DESIGN OF BEAM-COLUMNS IN STEEL SWAY FRAMES: FROM THE ACTUAL MEMBER TO A SIMPLY SUPPORTED EQUIVALENT ONE

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INTRODUCTION

Basic Concepts. A frame is formed by members (beams, columns, beam-columns) and joints. The design of the frame and its components aims at ensuring that they satisfy the ultimate and serviceability limit states. It consists of a two-step procedure involving a global frame analysis, followed by individual member design checks.

The frame global analysis is based on assumptions regarding the component behavior (elastic or elastic-plastic) and the frame geometric response (first or second-order theory). Once the global analysis is completed, *i.e.*, the relevant internal forces and moments are determined for the whole frame, one performs the design checks of the frame components (members and joints).

Frame Classification. Frames may be classified "braced" or "unbraced" and as "sway" or "non-sway", thus defining the four classes given in table 1.

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Table 1. Frame classification

		Bracing	
		Braced	*Unbraced*
Lateral Stiffness	*Sway*	*Braced – Sway*	*Unbraced – Sway*
	Non-sway	*Braced Non-sway*	*Unbraced Non-sway*

(white)	Second-order P-Δ effects must be considered
(shaded)	Second-order P-Δ effects need not be considered

The designation "non-sway frame" applies to a frame whose lateral response to in-plane horizontal forces is so stiff (due to the small magnitude of the acting compressive forces) that it is acceptable to neglect the additional forces or moments arising from establishing equilibrium at the frame "linearized deformed configuration" that includes the storey horizontal displacements (the so-called *P-Δ effects*). This means that the global second-order effects may be neglected. The frame is said to be a "sway frame" in the opposite case, *i.e.*, when the global second-order effects are not negligible.

In Eurocode 3 (CEN 2005), a frame is classified as sway or non-sway according to a criterion based on the value of the ratio $\alpha_{cr}=V_{cr}/V_{Ed}$, where V_{cr} is the elastic critical load associated with sway instability and V_{Ed} is the sum of all the applied design vertical loads. Then, the frame is said to be *non-sway* if

$$\alpha_{cr} = V_{cr}/V_{Ed} \geq 10 \text{ for elastic analysis} \quad (1)$$

$$\alpha_{cr} = V_{cr}/V_{Ed} \geq 15 \text{ for plastic analysis} \quad (2)$$

and *sway* if the appropriate above condition is not fulfilled. It is worth noting that the frame classification is associated with a particular load case, *i.e.*, the same frame can be classified differently for two load cases.

REVIEW OF ANALYSIS AND SAFETY CHECKING PROCEDURES

Besides the P-Δ effects, which are associated with global sway instability, the frame response may also be affected by the so-called second-order P-δ effects, which are associated with member (local) instability. In general, both the P-Δ effects (sway frames only) and the P-δ effects (all frames) must be taken into account in the design or safety checking of a frame – one or both of these second-order effects may be neglected in specific cases (CEN 2005).

As mentioned earlier, the design or safety checking of a given frame is based on a two-step procedure involving a global analysis followed by individual member/joint design checks. However, the frame global analysis must be preceded by the frame definition (layout, support conditions, load cases, member dimensions, joint configurations, etc.). Moreover, imperfections have to be considered (included in the analysis) – two main imperfections types are relevant are in plane frames: the *frame* (global) imperfections (out-of-plumb or equivalent/notional horizontal loads associated with the P-Δ effects) and the *member* (local) geometrical and material imperfections (initial curvature and residual stresses, often replaced by an "equivalent" initial deformed configuration associated with the P-δ effects). Finally, an appropriate method of global analysis has to be selected.

Obviously, the most accurate method is an elastic-plastic second-order analysis that accounts for both P-Δ and P-δ effects, as it reflects much more closely the actual frame response than any simplified method – note, however, that one speaks about *in-plane* second-order analysis, *i.e.*, no out-of-plane behavior is captured. Therefore, the safety checks required after performing the frame analysis are significantly reduced, since the all in-plane effects are already included in the frame internal forces and moments – only the safety checks related to out-of-plane instability need to be done.

In practice however, the member imperfections are usually not included in the frame global analysis accordingly, which means that their effects must be taken into account afterwards, through the use of appropriate member design formulae, such as the beam and beam-column ones included in the most recent Eurocode 3 version (Maquoi *et al.* 2001, CEN 2005, Boissonnade *et al.* 2006) – this approach is systematically followed in the present work. At this stage, it is worth mentioning that beam finite elements able to handle lateral-torsional buckling effects are not yet routinely used in global frame analysis – in Europe, this situation is not expected to change in the near future.

Whereas the performance of a second-order analysis that captures the P-Δ effects "exactly" is always possible, less sophisticated approximate analyses often provide accurate enough internal forces and moments to the designer (Hansoulle 2006). Amongst the various simplified second-order analyses available in the literature (*e.g.*, Chen & Lui 1991), four are described next.

Canadian Amplification Method (CAM). The application of this method involves performing the following tasks:

- Conducting a first-order analysis of the frame subjected to all the applied vertical (gravity) loads, assuming all storeys to be fully restrained against sway displacements. The internal forces and moments (IFM) obtained are termed "gravity IFM" and the horizontal reactions that appear in the fictitious supports are designated as R (see figure 1).
- Conducting a first-order analysis of the frame subjected to all the horizontal loads: applied loads, notional loads (replacing the initial imperfections) and the horizontal reactions R (with opposite direction). The internal forces and moments obtained are termed "sway IFM".
- Amplifying the "sway IFM", in order to account for the second-order P-Δ effects that were not included in the global (first-order) analysis, by means of the amplification factor

$$\frac{1}{1-1/\alpha_{cr}} \tag{3}$$

- Determining the frame second-order internal forces and moments, which are the sum of the "gravity" and amplified "sway" IFM.

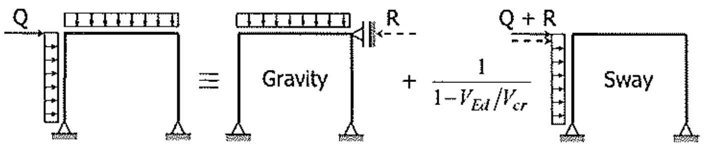

Figure 1. Schematic representation of the Canadian Amplification Method

Naturally, this simplified method is recommended in the Canadian code for the design of steel structure (CSA 2001) – however, its use is limited to frames with $\alpha_{cr} \geq 3.5$.

After determining the design values of the internal forces and moments, one must check the strength of the member cross-sections and joints, as well as the buckling strength of the columns, beams and beam-columns. For the in-plane buckling behavior of compressed members (columns or beam-columns), one may use the so-called "non-sway effective/buckling lengths", since all the P-Δ (sway) second-order effects have already been included in the design IFM values (obtained by means of either an "exact" second-order analysis or the amplification of the first-order sway IFM values). For the sake of simplicity, the above buckling lengths may be taken equal to the corresponding physical (system) lengths – this simple approach leads almost always to safe designs[1].

Amplified Sway Moment Method (ASMM)[2]. This method is very similar to the CAM – indeed, it only involves a slightly different interpretation of the same concepts. It has been routinely used in Europe for a long time and is included in current version of Eurocode 3 (CEN 2005). Its application is schematically represented in figure 2 and the differences with respect to the CAM are the following:

- *All* applied loads (vertical and horizontal, including the notional loads replacing the initial imperfections) are considered in the first-order analysis of the frame with the storeys restrained against sway displacements. The internal forces and moments obtained are termed "non-sway IFM" and one still designates the horizontal reactions appearing in the fictitious supports as R (see figure 2).
- The sway IFM are obtained by applying just the horizontal reactions R, now including the influence of *all* applied loads – of course, the ASMM and CAM coincide if there are no applied distributed horizontal loads.
- The frame second-order internal forces and moments are now the sum of the "non-sway" and amplified "sway" IFM – amplification factor still given by eq. (3).

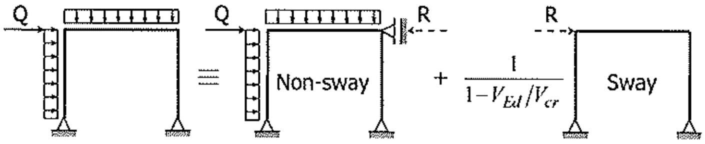

Figure 2. Schematic representation of the Amplified Sway Moment Method

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[1] This may not be the case if the frame contains columns with very different axial compression levels – *e.g.*, a member subjected to a small compressive force and large bending moments.

[2] Although the method designation and description often mentions only "moments", it really prescribes the amplification of *all* internal forces and moments associated with sway displacements.

Eurocode 3 only allows employing the ASMM provided that one has $\alpha_{cr} \geq 3.0$. Moreover, additional restrictions to the use of this method are specified – however, they are not particularly limiting for regular building frames.

"B_1–B_2" Method (B_1B_2M). This method is prescribed by the North American Specification (AISC 2006) and differs from the CAM (and also the ASMM) in the following two aspects:

- The sway IFM amplification factor given in eq. (3) is replaced by a set of approximate values termed B_2 – they must be evaluated separately for each frame storey.
- The gravity IFM are multiplied by a factor termed B_1 and given by

$$B_1 = \frac{C_m}{1 - N_{Ed}/N_{cr}} \quad (4)$$

  where N_{Ed} is the axial force acting on the compressed member under consideration and N_{cr} is its critical elastic buckling load. As for C_m, it stands for a "coefficient of equivalence" that makes it possible to take into account the effect of the column bending moment diagram.

Therefore, this method evaluates the second-order moments due to the P-Δ effects on a storey-by-storey basis (and not simultaneously for the whole frame, as done in the CAM and ASMM) and also includes the P-δ effects, through the B_1 factor. Since both the frame and member buckling are taken into account when determining the IFM design values, the final safety checks concern only the member cross-section strength (AISC 2006).

"Sway Buckling Length" Method (SBLM). This is an alternative approach to the frame design, which can be found in some steel design codes and appeared also in the Eurocode 3 ENV version (CEN 1992)[1]. Its application involves performing the following tasks:

- Conducting a first-order analysis of the frame.

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[1] This method no longer appears in the current EN version of Eurocode 3 (CEN 2005) – no reasoning or explanation has been provided for this surprising absence.

- Checking the in-plane frame overall and member (local) stability by considering "equivalent member" lengths equal to those of their *sway buckling modes*, obtained from the stability analysis of the whole frame. These buckling lengths may be obtained by means of simplified methods, based on two safe assumptions: (i) the frame sway buckling load is taken as that of its weakest storey and (ii) all the columns in a storey buckle simultaneously (when the weaker one does). One should note that this method does not address rationally the influence of the second-order effects in the IFM values at the beams and beam-to-column joints – *e.g.*, the Eurocode 3 ENV version stipulated a constant amplification factor equal to 1.2 (20% flat increase due to the second-order effects).

Comparison of the Methods. In order to compare the four methods just described, one considers a simple rectangular portal frame with a rigid beam and acted by two vertical point loads and two horizontal distributed loads q (Frame 1 in table 2). Figure 3 shows the column bending moment diagrams yielded by (i) a first-order analysis, (ii) a second-order analysis that accounts only for the P-Δ effects and (iii) an second-order analysis accounting for both the P-Δ and P-δ effects (exact P-Δ/δ analysis) (Hansoulle 2006). The column top moment values are precisely the target of the various methods under comparison: (i) SBLM, (ii) CAM, (iii) ASMM and (iv) B_1B_2M.

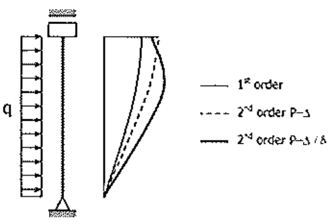


Figure 3. First and second-order bending moments (P-Δ and P-Δ/δ)

Obviously, the SBLM prediction is exact, as it coincides with the first-order bending moment. The application of the CAM or ASMM leads to the following second-order P-Δ bending moment at the top of the column (M_{top}^{II}):

$$M_{top}^{II} = \frac{qH^2}{2}\frac{1}{1-P/P_{cr}} = M_{top}^{I}\frac{1}{1-P/P_{cr}} \tag{5}$$

$$M_{top}^{II} = -\frac{qH^2}{8} + \frac{5qH^2}{8} \frac{1}{1 - P / P_{cr}} \tag{6}$$

Since there are no non-sway moments (in this particular case), the B_1B_2M yields the same results as the CAM. The analytical (exact) solution of this problem (bending moments including the P-δ effects) is given by

$$M_{top}^{II} = M_{top}^{I} \frac{2}{\varepsilon^2} \frac{1 - \cos\varepsilon}{\cos\varepsilon} = M_{top}^{I} \frac{C_m}{1 - P / N_{cr}} \tag{7}$$

$$with\ C_m \approx 1 + 0{,}03 P / N_{cr}\ and\ \varepsilon = H\sqrt{P / EI} \tag{8}$$

The comparison between eqs. (5), (6) and (7) shows that, for realistic values of the different parameters involved, both the CAM and ASMM yield reasonably accurate estimates, provided that the P-δ effects are not relevant. In addition, it is worth pointing out that only the CAM solution has a format that may easily be modified to replicate the exact solution.

Generally speaking, one may draw the following concerning the performance of the methods under scrutiny (Moszkowicz 1998, Hansoulle 2006):

- The SBLM disregards the second-order effects in the analysis and takes them into account by checking the member stability using a beam-column formula (P-δ effects) with a *sway* buckling mode (P-Δ effects).
- The CAM and ASMM usually lead to quite good predictions of the second-order P-Δ IFM, through the amplification of the first-order ones. The member stability is checked by means of beam-column interaction formulae (P-δ effects), using *non-sway* buckling lengths. The accuracy and validity of these methods, in terms of ultimate strength and serviceability limit state displacements, have been demonstrated by Moszkowicz (1998) and Hansoulle (2006), especially in frames with no relevant P-δ effects.
- There is no good agreement between the exact second-order P-Δ/δ bending moment and the B_1B_2M prediction – this method often provides overly conservative estimates, mainly due to the B_1 factor adopted in the current AISC approach.

Choice of a Method. In order to obtain an accurate prediction of the second-order P-Δ/δ moments, one must adopt a "B_1-B_2"-type approach, in which both the P-Δ and P-δ effects are integrated into the analysis. However, since the B_1B_2M often leads to unsatisfactory estimates, it was decided to retain the CAM or ASMM-type format and to try to improve it, by means of appropriate equivalence factors C_m (termed here $C_{m,end}$ and $C_{m,end}^*$) that take into account the P-δ effects. The CAM format was finally chose, because of its more obvious suitability to replicate exact solutions. Then, one expresses the approximate the second-order M^{II} P-Δ/δ bending moments in the form

$$M_{CAM,mod}^{II} = C_{m,end}^{*} M_{Gravity} + \frac{C_{m,end}}{1 - 1/\alpha_{cr}} M_{Sway} \tag{9}$$

Once the second-order bending moment (M^{II}) is determined, the remaining tasks of the design procedure can be performed – *e.g.*, checking the member cross-section resistances and joint strengths, or the stability of the columns, beams and beam-columns. In the case of beam-columns, an interaction formula (in-span P-δ effects) is used, together with (in-plane) non-sway buckling lengths. Moreover, a classical C_m value (termed here $C_{m,span}$) must be defined to handle the bending moment variation along the member length.

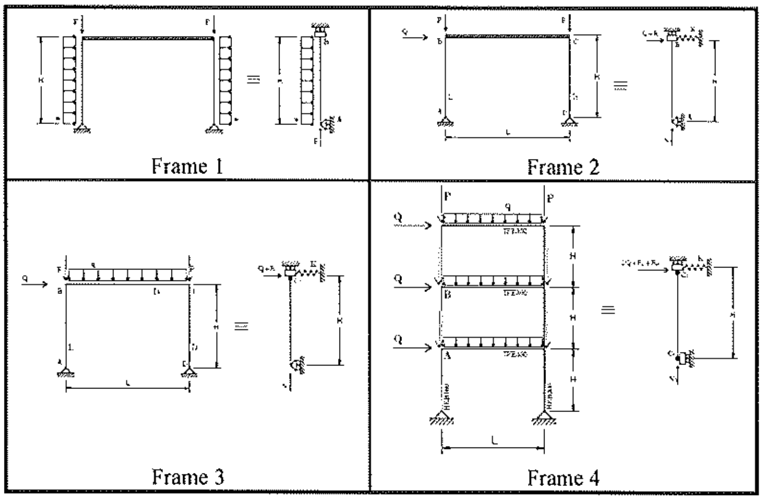
IMPROVEMENT OF THE CAM

Strategy. In order to improve the CAM, one analyzes the last three frames in table 2, which exhibit a growing complexity – from the academic Frame 2 to the more realistic Frame 4 (recall that Frame 1 was already studied). All the modifications made to the CAM are then validated through the comparison with "exact" numerical results. In these frames, α_{cr} varies from 3 to 10.

Asymmetric Portal Frame with Rigid Beam (Frame 2). While the (weaker) left column is built from a HE160B profile, different cross-sections shapes are adopted for the right column, so that the inertia ratio $I_{y,right}/I_{y,left}$ varies between 1 and 20. The frame spans over 5 m, is 4 m high, is acted by a horizontal point load Q=12 kN and the vertical load P is such that the ratio V_{Ed}/V_{cr} remains within the range of application of the CAM.

Since the frame is not symmetric, a clear distinction must be made between its *global sway* buckling mode (occurring for a load factor $\lambda_{cr,gl}$) and the weaker (left) column *local* buckling (occurring for a load factor $\lambda_{cr,loc}$). In this particular case, $\lambda_{cr,gl}$ is given approximately by

Table 2. Frames investigated in this work and associated equivalent systems

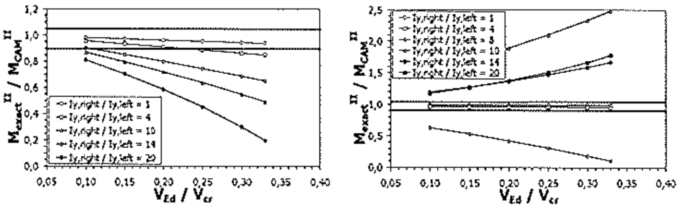


$$\lambda_{cr,gl} = \frac{1}{V_{Ed}} \frac{\pi^2 E\left(I_{y,left} + I_{y,right}\right)}{4H^2} = \frac{N_{Ed,1}}{V_{Ed}} \lambda^{*}_{cr,gl} \qquad (10)$$

where $N_{Ed,1}$ is the axial force acting on the left column[1]. It can be shown that both the P-Δ (global) and P-δ (local) effects influence the in-span and end section bending moments, which means that, rigorously speaking, they cannot be treated independently, as assumed in the CAM (and also the ASMM) – the P-Δ effects are handled in the global analysis and the P-δ ones in the member safety checks. The application of the CAM leads to the results presented in figures 4(a)-(b), which concern two different horizontal loadings: a point load applied at the left column top and identical uniformly distributed loads applied at the two columns (as in Frame 1). These results consist of curves providing the variation of (i) the ratio between the

[1] This expression only yields exact results if all the frame columns share the same N_{Ed}/EI ratio. Although this is clearly not the case in the frames analyzed here, it was found that the results provided by eq. (10) are fairly accurate (Hansoulle 2006).

exact and approximate moments at the left column top ($M_{exact}^{II} / M_{CAM}^{II}$) with (ii) the vertical load ratio V_{Ed}/V_{cr}, for various column stiffness ratios $I_{y,right}/I_{y,left}$. As for the M_{exact}^{II} values, they were yielded by accurate beam finite element analyses (Hansoulle 2006) – this applies to all the frames analyzed in this work.



(a) (b)

Figure 3. Application of the CAM to Frame 2 for (a) a top point load and (b) two uniformly distributed horizontal loads[1]

The observation of the above curves provides clear evidence that, for both loadings, the CAM only leads to "acceptable" moment estimates (errors below 10% and 5% on the safe or unsafe sides) in frames with $I_{y,right}/I_{y,left}$ equal to 1 (symmetric) or 4 ("weakly" asymmetric). For the more asymmetric frames, one obtains either excessively conservative or too unsafe moment estimates.

In order to try to improve the CAM performance, one considers the structural model equivalent to Frame 2, shown in table 2. It is intended to simulate the behavior of the frame left column, which is subjected to an axial force $N_{Ed,1}$ and laterally restrained by a spring that "replaces" the frame right column and ensures that the model and frame share the same first-order sway stiffness – thus, the spring stiffness value K is given by

$$K = 3EI_{y,right} / H^3 \tag{11}$$

The first-order (M_B^I) and second-order ($M_B^{II,P\Delta\delta}$) bending moments values at the top of the structural model are given by the expressions

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[1] Obviously, the results presented in figure 4(a) (horizontal point load) also correspond to the application of the ASMM. Recall that the the CAM and ASMM only differ in the presence of horizontal *distributed* loads (as is the case in figure 4(b)).

$$M_B^I = \frac{QH}{1+\frac{2}{3}\frac{KH^3}{2EI}} \; and \; M_B^{II,P\Delta/\delta} = \frac{QH}{1+\frac{\alpha_1 H\left(KH^2 - N_{Ed,1}H\right)}{2EI\left(\alpha_1^2-\beta_1^2\right)}} \tag{12}$$

$$\alpha_1 = \frac{\varepsilon_1\left(\sin\varepsilon_1 - \varepsilon_1\cos\varepsilon_1\right)}{2\left(2-2\cos\varepsilon_1 - \varepsilon_1\sin\varepsilon_1\right)} \; and \; \beta_1 = \frac{\varepsilon_1\left(\varepsilon_1 - \sin\varepsilon_1\right)}{2\left(2-2\cos\varepsilon_1 - \varepsilon_1\sin\varepsilon_1\right)} \tag{13}$$

where (i) α_1 and β_1 are "stability functions" (*e.g.*, Chen & Lui 1987) and the parameter ε_1 is a function of the axial force $N_{Ed,1}$ (see eq. (8)). Eq. (12) may be used to derive the amplification factor S, defined as

$$S = \frac{M_B^{II,P\Delta/\delta}}{M_B^I} = \frac{1+KH^3/(3EI)}{1+\alpha_1 H\left(KH^2 - N_{Ed,1}H\right)/\left(2EI(\alpha_1^2-\beta_1^2)\right)} \tag{14}$$

By looking at the second part of eq. (8), one readily recognizes that the equivalent moment factor $C_{m,end}$ may be written as

$$C_{m,end} = S\left(1-\frac{1}{\lambda_{cr,gl}^*}\right) = \frac{1+KH^3/(3EI)}{1+\frac{\alpha_1 H\left(KH^2 - N_{Ed,1}H\right)}{2EI\left(\alpha_1^2-\beta_1^2\right)}}\left(1-\frac{N_{Ed,1}}{V_{Ed}}\frac{1}{\lambda_{cr,gl}}\right) \tag{15}$$

where $\lambda_{cr,gl}^*$ is the critical load factor of the structural model, approximately related to the frame *global* sway critical load factor $\lambda_{cr,gl}$ by means of eq. (10). The inclusion of the above $C_{m,end}$ expression in the CAM leads to estimates of the second-order bending moment at the frame left column top given by

$$M_{CAM,mod}^{II} = M_{Gravity} + \frac{C_{m,end}}{1-1/\lambda_{cr,gl}} M_{Sway} = 0 + \frac{C_{m,end}}{1-1/\lambda_{cr,gl}} M_{Sway} \tag{16}$$

This modification (improvement) of the CAM will be hereafter designated as "modified CAM" (CAM_{mod}). Figure 5 shows how including the new $C_{m,end}$ improves the quality of the end moment estimates (now termed $M_{CAM,mod}{}^{II}$). Indeed, all curves providing the variation of $M_{exact}{}^{II}/M_{CAM,mod}{}^{II}$ with V_{Ed}/V_{cr} are nearly horizontal and extremely close to the unit value – in order to assess the drastic improvement due to the modification, it suffices to compare the curves associated with $I_{y,right}/I_{y,left}\geq 10$ in figures 4(a) and 5.

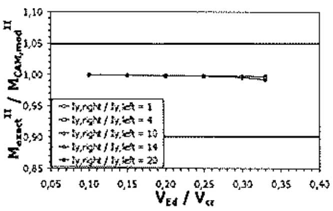


Figure 5. Application of the modified CAM to Frame 2 for a top point load

Once the value of the second-order bending moment at the left columns top is evaluated, it is possible to further analyze this column by treating it as being simply supported – indeed, $M_{CAM,mod}^{II}$ accounts for the influence of the P-Δ and P-δ effects and also of the other frame members. Then, since no additional transverse loads are applied within the member span, the maximum second-order "span bending moment" M_{max}^{II} is given by

$$M_{max}^{II} = \frac{C_{m,span}}{1 - N_{Ed,1}/N_{cr}} M_{CAM,mod} \tag{17}$$

where $M_{CAM,mod}$ is the previously obtained second-order bending moment at the column top, $C_{m,span}$ is a classical equivalent moment factor (Villette 2004) associated with the determination of second-order bending moments along the length of *simply supported* compressed members and N_E is the member Euler buckling load. At this stage, is worth pointing out that the use of N_E (instead of the column non-sway critical load N_{cr}) is due to the fact that one is dealing with the additional moments stemming from the difference between the member actual and linearized (chord) deformed configurations – of course, it is implicitly assumed that one has $N_{Ed,1} < N_E$. In this particular case (triangular bending moment diagram), the value of $C_{m,span}$ is yielded by

$$C_{m,span} = \begin{cases} \left(1 - N_{Ed,1}/N_E\right) \dfrac{1}{\sin\left(\pi\sqrt{N_{Ed,1}/N_E}\right)} & if\ N_{Ed,1} \geq N_{lim} \\ \left(1 - N_{Ed,1}/N_E\right) \quad if\ N_{Ed,1} \leq N_{lim} & \end{cases} \tag{18}$$

$$with\ N_E = \pi^2 EI_{y,left}/H^2 \tag{19}$$

where N_{lim} is the axial load for which the end and span maximum second-order moments have the same value – one has $N_{lim}=N_E$ in this particular case (*e.g.*, Chen & Lui 1987, Villette 2004). Figure 6(a) shows curves that provide the variation of the span moment ratio $M_{exact}^{II}/M_{CAM,mod}^{II}$ with V_{Ed}/V_{cr} and one notices that they are again (even more than in figure 5) practically coincident with the unit value horizontal line. Just for comparison purposes, figure 6(b) displays $M_{exact}^{II}/M_{CAM,mod}^{II}$ vs. V_{Ed}/V_{cr} curves evaluated with N_{cr} (*i.e.*, taking the column buckling length equal to $0.7H$) – naturally, several CAM_{mod} predictions are now excessively conservative.

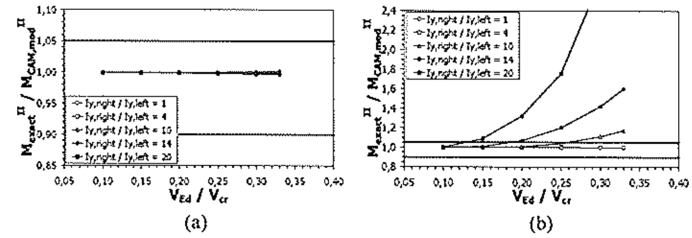


Figure 6. Application of the CAM_{mod} to Frame 2 with (a) N_E and (b) N_{cr}

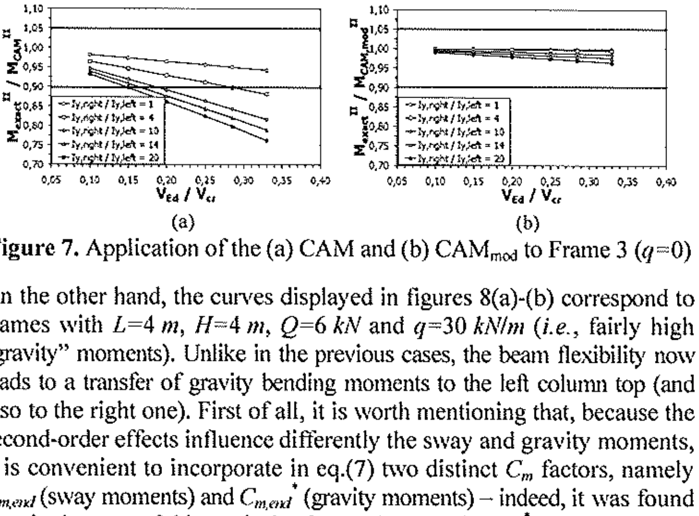
Asymmetric Portal Frame with Flexible Beam (Frame 3). Next, one looks at an asymmetric portal frame with a *flexible* beam, a feature that affects both the global (frame) and local (member) buckling behaviors. The frame is subjected to the loading indicated – notice that it includes a uniformly distributed vertical load q applied on the beam. As in the case of Frame 2, the left column is built from a HE160B profile and different cross-sections shapes are adopted for the right column ($I_{y,right}/I_{y,left}$ varies again from 1 to 20). The beam is always chosen as an IPE400 profile. Table 2 also shows a structural model equivalent to Frame 3, with (i) a translational spring with stiffness K and (ii) a rotational spring with stiffness C_2 that must account for the beam flexibility. In order to ensure that the model and frame share the same first-order sway and rotation stiffness values (when subjected to a horizontal point load applied at the storey level), K and C_2 need to satisfy the conditions

$$C_2 = 3EI\left(\Delta / H - \phi_B\right)/\left(\phi_B H\right) \; and \; K = \left(Q + R - C_2\phi_B / H\right)/\Delta \qquad (20)$$

where $Q+R$ stands for an arbitrary horizontal point load (see table 2). By adopting a procedure similar to the one employed for Frame 2 (but a bit more complex – Hansoulle 2006), it is possible to obtain an analytical expression to evaluate $C_{m,end}$, which reads

$$C_{m,end} = \frac{1 + KH^2\left(H/3EI + 1/C_2\right)}{1 + \left(KH^2 - N_{Ed,1}H\right)\left(\dfrac{\alpha_1 H}{2EI\left(\alpha_1^2 - \beta_1^2\right)} + \dfrac{1}{C_2}\right)}\left(1 - \frac{N_{Ed,1}}{V_{Ed}}\frac{1}{\lambda_{cr,gl}}\right) \quad (21)$$

The curves $M_{exact}{}^{II}/M_{CAM}{}^{II}$ vs. V_{Ed}/V_{cr} and $M_{exact}{}^{II}/M_{CAM,mod}{}^{II}$ vs. V_{Ed}/V_{cr} shown in figures 7(a)-(b) concern frames having L=4 m, H=4 m, Q=12 kN and q=0 kN/m (*i.e.*, no "gravity" moments). By comparing these curves with the ones in figures 4(a) and 5, one readily notices that the beam flexibility does not alter the "quality" of the CAM and CAM_{mod} moment estimates – indeed, while the majority of the former are too unsafe, all the latter are very accurate.

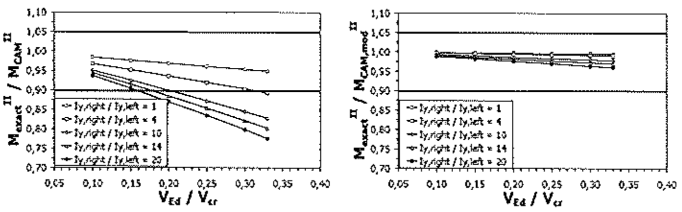


(a) (b)

Figure 7. Application of the (a) CAM and (b) CAM_{mod} to Frame 3 (q=0)

On the other hand, the curves displayed in figures 8(a)-(b) correspond to frames with L=4 m, H=4 m, Q=6 kN and q=30 kN/m (*i.e.*, fairly high "gravity" moments). Unlike in the previous cases, the beam flexibility now leads to a transfer of gravity bending moments to the left column top (and also to the right one). First of all, it is worth mentioning that, because the second-order effects influence differently the sway and gravity moments, it is convenient to incorporate in eq.(7) two distinct C_m factors, namely $C_{m,end}$ (sway moments) and $C_{m,end}^*$ (gravity moments) – indeed, it was found that, in the case of this particular frame, the use of $C_{m,end}^*$ reduces to about half the errors of the CAM and CAM_{mod} predictions. The determination of $C_{m,end}^*$ is addressed in detail in the next section (devoted to Frame 4).

The observation of the curves presented in figures 8(a)-(b) shows that, once more, (i) the CAM_{mod} estimates of the left column end moments are very accurate and (ii) several of their CAM counterparts are too unsafe.



(a) (b)

Figure 8. Application of the (a) CAM and (b) CAM_{mod} to Frame 3 ($q \neq 0$)

Asymmetric Three-Storey Frame (Frame 4). Finally, one considers a more complex frame, which (i) has a 4 *m* single bay and three 4 *m* high storeys, (ii) is formed by IPE300 beams, HEB160 left columns and HEB300 right columns (*i.e.*, is asymmetric – see table 2) and (iii) is acted by three identical (iii$_1$) uniformly distributed vertical loads q=10 kN/m applied on each beam and (iii$_2$) horizontal point loads Q=10kN applied at each floor level.

Attention is focused on the intermediate storey column AB, the behavior of which can be simulated by the structural model depicted in table 2 – the influence of the remaining frame members and loads is taken into account by means of one translational and two rotational springs. As in the previous cases, their stiffness values (K, C_1 and C_2) can be obtained by imposing the equivalence between the model and frame first-order sway and rotation stiffness values (when subjected to horizontal point loads applied at the storey levels) – obviously, the complexity of this procedure increases with the number of frame members. One is then led to

$$C_1 = \frac{2EI\left(3\Delta_{rel}/H - 2\phi_A - \phi_B\right)}{\phi_A H} \quad and \quad C_2 = \frac{2EI\left(3\Delta_{rel}/H - 2\phi_B - \phi_A\right)}{\phi_B H} \tag{22}$$

$$K = \frac{\sum Q}{\Delta_{rel}} - \frac{C_1\phi_A + C_2\phi_B}{H\Delta_{rel}} \tag{23}$$

where Δ_{rel} is the column *relative* sway displacement and the summation ΣQ is extended to all storeys located above column AB. The corresponding $C_{m,end}$ factor (concerning the sway moments) is given by

$$C_{m,end} = S\left(1 - \frac{N_{Ed,1}}{V_{Ed}}\frac{1}{\lambda_{cr}}\right) \tag{24}$$

$$S = \frac{1 + \frac{C_2 + 2EI/H}{C_1 + 2EI/H}\frac{C_1}{C_2} + \frac{KH^3}{6EI}\frac{C_2 + 2EI/H}{C_1 + 2EI/H}\frac{C_1 + 4EI/H}{C_2} + \frac{KH^2}{3C_2}}{1 + \left[1 + \frac{\left(KH^2 - N_{Ed,1}\right)}{\alpha_1 + \beta_1}\left(\frac{\alpha}{C_1} + \frac{H}{2EI}\right)\right]\frac{C_1}{C_2}\frac{C_2 + 2EI/H\left(\alpha_1 - \beta_1\right)}{C_1 + 2EI/H\left(\alpha_1 - \beta_1\right)} + \frac{\left(KH^2 - N_{Ed,1}\right)}{\alpha_1 + \beta_1}\frac{\beta_1}{C_2}} \tag{25}$$

where V_{Ed} corresponds to the sum of the vertical loads acting on the storeys located above column AB. In order to account properly for the P-δ effects associated with the gravity bending moments, one must derive a $C_{m,end}^*$ factor (already mentioned, with respect to Frame 3), a task performed by considering the structural model shown in figure 9, which concerns the frame non-sway behavior – the remainder of the frame is now replaced by only two rotational springs. After equating the model and frame first-order rotation stiffness values (when subjected to a top end moment M – see figure 9), one obtains, sequentially, expressions for (i) C_1^* and C_2^*, and (ii) $C_{m,end}^*$, which are

$$C_1^* = \frac{2EI\left(2\phi_A + \phi_B\right)}{\phi_A H} \quad and \quad C_2^* = \frac{2EI\left(2\phi_B + \phi_A\right)}{\phi_B H} \tag{26}$$

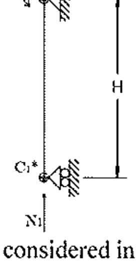


Figure 9. Structural model considered in the determination of $C_{m,end}^*$

$$C^{*}_{m,end} = \frac{\alpha_1 - \dfrac{\beta_1^2}{\alpha_1 + C_1^{*}H/2EI} \quad 2 - \dfrac{1}{2 + C_1^{*}H/2EI} + \dfrac{C_2^{*}H}{2EI}}{2 - \dfrac{1}{2 + C_1^{*}H/2EI} \quad \alpha_1 - \dfrac{\beta^2}{\alpha_1 + C_1^{*}H/2EI} + \dfrac{C_2^{*}H}{2EI}} \tag{27}$$

Table 3 presents the exact ($M_{B,exact}^{II}$) and approximate ($M_{B,CAN,mod}^{II}$) second-order moments at the column end B, for several values of P – $M_{B,CAN,mod}^{II}$ is evaluated through eq. (9)[1]. Once more, the moment estimates yielded by the CAM_{mod} are found to be very accurate – all errors are below 8%.

Table 3. Comparison of the actual and modified CAN and ASMM

P [kN]	$M_{B,exact}^{II}$ [kNm]	$M_{B,CAM}^{II}$ [kNm]	$Error_{CAM}$ [%]	$M_{B,CAM,mod}^{II}$ [kNm]	$Error_{CAM,mod}$ [%]
100	19.93	20.58	3.2	19.74	1.0
150	21.60	22.57	4.5	21.20	1.9
200	23.58	25.45	7.9	22.93	2.8
225	24.73	26.27	6.2	23.89	3.4

Concluding Remarks. The modified (improved) CAM, which is based on the analysis of structural models consisting of elastically restrained isolated members, has been shown to provide quite accurate bending moment estimates in the four frames considered – in theory, it should perform equally well in more general situations (frame geometries and loadings). Since the CAM_{mod} is based on the equality of the model and frame first-order stiffness (to characterize the model elastic springs), it exhibits a strong physical background, responsible for the high accuracy of the estimates obtained. In particular, the "quality" of these estimates is much better than that of their CAM counterparts.

It is important to stress the fact that, after estimating the second-order *end* moments (which include both the P-Δ and P-δ effects), the subsequent safety checking of the "equivalent simply supported member" must be

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[1] In order to be able to evaluate second-order span moments, one must first determine an appropriate $C_{m,span}$ factor (*e.g.*, Villette 2004).

carried out with a buckling length equal to the member physical length (*i.e.*, taking $N_{cr}=N_E$) – failing to do this will often lead to rather inaccurate second-order span moment estimates.

PRACTICAL DESIGN EXAMPLES

In this section, one applies and compares the performance of several design methods (evaluation of the second-order elastic internal forces and moments, followed by member strength and stability safety checks). The analysis by means of the CAM and CAM_{mod} is combined with the use of the beam-column interaction formulae included in Annex A of the EN version of Eurocode 3 (Boissonnade *et al.* 2006, Maquoi *et al.* 2001). As for the B_1B_2M, it is applied together with the AISC specifications (AISC 2005).

In order to assess the efficiency of the above design methods, one uses "exact (i) second-order elastic bending moments at frame member ends and (ii) elastic-plastic frame ultimate strength values, both obtained by means of rigorous finite element analyses – in the latter case, the non-linear analyses include appropriate initial geometrical imperfections and residual stresses.

Sway Frame 1. The results reported in table 4 concern the three-bay and three-storey frame shown in figure 10, made of S235 steel, and consist of (i) second-order bending moments at the top of the lower central columns (M_{top}^{II}) yielded by an "exact" elastic frame analysis and the CAM, CAM_{mod} and B_1B_2M, and (ii) frame collapse load parameters (λ_u – failure mechanism in figure 10) yielded by an "exact" elastic-plastic frame analysis and design approaches based on the CAM (Eurocode 3), CAM_{mod} (Eurocode 3) and B_1B_2M (AISC Manual)[1]. Although all M_{top}^{II} and λ_u estimates are quite accurate, it is clear that the CAM_{mod} ones exhibit a "higher" quality – indeed, they are virtually "exact". It is worth noting that all column buckling reduction factors are very close to unity, which means that the frame collapse is mostly due to plasticity effects – this fact also contributes to the accuracy of all the λ_u estimates.

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[1] Note that, since there are no distributed horizontal loads, the CAM_{mod} (Eurocode 3) and ASMM (Eurocode 3) design approaches yield exactly the same estimates – this statement applies also to the two frames considered next.

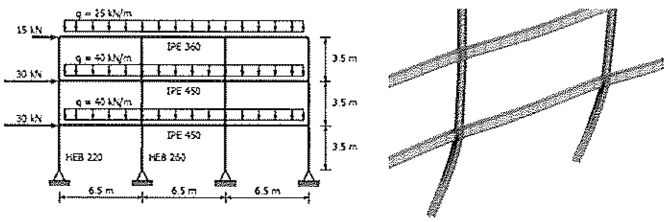

Figure 10. Sway Frame 1 data and failure mechanism (with yield pattern)

Table 4. Sway Frame 1: comparison between the analysis/design methods

Method	M_{top}^{II} [kNm]	*Error* [%]	λ_u	*Error* [%]
Numerical	102.89	–	1.669	–
CAM_{mod}	102.63	-0.3	1.661	-0.5
CAM	106.80	3.8	1.603	-4.0
B_1B_2M	105.58	2.6	1.650	-1.1

Sway Frame 2. This second sway frame, shown in figure 11 and made of a steel with E=200 *GPa* and f_y=248.2 *MPa*, was first investigated by Ziemian & Martinez-Garcia (2006) and has a highly asymmetric geometry, due to the fact that its member dimensions were selected to optimize the frame response under the particular load case indicated. The results presented in table 5 consist of (i) second-order bending moments at the top of lower left column (M_{top}^{II}) and again (ii) frame collapse load parameters (λ_u – failure mechanism in figure 11) – one employs the same analyses and design approaches as in the previous case. Even if the frame collapse appears to stem mostly from the yielding of the beams, it should be pointed out that plasticity also develops in several column zones. Moreover, it is worth mentioning that, according to the three design approaches, the frame failure load is governed by the safety checking of the lower central column.

Concerning the M_{top}^{II} values, one observes that both the CAM_{mod} and the B_1-B_2M yield very good (safe) estimates, a bit more accurate than the (unsafe) CAM one. On the other hand, all design approaches predict the same (very conservative) ultimate load parameter λ_u – the development of a plastic hinge in the W27x84 beam for λ=0.748, which obviously limits the application of the design procedures (based on elastic analysis).

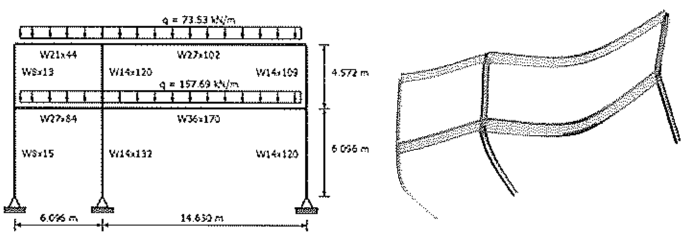

Figure 11. Sway Frame 2 data and failure mechanism (with yield pattern)

Table 5. Sway Frame 2: comparison between the analysis/design methods

Method	M_{top}^{II} [kNm]	*Error* [%]	λ_u	*Error* [%]
Numerical	451.30	—	1.029	—
CAM_{mod}	442.66	-1.9	0.748	37.6
CAM	473.68	5.0	0.748	37.6
B_1B_2M	442.99	-1.8	0.748	37.6

Non-Sway Frame. Since the modifications incorporated in the CAM remain valid for non-sway frames, due to the consideration of the $C_{m,end}^*$ factor, one applies now the CAM_{mod} to the frame displayed in figure 12, which has semi-rigid joints (their stiffness values are indicated) and is made of S235 steel.

The results presented in table 6 consist, once more, of M_{top}^{II} and λ_u values – the former concern the top of the lower central column and the failure mechanism associated with the latter is depicted in figure 12. Since (local) buckling effects are now relevant in the column under consideration, the difference between the CAM and CAM_{mod} bending moment estimates (both unsafe) is much more pronounced[1] – indeed, while the error of the former is a little over 7%, the former is off by more than 15% (the error of the B_1B_2M is precisely the same). As in the previous case (but to a much lesser extent), the three design procedures underestimate the ultimate load parameter λ_u=1.317 – one obtains either λ_u=1.225 (designs based on the CAM or CAM_{mod}) or λ_u=1.221 (design based on the B_1B_2M). The equality of the λ_u values yielded

[1] Note that the two methods only differ in the fact that the CAM_{mod} adopts a $C_{m,end}^*$ factor.

by the CAM and CAM_{mod} design procedures is due to the fact that they are associated with the safety checking of a beam (and not of the lower central column, where M_{top}^{II} acts). Moreover, the marginally lower λ_u value provided by the AISC specification is associated with the collapse of a column (and not a beam, as predicted by the Eurocode 3 provisions).

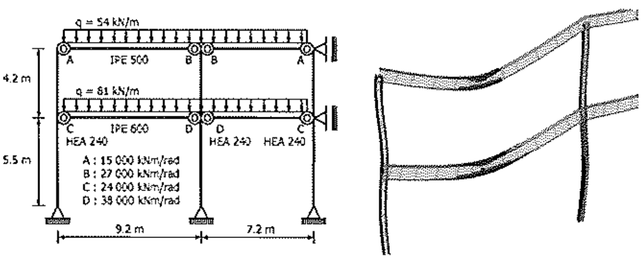

Figure 12. Non-Sway Frame data and failure mechanism (with yield pattern)

Table 6. Non-Sway Frame: comparison between the analysis/design methods

Method	M_{top}^{II} [kNm]	*Error* [%]	λ_u	*Error* [%]
Numerical	13.14	–	1.317	–
CAM_{mod}	14.09	7.2	1.225	-7.5
CAM	15.15	15.3	1.225	-7.5
B_1B_2M	15.15	15.3	1.221	-7.9

CONCLUSION

This paper proposed and illustrated the application and capabilities of a design approach for sway steel frames that (i) is based on a modification of the Canadian Amplification Method (CAM) and (ii) incorporates the influence of both the P-Δ and P-δ effects on the member end internal forces and moments yielded by a frame first-order analysis. This ensures that it is possible to perform the individual member strength and stability checks by considering "simply supported equivalent members" subjected to *fixed* (*i.e.*, no longer affected by P-δ effects) end moments – a feature that is particularly helpful, since it becomes possible to use a large number of available equivalent moment (C_m) factors, developed in the context of simply supported beam-columns.

Because the above modification is based on the frame physical behavior, the modified method (CAM_{mod}) yields more accurate IFM values and leads to more efficient design solutions – these assertions have been illustrated by means of the analysis and collapse load evaluation of several frames with different configurations. Moreover, it was also shown that the application of the CAM_{mod} to non-sway frames also yields accurate estimates.

Finally, one last word to mention that the proposed design approach is still under investigation, aimed at (i) obtaining a more in-depth assessment of its validity and efficiency, and (ii) establishing a well defined range of application.

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KEYWORDS

Steel frame design – pages 1, 2

Sway frames – pages 2, 6, 20

Beam-column design – page 3

Methods of frame analysis – page 4

Simply supported equivalent member – page 1