out-of-plane buckling and lateral torsional buckling into consideration, but in a too conservative way. The necessity to check separately the instability and the resistance allows however, contrary to DIN, EC3 and LRFD approaches, a reasonable evaluation of the collapse load.

6. Conclusions

Some important conclusions have to be drawn from this comparative study in view of the improvement of the existing beam-column interaction formulae.

- LRFD and EC3 codes lead to too conservative assessments of the carrying capacity of beam-columns when the resistance is controlling (low N/M ratios). These two approaches are also too conservative when considering interaction between in-plane and out-of-plane instabilities.

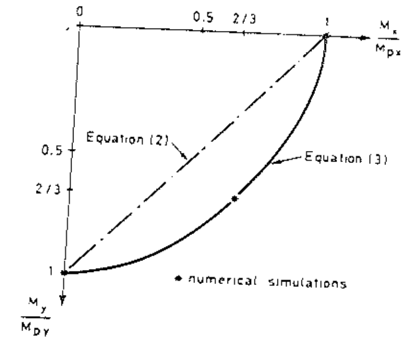


Figure 5 - Cross-section resistance under biaxial bending

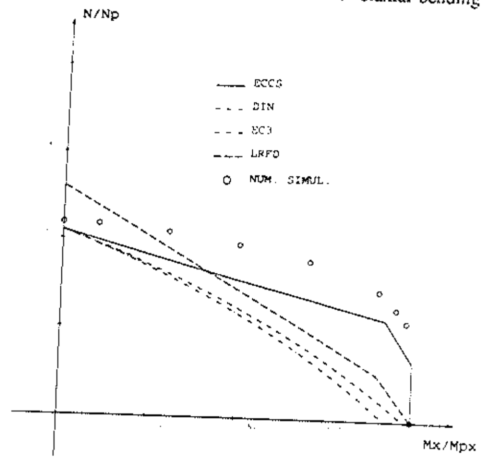


Figure 6 - Comparisons between numerical simulations and the four considered design approaches for a HE160B beam-column under uniaxial bending: lateral displacements allowed.

Simply replace $\overline{N}_y$ by $\overline{N}_x$ in the interaction formula proposed for in plane instability when the out-of-plane flexural buckling of the beam-column is likely to occur can not be considered as a consistant way to account for the actual amplification of the bending moment about strong and weak axes.
ECCS and DIN approaches consider also (and in a similar way) the interaction between in-plane and out-of-plane instabilities except when the lateral torsional

buckling is not likely to occur.
This results unfortunately in a non-smooth transition from one formula (with lateral torsional buckling) to the other (without lateral torsional buckling).
The ECCS formula is also characterized by a non-smooth transition from the case of bending about strong axis (collapse by lateral torsional buckling) to that of combined compression and uniaxial bending under strong axis (spatial instability).

7. References

[1] ECCS TC 8.1./8.2. Working documents on beam-columns, not published, 1985. (content introduced in the "Steel Structures" course of Prof. R. MAQUOI, Department MSM, University of Liège, 1988).

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[5] "Construction with hollow steel sections", British Steel Corporation, First Edition, December 1984.

A STUDY ON ULTIMATE STRENGTH OF STIFFENED PLATES IN STEEL BRIDGES SUBJECTED TO BIAXIAL IN-PLANE FORCES

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ABSTRACT

This paper deals with the ultimate strength of longitudinally stiffened plates subjected to biaxial in-plane forces. Firstly, the residual stress and initial deflections of steel plates with the closed cross-sectional stiffeners are predicated. Then, the ultimate strength of the stiffened plates is investigated through elasto-plastic and finite displacement analyses. Also formulated are the interaction curves on the longitudinal and transverse ultimate stresses of stiffened plates. An approximate interaction curve for the ultimate stresses, which is the function of the ultimate longitudinal and transverse compressive stresses of the stiffened plates subjected to longitudinal and transverse compressions separately, is proposed for predicting the ultimate stresses of the unstiffened and stiffened plates subjected to biaxial in-plane forces. Finally, the test results carried out in the study are compared with the approximate interaction curve for the ultimate stresses.

1. INTRODUCTION

Recently, the number of plate elements to be designed as the unstiffened or stiffened plates subjected to biaxial in-plane forces (hereafter called as the biaxially loaded plates or stiffened plates) , as illustrated in Fig.1, are gradually increasing in Japan in accordance with the increase of length of span or clear width of steel bridges. However, the current Japanese Specifications for Highway Bridges[1] (JSHB) does not codify any design criteria for the biaxially loaded plates. Accordingly, in designing such a plate, the buckling stability must be investigated through a theoretical or experimental study for checking the safety against their ultimate limit state in the case where the transverse compression as well as the longitudinal in-plane stress