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Lateral torsional buckling resistance of web-tapered I-beams under linear bending moments

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Abstract

Present paper focuses on the practical design of web-tapered I-beams in bending. Both linear stability and non-linear behavior are studied, by means of Finite Element Method (FEM) calculations. Practical examples demonstrate the suitability of the Ayrton-Perry concept for the design of tapered members.

1. Introduction

This paper focuses on the stability behavior of steel web-tapered I-members. They are widely used in medium and long span industrial portal frames, since they allow for significant material savings and for a more consistent design. However, their use is limited by the lack of specific accurate design recommendations, which often consist in rough elastic formulae, where taper effects are rather badly accounted for. As a consequence, design solutions involving tapered elements are sometimes neglected, the benefits brought by the tapering being withdrawn by the roughness of the design rules.

In design practice, tapered members are mainly submitted to axial forces and end moments arising from adjacent members; the problem of flexural buckling and lateral torsional buckling then become of prime importance. Whenever the first have received specific attention and acceptable solutions (see [1] for instance), it seems that no fully satisfactory design method for lateral torsional buckling is available in either design codes or in the literature. One may however refer to the works of Bradford [2], Braham [3] or Andrade [4].

Present paper aims at providing additional information on the behavior of tapered beams under linear bending moments. It shows that simple design rules based on the Ayrton-Perry concept of relative slenderness, widely used in Eurocode 3 [5], can provide sufficiently accurate results, in comparison with Finite Element Method (FEM) numerical results. The determination of the relative slenderness to lateral torsional buckling $\overline{\lambda}_{LT}$ requires on one hand the determination of a critical load multiplier, and on the other hand the calculation of a resistance load multiplier (either elastic or plastic):

$$\overline{\lambda}_{LT} = \sqrt{\frac{\lambda_{Rk}}{\lambda_{cr}}} \qquad (1)$$

where λ_{Rk} corresponds to the maximum load factor that makes the member fail by lack of resistance (see section 4), and λ_{cr} corresponds to the critical load factor (section 3). Then, use of a buckling curve allows the determination of the ultimate design of the member:

$$M_{Rd} = \frac{\chi_{LT} \lambda_{Rk} M_{Ed\,max}}{\gamma_M} \text{ with } \chi_{LT} = f^{\circ}\left(\bar{\lambda}_{LT}\right) \quad (2)$$

$M_{Ed\,max}$ being the maximum bending moment applied on the member and γ_M the safety factor.

First, the paper briefly describes FEM numerical models used to provide reference results; then, the particular case of linear stability analysis is studied in section 3. Section 4 presents results obtained for non-linear calculations, while section 5 plots the results under so-called "lateral torsional buckling curves".

2. Numerical models

The validation of the concepts and accuracy of the proposed method implies the availability of reference results. To the writers' knowledge, very few well-documented test results on tapered members are available in the literature. Therefore, the sole way to get detailed information on the behavior of such members consists in numerical simulations. They are especially suitable for testing the proposed concepts since they allow for an easy isolation of the different parameters and rather quick computations.

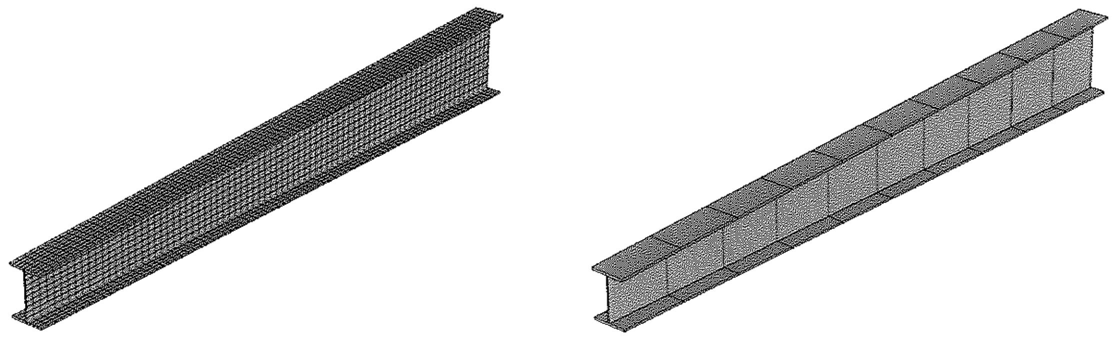

Figure 1. Shell FEM model (14 000 degees of freedom (dof's)) and tapered beam model (84 dof's)

Either FEM-shell or FEM-beam models may be used, see figure 1. Shell modeling allows covering a wide field of phenomena, from member instability to cross-section deformations (i.e. local buckling and/or distortion). However, shell models often require important data preparation and computation times, due to the important number of degrees of freedom involved.

On the contrary, beam modeling is much faster, and provides simple results. Nevertheless, for the specific case of tapered members, it has been previously shown ([6], [7], [8] and [9]) that an adequate beam element is needed when torsional deformations are of concern. In particular, it has been demonstrated that a simplified beam modeling resorting to stepped beam elements cannot lead to the correct solution. Therefore, such a purposely-derived beam element has been used to provide the "FEM-beam" results ([7] and [8]). Both "shell" and "beam" models have been used in present study.

Two different web-tapered I-beams under linear bending moments have been investigated in the following, see table 1. The supports at both ends are supposed to fulfill the so-called "fork" boundary conditions, and the material is assumed to be elastic-perfectly plastic.

In both shell and beams models, initial geometrical imperfections have been taken into account for non-linear computations, in both strong and weak axis directions; in addition, an initial torsional twist with maximum amplitude at mid-span has been introduced, in order to initiate the lateral torsional buckling phenomenon [10]; initial residual stresses have not been taken into account. All computations have been performed by means of home-made software FINELg [11].

Table 1. Geometrical definition of investigated members (dimensions in mm)

	$h_{w\,max}$	$h_{w\,min}$	t_w	b	t_f	L
Beam n°1	476	276	8	200	12	4000
Beam n°2	1176	176	8	200	12	6000

3. Linear stability analysis

The first type of analysis studied here is linear stability analysis; the determination of the critical load multiplier λ_{cr} is indeed needed for the determination of the relative slenderness to lateral torsional buckling, see eq. (1). Figure 2 reports the obtained results for $M_{cr} = \lambda_{cr} M_{Ed}$ (beam n°2), where the applied bending moment M_{Ed} at the largest end section is kept constant, while the other end is submitted to ψM_{Ed} ($-1 \leq \psi \leq 1$). As can be seen, the numerical results provided by the shell and beam models are in quite good agreement, as long as $\psi > -0.2$. The shell models give lower results than their beam counterparts when $\psi < -0.2$, due to the higher level of stresses required to reach the critical state, where the cross-section experiences distortion. In addition, figure 2 also reports the results provided by the analytic proposal of Andrade [4], which is amongst the most recent ones. Despite a discontinuity at $\psi = -0.6$, the correspondence with FEM results is rather good; this approach has been adopted in the following.

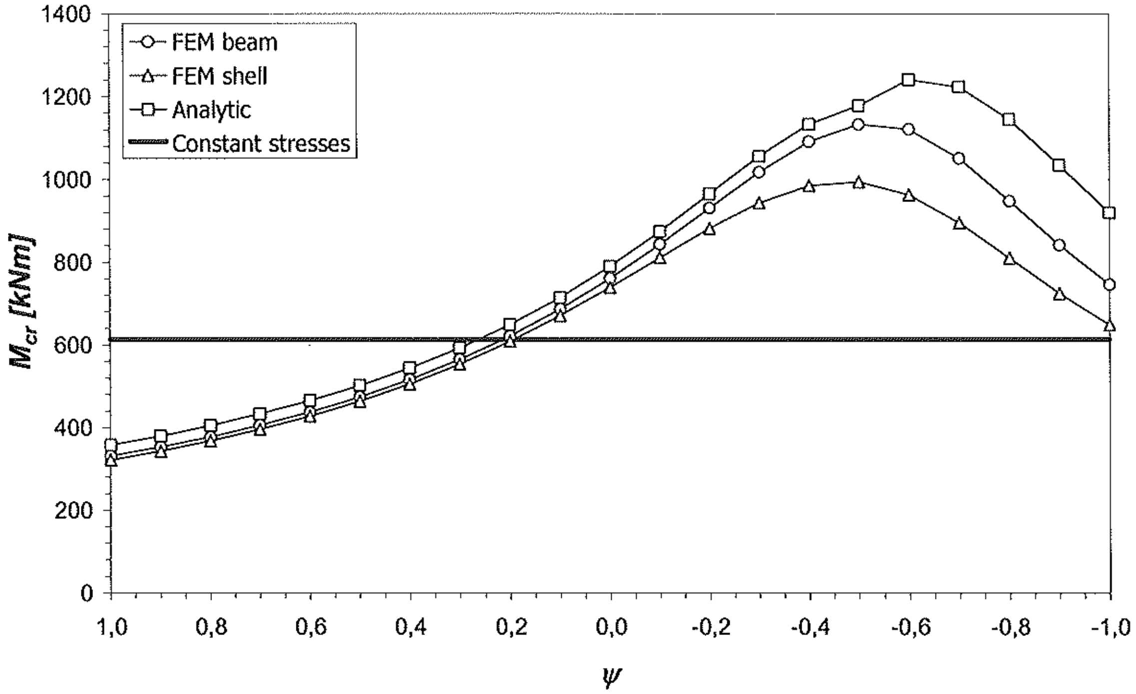

Figure 2. Results for linear stability analysis (beam n°2)

Similarly to what is usual for prismatic members (cf. [12]), it may be interesting to determine a loading situation leading to the smallest value of M_{cr}, for a given geometry; the calculation of the critical bending moment for any other bending moment distribution on the member would then be relative to this reference case. As for prismatic members, this may be achieved through a constant level of axial stresses along the member; a combination of end moments plus linearly distributed load allows meeting this case, and the corresponding value of M_{cr} is reported on Figure 2.

It appears that ensuring such a constant level of stresses may not lead to the worse loading situation, as a case $\psi = 1$ is seen to lead to the lower value of M_{cr}. This is due to the variations of lateral and torsional rigidities along the member, which provide a different level of restraint to each cross-section. As a consequence, the determination of the reference case (e.g. worst) must combine both the level of stresses and the distribution of restraints. As a consequence, it appears very difficult to propose a simple and practical way to define such a reference situation; this specific point would require further developments.

4. Geometrically and materially non linear analysis

Materially Non-linear Analyses (MNA) have also been performed in order to determine the "resistance load multiplier" λ_{Rk}; the latter corresponds to either the plastic, elastic or effective resistance of the *member* (prevented from any member instability phenomenon), since the resistance behavior cannot be reduced to a *cross-sectional* one, like for prismatic members. Indeed, while the cross-section resistance varies from $M_{pl.min}$ in the smaller section to $M_{pl.max}$ in the larger one, the distribution of stresses also changes along the member length, for a given value of ψ. "MNA results" for beam n°2 are plotted in figure 3; one can see that depending on the value of ψ, the maximum applied bending moment $M_{Ed\,max}$ varies between $M_{pl.min}$ and $M_{pl.max}$. A deep analysis of the results show that the differences in $M_{Ed\,max}$ are mainly due to the modifications of the loading situation, since the smallest section is almost always the yielded one. This indicates a low level of plastic redistribution between sections.

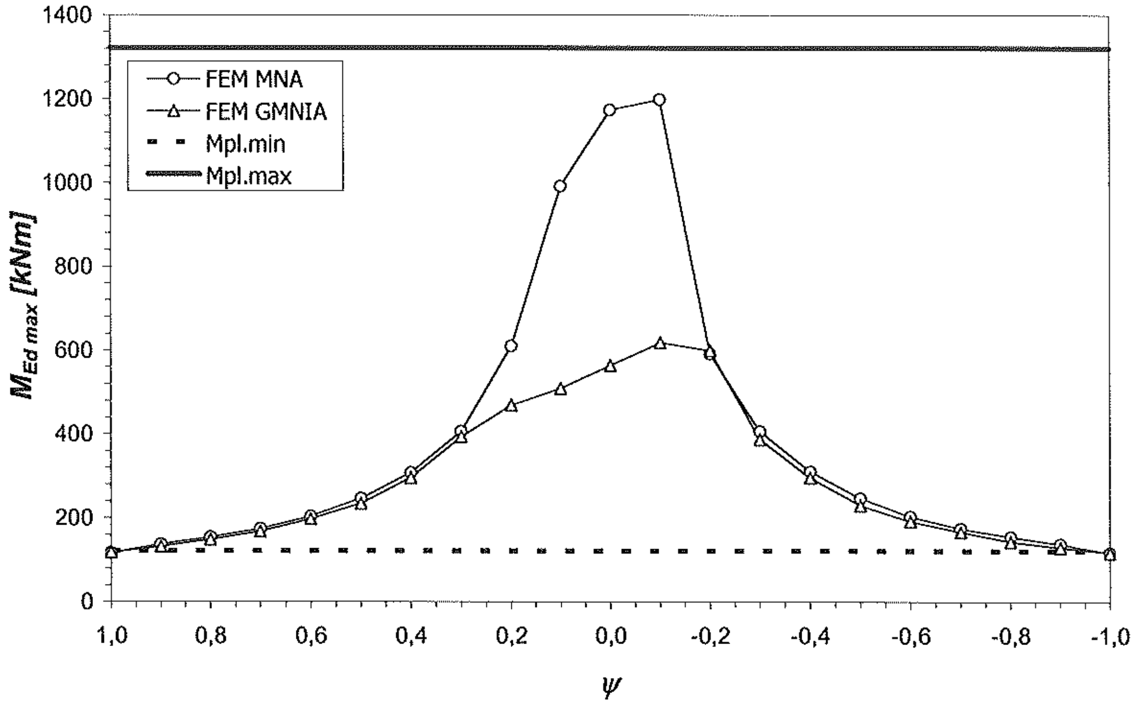

Figure 3. Results for non-linear analysis (beam n°2)

Figure 3 also reports on the ultimate resistance of the member to lateral torsional buckling, by means of the "GMNIA" curve (Geometrically and Materially Non-linear with Imperfections Analysis). As can be seen, when the bending moment on the member is nearly constant (ψ is close to 1) or highly variable (ψ is close to -1), the determination of $M_{Ed\,max}$ is mainly ruled by *resistance* aspects, since the GMNIA curve fits the MNA one; in such situations, yielding of the smallest cross-section governs the behavior of the member.

On the contrary, in the range $-0.2 \leq \psi \leq 0.3$, *instability* aspects are seen to have a strong influence on the member's resistance, regarding the significant differences between GMNIA and MNA results in this range.

5. Lateral torsional buckling curves

Figure 4 plots the results got for beams n°1 and 2 under the so-called "buckling curves" non-dimensional format. First, it shows that the "shell" results are in quite agreement with their "beam" counterparts, for beam n°1 as well as for beam n°2. It also confirms the ability of the Ayrton-Perry concept to successfully characterize the behavior of tapered members, as the results are seen to follow certain regularity. This demonstrates the possibility to derive analytical expressions for both the determination of λ_{LT} and χ_{LT}, for each set of curves. This would require further developments and research efforts, namely extensive parametric studies.

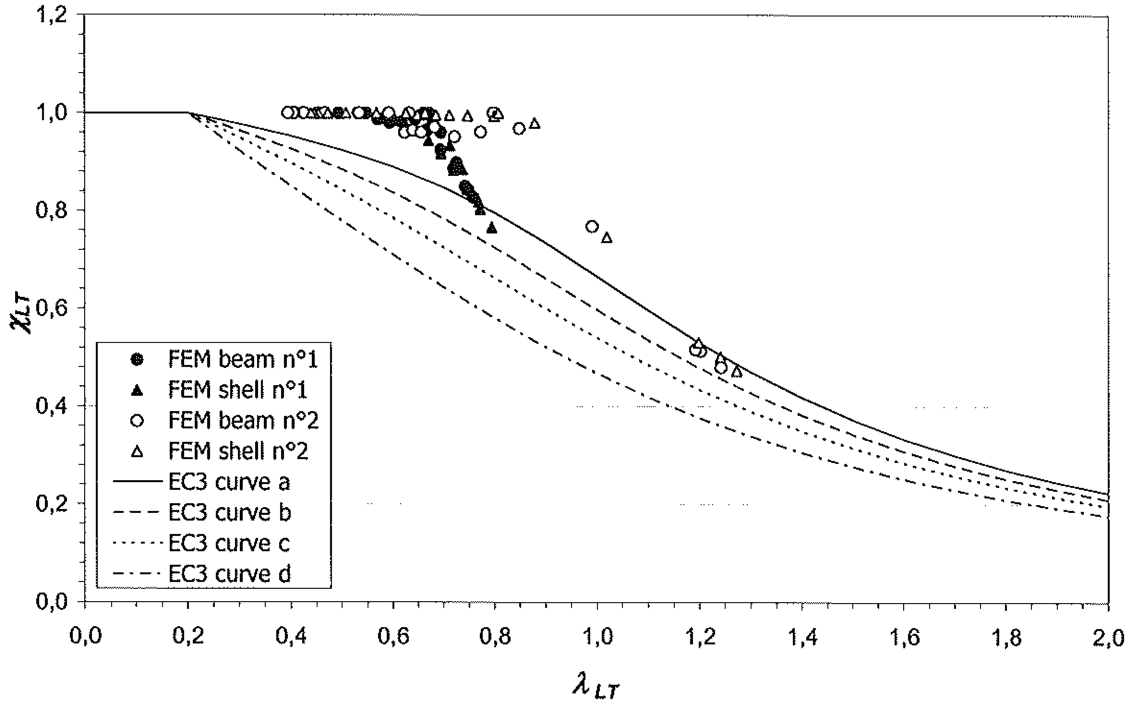

Figure 4. Results for lateral torsional buckling resistance

6. Conclusion

The paper first demonstrates the possibility to resort to an analytical determination of the critical bending moment of web-tapered I-beams, such as the one proposed by Andrade [4] for example. However, the determination of a reference loading case (e.g. worst) may be much more complicated than for prismatic members, due to the varying loading and geometry along the member.

In the same way, the determination of the characteristic resistance was seen to be highly depending on the loading distribution on the tapered beam. A low level of plastic redistribution between adjacent sections was also shown, except for very specific load cases.

Examples also illustrate the significant influence of lateral torsional buckling on the load carrying capacity of such tapered beams. Finally, the possibility to generalize the Ayrton-Perry approach to lateral torsional buckling of tapered members was demonstrated.

7. References

[1] Galéa Y. (1981), *Flambement des poteaux à inertie variable, Construction Métallique,* n°1, CTICM, Paris, France.

[2] Bradford M. A. (1988), *Stability of tapered I-beams, Journal of Constructional Steel Research*, n°9, pp. 195-216.

[3] Braham M. (2005), *Elastic lateral torsional buckling of web tapered I-beams subjected to end moments, Proceedings of the Steel Structures and Bridges Conference*, Brno, Czech Republic.

[4] Andradre A., Camotin D., Providência P.(2005), *Critical moment formulae for doubly symmetric web-tapered I-section steel beams acted by end moments, Proceedings of the 4th European Conference on Steel and Composite Structures "Eurosteel 2005"*, pp. 1.2-135-1.2-143, Maastricht, The Netherlands.

[5] Eurocode 3 EN 1993-1-1 (2004), *Calcul des structures en acier. Partie 1-1 : Règles générales et règles pour les bâtiments.*

[6] Boissonnade N. (2002), *Mise au point d'un élément fini de type poutre à section variable et autres applications à la construction métallique*, PhD Thesis, Blaise Pascal University, Clermont-Ferrand, France.

[7] Boissonnade N., Maquoi R. (2005), *A geometrically and materially non-linear 3-D beam finite element for the analysis of tapered steel members, International Journal of Steel Structures*, Vol. 5 n°5, pp. 413-419.

[8] Wekezer J.W. (1985), *Instability of thin-walled bars, Journal of Engineering Mechanics*, ASCE, Vol. 111 n°7, pp. 923-935.

[9] Kitipornchai S., Trahair N.S. (1972). *Elastic stability of tapered I-beams, Journal of the Structural Division*, ASCE, Vol. 98 n°3, pp. 713-728.

[10] Vandermeulen T. (2007), *Le déversement des poutres métalliques en I à hauteur d'âme linéairement variable sous moments d'extrémité*, Diploma Work, ArGEnCo Department, University of Liège, Belgium.

[11] Non-linear finite element analysis program "FINELg" (2002), *User's Manual, version 8.5*, ArGEnCo Department, University of Liège, Greisch Info S.A., Liège, Belgium.

[12] Braham M. (2001), *Le déversement élastique des poutres en I à section monosymétrique soumises à un gradient de moment de flexion, Construction Métallique*, n°1, CTICM, Paris, France.